Enhanced global symmetries and the chiral phase transition

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We examine the possibility that the physical spectrum of a vectorlike gauge field theory exhibits an enhanced global symmetry near a chiral phase transition. A transition from the Goldstone phase to the symmetric phase is expected as the number of fermions N_f is increased to some critical value. Various investigations have suggested that a parity-doubled spectrum develops as the critical value is approached. Using an effective Lagrangian as a guide, we note that parity doubling is associated with the appearance of an enhanced global symmetry in the spectrum of the theory. The enhanced symmetry would develop as the spectrum splits into two sectors, with the first exhibiting the usual pattern of a spontaneously broken chiral symmetry, and the second exhibiting an additional, unbroken symmetry and parity doubling. The first sector includes the Goldstone bosons and other states such as massive scalar partners. The second includes a parity-degenerate vector and axial vector along with other possible parity partners. We note that if such a near-critical theory describes symmetry breaking in the electroweak theory, the additional symmetry suppresses the contribution of the parity-doubled sector to the S parameter. [S0556-2821(99)01421-6]

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I. INTRODUCTION

Gauge-field theories exhibit many different patterns of infrared behavior. During the past few years, there has been much progress in understanding the possibilities in supersymmetric gauge theories [1]. For nonsupersymmetric gauge theories, less is known, but it is expected that the infrared behavior will vary according to the number of massless fermions (N_f) coupled to the gauge fields. For a vectorlike theory such as QCD, it is known that for low values of N_f, the theory confines and chiral symmetry breaking occurs. On the other hand, for large N_f the theory loses asymptotic freedom. In between, there is a conformal window where the theory does not confine, chiral symmetry is restored, and the theory acquires a long-range conformal symmetry. It has been proposed that for an SU(N) gauge theory, there is a transition from the confining, chirally broken theory to the chirally symmetric theory at $N_f \approx 4N$ [2,3]. Recent lattice simulations, however, seem to indicate [4] that the amount of chiral symmetry breaking decreases substantially (for $N=3$) when N_f is only about 4.

Assuming that a single transition takes place at some critical value of N_f, we can ask questions about the spectrum of the theory near the transition. In Ref. [5], it was argued by studying Weinberg spectral function sum rules that for near-critical theories parity partners become more degenerate than in QCD-like theories. This leads naturally to the idea that parity doublets might form as chiral symmetry is being restored. Lattice studies also indicate such a possibility [4].

In this paper we observe using an effective-Lagrangian as a guide, that the formation of degenerate parity partners is associated with the appearance of an enhanced global symmetry in the spectrum of states. We also note that this new symmetry could play a key role in describing a possible strong electroweak Higgs sector. Whether the new symmetry can be shown to emerge dynamically from an underlying gauge theory with N_f near a critical value remains an open question.

It is worth noting that there exist examples of extra symmetries, not manifestly present in the underlying theory, but dynamically generated at low energies. For instance, by using duality arguments, it has been argued [6] that a supersymmetric SU(2) gauge theory with N_f matter fields and global symmetry SU($2N_f$) is dual to a SU(N_f-2) gauge theory with N_f matter fields. For $N_f \geqslant 5$, the ultraviolet flavor symmetry of the latter theory is $SU_L(N_f)\times SU_R(N_f)\times U_B(1)$. Since its infrared global symmetry must be SU($2N_f$) (that of the dual), its infrared symmetry is enhanced.

In Sec. II we discuss the appearance of enhanced global symmetry. Confinement is assumed and the symmetry of the underlying gauge theory, $SU_L(N_f)\times SU_R(N_f)$, is built into an effective Lagrangian describing the physical states of the theory. Parity invariance is imposed and the usual pattern of chiral symmetry breaking $[SU_L(N_f)\times SU_R(N_f)\to SU_V(N_f)]$ is assumed. The N_f^2-1 Goldstone bosons appear together with scalar chiral partners. We augment the spectrum with a set of vector fields for both the $SU_L(N_f)$ and $SU_R(N_f)$ symmetry groups. The Lagrangian thus takes the form of a linear σ model coupled to vectors. It could be expanded to include fields corresponding to other states as well. The natural mass scale of this strongly interacting system is expected to be of order $2\pi v$, where v is the vacuum expectation value.

We examine the spectrum and recognize that there is a particular choice of the parameters that allows for a degenerate vector and axial vector, while enlarging the global symmetry to include an additional (unbroken) $SU_L(N_f)\times SU_R(N_f)$. This happens as the spectrum of the theory splits into two sectors with one displaying the additional symmetry. We then briefly review the arguments (see Ref.

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[5]) that an underlying near-critical SU(N) gauge theory might naturally lead to a more degenerate vector-axial spectrum than in QCD, and to an enhanced symmetry. Finally we note that even a discrete additional symmetry, $Z_{2L}\times Z_{2R}$, of the effective theory is adequate to insure the mass degeneracy of the vector and axial vector.[1]

The possible appearance of an additional, continuous symmetry was considered by Casalbuoni *et al.* in Refs. [7,8]. These papers were restricted to the case $N_f=2$ and did not include discussion of the possible connection to a near-critical underlying theory. The treatment in Ref. [7] made use of a nonlinear realization for the Goldstone degrees of freedom, using hidden gauge symmetry methods [9]. We could generate the effective Lagrangian of Ref. [7] by integrating out the massive scalar degrees of freedom, but that would keep some massive degrees of freedom (the vectors fields) and neglect others. When we focus on low-energy consequences (in Sec. III), we will integrate out all the massive degrees of freedom leading to the electroweak chiral Lagrangian. The treatment of Ref. [8] utilized a linear realization for the scalars and focused on the decoupling of the vectors as they are made heavy relative to the weak scale. We do not take this limit here since we assume the vector and scalar masses to be of the same order.

In Sec. III we embed the electroweak gauge group within the global symmetry group. We observe that the enhanced symmetry of the strongly interacting sector, which now provides electroweak symmetry breaking, plays an important role. The additional symmetry is a partial custodial symmetry for the electroweak S parameter, in the sense that the parity doubled part of the strong sector, by itself, makes no contribution to S. This is shown by integrating out the massive physics to construct the terms in the low-energy electroweak chiral Lagrangian. The S parameter corresponds to one such term.

