$\bar{B}\to K^*\gamma$ from $D\to K^*\bar{l}\nu$

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(Received 14 May 1999; published 8 November 1999)

The $\bar{B}\to K^*\gamma$ branching fraction is predicted using heavy quark spin symmetry at large recoil to relate the tensor and (axial-)vector form factors, using heavy quark flavor symmetry to relate the B decay form factors to the measured $D\to K^*\bar{l}\nu$ form factors, and extrapolating the semileptonic B decay form factors to large recoil assuming nearest pole dominance. This prediction agrees with data surprisingly well, and we comment on its implications for the extraction of $|V_{ub}|$ from $\bar{B}\to\rho l\bar{\nu}$. [S0556-2821(99)07023-X]

PACS number(s): 12.39.Hg, 12.15.Hh, 13.20.He

The next generation of B decay experiments will test the Cabibbo-Kobayashi-Maskawa (CKM) picture of quark mixing and CP violation with high precision. The basic approach is to determine the sides and angles of the unitarity triangle, and then check for the consistency of these results. A precise and model independent determination of the magnitude of the $b\to u$ CKM matrix element, $|V_{ub}|$, is particularly important. It is one of the least precisely known elements of the CKM matrix. At the present time the uncertainty of the standard model expectation for $\sin(2\beta)$, the CP asymmetry in $B\to J/\psi K_S$, depends strongly on the uncertainty of $|V_{ub}|$.

Currently, most determinations of $|V_{ub}|$ rely on phenomenological models [1]. The more promising model independent approaches for the future include studying the hadronic invariant mass distribution in inclusive semileptonic $\bar{B}\to X_u e\bar{\nu}$ decay [2], measuring the inclusive $\bar{B}\to X_{u\bar{c}d}$ nonleptonic decay rate [3], and comparing the exclusive $\bar{B}\to\rho l\bar{\nu}$ and $\bar{B}\to\pi l\bar{\nu}$ decay rates in the large q^2 region with lattice results [4] or predictions based on heavy quark symmetry and chiral symmetry [5–7]. A major uncertainty in the latter method is the size of the symmetry breaking corrections. Another question for this approach is whether the $D\to K^*\bar{l}\nu$ (or $D\to\rho\bar{l}\nu$) form factors can be extrapolated to cover a larger fraction of the $\bar{B}\to\rho l\bar{\nu}$ phase space.

In this Brief Report some of these ingredients are tested by comparing the measured $\bar{B}\to K^*\gamma$ branching fraction with a prediction relying on b quark spin symmetry at large recoil to relate the tensor and (axial-)vector form factors, heavy quark flavor symmetry to relate the B decay form factors to the measured $D\to K^*\bar{l}\nu$ form factors, and an extrapolation of the semileptonic B decay form factors assuming nearest pole dominance. We denote by a superscript $(H\to V)$ the form factors relevant for transitions between a pseudoscalar meson H containing a heavy quark, Q, and a member of the lowest lying multiplet of vector mesons, V. We view the form factors as functions of the dimensionless variable $y=v\cdot v'$, where $p=m_H v$, $p'=m_V v'$, and $q^2=(p-p')^2=m_H^2+m_V^2-2m_H m_V y$. (Note that even though we are using the variable $v\cdot v'$, we are not treating the quarks in V as heavy.) An approach with some similarities to the one presented here can be found in Ref. [8]. This decay has also been considered in Refs. [9,10].

The $\bar{B}\to K^*\gamma$ transition arises from a matrix element of the effective Hamiltonian

$$H_{\rm eff}=-\frac{4G_F}{\sqrt{2}}V_{ts}^*V_{tb}\sum_{i=1}^{8}C_i(\mu)O_i(\mu), \tag{1}$$

where G_F is the Fermi constant, and $C_i(\mu)$ are Wilson coefficients evaluated at a subtraction point μ. The $\bar{B}\to K^*\gamma$ matrix element of $H_{\rm eff}$ is thought to be dominated by the operator

$$O_7=\frac{e}{16\pi^2}\,\bar{m}_b\bar{s}_L\sigma^{\mu\nu}F_{\mu\nu}b_R, \tag{2}$$

where e is the electromagnetic coupling, $\bar{m}_b$ is the modified minimal subtraction scheme $\overline{\rm (MS)}$ b quark mass, and $F_{\mu\nu}$ is the electromagnetic field strength tensor. O_1–O_6 are four-quark operators and O_8 involves the gluon field strength tensor.

