Monopole clusters, $Z(2)$ vortices, and confinement in SU(2)

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We extend our previous study of magnetic monopole currents in the maximally Abelian gauge to larger lattices at small lattice spacings (20^4 at $\beta=2.5$ and 32^4 at $\beta=2.5115$). We confirm that at these weak couplings there continues to be one monopole cluster that is very much longer than the rest and that the string tension, K, is entirely due to it. The remaining clusters are compact objects whose population as a function of radius follows a power law that deviates from the scale invariant form, but much too weakly to suggest a link with the analytically calculable size distribution of small instantons. We also search for traces of $Z(2)$ vortices in the Abelian projected fields, either as closed loops of ''magnetic'' flux or through appropriate correlations among the monopoles. We find, by direct calculation, that there is no confining condensate of such flux loops. We also find, through the calculation of doubly charged Wilson loops within the monopole fields, that there is no suppression of the $q=2$ effective string tension out to distances of at least $r\simeq 1.6/\sqrt{K}$, suggesting that if there are any vortices they are not encoded in the monopole fields. [S0556-2821(99)04821-3]

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I. INTRODUCTION

Many recent efforts to elucidate the mechanism of confinement in QCD and non-Abelian gauge theories have focused on isolating a reduced set of variables that are responsible for the confining behavior. In the dual superconducting vacuum hypothesis [1,2] the crucial degrees of freedom are the magnetic monopoles revealed after Abelian projection. In the maximally Abelian gauge [2,3] one finds that the extracted U(1) fields possess a string tension that approximately equals the original SU(2) string tension (''Abelian dominance'') [4], and that this is almost entirely due to monopole currents in these Abelian fields (''monopole dominance'') [5,6]. The magnetic currents observed in the maximally Abelian gauge are found to have non-trivial correlations with gauge-invariant quantities such as the action and topological charge densities (see for example [7,8] and references therein) and this invites the hypothesis that the structures formed by the magnetic monopoles correspond to similar objects in the SU(2) vacuum, seen after gauge fixing and Abelian projection. If the magnetic monopoles truly reflect the otherwise unknown infrared physics of the SU(2) vacuum, analysis of these structures may provide important information about the confinement mechanism.

The main purpose of this paper is to extend our previous study [9] of monopole currents to lattices that are larger in physical units at the smallest lattice spacings. As reviewed in Sec. II, we obtained in [9] a strikingly simple monopole picture at $\beta=2.3$, 2.4. When the magnetic monopole currents are organized into separate clusters, one finds in each field configuration one and only one cluster which is very much larger than the rest and which percolates throughout the entire lattice volume. Moreover, this largest cluster is alone responsible for infrared physics such as the string tension. The remaining clusters are compact objects with radii varying with length roughly as $r\propto\sqrt{l}$ and with a population that follows a power law as a function of length. We found the exponent of this power law to be consistent with a universal value of 3. This simple pattern became more confused at $\beta=2.5$. The scaling relations for cluster size that we established in [9] suggested that our $L=16$ lattice at $\beta=2.5$ was simply too small. There was of course an alternative possibility: that the simple picture we found at lower β failed as one approached the continuum limit. Clearly it is important to distinguish between these two possibilities, and this is what we propose to do in this paper. The cluster size scaling relations referred to above imply that an $L=32$ lattice at $\beta=2.5115$ should have a large enough volume. Such gauge fixed lattice fields were made available to us by Bali (private communication) and we have used them, supplemented by calculations on an intermediate $L=20$ volume at $\beta=2.5$, to obtain evidence, as described in Sec. II and III, that the monopole picture we found previously is in fact valid at these lattice spacings and that the deviations we found previously were due to too small a lattice size.

The fact that one has to go to space-time volumes that are ever larger, in physical units, as the lattice spacing decreases, hints at some kind of breakdown of ''monopole dominance'' in the continuum limit. We finish Sec. II with a discussion of the form that this breakdown might take.

An attractive alternative to the dual superconducting vacuum as a mechanism for confinement is vortex condensation [10–15]. Here the confining degrees of freedom are the vortices created by the 't Hooft dual disorder loops [10]

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and the confining disorder is located in the center $Z(N)$ of the SU(N) gauge group. When such a vortex intertwines a Wilson loop, the fields along the loop undergo a gauge transformation that varies from unity to a non-trivial element of the center as one goes once around the Wilson loop. For SU(2) this means that the Wilson loop acquires a factor of -1. A condensate of such vortices will therefore completely disorder the Wilson loop and will lead to linear confinement. At the center of the vortex, which will be a line in $D=2+1$ and a sheet in $D=3+1$, the fields are clearly singular (multivalued) if we demand that the vortex correspond to a gauge transformation almost everywhere. In a properly regularized and renormalized theory, this singularity will be smoothed out [10] into a core of finite size in which there is a non-trivial but finite action density, and whose size will be $O(1)$ in units of the physical length scale of the theory. One can either try to study these vortices directly in the SU(2) gauge fields or one can go to the center gauge [14,15], where one makes the gauge links as close to $+1$ or -1 as possible, and construct the corresponding fields where the link matrices take values in $Z(2)$ (''center projection'') and where the only nontrivial fluctuations are singular $Z(2)$ vortices. Just as a 't Hooft–Polyakov monopole will appear as a singular Dirac monopole in the Abelian fields that one obtains after Abelian projection, one would expect the presence of a vortex in the SU(2) fields to appear as a singular $Z(2)$ vortex after center projection. This picture has received increasing attention recently and has, for example, proved successful in reproducing the static quark potential [13,14] (''center dominance''). Our ability, in this paper, to address the question of how important are such vortices is constrained by the fact that we only work with Abelian projected SU(2) fields. So first we need to clarify how such vortices might be encoded in these Abelian fields and only then can we perform numerical tests to see whether there is any sign of their presence. This is the content of Sec. III.

Finally there is a summary of the results in Sec. IV.

II. MONOPOLE CLUSTER STRUCTURE

A. Background

Fixing to the maximally Abelian gauge of SU(2) amounts to maximizing with respect to gauge transformations the Morse functional

$$R=-\sum_{n,\mu} \mathrm{Tr}[U_\mu(n)\cdot i\sigma_3\cdot U_\mu^\dagger(n)\cdot i\sigma_3]. \tag{1}$$

It is easy to see that this maximizes the matrix elements $|[U_\mu(n)]_{11}|^2$ summed over all links. That is to say, it is the gauge in which the SU(2) link matrices are made to look as diagonal, and as Abelian, as possible—hence the name. Having fixed to this gauge, the link matrices are then written in a factored form and the U(1) link angle (just the phase of $[U_\mu(n)]_{11}$) is extracted. The U(1) field contains integer valued monopole currents [17], $\{j_\mu(n)\}$, which satisfy a continuity relation, $\Delta_\mu j_\mu(n)=0$, and may be unambiguously assigned to one of a set of mutually disconnected closed networks or ''clusters.''