We extend the study to fermions in a pseudoreal representation of the underlying gauge group in Sec. IV. In this case parity is automatically enforced. The pseudoreal representations allow for the lowest number of colors (i.e., $N=2$) and consequently for the lowest possible number of flavors for which the theory might show a dynamically enhanced symmetry. The enhanced global symmetry is $[\mathrm{SU}(2N_f)]^2$ spontaneously broken to $\mathrm{Sp}(2N_f)\times\mathrm{SU}(2N_f)$.

In Sec. V we conclude and suggest some directions for future work. In the Appendix we provide an explicit representation for the Sp(4) generators.

II. EFFECTIVE LAGRANGIAN FOR $\mathrm{SU}_L(N_F)\times\mathrm{SU}_R(N_F)$ GLOBAL SYMMETRY

To discuss the possible appearance of enhanced symmetry in a strongly interacting spectrum, some description of the spectrum is needed. We will find it helpful to use an effective Lagrangian possessing $\mathrm{SU}_L(N_f)\times\mathrm{SU}_R(N_f)$ symmetry, the global invariance of the underlying gauge theory. We assume that chiral symmetry is broken according to the standard pattern $\mathrm{SU}_L(N_f)\times\mathrm{SU}_R(N_f)\to\mathrm{SU}_V(N_f)$. The N_f^2-1 Goldstone bosons are encoded in the $N_f\times N_f$ real traceless matrix Φ^i_j with $i,j=1,...,N_f$. The complex matrix $M=S+i\Phi$ describes both the Goldstone bosons as well as associated scalar partners S. It transforms linearly under a chiral rotation:

$$M\to u_L M u_R^\dagger, \tag{2.1}$$

with $u_{L/R}$ in $\mathrm{SU}_{L/R}(N_f)$.

To augment the massive spectrum, we introduce vector and axial vector fields following a method outlined in Ref. [10]. We first formally gauge the global chiral group introducing the covariant derivative

$$D^\mu M=\partial^\mu M-i\tilde{g}A_L^\mu M+i\tilde{g}MA_R^\mu, \tag{2.2}$$

where $A^\mu_{L/R}=A^{\mu,a}_{L/R}T^a$ and T^a are the generators of SU(N_f), with $a=1,...,N_f^2-1$ and $\mathrm{Tr}[T^aT^b]=(1/2)\delta^{ab}$. The left and right couplings are the same since we assume parity invariance. Under a chiral transformation

$$A^\mu_{L/R}=u_{L/R}A^\mu_L u^\dagger_{L/R}-\frac{i}{\tilde{g}}\partial^\mu u_{L/R}u^\dagger_{L/R}. \tag{2.3}$$

The effective Lagrangian needs only to be invariant under global chiral transformations. Including terms only up to mass dimension four, it may be written in the form

$$\begin{aligned}L=&\frac{1}{2}\mathrm{Tr}[D_\mu M D^\mu M^\dagger]+m^2\,\mathrm{Tr}[A_{L\mu}A_L^\mu+A_{R\mu}A_R^\mu]\\&+h\,\mathrm{Tr}[A_{L\mu}MA_R^\mu M^\dagger]+r\,\mathrm{Tr}[A_{L\mu}A_L^\mu MM^\dagger\\&+A_{R\mu}A_R^\mu M^\dagger M]+i\frac{s}{2}\mathrm{Tr}[A_{L\mu}(MD^\mu M^\dagger-D^\mu MM^\dagger)\\&+A_{R\mu}(M^\dagger D^\mu M-D^\mu M^\dagger M)].\end{aligned} \tag{2.4}$$

The parameters h, r, and s are dimensionless real parameters, while m^2 is a common mass term. To this, we may add a kinetic term for the vector fields

$$L_{\mathrm{Kin}}=-\frac{1}{2}\mathrm{Tr}[F_{L\mu\nu}F_L^{\mu\nu}+F_{R\mu\nu}F_R^{\mu\nu}], \tag{2.5}$$

where

$$F^{\mu\nu}_{L/R}=\partial^\mu A^\nu_{L/R}-\partial^\nu A^\mu_{L/R}-i\tilde{g}[A^\mu_{L/R},A^\nu_{L/R}], \tag{2.6}$$

along with vector-interaction terms respecting only the global symmetry. Finally, we may add the double trace term,

$$\mathrm{Tr}[MM^\dagger]\mathrm{Tr}[A_L^2+A_R^2], \tag{2.7}$$

at the dimension-four level. To arrange for symmetry breaking, a potential $V(M,M^\dagger)$ must be added. When the effective Lagrangian is extended to the dimension-six level and higher, many new operators enter.

[1]We thank Noriaki Kitazawa for suggesting the possibility of a discrete symmetry.

Parity is also a symmetry and it acts on the fields according to

$$PM(x)(P)^{-1}=M^{\dagger}(-x), \tag{2.8}$$

$$PA^{\mu}_{L/R}(x)(P)^{-1}=\epsilon(\mu)A^{\mu}_{R/L}(-x), \tag{2.9}$$

where $\epsilon(\mu)=1$ for $\mu=0$ and -1 for $\mu=1,2,3$.

The spectrum described by this effective Lagrangian consists of Goldstone bosons, a set of scalars, and massive vector and axial vectors. With its massive vectors and axial vectors, it is of course not renormalizable, but it can nevertheless provide a reasonable description of low-lying states. (It is worth noting that a Lagrangian of this type does this for the low-lying QCD resonances [11]). While it cannot be a complete description of the hadronic spectrum, it has sufficient content to guide a general discussion of enhanced symmetries.

Keeping only terms quadratic in the fields and temporarily neglecting the massive scalars, the Lagrangian Eq. (2.4) takes the form

$$L=\frac{1}{2}\mathrm{Tr}[\partial_{\mu}\Phi\partial^{\mu}\Phi]+\sqrt{2}(s-\tilde{g})v\,\mathrm{Tr}[\partial_{\mu}\Phi A^{\mu}] +M_A^2\,\mathrm{Tr}[A_{\mu}A^{\mu}]+M_V^2\,\mathrm{Tr}[V_{\mu}V^{\mu}], \tag{2.10}$$

where $M=v+i\Phi$, v is the vacuum expectation value and we have defined the new vector fields

$$V=\frac{A_L+A_R}{\sqrt{2}},\quad A=\frac{A_L-A_R}{\sqrt{2}}. \tag{2.11}$$

The vector and axial masses are related to the effective Lagrangian parameters via

$$M_A^2=m^2+v^2\left[r+\tilde{g}^2-2s\tilde{g}-\frac{h}{2}\right],$$

$$M_V^2=m^2+v^2\left[r+\frac{h}{2}\right], \tag{2.12}$$

where the contribution from Eq. (2.7) has been absorbed into m^2. The terms proportional to v^2 are Higgs-like contributions, arising from the spontaneous breaking.