The $\bar{B}\to K^*\gamma$ matrix element of O_7 can be expressed in terms of hadronic form factors, $g_\pm$ and h, defined by

$$\begin{aligned}
&\langle V(p',\epsilon)|\bar{q}\sigma_{\mu\nu}Q|H(p)\rangle\\
&\quad=g_+^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p+p')^\sigma\\
&\quad+g_-^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p-p')^\sigma\\
&\quad+h^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}(p+p')^\lambda(p-p')^\sigma(\epsilon^*\cdot p),\\
&\langle V(p',\epsilon)|\bar{q}\sigma_{\mu\nu}\gamma_5 Q|H(p)\rangle\\
&\quad=ig_+^{(H\to V)}[\epsilon_\nu^*(p+p')_\mu-\epsilon_\mu^*(p+p')_\nu]+ig_-^{(H\to V)}\\
&\qquad\times[\epsilon_\nu^*(p-p')_\mu-\epsilon_\mu^*(p-p')_\nu]+ih^{(H\to V)}\\
&\qquad\times[(p+p')_\nu(p-p')_\mu-(p+p')_\mu(p-p')_\nu](\epsilon^*\cdot p).
\end{aligned} \tag{3}$$

The second relation follows from the first one using the identity $\sigma^{\mu\nu}=(i/2)\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}\gamma_5$. We use the convention $\varepsilon^{0123}=-\varepsilon_{0123}=1$. The $\overline{B}\to K^*\gamma$ decay rate is then given by

$$\Gamma(\overline{B}\to K^*\gamma)=\frac{G_F^2\alpha|V_{ts}^*V_{tb}|^2}{32\pi^4}\overline{m}_b^2 m_B^3 \times\left(1-\frac{m_{K^*}^2}{m_B^2}\right)^3|C_7|^2|g_+^{(B\to K^*)}(y_0)|^2, \quad (4)$$

where $y_0=(m_B^2+m_{K^*}^2)/(2m_B m_{K^*})=3.05$.

In semileptonic decays such as $D\to K^*\overline{l}\nu$ or $\overline{B}\to\rho l\overline{\nu}$ another set of form factors occurs, g, f, and $a_\pm$, defined by

$$\langle V(p',\epsilon)|\overline{q}\gamma_\mu Q|H(p)\rangle=ig^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}(p+p')^\lambda \times(p-p')^\sigma,$$

$$\langle V(p',\epsilon)|\overline{q}\gamma_\mu\gamma_5 Q|H(p)\rangle=f^{(H\to V)}\epsilon^*_\mu +a_+^{(H\to V)}(\epsilon^*\cdot p)(p+p')_\mu +a_-^{(H\to V)}(\epsilon^*\cdot p)(p-p')_\mu. \quad (5)$$

The experimental values for the $D\to K^*\overline{l}\nu$ form factors assuming nearest pole dominance for the q^2 dependences are [11]

$$f^{(D\to K^*)}(y)=\frac{(1.9\pm0.1)\mathrm{GeV}}{1+0.63(y-1)},$$

$$a_+^{(D\to K^*)}(y)=-\frac{(0.18\pm0.03)\mathrm{GeV}^{-1}}{1+0.63(y-1)}, \quad (6)$$

$$g^{(D\to K^*)}(y)=-\frac{(0.49\pm0.04)\mathrm{GeV}^{-1}}{1+0.96(y-1)}.$$

The shapes of these form factors are beginning to be probed experimentally and the pole form is consistent with data [11]. The form factor a_- is not measured because its contribution to the $D\to K^*\overline{l}\nu$ decay amplitude is suppressed by the lepton mass. The minimal value of y is unity (corresponding to the zero recoil point) and the maximum value of y is $(m_D^2+m_{K^*}^2)/(2m_D m_{K^*})\simeq1.3$ (corresponding to $q^2=0$). In comparison, the allowed kinematic region for $\overline{B}\to\rho l\overline{\nu}$ is $1<y<3.5$.