In [9] we found that the clusters may be divided into two classes on the basis of their length, where the length is obtained by simply summing the current in the cluster:

$$l=\sum_{n,\mu\varepsilon \mathrm{cl}} |j_\mu(n)|. \tag{2}$$

The first class comprises the largest cluster, which is physically the most interesting. It percolates the whole lattice volume and its length $l_{\max}$ is simply proportional to the volume L^4 (at least in the interval $2.3\leqslant\beta\leqslant2.5$) when these are reexpressed in physical units, i.e. $l_{\max}\sqrt{K}\propto(L\sqrt{K})^4$, where K is the SU(2) lattice string tension in lattice units and we use $1/\sqrt{K}$ to set our physical length scale. We remark that over this range in β there is a factor of 2 change in the lattice spacing, and so one might have expected that the extra ultraviolet fluctuations on the finer lattice would lead to significant violations of the naive scaling relation. That is to say, one might expect to need to coarse grain the currents at larger β to obtain reasonable scaling. That this is not required is perhaps surprising.

The remaining clusters were found to be much shorter. Their population as a function of length (the ''length spectrum'') is described by a power law

$$N(l)=\frac{c_l(\beta)}{l^\gamma}, \tag{3}$$

where $\gamma\approx3$ for all lattice spacings and sizes tested and the coefficient $c_l(\beta)$ is proportional to the lattice volume, L^4, and depends weakly on β. The radius of gyration of these clusters is small and approximately proportional to the square root of the cluster length, just like a random walk. When folded with the length spectrum, this suggests [9] that the ''radius spectrum'' should also be described by a power law

$$N(r)=\frac{c_r(\beta)}{r^\eta}, \tag{4}$$

with $\eta\approx5$ and $c_r(\beta)$ weakly dependent on β. Such a spectrum is close to the scale invariant spectrum of 4-dimensional balls of radius ρ, $N(\rho)d\rho\sim d\rho/\rho\times1/\rho^4$, and so one might try and relate these clusters to the SU(2) instantons in the theory, which classically also possess a scale-invariant spectrum. It is well known, however, that the inclusion of quantum corrections renders the spectrum of the latter far from scale invariant, at least for the small instantons where perturbation theory can be trusted, and so such a connexion does not seem to be possible [9].

On sufficiently large volumes the difference in length between the largest and second largest clusters is very marked, and where this gulf is clear one finds that the long range physics such as the monopole string tension arises solely from the largest cluster. This is the case at $\beta=2.3, L\geqslant10$ and at $\beta=2.4, L\geqslant16$. On moving to a finer $L=16$ lattice at $\beta=2.5$ the gulf was found to disappear and the origin of the long range physics was no longer so clear-cut. This could be a mere finite volume effect, or, much more seriously, it might signal the breakdown of this monopole picture in the weak coupling, continuum limit. Clearly this needs to be resolved and the only unambiguous way to do so is by performing the calculations on large enough lattices.

B. This calculation

The direct way to estimate the lattice size necessary at $\beta=2.5$ to restore (if that is possible) our picture is as follows. Suppose that the average size of the second largest cluster scales approximately as

$$l_{2nd} \propto L^{\alpha}(\sqrt{K})^{\delta}. \tag{5}$$

We know that $l_{max} \propto L^4(\sqrt{K})^3$ to a good approximation for the largest cluster. So we will maintain the same ratio of lengths l_{2nd}/l_{max}, and a gulf between these, if

$$\frac{L_1}{L_2}=\left(\frac{\sqrt{K_1}}{\sqrt{K_2}}\right)^{-[(3-\delta)/(4-\alpha)]}. \tag{6}$$

If we take our directly calculated values of l_{2nd}, they seem to give roughly $\alpha\simeq 1$ and $\delta\simeq -2$. This suggests that we need to scale our lattice size with β so as to keep $L(\sqrt{K})^{5/3}$ constant. This estimate is not entirely reliable because, on smaller lattices, the distributions of the ''largest'' and ''second largest'' clusters overlap so that they exchange roles. An alternative estimate can be obtained from the tail of the distribution in Eq. (3) that integrates to unity. Doing so [9] one obtains $\alpha\simeq 2$ and $0<\delta<0.25$. This suggests that we scale our lattice size so as to keep $L(\sqrt{K})^{\{1.4\to 1.5\}}$ constant. This estimate is also not very reliable, since it assumes that the distribution of secondary cluster sizes on different field configurations fluctuates no more than mildly about the average distribution given in Eq. (3). In fact the fluctuations are very large. [As we can see immediately when we try to calculate $\langle l^2\rangle$ in order to obtain a standard fluctuation—it diverges for a length spectrum with $N(l)\propto dl/l^3$.] Nonetheless, the two very different estimates we have given above produce a very similar final criterion: to maintain the same gap between the largest and second largest clusters as β is varied, one should choose L so as to keep $L(\sqrt{K})^{\sim 1.5}$ constant.

So if we wish to match the clear picture on an $L=10$ lattice at $\beta=2.3$ [where $K=0.136\ (2)$], we should work on a lattice that is roughly $L=28$ at $\beta=2.5$ [where $K=0.0346\ (8)$]. In particular we note that an $L=32$ lattice at $\beta=2.5115$ [where $K=0.0324\ (10)$] is more than large enough and an ensemble of 100 such configurations, already gauge fixed [6], has been made available to us by the authors. The gauge fixing procedure used in obtaining these is somewhat different from the one we have used in our previous calculations (in its treatment of the Gribov copies—see below), and although this is not expected to affect the qualitative features that are our primary interest here, it will have some effect on detailed questions of scaling, etc. We have therefore also performed a calculation on an ensemble of 100 gauge fixed $L=20$ field configurations at $\beta=2.5$. While the latter volume is not expected to be large enough to recreate a clear gulf between the largest and remaining clusters, we would expect to find smaller finite size corrections than with the $L=16$ lattice we used previously.

In gauge fixing a configuration we select a local maximum of the Morse functional, R, of which on lattices large enough to support non-perturbative physics there is typically a very large number [18]. These correspond to the (lattice) Gribov copies. Gauge dependent quantities appear to vary by $O(10\%)$ depending upon the Gribov copy chosen; this is true not only of local quantities such as the magnetic current density [19] but also of supposedly long range, physical numbers such as the Abelian and monopole string tensions [6,18]. Some criterion must be employed for the selection of the maxima of R, and in the absence of a clear understanding of which maximum, if any, is the most ''physical,'' one maximum was selected at random in [9]. An alternative strategy, used in gauge fixing the $L=32$ lattices at $\beta=2.5115$, is to pursue the global maximum of R [6]. Each field configuration is fixed to the maximally Abelian gauge 10 times using a simulated annealing algorithm that already weights the distribution of maxima so selected towards those of higher R. The solution with the largest R from these is selected. Details of this method are discussed in [6]. The difference in procedures invites caution in comparing exact numbers between this ensemble and those studied previously; for example a $O(10\%)$ suppression in the string tension is observed. It is likely that cluster lengths will differ by a corresponding amount and this will prevent a quantitative scaling analysis using this ensemble. The power law indices do appear, however, to be robust [20] and it also seems likely that ratios of string tensions obtained on the same ensemble can be reliably compared with other ratios.