The second term in Eq. (2.10) mixes the axial vector with the Goldstone bosons. This kinetic mixing may be diagonalized away by the field redefinition

$$A\rightarrow A+v\frac{\tilde{g}-s}{\sqrt{2}M_A^2}\partial\Phi, \tag{2.13}$$

leaving the mass spectrum unchanged [10]. The vector-axial vector mass difference is given by

$$M_A^2-M_V^2=v^2[\tilde{g}^2-2\tilde{g}s-h]. \tag{2.14}$$

In QCD this difference is known experimentally to be positive, a fact that can be understood by examining the Weinberg spectral function sum rules (see Ref. [12], and references therein). The effective Lagrangian description is of course unrestrictive. Depending on the values of the $\tilde{g}$, s, and h parameters, one can have a degenerate or even inverted mass spectrum.

What kind of underlying gauge theory might provide a degenerate or inverted spectrum? Clearly, it has to be different from QCD, allowing for a modification of the spectral function sum rules. In Ref. [5], an SU(N) gauge theory (with $N>2$) and N_f flavors was considered. If N_f is large enough but below $11N/2$, an infrared fixed point of the gauge coupling α_* exists, determined by the first two terms in the β function. For N_f near $11N/2$, α_* is small and the global symmetry group remains unbroken. For small N_f, on the other hand, the chiral symmetry group $\mathrm{SU}_L(N_f)\times\mathrm{SU}_R(N_f)$ breaks to its diagonal subgroup. One possibility is that the transition out of the broken phase takes place at a relatively large value of $N_f/N(\approx 4)$, corresponding to a relatively weak infrared fixed point [2,3]. An alternate possibility is that the transition takes place in the strong-coupling regime, corresponding to a small value of N_f/N [4]. The larger value emerges from the renormalization-group improved gap equation, as well as from instanton effects [13], and saturates a recently conjectured upper limit [14]. It corresponds to the perturbative infrared fixed point α_* reaching a certain critical value α_c. A similar result has also been obtained by using a suitable effective Lagrangian [3].

These studies also suggest that the order parameter, for example the Goldstone boson decay constant $F_{\pi}\equiv v$, vanishes continuously at the transition relative to the intrinsic renormalization scale Λ of the gauge theory. In the broken phase near the transition, the fact that one is approaching a phase with long-range conformal symmetry suggests that all massive states scale to zero with the order parameter relative to Λ [15].

In Ref. [5] the spectrum of states in the broken phase near a large-N_f/N transition was investigated using the spectral function sum rules. It was shown that the ordering pattern for vector-axial hadronic states need not be the same as in QCD-like theories (small N_f/N). The crucial ingredient is that these theories contain an extended ‘‘conformal region’’ extending from roughly $2\pi F_{\pi}$ to the scale Λ where asymptotic freedom sets in. In this region, the coupling remains close to an approximate infrared fixed point and the theory has an approximate long-range conformal symmetry. It was argued that this leads to a reduced vector-axial mass splitting, compared to QCD-like theories. This suggests the interesting possibility that parity doublets begin to form as chiral symmetry is being restored. That is, the vector-axial mass ratio approaches unity as the masses decrease relative to Λ. Lattice results seem to provide supporting evidence for such a possibility [4], although at smaller values of N_f/N.

If a parity-doubled spectrum does appear, it is natural to expect it to be associated with some new global symmetry. While we have not demonstrated the appearance of a new global symmetry using the underlying degrees of freedom, we can explore aspects of parity doubling at the effective Lagrangian level. Returning to this description, we note that vector-axial parity doubling corresponds to the parameter choice [see Eq. (2.14)],

$$\tilde{g}^2 = 2\tilde{g}s + h. \tag{2.15}$$

This condition does not yet reveal an additional symmetry and therefore there is no reason to expect parity degeneracy to be stable in the presence of quantum corrections and the many higher dimensional operators that can be added to the effective Lagrangian in Eq. (2.4).

However, for the special choice $s=\tilde{g}$, $r=\tilde{g}^2/2$ and $h=-\tilde{g}^2$, the effective Lagrangian acquires a new continuous global symmetry that protects the vector-axial mass difference. The effective Lagrangian at the dimension-four level takes the simple form

$$L = \frac{1}{2}\mathrm{Tr}[\partial_\mu M \partial^\mu M^\dagger] + m^2\,\mathrm{Tr}[A_{L\mu}A_L^\mu + A_{R\mu}A_R^\mu], \tag{2.16}$$

along with vector kinetic and interaction terms, the interaction term Eq. (2.7), and the symmetry-breaking potential $V(M,M^\dagger)$. The theory now has two sectors, with the vector and axial vector having their own unbroken global $\mathrm{SU}_L(N_f)\times \mathrm{SU}_R(N_f)$. The two sectors interact only through the product of singlet operators. The full global symmetry is $[\mathrm{SU}_L(N_f)\times \mathrm{SU}_R(N_f)]^2\times \mathrm{U}_V(1)$ spontaneously broken to $\mathrm{SU}_V(N_f)\times \mathrm{U}_V(1)\times[\mathrm{SU}_L(N_f)\times \mathrm{SU}_R(N_f)]$. The vector and axial vector become stable due to the emergence of a new conservation law. This enhanced symmetry would become exact only in the chiral limit. For finite but small (relative to Λ) values of the mass scales in Eq. (2.16), there are additional, smaller terms giving smaller mass splittings and small width-to-mass ratios.

It is, of course, a simple observation that a new symmetry and conservation law emerge if a theory is split into two sectors by setting certain combinations of parameters to zero. But here we were led to this possibility by looking for a symmetry basis for the parity doubling that has been hinted at by analyses of the underlying gauge theory. Although we have used a relatively simple effective Lagrangian, we anticipate that the conclusion is true in general, that is, that parity doublets form in the spectrum of a strongly interacting theory with chiral symmetry breaking only if the spectrum splits into two sectors, one exhibiting the spontaneous breaking and the other, parity-doubled, sector exhibiting an unbroken additional symmetry.