A prediction for the $\overline{B}\to K^*\gamma$ decay rate can be made using heavy quark spin symmetry, which implies relations between the tensor and (axial-)vector form factors in the $m_b\to\infty$ limit [5,6]

$$g_+^{(B\to K^*)}+g_-^{(B\to K^*)}=\frac{f^{(B\to K^*)}+2g^{(B\to K^*)}m_B m_{K^*}y}{m_B}, \quad (7)$$

$$g_+^{(B\to K^*)}-g_-^{(B\to K^*)}=-2m_B g^{(B\to K^*)},$$

$$h^{(B\to K^*)}=\frac{a_+^{(B\to K^*)}-a_-^{(B\to K^*)}-2g^{(B\to K^*)}}{2m_B},$$

and, therefore,

$$g_+^{(B\to K^*)}=-g^{(B\to K^*)}(m_B-m_{K^*}y)+f^{(B\to K^*)}/(2m_B). \quad (8)$$

We use heavy quark symmetry again to obtain $g^{(B\to K^*)}$ and $f^{(B\to K^*)}$ from the measured $D\to K^*\overline{l}\nu$ form factors given in Eq. (6) [5]:

$$f^{(B\to K^*)}(y)=\left(\frac{m_B}{m_D}\right)^{1/2}f^{(D\to K^*)}(y),$$

$$g^{(B\to K^*)}(y)=\left(\frac{m_D}{m_B}\right)^{1/2}g^{(D\to K^*)}(y). \quad (9)$$

For y not too large, Eq. (7) has order $1/m_b$ corrections, whereas Eq. (9) receives both order $1/m_b$ and $1/m_c$ corrections.

The model dependence in our prediction of $\Gamma(\overline{B}\to K^*\gamma)$ arises from the use of b quark spin symmetry at large recoil and due to the fact that the B decay form factors are extrapolated beyond $y=1.3$. In Ref. [12] it was argued that the heavy quark spin symmetry relations in Eq. (7) should hold over the entire phase space without unusually large corrections. To extrapolate $f^{(B\to K^*)}$ and $g^{(B\to K^*)}$ to values of $y>1.3$ we assume the pole form; i.e., we simply use Eqs. (6) and (9) evaluated at $y_0=3.05$.[1] Although this is not a controlled approximation, it would not be surprising if the y dependence of $f^{(B\to K^*)}$ and $g^{(B\to K^*)}$ was consistent with a simple pole in this region. Between $y=1$ and $y=3.05$ the form factor $g^{(B\to K^*)}$ falls by roughly a factor of 3. In the spacelike region $0<-Q^2<1\ \mathrm{GeV}^2$, over which the pion electromagnetic form factor falls by a factor of 2.7, its measured Q^2 dependence is consistent with a simple ρ pole [13].[2] Note also that if $g^{(B\to K^*)}$ and $f^{(B\to K^*)}$ have pole forms, then the y dependence of $g_+^{(B\to K^*)}$ given by Eq. (8) does not correspond to a simple pole.

Using Eqs. (6), (8), and (9) we obtain $g_+^{(B\to K^*)}(3.05)=0.38$. Then Eq. (4) gives the following prediction for the $\overline{B}\to K^*\gamma$ branching fraction:

$$\mathcal{B}(\overline{B}\to K^*\gamma)=4.1\times10^{-5}. \quad (10)$$

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[1]The y dependence of the nearest pole dominated form factors for B decay are expected to be almost the same as for D decay, so we continue to use Eq. (6) for $y>1.3$. For example, with $m_{B_s^*}=5.42$ GeV the ‘‘slope’’ of $g^{(B\to K^*)}$ is 0.94 (instead of 0.96), and with $m_{B_s^{**}}=5.87$ GeV the ‘‘slope’’ of the axial form factors is 0.62 (instead of 0.63).