C. Cluster properties

The fact that the largest cluster does not belong to the same distribution as the smaller clusters is seen from the very different scaling properties of these clusters with volume [9]. It is also apparent from the fact that the largest cluster is very much longer than the second largest cluster. Indeed for a large enough volume and for a reasonable size of the configuration ensemble, there will be a substantial gulf between the distribution of largest cluster lengths and that of the second largest clusters. By contrast the length distributions of the second and third largest clusters strongly overlap. This is the situation that prevailed for the larger lattices at $\beta=2.3$ and 2.4 but which broke down on the $L=16$ lattice at $\beta=2.5$. We can now compare what we find on our $L=20$ and $L=32$ lattices with the latter. This is done in Table I. There we show the longest and shortest cluster lengths for the largest, second largest and third largest clusters respectively over the ensemble. The ensemble sizes are not exactly the same, but it is nonetheless clear that there is a real gulf between the largest and second largest clusters on the $L=32$ lattice while there is significant overlap in the $L=16$ case. The $L=20$ lattice is a marginal case. We conclude from this that the apparent loss of a well-separated largest cluster as seen in [9] at $\beta=2.5$ was in fact a finite volume effect, and that our

TABLE I. Range of lengths found for the largest, the second largest, their difference and for the third largest clusters for the ensembles of N configurations shown.

L	β	N	$l_{\max}$	l_{2nd}	$l_{\max}-l_{\text{2nd}}$	l_{3rd}
12	2.3	500	2358–3970	18–220	2172–3930	
14	2.4	500	894–3436	22–1112	28–3400	
16	2.5	500	268–2462	22–910	4–2414	
20	2.5	100	1718–5050	50–1644	318–4964	36–684
32	2.5115	100	11872–20040	114–4676	9066–19886	92–2476

scaling analysis has proved reliable in predicting what volume one needs to use in order to regain the simple picture.

In Fig. 1 we show how the length of the largest cluster varies with the lattice volume when both are expressed in physical units (set by $\sqrt{K}$). To be specific, we have divided $l_{\max}\sqrt{K}$ by $(L\sqrt{K})^4$ and plotted the resulting numbers against $L\sqrt{K}$ for both our new and our old calculations. The fact that at fixed β the values fall on a horizontal line tells us that that the length of the largest cluster is proportional to the volume at fixed lattice spacing: $l_{\max}\propto L^4$. The fact that the various horizontal lines almost coincide tells us that the current density in the largest cluster is consistent with scaling. That is to say, it has a finite non-zero value in the continuum limit. Thus the monopole whose world line traces out this largest cluster percolates throughout the space-time volume and its world line is sufficiently smooth on short distance scales that its length does not show any sign of diverging as we take the continuum limit. We note that the $L=32$ lattice deviates by $\sim 10\%$ from the other values. This is consistent with what we might have expected from the different gauge fixing procedure used in that case.

Turning now to the secondary clusters, we display in Fig. 2 the length spectrum that we obtain at $\beta=2.5115$. It is clearly well described by a power law as in Eq. (3) and we fit the exponent to be $\gamma=3.01\ (8)$. This is in accordance with the universal value of 3 that was postulated in [9] on the basis of calculations on coarser lattices. The value one fits to

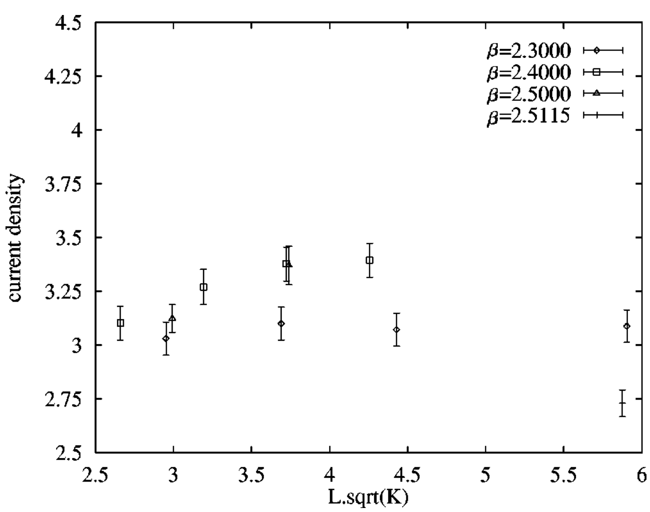

FIG. 1. The current density of the largest cluster as a function of lattice size in physical units for various β.

the spectrum obtained on the $L=20$ lattice at $\beta=2.5$ is $\gamma=2.98\ (7)$ and is equally consistent. We also examine the dependence on β of the coefficient $c_l(\beta)$ in Eq. (3), adding to the older work our calculations at $\beta=2.5$ on the $L=20$ lattice. (We do not use the $L=32$ lattice for this purpose because of the different gauge fixing procedure used.) If we assume a constant power (which is approximately the case), then $c_l(\beta)$ is just proportional to the total length of the secondary clusters. At fixed β we find this length to be proportional to L^4 just as one might expect. (Small clusters in very different parts of a large volume are presumably independent.) The dependence on β, on the other hand, is much less clear. Between $\beta=2.3$ and $\beta=2.4$ it varies weakly, roughly as $K^{0.12\pm0.13}$. Between $\beta=2.4$ and $\beta=2.5$ it varies more strongly, roughly as $K^{0.48\pm0.09}$. We can try to summarize this by saying that

$$c_l(\beta)=\text{const.}\times L^4\sqrt{K}^{\zeta} \tag{7}$$

where $\zeta=0.5\pm0.5$, which is consistent with what was found previously [9].

The smaller clusters are compact objects in $d=4$, and having determined the cluster spectrum as a function of length we can then ask what is the spectrum when re-expressed as a function of the radius (of gyration) of the cluster. In [9] we obtained this spectrum by determining the average radius as a function of length and folding that in with the number density as a function of length. This is an approximate procedure (forced upon us by the fact that we did not foresee the interest of this spectrum during the processing of the clusters) and one can obtain the spectrum more accurately by calculating r for each cluster and forming the spectrum directly. Doing so for the $L=32$ lattice at $\beta=2.5115$, also in Fig. 2, we find a power law as in Eq. (4) with $\eta=4.20\ (8)$. The spectrum on the $L=20$ lattice at $\beta=2.5$ yields $\eta=4.27\ (6)$. We recall that in [9] we claimed that the spectrum was consistent with the scale invariant result $dr/r\times 1/r^4$, i.e. $\eta=5$. This followed from the fact that we found the radius of the smaller clusters to vary with their length as $r(l)=s+t.l^{0.5}$, i.e. just what one would expect from a random walk. Folded with a length spectrum $N(l)\sim 1/l^3$, this gives $\eta=5$. On the $L=32$ lattice we still find that the random walk ansatz provides an acceptable fit but we also find that $r(l)=s+t.l^{0.65}$ works equally well over similar ranges. The latter, when folded with $\gamma=3$, gives $\eta=4.2$. It is clear that the direct calculation of $N(r)$ is much more accurate than the indirect approach.

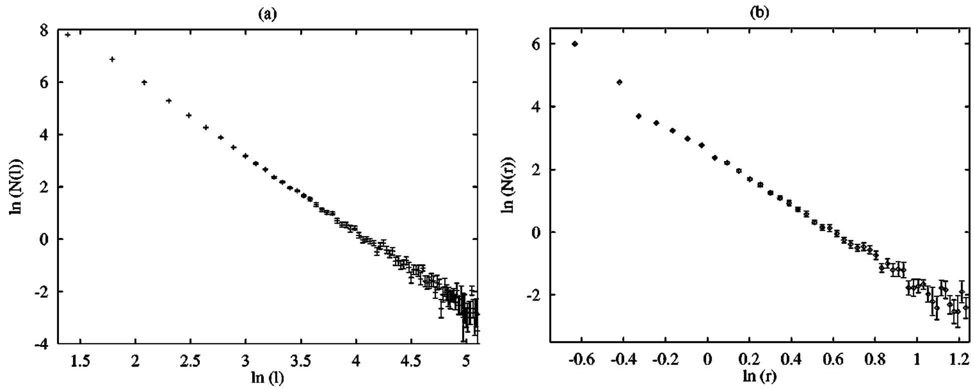

FIG. 2. The cluster spectra by (a) length and (b) radius at $\beta=2.5115$ on $L=32$.