We next observe that along with the additional global symmetry $\mathrm{SU}_L(N_f)\times \mathrm{SU}_R(N_f)$, the effective Lagrangian Eq. (2.16) possesses a discrete $Z_{2L}\times Z_{2R}$ symmetry. Under $Z_{2L}\times Z_{2R}$ the vector fields transform according to

$$A_L \to z_L A_L, \quad A_R \to z_R A_R, \tag{2.17}$$

with $z_{L/R}=1,-1$ and $z_{L/R}\in Z_{2L/R}$. Actually, the discrete symmetry alone is enough to insure vector-axial mass degeneracy and stability against decay. In that case, additional interaction terms, such as the single trace term

$$r\,\mathrm{Tr}[A_{\mu L}A_L^\mu M M^\dagger + A_{\mu R}A_R^\mu M^\dagger M], \tag{2.18}$$

are allowed, but degeneracy and stability are still insured. Of course, trilinear vector interactions will not respect this discrete symmetry. Nevertheless, one cannot rule out the possibility that it is only this smaller, discrete symmetry that appears as an effective infrared symmetry of an underlying gauge theory near the chiral and/or conformal transition.

From the point of view of the underlying theory, the appearance of any additional symmetry in the spectrum, at criticality, would seem mysterious. The composite degrees of freedom in both sectors are made of the same fundamental fermions with a single underlying $\mathrm{SU}_L(N_f)\times \mathrm{SU}_R(N_f)$ symmetry. If the symmetry of the parity-doubled sector is an unbroken $\mathrm{SU}_L(N_f)\times \mathrm{SU}_R(N_f)$, it would look as though the chiral symmetry is being realized there in the Wigner-Weyl mode. If that is the case, chiral dynamics would have to be influenced by confinement and bound-state formation in an interesting new way. Whether a near-critical gauge theory can lead to this behavior is an unresolved question.

III. STRONGLY INTERACTING ELECTROWEAK SECTOR

We next discuss the consequences of enhanced symmetry for a strong symmetry breaking sector of the standard electroweak theory, embedding the $\mathrm{SU}_L(2)\times \mathrm{U}_Y(1)$ gauge symmetry in the global $\mathrm{SU}_L(N_f)\times \mathrm{SU}_R(N_f)$ chiral group. In this section, for simplicity, we will restrict attention to the $\mathrm{SU}_L(2)\times \mathrm{SU}_R(2)$ subgroup of the full global group [8]. The electroweak gauge transformation then takes the form

$$M \to u_W M u_Y^\dagger, \tag{3.1}$$

where M is now a 2×2 matrix which can be written as $M=(1/\sqrt{2})[\sigma + i\vec{\tau}\cdot\vec{\pi}]$, where $u_W=u_L=\exp[(i/2)\epsilon^a\tau^a]$ with τ^a the Pauli matrices, and where $u_Y=\exp[(i/2)\epsilon_0\tau^3]$. The weak vector boson fields transform as

$$W^\mu \to u_L W^\mu u_L^\dagger - \frac{i}{g}\partial^\mu u_L u_L^\dagger, \tag{3.2}$$

$$B^\mu \to u_Y B^\mu u_Y^\dagger - \frac{i}{g'}\partial^\mu u_Y u_Y^\dagger, \tag{3.3}$$

where g and g' are the standard electroweak coupling constants, $W_\mu = W_\mu^a(\tau^a/2)$ and $B_\mu = B_\mu(\tau^3/2)$.

A convenient method of coupling the electroweak gauge fields to the globally invariant effective Lagrangian of Sec. II is to introduce a covariant derivative, which includes the W and B fields as well as the strong vector and axial-vector fields,

$$D^\mu M = \partial^\mu M - igW^\mu M + ig' M B^\mu - i\tilde{g}cC_L^\mu M + i\tilde{g}c' M C_R^\mu, \tag{3.4}$$

where we have defined the new vector fields

$$C_L^\mu = A_L^\mu - \frac{g}{\tilde{g}}W^\mu, \quad C_R^\mu = A_R^\mu - \frac{g'}{\tilde{g}}B^\mu, \tag{3.5}$$

and where c and c' are arbitrary real constants. Since the $A_{L/R}^\mu$ transform as Eq. (2.3), the $C_{L/R}^\mu$ transform under the electroweak transformations as

$$C_L^\mu \to u_L C_L^\mu u_L^\dagger, \quad C_R^\mu \to u_Y C_R^\mu u_Y^\dagger. \tag{3.6}$$

By requiring invariance under the parity operation exchanging the labels $L \leftrightarrow R$ we have the extra condition $c = c'$.

The effective Lagrangian is constructed to be invariant under a local $SU_L(2) \times U_Y(1)$ as well as *CP*. The *CP* transformation properties of the fields[2] insure that the covariant derivative transforms as M: i.e.,

$$CPD_\mu M(x)(CP)^{-1} = \eta [D^\mu M(-x)]^*. \tag{3.11}$$

The effective Lagrangian is then obtained by replacing in Eq. (2.4) the covariant derivative with the new one in Eq. (3.4). To make the theory electroweak gauge invariant, one substitutes the $A_{L/R}$ with the $C_{L/R}$, giving, through dimension four,

$$\begin{aligned} L = {} & \frac{1}{2} \mathrm{Tr}[D_\mu M D^\mu M^\dagger] + m^2 \mathrm{Tr}[C_{L\mu} C_L^\mu + C_{R\mu} C_R^\mu] \\ & + h\, \mathrm{Tr}[C_{L\mu} M C_R^\mu M^\dagger] + r\, \mathrm{Tr}[C_{L\mu} C_L^\mu M M^\dagger \\ & + C_{R\mu} C_R^\mu M^\dagger M] + i \frac{s}{2} \mathrm{Tr}[C_{L\mu}(M D^\mu M^\dagger - D^\mu M M^\dagger) \\ & + C_{R\mu}(M^\dagger D^\mu M - D^\mu M^\dagger M)]. \end{aligned} \tag{3.12}$$

To this we add a kinetic term

$$\begin{aligned} L_{\mathrm{Kin}} = {} & -\frac{1}{2} \mathrm{Tr}[F_{L\mu\nu} F_L^{\mu\nu} + F_{R\mu\nu} F_R^{\mu\nu}] - \frac{1}{2} \mathrm{Tr}[W_{\mu\nu} W^{\mu\nu}] \\ & - \frac{1}{2} \mathrm{Tr}[B_{\mu\nu} B^{\mu\nu}], \end{aligned} \tag{3.13}$$

where

$$\begin{aligned} W_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu], \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{3.14}$$

with the $F_{L/R}$ for the fields $A_{L/R}$ defined in Eq. (2.6), along with other interaction terms involving the $C_{L/R}$ fields, the interaction term

$$\mathrm{Tr}[M M^\dagger] \mathrm{Tr}[C_L^2 + C_R^2], \tag{3.15}$$

and a symmetry-breaking potential.