[2]At higher $-Q^2$, it does appear to be falling somewhat faster.

To evaluate Eq. (4), we used $\tau_B=1.6$ ps, $|C_7|=0.31$, $|V_{tb}V_{ts}^*|=0.04$, and $\overline{m}_b=4.2$ GeV. This result compares unexpectedly well with the CLEO measurement $\mathcal{B}(\overline{B}\to K^*\gamma)=(4.2\pm0.8\pm0.6)\times10^{-5}$ [14], and lends support to the validity of heavy quark symmetry relations between B and D semileptonic form factors and to the hypothesis that the pole form can be extended beyond $y=1.3$. Of course, it is also possible that the agreement between our prediction and data is a result of a cancellation between large corrections. Note that the sign of the form factor $g^{(D\to K^*)}(y)$, which only enters differential distributions but not the total $D\to K^*$ rate, is very important for the prediction in Eq. (10).

This set of approximations together with neglecting $SU(3)$ violation in the form factors $f^{(H\to V)}$ and $g^{(H\to V)}$ also implies that the short distance contribution to the $\overline{B}\to\rho\gamma$ branching ratio is $\mathcal{B}(\overline{B}\to\rho\gamma)=0.80|V_{td}/V_{ts}|^2\times\mathcal{B}(\overline{B}\to K^*\gamma)$.

Including perturbative strong interaction corrections, the right-hand side of Eq. (9) gets multiplied by $1+(\alpha_s/\pi)\ln(m_b/m_c)$, but Eqs. (7) and (8) remain unaffected. Evaluating α_s at the scale $\sqrt{m_b m_c}$, this gives a 10% increase in the prediction for $g_+^{(B\to K^*)}$ and a 20% increase in the prediction for the $\overline{B}\to K^*\gamma$ branching ratio in Eq. (10).

The factors of m_D and m_B in Eq. (9) are kinematical in origin. At y near 1, the validity of Eq. (9) relies partly on the charm quark being heavy enough that the B and D hadrons have similar configurations for the light degrees of freedom. Even though $m_{K^*}/m_D\sim1/2$, the typical momenta of the ''spectator'' light valence quark in the K^* meson is of order $\Lambda_{\rm QCD}$. Near $y=1$ the corrections to Eq. (9) need not be larger than the order $\Lambda_{\rm QCD}/m_{c,b}$ corrections that occur in some of the $B\to D^{(*)}$ or $\Lambda_b\to\Lambda_c$ semileptonic decay form factors. For example, the $1/m_c$ corrections in the matching of the full QCD weak current onto the current in the heavy quark effective theory (HQET) result in the following correction to the form factor $g^{(D\to K^*)}$:

$$\delta g^{(D\to K^*)}=\frac{1}{4m_c}\left[4c^{(D\to K^*)}+\left(1+\frac{\overline{\Lambda}}{m_D}\right)g_+^{(D\to K^*)}+\left(1-\frac{\overline{\Lambda}}{m_D}\right)g_-^{(D\to K^*)}\right], \tag{11}$$

where $c^{(H\to V)}$ is defined by the HQET matrix element

$$\langle V(p',\epsilon)|\overline{q}iD_\mu Q|H(p)\rangle=ic^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}(p+p')^\lambda\times(p-p')^\sigma. \tag{12}$$

The function $c^{(H\to V)}$ is not known, but it could be computed in lattice QCD. Neglecting it, and using Eqs. (6) and (7) with $B\to D$, we find that $\delta g^{(D\to K^*)}/g^{(D\to K^*)}$ is about $\{-0.20,-0.13\}$ at $y=\{1,1.3\}$. It is not surprising that heavy quark symmetry is useful near $y=1$, but at $y=y_0$ there is no obvious reason why the relation between $g^{(D\to K^*)}$ and $g^{(B\to K^*)}$ in Eq. (9) should be valid. Strictly speaking, our prediction for $\Gamma(\overline{B}\to K^*\gamma)$ does not depend on this assumption. As long as Eq. (9) holds for $1<y<1.3$ and the B decay form factors have the pole form for $y>1.3$, Eq. (10) follows. We do not need to assume that the D decay form factors also continue to be dominated by the nearest pole for $y>1.3$ (which is beyond the $D\to K^*\overline{l}\nu$ kinematic range). Nonetheless, under the assumption that the pole form continues to hold for the D decay form factors, the order $\Lambda_{\rm QCD}/m_c$ contribution to $\delta g^{(D\to K^*)}/g^{(D\to K^*)}$ from the last two terms in Eq. (11) is not anomalously large even at $y=y_0$.

