Effective Hamiltonian approach to hyperon beta decay with final-state baryon polarization

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Using an effective Hamiltonian approach, we obtain expressions for hyperon β decay final-state baryon polarization. Terms through second order in the energy release are retained. The resulting approximate expressions are much simpler and more compact than the exact expressions, and they agree closely with them. [S0556-2821(99)00223-4]

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In decays such as $\Xi^- \rightarrow \Lambda e^- \overline{\nu}$ or the recently observed [1] $\Xi^0 \rightarrow \Sigma^+ e^- \overline{\nu}$, the decay form factors can be probed by observing the parity-violating polarization of the final-state baryon via its two body decay mode ($\Lambda \rightarrow p\pi^-$ or $\Sigma^+ \rightarrow p\pi^0$ respectively). In addition, other kinematic distributions can be evaluated in the rest frame of the final baryon.

Early analyses of hyperon β decay with final-state polarization observed were restricted to the zero-recoil approximation [2] or were limited in scope [3]. More recent detailed treatments exist [4,5], but the resulting expressions are quite opaque, and, as a result, the physical content is hidden. Also, experiments are not likely to require exact formulas within the foreseeable future.

Using a method introduced by Primakoff for muon capture [6,7], we keep only terms through second order in the recoil velocity of the initial baryon (in the rest frame of the final baryon). A similar approach has been used to derive expressions for the case of a polarized initial baryon [8].

The most general $V-A$ transition matrix element for the generic hyperon β decay process $B \rightarrow b e^- \overline{\nu}$ can be written [9] in the form

$$\mathcal{M} = G_S \frac{\sqrt{2}}{2} \overline{u}_b (O^V_\alpha + O^A_\alpha) u_B \overline{u}_e \gamma^\alpha (1+\gamma_5) v_\nu + \text{H.c.}, \quad (1)$$

where

$$O^V_\alpha = f_1 \gamma_\alpha + \frac{f_2}{M_B} \sigma_{\alpha\beta} q^\beta + \frac{f_3}{M_B} q_\alpha,$$

$$O^A_\alpha = \left(g_1 \gamma_\alpha + \frac{g_2}{M_B} \sigma_{\alpha\beta} q^\beta + \frac{g_3}{M_B} q_\alpha \right) \gamma_5,$$

$$q^\alpha = (p_e + p_\nu)^\alpha = (p_B - p_b)^\alpha, \quad (2)$$

and

$$G_S = \begin{cases} G_F V_{us} & \text{for } |\Delta S| = 1, \\ G_F V_{ud} & \text{for } \Delta S = 0. \end{cases}$$

Here G_F is the Fermi coupling constant, V_{us} and V_{ud} are the appropriate Cabibbo-Kobayashi-Maskawa matrix elements, and ΔS denotes the strangeness change in the decay.

We relate the transition matrix element to an effective Hamiltonian by

$$\mathcal{M} = \langle be | \mathcal{H}_{\text{eff}} | B\nu \rangle \sqrt{2e 2\nu 2M_b (E_B + M_B)} \quad (3)$$

with

$$\frac{\sqrt{2}}{2} \mathcal{H}_{\text{eff}} = G_S \frac{1}{2} (1 - \boldsymbol{\sigma}_l \cdot \hat{e}) [G_V + G_A \boldsymbol{\sigma}_l \cdot \boldsymbol{\sigma}_b + G^e_P \boldsymbol{\sigma}_b \cdot \hat{e} + G^\nu_P \boldsymbol{\sigma}_b \cdot \hat{\nu}] \frac{1}{2} (1 - \boldsymbol{\sigma}_l \cdot \hat{\nu}). \quad (4)$$

Here $\hat{e}$ and $\hat{\nu}$ are unit vectors along the electron and antineutrino directions, while e, ν, and E_B are the energies of the electron, antineutrino, and initial baryon (all quantities are in the rest frame of b). The spin operators $\boldsymbol{\sigma}_l$ and $\boldsymbol{\sigma}_b$ act respectively on the lepton and baryon states (represented by two-component spinors).