One can show that this is the most general dimension-four, *CP*-invariant Lagrangian describing a strongly interacting set of scalars, vectors, and axial vectors with a spontaneously broken $SU_L(2) \times SU_R(2)$ symmetry, and possessing electroweak gauge invariance. It describes weak mixing between the $A_{L/R}$ fields and the W and Z, and, through the mixing, conventional electroweak charges for the $A_{L/R}$. The extension of this effective Lagrangian to the relevant case of the larger symmetry group $SU_L(N_f) \times SU_R(N_f)$ with $N_f > 2$, is straightforward.

Replacing M by its vacuum value $v/\sqrt{2}$, and keeping only terms quadratic in the fields, the Lagrangian Eq. (3.12) takes the form

$$\begin{aligned} L = {} & M_A^2 \mathrm{Tr}[A^2] + M_V^2 \mathrm{Tr}[V^2] - \frac{\sqrt{2}}{\tilde{g}} (1-\chi) M_A^2 \mathrm{Tr}[(gW \\ & - g'B)A] - \frac{\sqrt{2}}{\tilde{g}} M_V^2 \mathrm{Tr}[(gW + g'B)V] + \frac{M_V^2}{2\tilde{g}^2} \mathrm{Tr}[(gW \\ & + g'B)^2] + \frac{M_A^2}{2\tilde{g}^2} (1+\delta) \mathrm{Tr}[(gW - g'B)^2] + \cdots, \end{aligned} \tag{3.16}$$

where we have defined

$$\begin{aligned} M_V^2 &= m^2 + v^2 \left[r + \frac{h}{2} \right], \\ M_A^2 &= m^2 + v^2 \left[r + \tilde{g}^2 c^2 - 2s\tilde{g}c - \frac{h}{2} \right], \\ \chi &= \frac{v^2}{2M_A^2} \tilde{g} [\tilde{g}c - s], \\ \delta &= \frac{v^2}{2M_A^2} [\tilde{g}^2(1-2c) + 2s\tilde{g}]. \end{aligned} \tag{3.17}$$

The vector V and axial A fields are defined in Eq. (2.11). This quadratic Lagrangian describes masses for the V and A, weak mass mixing with the W and B, and a mass matrix for the W and B. There is no further, kinetic energy mixing among these fields. The vector and axial vector masses, M_V^2 and M_A^2, are arbitrary, depending on the choice of parameters, although generically we expect them and the scalar masses to be of order $4\pi^2 v^2$.

The weak mixing terms in Eq. (3.16) provide a contribution from physics beyond the standard model to the oblique electroweak corrections. These may be described by the S, T, and U parameters, but the last two vanish in the present model because there is no breaking of weak isospin in the strong sector. While this is not apparent in Eq. (3.16), it is insured by the Ward identities and easily revealed through the mixing effects. The S parameter receives contributions from all the physics beyond the standard model, including, in the model being used here, loops of pseudo Goldstone bosons (PGB's), the strongly interacting massive scalars, and the vector and axial vector. The direct, vector-dominance contribution of the vector and axial vector may be read off from Eq. (3.16) together with the kinetic term for the V and A. One finds

[2]Here we summarize the *CP* field transformations:

$$CPM(x)(CP)^{-1} = \eta M^*(-x), \tag{3.7}$$

$$CPA_{L/R\mu}(x)(CP)^{-1} = -A_{L/R}^\mu(-x), \tag{3.8}$$

$$CPW_\mu(x)(CP)^{-1} = -W^\mu(-x), \tag{3.9}$$

$$CPB_\mu(x)(CP)^{-1} = -B^\mu(-x), \tag{3.10}$$

where η is an arbitrary C phase.

$$S_{\text{vect-dom}}=\frac{8\pi}{\tilde{g}^2}\left[\frac{M_A^2(1-\chi)^2}{M_Z^2-M_A^2}-\frac{M_V^2}{M_Z^2-M_V^2}\right]$$
$$\approx\frac{8\pi}{\tilde{g}^2}[1-(1-\chi)^2]. \tag{3.18}$$

Clearly, this contribution to the S parameter can take on any value depending on the choice of parameters. Its typical order of magnitude, with the strong-coupling estimate $\tilde{g}^2\approx4\pi^2$, is expected to be $O(1)$. This expression can be seen to be equivalent to the familiar vector-dominance formula $S_{\text{vect-dom}}\approx4\pi[F_V^2/M_V^2-F_A^2/M_A^2]$ [16], with the identifications $F_V^2=(2/\tilde{g}^2)M_V^2$ and $F_A^2=(2/\tilde{g}^2)M_A^2(1-\chi)^2$.

We next observe that the choice

$$s=\tilde{g}c,\quad h=-\tilde{g}^2c^2 \tag{3.19}$$

gives $\chi=0$, leading immediately to the degeneracy of the vector and axial vector [see Eq. (3.17)], the relation $F_A=F_V$, and the vanishing of $S_{\text{vect-dom}}$. The further choice $r=\tilde{g}^2c^2/2$ leads to the collapse of the general effective Lagrangian into the simple form

$$L=\frac{1}{2}\text{Tr}[D_\mu MD^\mu M^\dagger]+m^2\,\text{Tr}[C_{L\mu}C_L^\mu+C_{R\mu}C_R^\mu], \tag{3.20}$$

along with the kinetic terms of Eq. (3.13), interactions among the $C_{L/R}^\mu$ fields, the interaction term Eq. (3.15), and a symmetry-breaking potential. Here, $DM=\partial M-igWM+ig'MB$ is the standard electroweak covariant derivative, and $C_{L/R}^\mu$ are given by Eq. (3.5).