If we take Eq. (10) as (circumstantial) evidence that heavy quark symmetry violation in scaling the g and f form factors from D to B decay is small, this has implications for extracting $|V_{ub}|$ from $\overline{B}\to\rho l\overline{\nu}$. The measurement $\mathcal{B}(D\to\rho^0\overline{l}\nu)/\mathcal{B}(D\to\overline{K}^{*0}\overline{l}\nu)=0.047\pm0.013$ [15] suggests that $SU(3)$ symmetry violation in the $D\to V$ form factors is also small. Assuming $SU(3)$ symmetry for these form factors, but keeping the explicit m_V dependence in the matrix element and in the phase space, the measured form factors in Eq. (6) imply $\mathcal{B}(D\to\rho^0\overline{l}\nu)/\mathcal{B}(D\to\overline{K}^{*0}\overline{l}\nu)=0.044$ [7].[3]

The differential decay rate for semileptonic B decay (neglecting the lepton mass, and not summing over the lepton type l) is

$$\frac{{\rm d}\Gamma(\overline{B}\to\rho l\overline{\nu})}{{\rm d}y}=\frac{G_F^2|V_{ub}|^2}{48\pi^3}m_B m_\rho^2 S^{(B\to\rho)}(y). \tag{13}$$

Here $S^{(H\to V)}(y)$ is the function

$$\begin{aligned}S^{(H\to V)}(y)=\sqrt{y^2-1}\{&|f^{(H\to V)}(y)|^2(2+y^2-6yr+3r^2)\\&+4\,{\rm Re}[a_+^{(H\to V)}(y)f^{*(H\to V)}(y)]m_H^2 r(y-r)\\&\times(y^2-1)+4|a_+^{(H\to V)}(y)|^2m_H^4r^2(y^2-1)^2\\&+8|g^{(H\to V)}(y)|^2m_H^4r^2(1+r^2-2yr)(y^2-1)\},\end{aligned} \tag{14}$$

with $r=m_V/m_H$. $S^{(B\to\rho)}(y)$ can be estimated using combinations of $SU(3)$ flavor symmetry and heavy quark symmetry. $SU(3)$ symmetry implies that the $\overline{B}^0\to\rho^+$ form factors are equal to the $B\to K^*$ form factors and the $B^-\to\rho^0$ form factors are equal to $1/\sqrt{2}$ times the $B\to K^*$ form factors. Heavy quark symmetry implies the relations in Eq. (9) and [5]

$$a_+^{(B\to K^*)}(y)=\frac{1}{2}\left(\frac{m_D}{m_B}\right)^{1/2}\left[a_+^{(D\to K^*)}(y)\left(1+\frac{m_D}{m_B}\right)-a_-^{(D\to K^*)}(y)\left(1-\frac{m_D}{m_B}\right)\right]. \tag{15}$$

[3]This prediction would be $|V_{cd}/V_{cs}|^2/2\simeq0.026$ with $m_\rho=m_{K^*}$. Phase space enhances $D\to\rho$ compared to $D\to K^*$ to yield the quoted prediction.

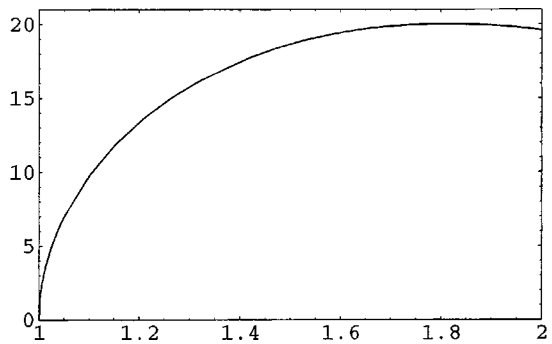

FIG. 1. $S^{(B\to\rho)}(y)$ defined in Eq. (13) using the measured $D \to K^* \bar{l} \nu$ form factors plus heavy quark and $SU(3)$ symmetry.