The effective coupling coefficients G_V, G_A, G^e_P, and G^ν_P are functions of the form factors in Eq. (2):

$$G_V = f_1 + \delta f_2 - \frac{\nu + e}{2M_B} (f_1 + \Delta f_2),$$

$$G_A = -g_1 + \delta g_2 + \frac{\nu - e}{2M_B} (f_1 + \Delta f_2),$$

$$G^e_P = \frac{e}{2M_B} (-(f_1 + \Delta f_2) - g_1 + \Delta g_2),$$

$$G^\nu_P = \frac{\nu}{2M_B} (f_1 + \Delta f_2 - g_1 + \Delta g_2), \quad (5)$$

where $\delta = (M_B - M_b)/M_B$ and $\Delta = (M_B + M_b)/M_B = 2 - \delta$. Since the form factors f_3 and g_3 always appear with a multiplier of the electron mass divided by M_B, they are neglected throughout. Note also that f_2 and g_2 always appear multiplied by a quantity of order δ, so their q^2 dependence is not relevant to our order δ^2 approximation. However, the q^2 dependence of f_1 and g_1 does need to be included [5] in calculations to maintain a completely consistent order of approximation.

Electron and antineutrino spins are not usually observed, and this analysis focuses on measurement of the final baryon polarization. We therefore sum over the electron and antineutrino spins and average over initial baryon spin:

$$\sum_{\nu\ \text{spins},\ B\ \text{spins}} |\langle be|\mathcal{H}_{\text{eff}}|B\nu\rangle|^2 = \langle be|\mathcal{H}_{\text{eff}}H_{\text{eff}}^{\dagger}|be\rangle \tag{6}$$

and

$$\sum_{e\ \text{spins}} \langle be|\mathcal{H}_{\text{eff}}H_{\text{eff}}^{\dagger}|be\rangle = \text{Tr}[(1+\boldsymbol{\sigma}_b\cdot\mathbf{P}_b)\mathcal{H}_{\text{eff}}H_{\text{eff}}^{\dagger}]. \tag{7}$$

By projecting out the spin of the final baryon and taking the trace, we obtain

$$\begin{aligned}
|\mathcal{M}|^2 = {}& \xi[1+a\hat{e}\cdot\hat{\nu}+A\mathbf{P}_b\cdot\hat{e}+B\mathbf{P}_b\cdot\hat{\nu} \\
&+A'(\mathbf{P}_b\cdot\hat{e})(\hat{e}\cdot\hat{\nu})+B'(\mathbf{P}_b\cdot\hat{\nu})(\hat{e}\cdot\hat{\nu}) \\
&+D\mathbf{P}_b\cdot(\hat{e}\times\hat{\nu})](2e)(2\nu)(2M_b)(E_B+M_B)G_S^2, \\
\xi = {}& |G_V|^2+3|G_A|^2-2\,\text{Re}[G_A^*(G_P^e+G_P^\nu)] \\
&+|G_P^e|^2+|G_P^\nu|^2, \\
\xi a = {}& |G_V|^2-|G_A|^2-2\,\text{Re}[G_A^*(G_P^e+G_P^\nu)] \\
&+|G_P^e|^2+|G_P^\nu|^2+2\,\text{Re}[(G_P^{e*}G_P^\nu)(1+\hat{e}\cdot\hat{\nu})], \\
\xi A = {}& -2\,\text{Re}(G_V^*G_A)+2|G_A|^2+2\,\text{Re}(G_V^*G_P^e-G_A^*G_P^\nu), \\
\xi B = {}& -2\,\text{Re}(G_V^*G_A)-2|G_A|^2+2\,\text{Re}(G_V^*G_P^\nu+G_A^*G_P^e), \\
\xi A' = {}& 2\,\text{Re}[G_P^{e*}(G_V-G_A)], \\
\xi B' = {}& 2\,\text{Re}[G_P^{\nu*}(G_V+G_A)], \\
\xi D = {}& 2\ \text{Im}(G_V^*G_A)+2\ \text{Im}(G_P^{e*}G_P^\nu)(1+\hat{e}\cdot\hat{\nu}) \\
&+2\ \text{Im}[G_A^*(G_P^e-G_P^\nu)].
\end{aligned} \tag{8}$$

The polarization of the final baryon may be expressed explicitly as

$$\mathbf{P}_b = \frac{(A+A'\hat{e}\cdot\hat{\nu})\hat{e}+(B+B'\hat{e}\cdot\hat{\nu})\hat{\nu}+D\hat{e}\times\hat{\nu}}{1+a\hat{e}\cdot\hat{\nu}}. \tag{9}$$

The components of this polarization can readily be measured when the outgoing baryon b is a hyperon which undergoes a subsequent weak decay $b\to b'\pi$ with a non-zero decay asymmetry parameter $\alpha_{b'}$. The distribution of the b' direction relative to any axis defined by a unit vector $\hat{i}$ is given by