Treating the power as a free parameter in the fit, $r(l)=s+t.l^u$, we find $u=0.57$ (3) on $L=32$ at $\beta=2.5115$, consistent with $u=0.58$ (4) on $L=20$ at $\beta=2.5$. Thus both $u=0.5$ and $u=0.65$ lie within about two standard deviations from the fitted value. Note that what the fitted powers γ and u parametrize are the means of the distributions of lengths and radii respectively. That combining these does not give the directly calculated value of η is not unexpected, and reflects the importance of fluctuations around the mean in the distributions.

If the secondary monopole clusters can be associated with localized excitations of the full SU(2) vacuum (''4-balls''), it would seem that such objects do not have an exactly scale invariant distribution in space-time, so that the number of larger radius objects is somewhat greater than would be expected were this the case. Now it is known that an isolated instanton (even with quantum fluctuations) is associated with a monopole cluster within its core (see [21,22] and references therein) and that the scale invariant semiclassical density of instantons acquires corrections due to quantum fluctuations. These corrections are, however, very large; in SU(2) the spectrum of small instantons (where perturbation theory is reliable) goes as $N(\rho)d\rho\propto d\rho/\rho\times\rho^{10/3}$, where ρ is the core size. The scale breaking we have observed for monopole clusters is negligible in comparison. Thus we cannot identify the ''4-balls'' with instantons. Indeed, the fact that the monopole spectrum is so close to being scale invariant strongly suggests that these secondary clusters have no physical significance. In the next section we shall show explicitly that, in the large volume limit, they do not play any part in the long range confining physics.

D. Breakdown of ''monopole dominance''?

We finish this section by asking if there are hints from our cluster analysis that ''monopole dominance'' might be breaking down as we approach the continuum limit. This question is motivated by the observation that the monopoles are identified by a gauge fixing procedure which involves making the bare SU(2) fields as diagonal as possible. Since the theory is renormalizable, the long distance physics increasingly decouples from the fluctuations of the ultraviolet bare fields as we approach the continuum limit. For example, the ultraviolet contribution to the action density is $O(1/\beta)$ while the long distance contribution is $O(e^{-c\beta})$. Thus as $a\to0$ the maximally Abelian gauge will be overwhelmingly driven by ultraviolet rather than by physical fluctuations. Moreover, at the location of the monopoles the Abelian fields are far from unity and so one would expect the SU(2) fields also to be far from unity. Thus the number of monopoles would seem to be constrained by the probability of finding corresponding clumps of SU(2) fields with large plaquette values. This probability depends on the detailed form of the SU(2) lattice action far from the Gaussian minimum and one could easily choose an action where it is completely suppressed and yet which one would expect to be in the usual universality class. None of the above arguments are completely compelling of course. In the Gaussian approximation, for example, the $O(1/\beta)$ ultraviolet fluctuations would not generate any monopoles at all, and in that case there would be no reason to expect any breakdown of monopole dominance. Nonetheless, the arguments do suggest that it would be surprising if the long distance physics were to be usefully and simply encoded in the monopole structure (as defined on the smallest ultraviolet scales) all the way to the continuum limit.

There are different ways in which monopole dominance could be lost. The most extreme possibility is that as $a\to0$ the fields simply cease to contain monopole clusters that are large enough to disorder large Wilson loops. That this is indeed so has been argued in [23] where it has been claimed that the exponent γ in our Eq. (3) (but defined for loops rather than for clusters) increases rapidly with decreasing a. Of course this would not in itself preclude the existence of a large percolating cluster, as long as this cluster could be decomposed into a large number of small and correlated intersecting loops. Irrespective of this, we also note that the volumes used in [23] are very small by the criterion given in Eq. (6). For example, from our scaling relations we would expect to need an $L\simeq46$ lattice at $\beta=2.6$ and an $L\simeq70$ lattice at $\beta=2.7$ in order to resolve our simple monopole picture, if it still holds at these values of β. This contrasts with the $L=12$ and $L=20$ lattices actually employed in [23]. So it appears to us that while the claims in [23] are certainly interesting, further calculations on much larger lattices are required.

Our work suggests a somewhat different form of the breakdown to the one above. We see from Eq. (7) that the ratio of the (total) monopole current residing in the physically irrelevant, smaller clusters to that residing in the large percolating cluster increases rapidly as $a \to 0$ as $1/\sqrt{K}^{3-\zeta} \propto 1/a^{3-\zeta}$. This suggests that as $a \to 0$ a calculation of Wilson loops will become increasingly dominated by the fluctuating contribution of the unphysical monopoles that are ever denser on physical length scales, and that this will eventually prevent us from extracting a potential or string tension. That is to say, calculations in the maximally Abelian gauge will eventually acquire a similar problem to that which typically afflicts Abelian projections using other gauges. In our case we can overcome this problem by going to a large enough volume that the physically relevant percolating cluster can be simply identified. (The reason this cannot be done with other typical Abelian gauge fixings is that there the unphysical monopoles are dense on lattice scales, making any meaningful separation into clusters impossible.) We can then extract the string tension using, in our Wilson loop calculation, only this largest monopole cluster. The fact that the length of this cluster scales in physical units, with apparently no significant anomalous dimension, tells us that this calculation will not be drowned in ultraviolet ''noise'' as we approach the continuum limit. Of course, the fact that we can only do this for volumes that diverge in physical units as $a \to 0$ is a symptom of the underlying breakdown of the Abelian projection.