The strongly interacting sector has split into two subsectors, communicating only through the electroweak interactions. One subsector consists of the Goldstone bosons together with their massive scalar partners. The other consists of the degenerate vector and axial vector described by the $A_{L/R}^\mu$ fields. The mass mixing in Eq. (3.12) insures that they have conventional electroweak couplings. In the absence of electroweak interactions, there is an enhanced symmetry $[\text{SU}_L(2)\times\text{SU}_R(2)]\times[\text{SU}_L(2)\times\text{SU}_R(2)]$, breaking spontaneously to $\text{SU}_V(2)\times[\text{SU}_L(2)\times\text{SU}_R(2)]$. The electroweak interactions explicitly break the enhanced symmetry to $\text{SU}_L(2)\times\text{U}_Y(1)$. All of this may be generalized to $N_f>2$, necessary to yield a near-critical theory.

The additional symmetry has an important effect on the S parameter, suppressing contributions that are typically large in QCD-like theories. It does not suppress all contributions, of course, since the symmetry-breaking subsector gives contributions that are expected to be of order unity. The parity-doubled subsector, however, cannot by itself contribute to S, because S relies on electroweak symmetry breaking for its existence. It is the coefficient of an operator in the low-energy electroweak chiral Lagrangian (L_1 in Ref. [17]), which may be written in the form $\text{Tr}\,W^{\mu\nu}UB_{\mu\nu}U^\dagger$, where $W^{\mu\nu}$ and $B_{\mu\nu}$ are defined in Eq. (3.14) and U is the Goldstone matrix field satisfying the nonlinear constraint $UU^\dagger=U^\dagger U=1$. Clearly the U operator, with its vacuum value $U=1$, is necessary to couple $W^{\mu\nu}$ to $B_{\mu\nu}$.

Among the contributions to S remaining in the limit of enhanced symmetry, are loops of pseudo-Goldstone bosons, present when $N_f>2$. They may be estimated using chiral perturbation theory, with the standard-model corrections removed by convention. While they typically give contributions to S of order unity, their specific value depends on details such as mass estimates for the PGB's that arise from electroweak, QCD, and other interactions [18]. An interesting new feature in the limit of enhanced symmetry is that the PGB contribution is not related to a direct, vector-dominance effect (which is now zero). There will also be contributions from the strongly interacting TeV physics, represented in our effective Lagrangian by the massive scalars. Our purpose here is not to make these estimates, but only to point out that an enhanced symmetry, leading to vector-axial vector degeneracy, will suppress contributions to S purely from the parity-doubled sector. These include the typically large vector dominance contribution discussed above.

Finally we note that, as we discussed at the end of Sec. III, it could be that only a lesser, discrete symmetry emerges in the physical spectrum. Even this would be sufficient to insure vector-axial degeneracy and the vanishing of the vector dominance contribution to the S parameter. The discrete symmetry of Sec. III would only be possible if trilinear vector interactions are somehow suppressed. It will be interesting to explore the phenomenology of this possibility, in particular the effect on the self interactions of the W and Z.

IV. SU($2N_F$) GLOBAL SYMMETRY

In this section we adapt the above discussion to the interesting case of fermions in pseudoreal representations of the gauge group. The simplest example is provided by an underlying SU(2) gauge theory, a choice that will also offer the smallest value for the critical N_f [19]. Such theories are currently being investigated on the lattice (see Ref. [20]). The quantum global symmetry for N_f matter fields in the pseudoreal representation of the gauge group [21] is $\text{SU}(2N_f)$. We expect the gauge dynamics to create a nonvanishing fermion-antifermion condensate which breaks the global symmetry to $\text{Sp}(2N_f)$. Since $\text{SU}(2N_f)\supset\text{SU}_L(N_f)\times\text{SU}_R(N_f)$, the left-right independent groups are unified and parity invariance is automatic.

This breaking pattern gives $2N_f^2-N_f-1$ Goldstone bosons which are contained in the antisymmetric matrix M^{ij} and $i,j=1,...,2N_f$. With $u\in\text{SU}(2N_f)$ we have

$$M\rightarrow uMu^T. \tag{4.1}$$

We associate a vector field $A_\mu=A_\mu^aT^a$ with T^a, a generic generator of $\text{SU}(2N_f)$, $(a=1,...,4N_f^2-1)$, and $\text{Tr}[T^aT^b]=(1/2)\delta^{ab}$. Following the procedure outlined in the previous sections, we define a formal covariant derivative as

$$D_\mu M=\partial_\mu M-i\tilde{g}A_\mu M-i\tilde{g}MA_\mu^T, \tag{4.2}$$

where A transforms as

$$A_\mu\rightarrow uA_\mu u^\dagger-\frac{i}{\tilde{g}}\partial_\mu uu^\dagger. \tag{4.3}$$

With electroweak interactions turned off, the effective Lagrangian reads

$$L=\frac{1}{2}\mathrm{Tr}[DMDM^{\dagger}]+m^{2}\,\mathrm{Tr}[A^{2}]+r\,\mathrm{Tr}[A^{2}MM^{\dagger}]+h\,\mathrm{Tr}[AMA^{T}M^{\dagger}]+is\,\mathrm{Tr}[A(MDM^{\dagger}-DMM^{\dagger})], \tag{4.4}$$

together with the kinetic term

$$L_{\mathrm{Kin}}=-\frac{1}{2}\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}], \tag{4.5}$$

where $F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}-i\tilde{g}[A_{\mu},A_{\nu}]$, along with globally invariant vector interaction terms, an interaction term proportional to $\mathrm{Tr}\,A^{2}\,\mathrm{Tr}\,MM^{\dagger}$, and a symmetry-breaking potential.

The global symmetry is enhanced to $\mathrm{SU}(2N_f)\times\mathrm{SU}(2N_f)$ for the parameter choice $s=\tilde{g}$, $h=\tilde{g}^{2}$ and $r=\tilde{g}^{2}$. The effective Lagrangian then takes the form

$$L=\frac{1}{2}\mathrm{Tr}[\partial M\partial M^{\dagger}]+m^{2}\,\mathrm{Tr}[A^{2}] \tag{4.6}$$

together with the same terms as above. The spontaneous breaking leads to the vacuum symmetry $\mathrm{Sp}(2N_f)\times\mathrm{SU}(2N_f)$.