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_{b'}} = \frac{1}{4\pi}(1+\mathsf{S}_i\alpha_{b'}\hat{i}\cdot\hat{b'}), \tag{10}$$

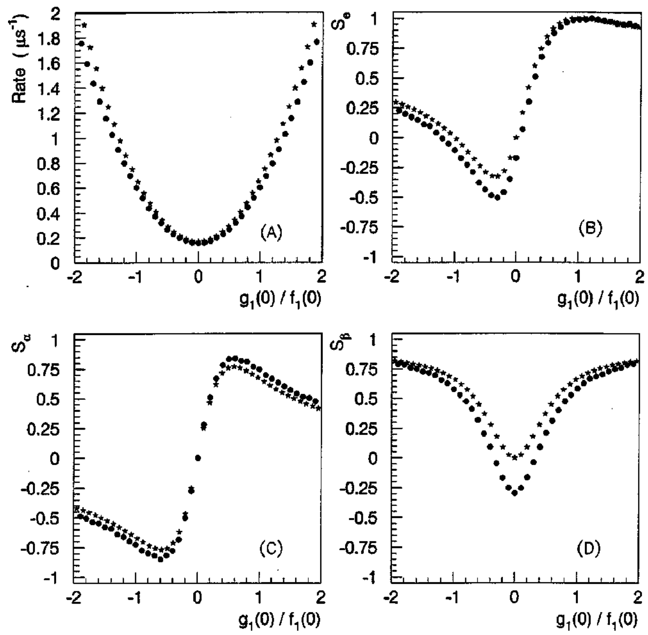

FIG. 1. Integrated observable quantities for the decay $\Xi^0\to\Sigma^+e^-\overline{\nu}$ as a function of g_1/f_1: (A) the total decay rate (μs^{-1}), (B) the polarization of the Σ^+ in the e^- direction ($\mathsf{S}_e=\langle\mathbf{P}_b\cdot\hat{e}\rangle$), (C) the polarization of the Σ^+ in the α direction ($\mathsf{S}_\alpha=\langle\mathbf{P}_b\cdot\hat{\alpha}\rangle$), (D) the polarization of the Σ^+ in the β direction ($\mathsf{S}_\beta=\langle\mathbf{P}_b\cdot\hat{\beta}\rangle$). The stars ($\star$) are zero recoil values, and circles (●) are values obtained by numerical integration of appropriate parts of Eqs. (8) and (9) (correct to order δ^2).

where $\mathsf{S}_i=\langle\mathbf{P}_b\cdot\hat{i}\rangle$ is the average polarization of b in the $\hat{i}$ direction. Conceptually, it is advantageous to employ the orthonormal basis

$$\begin{aligned}
\hat{\alpha} &= \frac{\hat{e}+\hat{\nu}}{\sqrt{2(1+\hat{e}\cdot\hat{\nu})}}, \\
\hat{\beta} &= \frac{\hat{e}-\hat{\nu}}{\sqrt{2(1-\hat{e}\cdot\hat{\nu})}}, \\
\hat{\gamma} &= \hat{\alpha}\times\hat{\beta}.
\end{aligned} \tag{11}$$

Experimentally, it may be more advantageous to determine the polarization components along one or more of the outgoing particle directions ($\hat{e},\hat{\nu},\hat{b}$).

To gauge the importance of the recoil contributions, in Fig. 1 we compare values of several integrated observables calculated from our expressions with the corresponding zero-recoil values for the decay $\Xi^0\to\Sigma^+e^-\overline{\nu}$. For these calculations, we assumed $V_{us}=0.2205$, $f_1(0)=1.0$, $f_2=2.6$, and $g_2=0.0$. Comparing values of integrated observables obtained from our expressions with exact values from tables in Ref. [5], we find that the decay rates agree to better than 1%, and that polarizations and asymmetries agree to better than 0.004, the differences being almost entirely due to terms of order δ^3. We have not included electromagnetic corrections, which are discussed in Ref. [5].