The qualitative discussion in the previous paragraph overestimates the effect of the secondary clusters; for example, the contribution that a cluster of fixed size in lattice units makes to a Wilson loop of a fixed physical size will clearly go to zero as $a \to 0$. So it is useful to ask how Wilson loops are affected by the secondary clusters, and to do so using approximations that underestimate the effect of these smaller clusters. Consider an $R \times R$ Wilson loop. A monopole cluster that has an extent r that is smaller than R will affect it only weakly through higher multipole fields which cannot on their own give rise to an area law decay and a string tension. So we neglect such clusters and consider only those larger than R. Let us first neglect the observed breaking of scale invariance and simply assume that $r \propto \sqrt{l}$ and that $\gamma = 3$. We then find, by integrating Eq. (3) and using Eq. (7), that the number of secondary clusters with $r > R$ is proportional to $L^4 \sqrt{K}^{\zeta}/R^4$. We further assume that such clusters must be within a distance ξ from the minimal surface of the Wilson loop, where ξ is the screening length, if they are to disorder that loop significantly. The lattice volume this encompasses is the area of the planar loop, R^2, multiplied by a factor of ξ for each of the two orthogonal directions in $d = 4$. So the probability for this Wilson loop to be disordered thus decreases with R as $(R^2 \xi^2 / L^4 \times L^4 \sqrt{K}^{\zeta}/R^4) \sim \sqrt{K}^{\zeta} (\xi/R)^2$. So if we look at a Wilson loop that is of a fixed size in physical units, i.e. R/ξ fixed assuming ξ scales as a physical quantity [9], then the influence of the secondary clusters will decrease to zero as $a \to 0$ as long as $\zeta > 0$. If $\zeta < 0$, however, then we would have to go to Wilson loops that were ever larger in physical units as we approached the continuum limit, in order that the physical contribution from the percolating cluster should not be swamped by the unphysical contribution of the secondary clusters. Of course this calculation uses a scale invariant $dr/r \times 1/r^4$ spectrum, whereas, as we have seen, there is significant scale breaking and the actual spectrum is closer to $dr/r \times 1/r^{3.2}$. If we redo the above analysis with the latter spectrum, we see that we are only guaranteed to preserve this aspect of ''monopole dominance'' if $\zeta > 0.8$. As demonstrated in Eq. (7), there is some evidence that $\zeta > 0$ but it is not at all clear that $\zeta > 0.8$. All this indicates that even in a calculation that errs on the side of neglecting the effect of the smaller clusters, they nonetheless will most likely dominate the values of Wilson loops on fixed physical length scales. It is only by separating the percolating cluster from the other smaller clusters, and calculating Wilson loops just using that largest cluster, that we can hope to be able to extract the string tension as $a \to 0$.

III. MONOPOLES, VORTICES AND THE STRING TENSION

In this section we begin by describing how we calculate the string tension from an arbitrary set of monopole currents. We then go on to show that even at the smallest lattice spacings, the string tension arises essentially entirely from the largest cluster, as long as we use a sufficiently large volume. We then calculate the string tension for sources that have a charge of $q = 2$, 3 and 4 times the basic charge, and compare these results to a simple toy model calculation. Finally we discuss the implications of our calculations for the question whether it is really monopoles or vortices that drive the confining physics.

A. Monopole Wilson loops

The monopole contribution to the string tension may be estimated using Wilson loops. If the magnetic flux due to the monopole currents through a surface spanning the Wilson loop, $\mathcal{C}$ (by default the minimal one), is $\Phi(\mathcal{S})$, then the charge q Wilson loop has value

$$W(\mathcal{C}) = \exp[iq\Phi(\mathcal{S})]. \tag{8}$$

We may obtain the static potential from the rectangular Wilson loops:

$$V(r) = \lim_{t \to \infty} V_{\text{eff}}(r,t) \equiv \lim_{t \to \infty} \ln\left[\frac{\langle W(r,t)\rangle}{\langle W(r,t+a)\rangle}\right]. \tag{9}$$

The string tension, K, may then be obtained from the long range behavior of this potential, $V(r) \simeq Kr$. The string tension may also be found from the Creutz ratios

$$K = \lim_{r \to \infty} K_{\text{eff}}(r) \equiv \lim_{r \to \infty} \ln\left[\frac{\langle W(r+a,r)\rangle \langle W(r,r+a)\rangle}{\langle W(r,r)\rangle \langle W(r+a,r+a)\rangle}\right]. \tag{10}$$

Square Creutz ratios at a given r are useful because they provide a relatively precise probe for the existence of confining physics on that length scale. In addition Creutz ratios are useful where the quality of the ''data'' precludes the

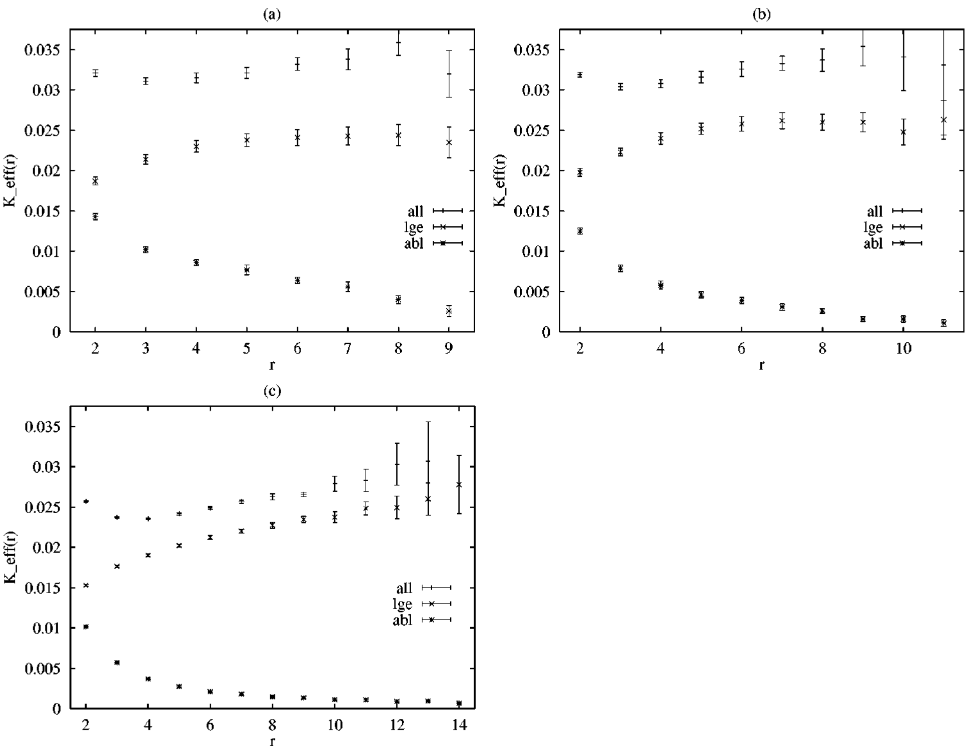

FIG. 3. The effective string tension for all clusters (''all''), the largest cluster alone (''lge'') and the remaining clusters (''abl'') on lattices (a) $\beta=2.5,L=16$, (b) $\beta=2.5,L=20$ and (c) $\beta=2.5115,L=32$.

double limit of the potential fit. This is so particularly when positivity is badly broken as it frequently is for our gauge dependent correlators.

The magnetic flux due to the monopole currents is found by solving a set of Maxwell equations with a dual vector potential reflecting the exclusively magnetic source terms. An iterative algorithm being prohibitively slow on $L=32$, we utilized a fast Fourier transform method to evaluate an approximate solution as the convolution of the periodic lattice Coulomb propagator and the magnetic current sources [24]. The error in this solution was then reduced to an acceptable level by using it as the starting point for the over-relaxed, iterative method.

We may use any subset of the monopole currents as the source term to calculate the contribution to the Wilson loops and potential of those currents, provided that they (i) are locally conserved and (ii) have net zero winding number around the periodic lattice in all directions, e.g.

$$Q_{\mu=4}\equiv\sum_{x,y,z} j_4(x,y,z,t=1)=0. \tag{11}$$

If we choose complete clusters, then the first condition is always satisfied but the second condition is often not met (even though the winding number for all the clusters together must be zero). In such cases we introduce a ''fix'' as follows. At random sites in the lattice we introduce a Polyakov-like straight line of magnetic current of corrective charge $-Q_\mu$ for each direction, and use these as sources for a dual vector potential. Such lines represent static monopoles and a random gas of these can lead to a string tension. This introduces a systematic error to the monopole string tension that we need to estimate. We do so by placing the same corrective loop on an otherwise empty lattice, along with a second loop of charge $+Q_\mu$ at another random site. From this new ensemble we calculate the string tension from Creutz ratios. One-half of this is a crude estimate of the bias introduced in correcting the original configurations, and this is quoted as a second error on our string tension values, as appropriate.