To proceed further, we simplify the notation by choosing $N_f=2$. We divide the generators $\{T\}$ of SU(4) into two classes, calling the generators of Sp(4) $\{S^{a}\}$ with $a=1,...,10$ and the broken generators $\{X^{i}\}$ with $i=1,...,5$. We have

$$S^{T}E+ES=0, \tag{4.7}$$

with

$$E=\frac{1}{2\sqrt{2}}\begin{pmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{-1} & \mathbf{0}\end{pmatrix}. \tag{4.8}$$

In the Appendix we provide a convenient representation for the $\{S\}$ and $\{X\}$ generators. We define the antisymmetric meson matrix $M=(-M^{T})$ as

$$M=\sqrt{2}[\sigma+i2\sqrt{2}X^{i}\Pi^{i}]E, \tag{4.9}$$

where the five Π^{i} fields are the Goldstone bosons associated with the breaking of SU(4)$\rightarrow$Sp(4).

It is convenient to divide the vector field A in the following way:

$$A=A_X+A_S, \tag{4.10}$$

where $A_X=A_X^{i}X^{i}$, and $A_S=A_S^{a}S^{a}$. The A_X are the axial vector fields while the A_S are the vectors. Then expanding M around its vacuum value $\sqrt{2}vE$ and keeping only terms quadratic in the fields, the Lagrangian Eq. (4.4) takes the form

$$L=\frac{1}{2}[\partial\sigma\partial\sigma+\partial\Pi^{i}\partial\Pi^{i}]-v\frac{(\tilde{g}-s)}{\sqrt{2}}\partial\Pi^{i}A_X^{i}+M_X^{2}\,\mathrm{Tr}[A_X^{2}]+M_S^{2}\,\mathrm{Tr}[A_S^{2}] \tag{4.11}$$

with

$$M_S^{2}=m^{2}+\frac{v^{2}}{4}[r-h],$$
$$M_X^{2}=m^{2}+\frac{v^{2}}{4}[2\tilde{g}^{2}+r-4\tilde{g}s+h]. \tag{4.12}$$

For the choice of parameters associated with an additional SU(4) global symmetry (i.e., $s=\tilde{g}$, $h=\tilde{g}^{2}$ and $r=\tilde{g}^{2}$) the vector-axial vector mass difference vanishes, as do the width to mass ratios.

We next treat the above theory as an electroweak symmetry-breaking sector by gauging the $\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$ subgroup. It is convenient to introduce a vector field G_{μ}. If we were to gauge the entire SU(4) flavor symmetry then G_{μ} would transform under chiral rotations in the standard way

$$G_{\mu}\rightarrow uG_{\mu}u^{\dagger}-\frac{i}{g}\partial_{\mu}uu^{\dagger}. \tag{4.13}$$

We identify the electroweak gauge transformations in the following way:

$$u=\begin{pmatrix}u_L & \mathbf{0}\\ \mathbf{0} & u_R^{*}\end{pmatrix}, \tag{4.14}$$

with $u_{L/R}\in\mathrm{SU}_{L/R}(2)$. Then

$$G_{\mu}=\begin{pmatrix}W_{\mu} & \mathbf{0}\\ \mathbf{0} & -\frac{g'}{g}B_{\mu}^{T}\end{pmatrix}, \tag{4.15}$$

where $W_{\mu}=W_{\mu}^{a}(\tau^{a}/2)$ and $B_{\mu}=B_{\mu}(\tau^{3}/2)$, and g and g' are the electroweak couplings. It is easy to verify that the electroweak transformation properties of the gauge bosons are respected [see Eq. (3.3)].

Using the left-right generators defined in Eq. (A10) we have

$$G=W^{a}L^{a}-\frac{g'}{g}B^{3}R^{3T}, \tag{4.16}$$

with $a=1,2,3$. In terms of the axial and vector type generators we have

$$G=G_X+G_S \tag{4.17}$$

with

$$G_X=\frac{1}{\sqrt{2}}\left(W^{a}-\frac{g'}{g}B^{a}\right)X^{a},\quad G_S=\frac{1}{\sqrt{2}}\left(W^{a}+\frac{g'}{g}B^{a}\right)S^{a}. \tag{4.18}$$

The covariant derivative including the weak vector bosons and the composite vector fields is

$$D_\mu M = \partial_\mu M - ig(G_\mu M + M G_\mu^T) - i\tilde{g}c(C_\mu M + M C_\mu^T), \tag{4.19}$$

where c is a real coefficient and we have introduced the vector C_μ

$$C = A - \frac{g}{\tilde{g}} G, \tag{4.20}$$

transforming covariantly under electroweak rotations.

We extend the effective Lagrangian of Eq. (4.4) to include electroweak interactions by replacing the old covariant derivative with the one in Eq. (4.19). To render the full theory invariant under electroweak transformations we also substitute A with C giving

$$L = \frac{1}{2}\mathrm{Tr}[DMDM^\dagger] + m^2\,\mathrm{Tr}[C^2] + r\,\mathrm{Tr}[C^2 M M^\dagger] + h\,\mathrm{Tr}[CMC^T M^\dagger] + is\,\mathrm{Tr}[C(MDM^\dagger - DMM^\dagger)]. \tag{4.21}$$

To this we add kinetic terms, interaction terms involving the C fields, the interaction term $\mathrm{Tr}\,C^2\,\mathrm{Tr}\,MM^\dagger$, and a symmetry-breaking potential.