Finally, we present the analytic expressions for the integrated final state polarization to order δ in the final state rest frame, assuming real form factors. The order δ^2 expressions, except for the q^2 dependence of the form factors, can be obtained by adding the $\mathcal{O}(\delta^2)$ terms given in [11]:

$$R=R_0\left[\left(1-\frac{3}{2}\delta\right)f_1^2+\left(3-\frac{9}{2}\delta\right)g_1^2-(4\delta)g_1g_2\right]+R(\delta^2),$$

$$RS_e=R_0\left[\left(2-\frac{10}{3}\delta\right)g_1^2+\left(2-\frac{7}{3}\delta\right)f_1g_1-\left(\frac{1}{3}\delta\right)f_1^2-\left(\frac{2}{3}\delta\right)f_1f_2+\left(\frac{2}{3}\delta\right)f_2g_1-\left(\frac{2}{3}\delta\right)f_1g_2-\left(\frac{10}{3}\delta\right)g_1g_2\right]+RS_e(\delta^2),$$

$$RS_\nu=R_0\left[\left(-2+\frac{10}{3}\delta\right)g_1^2+\left(2-\frac{7}{3}\delta\right)f_1g_1+\left(\frac{1}{3}\delta\right)f_1^2+\left(\frac{2}{3}\delta\right)f_1f_2+\left(\frac{2}{3}\delta\right)f_2g_1-\left(\frac{2}{3}\delta\right)f_1g_2+\left(\frac{10}{3}\delta\right)g_1g_2\right]+RS_\nu(\delta^2),$$

$$RS_\beta=\left[\left(\frac{8}{3}-\frac{52}{15}\delta\right)f_1g_1+\left(\frac{16}{15}\delta\right)f_2g_1-\left(\frac{16}{15}\delta\right)f_1g_2\right]+RS_\alpha(\delta^2),$$

$$RS_\beta=R_0\left[\left(\frac{8}{3}-4\delta\right)g_1^2-\left(\frac{8}{15}\delta\right)f_1^2-\left(\frac{16}{15}\delta\right)f_1f_2-\left(\frac{64}{15}\delta\right)g_1g_2\right]+RS_\beta(\delta^2), \quad (12)$$

where

$$R_0=\frac{G_S^2(\delta M_B)^5}{60\pi^3}.$$

As can be seen in Ref. [2], the zero-recoil ($\delta=0$) expression for S_e (S_ν) is the same as that for the neutrino (electron) asymmetry for a polarized initial baryon [5]. Also, RS_α depends only on $V\times A$ cross terms, and RS_β depends only on $V\times V$ and $A\times A$ terms, as required by a theorem due to Weinberg [10].

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[9] We employ the metric and γ matrix conventions of Ref. [5], so that g_1/f_1 is *positive* for neutron beta decay. They differ from those of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964) only in that γ_5 has the opposite sign, and that there is no i in the definition of $\sigma_{\alpha\beta}$.
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[11] The correct order δ^2 expressions are obtained by adding

$$R(\delta^2)=R_0\delta^2\left(\frac{6}{7}f_1^2+\frac{12}{7}g_1^2+6g_1g_2+\frac{6}{7}f_1f_2+\frac{4}{7}f_2^2+\frac{12}{7}g_2^2\right),$$

$$RS_e(\delta^2)=R_0\delta^2\left(\frac{55}{42}g_1^2+\frac{17}{21}f_1g_1+\frac{19}{42}f_1^2+\frac{4}{3}f_1f_2-\frac{10}{21}f_2g_1+\frac{10}{21}f_1g_2+\frac{116}{21}g_1g_2+\frac{4}{21}f_2^2+\frac{4}{3}g_2^2-\frac{16}{21}f_2g_2\right),$$

$$RS_\nu(\delta^2)=R_0\delta^2\left(-\frac{55}{42}g_1^2+\frac{17}{21}f_1g_1-\frac{19}{42}f_1^2-\frac{4}{3}f_1f_2-\frac{10}{21}f_2g_1+\frac{10}{21}f_1g_2-\frac{116}{21}g_1g_2-\frac{4}{21}f_2^2-\frac{4}{3}g_2^2-\frac{16}{21}f_2g_2\right),$$

$$RS_\alpha(\delta^2)=R_0\delta^2\left(\frac{316}{245}f_1g_1-\frac{752}{735}f_2g_1+\frac{752}{735}f_1g_2-\frac{128}{105}f_2g_2\right),$$

$$RS_\beta(\delta^2)=R_0\delta^2\left(\frac{422}{735}f_1^2+\frac{88}{49}f_1f_2+\frac{8}{35}f_2^2+\frac{362}{245}g_1^2+\frac{1576}{245}g_1g_2+\frac{8}{5}g_2^2\right)$$

to $R, RS_e, RS_\nu, RS_\alpha$ and RS_β, respectively, in Eq. (12).