B. Largest cluster

In [9] we observed that at $\beta=2.3,L\geqslant10$ and at $\beta=2.4,L\geqslant14$ the $q=1$ monopole string tension was produced almost entirely by the largest cluster, and the other clusters had a string tension near zero. At $\beta=2.5,L=16$ the situation was more confused; the smaller, power law clusters still had a very low string tension, but that of the largest cluster alone was substantially less than the full monopole string tension. This suggested some kind of constructive correlation between the two sets of clusters. In our new calculation on an $L=20$ lattice at $\beta=2.5$, we still find a situation that is confused, although somewhat less so than on the $L=16$ lattice, while on the $L=32$ lattice at $\beta=2.5115$ the clear picture seen at $\beta=2.3$ re-emerges, with nearly all the string tension being produced by the largest cluster, and the remaining clusters having a negligible contribution. To illustrate this we display in Fig. 3 the effective string tensions as a function of r for the lattices at $\beta=2.5$ and $\beta=2.5115$.

The confused roles of the clusters on finer lattices [9] are thus a finite volume effect and do not represent a breakdown of the monopole picture as we near the continuum limit. As a result of the differing scaling relations for the lengths of the two largest clusters, it is not enough to maintain a constant lattice volume in physical units to reproduce the physics as we reduce the lattice spacing. Rather the lattice must actually become larger even in physical units, as discussed in Sec. II B. The string tension arises from ''disordering''—i.e.

TABLE II. Monopole string tensions from Wilson loops of varying charge, using all current (''all''), current from the largest cluster alone (''lge'') and the remaining current (''abl'').

	q	K-all	K-lge	K-abl
$\beta=2.5,\ L=20$	1	0.035 (3)	0.026 (3) (1)	<0.0015
$\beta=2.5115,\ L=32$	1	0.0270 (10)	0.0240 (10) (3)	<0.0010
	2	0.0520 (10)	0.0450 (10) (4)	<0.0022
	3	0.075 (2)		
	4	0.103 (5)		

switches in sign—of the Wilson loop by the monopoles. A monopole that is sufficiently close to a large Wilson loop will multiply the loop by $\exp[iq\pi]$ which would naively suggest that even-charged loops are not disordered and have no string tension. In a screened monopole plasma, however, as the monopole is moved away from the loop, the flux falls and the possibility for disorder and a string tension exists. (This will also occur without screening, but only when the monopole is a distance away from the Wilson loop that is comparable to the size of the loop.) Clearly the exact value of the string tension will depend upon the details of the screening mechanism, especially as we increase q. This can be calculated in the usual saddle point approximation [16] where one finds that the string tension is proportional to q [15]. One can obtain a crude model estimate with much less effort, and this we do in the next subsection. Returning to our lattice calculations, we list in Table II the monopole string tensions that we obtain using charge q Wilson loops at $\beta=2.5115$ on the $L=32$ lattice. We see that they are indeed consistent with a scaling relation $R(q)=q$, at least up to $q=4$.

C. Simple model

It is useful to consider here a simple model for the disordering of Abelian Wilson loops of various charges by monopoles. We consider only static monopoles in $d=4$, with a mean field type of screening, assuming that the macroscopic, exponential falloff in the flux with screening length ξ could be applied on the microscopic scale also. For numerical reasons we also impose a cutoff: beyond N screening lengths the flux is set exactly zero. The magnetic field is thus

$$B(d)=\begin{cases}\dfrac{1}{2d^2}e^{-d/\xi}, & d\leqslant N\xi,\\ 0, & d>N\xi.\end{cases} \tag{12}$$

The flux from a monopole distance $z\leqslant N\xi$ above a large (spacelike) Wilson loop through that loop is

$$\Phi(N,z,\xi)=\pi\int_{z/N\xi}^{1}dy\,\exp\left(-\frac{z}{y\xi}\right). \tag{13}$$

Considering a slab of monopoles and antimonopoles all distance z above the loop (and similar below), the charge q Wilson loop gives a string tension [9]

TABLE III. Ratio of monopole string tensions from Wilson loops of varying charge, in the static plasma model and measured at $\beta=2.5115$ on $L=32$.

q	$R(q)$	$K(q)/K(q=1)$
2	1.827 (1)	2.00 (9)
3	2.192 (1)	2.88 (14)
4	2.526 (1)	3.96 (25)

$$\delta K(N,z,q)\propto\{1-\cos[q\Phi(N,z,\xi)]\}. \tag{14}$$

Integrating over all $|z|\leqslant N\xi$, the ratio of string tensions calculated using charge q and charge $q=1$ Wilson loops is

$$R(N,q)\equiv\frac{K(N,q)}{K(N,q=1)}=\frac{\int_0^N da\left(1-\cos\left[q\pi\int_{a/N}^1 dy\cdot e^{-a/y}\right]\right)}{\int_0^N da\left(1-\cos\left[\pi\int_{a/N}^1 dy\cdot e^{-a/y}\right]\right)} \tag{15}$$

for this static monopole assumption. This may be evaluated numerically, and extrapolated as $N\to\infty$, where there is a well-defined limit, $R(q)$. The results for small q are shown in Table III, where the error on $R(q)$ reflects the extrapolation uncertainty. Comparing these numbers to the actual ratio of string tensions, we find this simplistic model works remarkably well for $q=2$, but becomes less reliable as we increase q. This no doubt reflects the increasing importance of the neglected fluctuations of the flux away from the mean screened values.

D. Monopoles or vortices?

The fact that the Abelian fields that one extracts in the maximally Abelian gauge, and their corresponding monopoles, successfully reproduce the SU(2) fundamental string tension provides some evidence for the dual superconducter model of confinement. As we remarked in the Introduction, however, an attractive alternative picture exists, based on vortex condensation, and one has comparable evidence for that picture, obtained by going to the maximal center gauge and calculating Wilson loops using the singular vortices obtained after center projection.

Since the Abelian projected fields seem to contain the full string tension, it is reasonable to assume that they encode all the significant confining fluctuations in the SU(2) fields, even if these are vortices. How would one expect a vortex to be encoded in the Abelian fields? And how can we test for their presence?

Recall that the kind of vortex we are interested in has a smooth core and flips the sign of any Wilson loop that it threads. Consider now a space-like Wilson loop in some time slice of our Abelian projected lattice field. We observe that it will flip its sign if threaded by a loop of magnetic flux whose core contains a total flux equal to π. If the core size is not arbitrarily large, so that a (large enough) Wilson loop has negligible probability to overlap with the actual core, then a condensate of such fluxes will lead to linear confinement.