In the large m_c limit, $(a_+^{(D\to K^*)} + a_-^{(D\to K^*)})/(a_+^{(D\to K^*)} - a_-^{(D\to K^*)})$ is of order $\Lambda_{\rm QCD}/m_c$, so we can set $a_-^{(D\to K^*)} = -a_+^{(D\to K^*)}$, yielding

$$a_+^{(B\to K^*)}(y) = \left(\frac{m_D}{m_B}\right)^{1/2} a_+^{(D\to K^*)}(y). \tag{16}$$

Equation (16) may have significant corrections. In the large m_c limit, $(g_+^{(D\to K^*)} + g_-^{(D\to K^*)})/(g_+^{(D\to K^*)} - g_-^{(D\to K^*)})$ is also of order $\Lambda_{\rm QCD}/m_c$. From Eq. (7) with $B \to D$ and Eq. (6) we find that $g_-^{(D\to K^*)} = -\lambda g_+^{(D\to K^*)}$, where $\lambda = \{0.86, 1.04\}$ at $y = \{1, 1.3\}$.

Using Eqs. (9) and (16), and $SU(3)$ to get the $\bar{B}^0 \to \rho^+ l \bar{\nu}_l$ form factors from those for $D \to K^* \bar{l} \nu$ given in Eq. (6), yields $S^{(B\to\rho)}(y)$, plotted in Fig. 1 in the region $1 < y < 2$. In this region $a_+^{(B\to\rho)}$ and $g^{(B\to\rho)}$ make a modest contribution to the differential rate. For $y > 2$, $S^{(B\to\rho)}(y)$ is quite sensitive to the form of $a_+^{(B\to K^*)}$ in Eq. (16) which relies on setting $a_-^{(D\to K^*)} = -a_+^{(D\to K^*)}$. An extraction of $|V_{ub}|$ from $\bar{B} \to \rho l \bar{\nu}$ data using Fig. 1 in the limited range $1 < y < 1.3$ is model independent, with corrections to $|V_{ub}|$ first order in $SU(3)$ and heavy quark symmetry breaking. Extrapolation to a larger region increases the uncertainties both because the sensitivity to setting $a_-^{(D\to K^*)} = -a_+^{(D\to K^*)}$ increases and because the dependence on the functional form used for the extrapolation of the form factors increases.

In summary, we predicted in Eq. (10) the $\bar{B} \to K^* \gamma$ branching fraction in surprising agreement with CLEO data using b quark spin symmetry at large recoil to relate the tensor and (axial-)vector form factors, using heavy quark flavor symmetry to relate the B decay form factors to the measured $D \to K^* \bar{l} \nu$ form factors, and extrapolating the semileptonic B decay form factors to large recoil assuming nearest pole dominance. Although this agreement could be accidental, it suggests that heavy quark symmetry can be used to relate D and B semileptonic form factors and that $f^{(B\to K^*)}$ and $g^{(B\to K^*)}$ can be extrapolated to $y > 1.3$ using the pole form. This is encouraging for the extraction of $|V_{ub}|$ from $\bar{B} \to \rho l \bar{\nu}$ using Fig. 1. If experimental data on the $D \to \rho \bar{l} \nu$ and $\bar{B} \to K^* l \bar{l}$ differential decay rates become available, then a model independent determination of $|V_{ub}|$ can be made with corrections only of order $m_s/m_{c,b}$ (rather than $m_s/\Lambda_{\rm QCD}$ and $\Lambda_{\rm QCD}/m_{c,b}$) [6,7,16].

We thank Jeff Richman for a conversation that led to this paper and Adam Falk for useful remarks. M.B.W. was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-92-ER 40701. Fermilab is operated by Universities Research Association, Inc., under DOE Contract No. DE-AC02-76CH03000.

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