Replacing M by its vacuum value and retaining only quadratic mass terms for the vectors we have

$$L = M_X^2(1+\delta)\frac{g^2}{\tilde{g}^2}\mathrm{Tr}[G_X^2] + M_X^2\,\mathrm{Tr}[A_X^2] - 2\frac{g}{\tilde{g}}M_X^2(1-\chi) \times \mathrm{Tr}[G_X A_X] + M_S^2\,\mathrm{Tr}\left[A_S^2 + \frac{g^2}{\tilde{g}^2}G_S^2 - 2\frac{g}{\tilde{g}}G_S A_S\right] + \cdots, \tag{4.22}$$

where we have identified

$$M_S^2 = m^2 + \frac{v^2}{4}[r-h],$$

$$M_X^2 = m^2 + \frac{v^2}{4}[r+h+2\tilde{g}^2c^2 - 4s\tilde{g}c],$$

$$\delta = \frac{v^2}{2M_X^2}[\tilde{g}^2 - 2\tilde{g}(\tilde{g}c - s)],$$

$$\chi = \frac{v^2}{2M_X^2}\tilde{g}(\tilde{g}c - s). \tag{4.23}$$

The generalization of this discussion to the case $N_f > 2$, necessary for near criticality of the underlying gauge theory, is straightforward.

From this point on, the discussion of enhanced symmetry, parity degeneracy, and the estimate of the S parameter proceeds as in the previous section. The choice of parameters $s = \tilde{g}c$ and $h = r = \tilde{g}^2c^2$ leads to the enhanced symmetry of the strongly interacting sector and to parity degeneracy. The contribution to the S parameter from the parity-doubled sector by itself is zero. The contribution from the symmetry-breaking sector is modified by the presence of the larger number of pseudo-Goldstone bosons [19] associated with the $SU(2N_f)$ global symmetry.

V. CONCLUSIONS

In this paper we used an effective Lagrangian to explore some features that might arise in a strongly coupled gauge theory when the number of fermions N_f is near a critical value for the transition to chiral symmetry. It has been argued that this transition is second order or higher and that a long-range conformal symmetry also sets in at the transition. It has also been suggested that near the transition, parity doublets may begin to form [5,4].

We explored this possibility using as a guide an effective Lagrangian with a linear realization of the global chiral symmetry. The spectrum was taken to consist of a set of Goldstone particles, associated massive scalars, and a set of massive vectors and axial vectors. It was observed that parity doubling is associated with the appearance of an enhanced global symmetry, consisting of the spontaneously broken chiral symmetry of the underlying theory $[SU_L(N_f) \times SU_R(N_f)]$ together with an additional, unbroken symmetry, either continuous or discrete. The additional symmetry leads to the degeneracy of the vector and axial vector, and to their stability with respect to decay into the Goldstone bosons.

It is worth noting that the effective Lagrangian employed here, while describing the global symmetries, does not accurately describe the dynamics of a chiral and/or conformal transition. That is, it cannot be used directly as the basis for a Landau-Ginzburg theory of this transition with its expected nonanalytic behavior [2]. In Ref. [3], an approach to such a Landau-Ginzburg theory was developed, restricted to only the scalar degrees of freedom, and it described the usual global symmetries. This approach could perhaps be extended to include the vectors of the present effective Lagrangian. We expect that it would describe the same symmetries we have considered here, both the spontaneously broken symmetry and the additional, unbroken symmetry.

Despite the hints in Refs. [5, 4], it has not been established that an underlying gauge theory leads to these enhanced symmetries as N_f approaches a critical value for the chiral transition. If it is to happen, an unusual and interesting interplay between confinement and chiral symmetry breaking would have to develop at the transition.

We also noted, by electroweak gauging of a subgroup of the chiral flavor group, that the enhanced symmetry provides a partial custodial symmetry for the S parameter, in that there is no contribution from the parity-doubled sector by itself. It could be interesting to explore further the consequences of an enhanced symmetry for electroweak precision measurements.

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APPENDIX: EXPLICIT REALIZATION OF THE SP(4) GENERATORS

We conveniently represent the generators of SU(4) in the following way:

$$S^a=\begin{pmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B}^\dagger & -\mathbf{A}^T\end{pmatrix},\quad X^i=\begin{pmatrix}\mathbf{C} & \mathbf{D}\\ \mathbf{D}^\dagger & \mathbf{C}^T\end{pmatrix}, \tag{A1}$$

where A is Hermitian, C is Hermitian and traceless, $B=B^T$ and $D=-D^T$. The $\{S\}$ are also a representation of the Sp(4) generators since they obey the relation $S^TE+ES=0$. We define

$$S^a=\frac{1}{2\sqrt{2}}\begin{pmatrix}\tau^a & \mathbf{0}\\ \mathbf{0} & -\tau^{aT}\end{pmatrix},\quad a=1,2,3,4. \tag{A2}$$

For $a=1,2,3$ we have the standard Pauli matrices, while for $a=4$ we define $\tau^4=\mathbf{1}$. These are the generators for $SU_V(2)\times U_V(1)$. For $a=5,...,10$,

$$S^a=\frac{1}{2\sqrt{2}}\begin{pmatrix}\mathbf{0} & \mathbf{B}^a\\ \mathbf{B}^{a\dagger} & \mathbf{0}\end{pmatrix},\quad a=5,...,10 \tag{A3}$$

and

$$B^5=1\quad B^7=\tau^3\quad B^9=\tau^1,$$
$$B^6=i1\quad B^8=i\tau^3\quad B^{10}=i\tau^1. \tag{A4}$$

The five axial type generators $\{X^i\}$ are

$$X^i=\frac{1}{2\sqrt{2}}\begin{pmatrix}\tau^i & \mathbf{0}\\ \mathbf{0} & \tau^{iT}\end{pmatrix},\quad i=1,2,3. \tag{A5}$$

τ^i are the standard Pauli matrices. For $i=4,5$

$$X^i=\frac{1}{2\sqrt{2}}\begin{pmatrix}\mathbf{0} & \mathbf{D}^i\\ \mathbf{D}^{i\dagger} & \mathbf{0}\end{pmatrix},\quad i=4,5, \tag{A6}$$

and

$$D^4=\tau^2,\quad D^5=i\tau^2. \tag{A7}$$

The generators are normalized as follows:

$$\mathrm{Tr}[S^aS^b]=\mathrm{Tr}[X^aX^b]=\frac{1}{2}\delta^{ab},\quad \mathrm{Tr}[X^iS^a]=0. \tag{A8}$$

The $SU_{L/R}(2)$ generators are readily identified as

$$L^a\equiv\frac{S^a+X^a}{\sqrt{2}}, \tag{A9}$$

$$R^a\equiv\frac{X^{aT}-S^{aT}}{\sqrt{2}}, \tag{A10}$$

and $a=1,2,3$.

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