Since the original SU(2) vortex has a smooth core, the simplest expectation is that this flux, if it reflects the vortex, should not have a singular monopole source; rather it should be a closed loop of magnetic flux. If its length is much larger than the size of the Wilson loop, it can easily thread the loop an odd number of times and can disorder it. So the natural way for a $Z(2)$ vortex to be encoded in the Abelian projected fields is as a closed loop of magnetic flux, in roughly the same position and with a smooth core of roughly the same size. If this is so and if vortices are present in the SU(2) fields, we would expect that our Abelian fields contain two kind of confining fluctuations: singular magnetic monopoles and smooth closed loops of (π units of) magnetic flux. Since these closed loops of flux are smooth, they will be hard to identify individually in the midst of the magnetic fluxes generated by the monopoles. Their presence can however be easily tested for as follows. The flux in the U(1) fields is conserved and so any flux either originates on the monopoles or closes on itself as part of a closed flux loop. The monopoles are easy to identify and their flux can be calculated. So for any Wilson loop, $\mathcal{C}$, we can calculate the flux, $B_{\rm mon}(\mathcal{C})$, due to the monopoles and we can subtract it from the total flux, $B(\mathcal{C})$, so as to obtain the remaining flux,

$$B_{\delta}(\mathcal{C}) \equiv B(\mathcal{C}) - B_{\rm mon}(\mathcal{C}), \tag{16}$$

which comes from closed flux loops. The corresponding value of the Wilson loop will be $e^{-B_{\delta}(\mathcal{C})}$. In this way we can calculate the potential due to the non-monopole flux, and if we find a non-zero string tension, this demonstrates the existence of a condensate of such flux loops and provides evidence for corresponding $Z(2)$ vortices. If the flux loops carry π units of flux, Wilson loops corresponding to sources with an even charge will have zero string tension.

We remark here that in U(1) lattice gauge theories, such loops of magnetic flux are not usually discussed as significant degrees of freedom. That is not because they cannot exist but rather that the dynamics is such that they usually play no significant role. [One can always smoothly reduce the usual U(1) action by increasing the core size of such a loop. Ultimately they contribute a non-confining ''spin wave'' contribution to the interaction.] The Abelian projected fields, on the other hand, are not generated from some local U(1) action. They may possess any structures that are kinematically allowed.

Vortices can also be encoded in the Abelian fields in a more subtle way than the above. This involves long-distance correlations among the monopoles. In $d=2+1$ suppose that at least some of the monopoles lie along ''lines'' in such a way that each monopole is followed by an antimonopole (and vice versa) as we follow the line. This will generate an alternating flux of $\pm\pi$ along the line [14]. So a Wilson loop threaded by this line will acquire a factor of -1. Such correlated ensembles can therefore encode the vortices in the original three dimensional SU(2) fields. A similar restriction of monopole current world lines to two dimensional sheets can be envisaged in $d=3+1$. In both cases, their presence would be signalled by the fact that they do not disorder Wilson loops corresponding to an even charge (unlike a plasma of monopoles). So if we calculate the string tension due to the monopoles, and if we find a significant suppression of the $q=2$ string tension, then this will indicate the significant presence of such correlations and hence of vortices.

This latter way of encoding vortices in the Abelian projected fields might seem less natural given the smoothness of the underlying $Z(2)$ vortices. As pointed out in [15], however, such correlated monopole structures actually occur in what one usually regards as a standard example of a field theory that demonstrates linear confinement driven by monopole condensation: the Georgi-Glashow model in three dimensions. This model couples an SU(2) gauge field to a scalar Higgs field in the adjoint representation of the gauge group. The theory has a Higgs phase, and the Higgs field drives the gauge field into a vacuum state which has only U(1) gauge symmetry, save in the cores of extended topological objects. These 't Hooft–Polyakov monopoles are magnetically charged with respect to the U(1) fields, and give rise to the linear confining potential, at least in the semiclassical approximation [16] which holds good when the charged vector bosons are heavy. As pointed out in [15], however, this conventional picture cannot be true on large enough length scales since eventually the presence of the charged massive $W^{\pm}$ fields will lead to the breaking of strings between doubly charged sources (the $W^{\pm}$ possessing twice the fundamental unit of charge). A plasma of monopoles, on the other hand, will predict the linear confinement of such double charges. So it was argued that in this limit it is $Z(2)$ vortices, which do not disorder doubly charged Wilson loops, that drive the confinement [15]. The crossover between the two pictures, it is argued, would occur beyond a certain length scale dictated by the $W^{\pm}$ mass, where the distribution of monopole flux would no longer be purely Coulombic, but would be collimated into structures of lower dimension—essentially strings of alternating monopoles and antimonopoles—that reflect the $Z(2)$ vortices of the vacuum.

Of course one cannot carry this argument over in all its details to the case of the pure SU(2) gauge theory. Here there are no explicit Higgs or $W^{\pm}$ fields; any analogous objects would need to be composite. The theory also has only one scale, and so one would not expect an extended intermediate region between the onset of confining behavior and the collimation of the flux signalling $Z(2)$ disorder. But it does raise the possibility that the $Z(2)$ vortices in the SU(2) fields might be encoded, after Abelian projection, in such correlations among the monopoles rather than in separate smooth closed loops of magnetic flux.

To probe for the presence of smooth loops of magnetic flux in the Abelian projected fields, we have calculated the ''difference'' flux, as defined in Eq. (16), and the resulting string tension, and to probe for vortex-like ensembles of monopoles we have calculated the monopole string tension, $K(q)$, for various source charges, q.

We start with the latter. In Table IV we show the $q=1,2$ monopole effective string tensions that we have obtained from Creutz ratios on the $L=32$ lattice at $\beta=2.5115$. We see that for $q=2$, just as for $q=1$, there are very few transients at small r, and the extraction of an asymptotic

TABLE IV. Effective monopole string tensions from Creutz ratios for charges $q=1,2$ and from the difference of U(1) and monopole fluxes at $\beta=2.5115$ on $L=32$.

r	$K_{\rm eff}(r),\ q=1$	$K_{\rm eff}(r),\ q=2$	$K_{\rm eff}^{\rm diff}(r),\ q=1$
2	0.02572 (8)	0.0511 (2)	0.0799 (2)
3	0.02371 (9)	0.0497 (2)	0.0335 (2)
4	0.02355 (10)	0.0483 (3)	0.0176 (5)
5	0.02414 (14)	0.0492 (5)	0.0102 (9)
6	0.02487 (17)	0.0496 (7)	0.0082 (13)
7	0.02565 (20)	0.0501 (7)	0.0062 (21)
8	0.02628 (37)	0.0493 (25)	0.0076 (53)
9	0.02652 (25)	0.0556 (58)	−0.0076 (27)

string tension appears to be unambiguous. We have accurate calculations out to a distance of $r=9a$ which corresponds to

$$r=9a\sim\frac{1.6}{\sqrt{K}} \tag{17}$$

in physical units at this β. Out to this distance there is absolutely no hint of any reduction in the $q=2$ effective string tension. It has been pointed out [14] that when the Wilson loop is not much larger than the typical vortex core, it is not completely unnatural to obtain an effective string tension comparable to the one from a monopole plasma. Here the size is beginning to be large compared to the natural scale of the theory, however, and it is hard not to view the lack of any variation at all in the $q=2$ effective string tension as pointing to the absence of the kind of correlations among the monopoles that might be encoding $Z(2)$ vortices.

The second possibility is that the vortices might be encoded not in correlations among the monopoles but rather in closed loops carrying π units of magnetic flux. Such loops would contribute to $K(q=1)$ but not to $K(q=2)$. We recall that there has been a calculation of $K(q)$, calculated within the full Abelian fields at $\beta=2.5115$, and that there it was found [6] that there is a finite $q=2$ effective string tension that extends out to at least as far as $r=9a$, and that the ratio of the U(1) string tensions is $K(q=2)/K(q=1)=2.23\ (5)$. While this suggests that closed flux loops are not important, these string tensions necessarily include the contribution from monopoles, and it would be useful to have a calculation that excludes the latter. We have therefore calculated the effective string tension using only the flux that comes from closed flux loops, as defined in Eq. (16). The results of this calculation are listed in Table IV for $q=1$. We see that, within small errors, there is asymptotically no string tension from such loops (a potential fit to the Wilson loops yields $K<0.0025$). This shows in a direct way that there is no significant condensate of closed loops of flux in the Abelian projected fields.

In conclusion, our investigations here have shown no sign of vortices encoded in the Abelian projected fields in either of the two ways that one might plausibly have expected them to be.

It is worth stepping back at this point and reflecting upon the tentative nature of the above arguments. Our calculation of the monopole string tension takes each monopole to be a source of a simple Coulombic flux, as obtained by solving Maxwell's equations. Treating the monopoles as being ''isolated'' in this way is the obvious starting point if one wishes to ask what the physics is ''due to monopoles.'' But it is no guarantee that such a question makes any sense. Indeed it is only in the Villain model that one has the exact factorization of Wilson loops into monopole and non-monopole pieces that is needed for this question to be clearly unambiguous. For example, it is not *a priori* clear that the ensemble of monopoles one obtains in the maximally Abelian gauge is even qualitatively such as one would expect from a generic U(1) action. If it is not, then one must ask what are the fluctuations in the SU(2) fields that determine the nature of the monopole ensemble, and whether these features of the ensemble have a significant effect on the calculated string tension. If they do, then the question we are asking, whether the string tension is ''due to monopoles,'' becomes intrinsically ambiguous. Our demonstration that there is no suppression of the $q=2$ monopole string tension may be regarded as a first step, but only a first step, towards showing that the monopole ensemble does not possess such features that require additional explanation. One should also mention that the Abelian fields are periodic in 2π (in the sense that the number density of plaquette angles peaks at multiples of 2π), which is the requirement for Dirac strings to be invisible, and that they possess a screening length that is characteristic of plasmas [9]. Equally, if we had found a significant flux loop condensate in the Abelian fields, we would have had to study carefully the (presumably) non-trivial correlations between the monopoles and flux loops in order to determine if there was any sense in claiming that some physics was ''due to monopoles.'' The fact that we have not found any sign of such a flux loop condensate, or of any anomalous features of the monopole plasma, means that we are not yet forced to confront this quite general problem. But this question clearly needs systematic exploration.

IV. SUMMARY

We have studied the magnetic monopole currents obtained after fixing to the maximally Abelian gauge of SU(2), on lattices that are both large in physical units and have a relatively small lattice spacing. The monopole clusters are found to divide into two clear classes, both on the basis of their lengths and their physical properties. The smaller clusters have a distribution of lengths which follows a power law, and the exponent is consistent with 3, as was previously seen on coarser lattices [9]. These clusters are compact objects, and their radii also follow a power law whose exponent we found to be 4.2 (1). This is close to, but a little less than, the scale invariant value of 5, which indicates that if the smaller clusters correspond to objects in the SU(2) vacuum, these objects have a size distribution which yields slightly more large radius objects than would be expected in a purely scale invariant theory. This scale breaking is, however, far too weak to encourage the identification of such objects with the small instantons in the theory.

That is not to say that instantons are necessarily irrelevant; the correlations between the monopole currents and the action and topological charge densities ([7,8,21,22] and references therein) indicate some connection. It would be interesting to measure the correlations separately using the largest cluster and the remaining power law clusters.

The small clusters do not appear relevant to the long range physics; they produce a zero, or at most a very small, contribution to the string tension. Indeed the string tension is consistent with being produced by the largest cluster alone. The fact that there should be a large percolating monopole cluster associated with the long-distance physics is an old idea (see [25] for an early reference). The properties that we find for this cluster, however, are certainly not those associated with naive percolation. In particular, as we approach the continuum limit the density of monopoles belonging to this cluster goes to zero. And indeed the fraction of the total monopole current that arises from this largest cluster also appears to go to zero. This is because this single very large cluster seems to percolate on physical and not on lattice length scales, while the physically unimportant secondary clusters have an approximatley constant density in lattice units. All this reproduces the properties that we previously obtained on coarser lattices, but which seemed to be lost when going to finer lattice spacings, albeit on volumes of a smaller physical extent. This study demonstrates that the breakdown was a finite volume effect, rather than a failure of the monopole picture in the weak coupling limit. The volume at which the picture was restored was as predicted by the scaling relations derived from the coarser lattices.

The fact that one has to go to volumes that are ever larger as $a\rightarrow 0$ can be interpreted as a breakdown of the Abelian projection. As we remarked, something like this is not unexpected: as $a\rightarrow 0$ the Abelian projection will presumably be increasingly driven by the irrelevant ultraviolet fluctuations of the SU(2) link matrices. This leads to an increasing fraction of the monopole current — that belonging to the smaller clusters — containing no physics and this contributes an increasing background ''noise'' to attempts at extracting physical observables as we approach the continuum limit. Fortunately the unphysical gas of monopoles that one obtains by Abelian projection within the maximally Abelian gauge is sufficiently dilute that one can isolate the physically relevant ''percolating'' cluster, even if the price is that one has to work with ever larger volumes.

We also calculated the monopole contribution to Wilson loops of higher charges, and found that the corresponding monopole string tensions appear to be simply proportional to the charge, at least up to $q=4$. This is what is predicted by a saddle point treatment of the U(1) lattice gauge theory [15] as can be seen more simply, if more approximately, within our simplistic charge plasma model.

Our main reason for studying these higher-q string tensions was to probe for any sign of a condensate of $Z(2)$ vortices in the Abelian projected fields. It might, of course, be that such vortices are simply not encoded in the Abelian fields. It is plausible, however, to infer from the observed monopole and center dominance that both when we force the SU(2) link matrices to be as Abelian as possible and when we force them to be as close to ± 1 as possible, the resulting Abelian and $Z(2)$ fields capture essentially all the long range confining disorder present in the original SU(2) fields. In the case of $Z(2)$ fields the disorder must be encoded by vortices (there is nothing else). In the Abelian case however the disorder can be carried either by monopoles or by closed loops of ''magnetic'' flux. We argued that such a closed loop, carrying a net magnetic flux of π units, provides a plausible way for the Abelian fields to encode the presence of an underlying $Z(2)$ vortex. Our study of the monopole-U(1) ''difference gas'' showed, however, that there is no significant contribution to confinement from such loops of magnetic flux. An alternative [14,15] is that the $Z(2)$ disorder is encoded in correlated strings of (anti)monopoles. If such correlations were important, however, they would lead to a significant suppression of the $q=2$ string tension, and this we do not observe. Instead we find that the effective monopole string tensions satisfy $K(q=2)=2K(q=1)$ very accurately to distances that are quite substantial in physical units. While there is a limit to what one can conclude about $Z(2)$ vortices in a study that focuses solely on the Abelian projected fields, the fact that they do not manifest themselves in any of the ways that one might expect must cast some doubt on their importance in the SU(2) vacuum.

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