Long-range forces of QCD

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We consider the scattering of two color dipoles (e.g., heavy quarkonium states) at low energy—a QCD analogue of van der Waals interaction. Even though the couplings of the dipoles to the gluon field can be described in perturbation theory, which leads to a potential proportional to $(N_c^2-1)/R^7$, at large distances R the interaction becomes totally nonperturbative. Low-energy QCD theorems are used to evaluate the leading long-distance contribution $\sim(N_f^2-1)/(11N_c-2N_f)^2\ R^{-5/2}\exp(-2\mu R)$ (μ is the Goldstone boson mass), which is shown to arise from the correlated two-boson exchange. The sum rule which relates the overall strength of the interaction to the energy density of the QCD vacuum is derived. Surprisingly, we find that when the size of the dipoles shrinks to zero (the heavy quark limit in the case of quarkonia), the nonperturbative part of the interaction vanishes more slowly than the perturbative part as a consequence of the scale anomaly. As an application, we evaluate elastic $\pi J/\psi$ and $\pi J/\psi\to\pi\psi'$ cross sections. [S0556-2821(99)01123-6]

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I. INTRODUCTION

The interaction between small color dipoles[1] provides an interesting theoretical laboratory for the study of QCD and its applications in nuclear physics. Indeed, asymptotic freedom dictates that the coupling of strong interactions becomes weak at short distances, and since the small size of the dipoles introduces a natural infrared cutoff, one can hope that their interactions can be systematically treated in perturbation theory [1–6].

One could therefore expect that at low energy the interaction between the dipoles in SU(N) gauge theory would be of van der Waals type

$$V_{\rm pert}(R)\sim -g^4(N^2-1)\ \frac{1}{R^n}, \tag{1}$$

where $n=6$ in the original van der Waals potential, and g is the gluon coupling evaluated at the scale of quarkonium size. Indeed, this behavior was established by Appelquist and Fischler [1], who studied the interactions of static color dipoles described by Wilson loops. These authors also explored the breakdown of the perturbative expansion in the static potential [2], and pointed to the possibility that retardation effects can modify the $1/R^6$ dependence once the spatial motion of the quarks is considered. In this paper, we take this effect into account and argue that, in the limit of the small size of the dipoles, the potential (1) is actually of Casimir-Polder [7] type, with $n=7$. On the other hand, gluons cannot propagate at large distances, where the dominant degrees of freedom are the lightest hadronic states. In the chiral limit, the theory with spontaneously broken $\mathrm{SU}_L(N_f)\times\mathrm{SU}_R(N_f)$ symmetry contains (N_f^2-1) Goldstone bosons, and the number of flavors N_f should effectively replace the number of colors in the coefficient of Eq. (1) at large distances:

$$V_{\rm chiral}(R)\sim -(N_f^2-1)\ \frac{1}{R^n}. \tag{2}$$

In the real world where the masses μ of Goldstone bosons are not equal to zero, instead of Eq. (2) at large distances one expects to find the potential of Yukawa form

$$V(R)\sim -(N_f^2-1)\ \frac{e^{-2\mu R}}{R}. \tag{3}$$

[We will show that the actual form of the long–distance potential is different from Eq. (3)—see Eq. (31).] How does the transition between the behavior at short and long distances occur? Can one explicitly, from the first principles, evaluate the long-distance potential?

In this paper we address these questions, and argue that the interaction between small color dipoles (heavy quarkonium states in our example) at large distances can be reliably evaluated. Our analysis is based on the following two properties of QCD: (1) the scale invariance which is present at the tree level in QCD with massless quarks is broken by interactions; this is reflected in the nonzero divergence of scale current, and hence non-vanishing trace of the energy-momentum tensor [8,9]; (2) the chiral symmetry is broken spontaneously, which implies the existence of Goldstone bosons; being the lightest of all hadrons, they are the relevant degrees of freedom at large distances.

These two properties of QCD are beautifully linked by the low-energy theorem derived by Voloshin and Zakharov [10], which we discuss below. The first of these properties was previously exploited to derive the low-energy amplitude of quarkonium-nucleon scattering [11,12] (for recent work, see Refs. [13,14]). van der Waals interactions of quarkonium with nucleons and nuclei were discussed in Refs. [15–18]. For applications to the low-energy quarkonium-pion scattering and the structure of quarkonium, see Refs. [19–22]. Quarkonium dissociation cross sections in interactions with

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[1]Small color dipoles can be realized in the real word as heavy quarkonium states or as virtual quark-antiquark pairs in deep inelastic scattering.

light hadrons were evaluated in Refs. [5,23,24]. Some of the results of this study were previously reported in Ref. [25].

The picture which emerges from our approach is the following. The heavy ''onia'' couple perturbatively to the gluon field; at small distances, the entire interaction can be evaluated perturbatively. At larger distances, however, the interaction becomes grossly modified by the coupling to pion fields, which is fixed by low-energy theorems. The dominance of the nonperturbative interaction at large distances in this case will be shown to be a consequence of the finite energy density of QCD vacuum.[2] We also find that when the size of the dipoles shrinks to zero (which is what happens in the heavy quark limit with quarkonia), the nonperturbative part of the interaction vanishes more slowly than the perturbative part — in other words, the interaction between very small dipoles becomes totally nonperturbative. This surprising result will be shown to be a natural consequence of scale anomaly in QCD.

In this paper, we limit ourselves to the interaction at small energies; however we hope that some of our results may be extended to the case of dipole scattering at high energies [26], where the broken chiral symmetry can also play a substantial role, as discussed by Anselm and Gribov [27]. The paper is organized as follows. In Sec. II, we give the general expression for the scattering amplitude of two color dipoles in the framework of the operator product expansion (OPE), introduce the spectral representation method for the evaluation of this amplitude, and use this method to rederive the perturbative expression [5] for the low-energy scattering amplitude (or potential). In Sec. III, we discuss the scattering amplitude of color dipoles beyond the perturbation theory, derive the leading long-distance behavior of the potential, and discuss the relative strength of perturbative and nonperturbative contributions. In Sec. IV, we evaluate the potential acting between two J/ψ's. In Sec. V we use the low energy theorems [10,29–31] to derive the sum rule relating the strength of the potential to the energy density of QCD vacuum. In Sec. VI, we evaluate the cross sections of J/ψ interactions with pions, relevant for the problem of J/ψ suppression in heavy ion collisions [32,33]. The final Sec. VII is devoted to summary and discussion.

II. INTERACTION OF COLOR DIPOLES IN PERTURBATION THEORY

The small size of the heavy quarkonium Φ allows us to expand the amplitude of its interaction with hadrons (h,h') at low energy in the form of multipole, or operator product, expansion [4,5]:

$$\mathcal{M}=\sum_i c_i\langle h'|O_i(0)|h\rangle, \tag{4}$$

where $O_i(x)$ are the gauge-invariant local operators and c_i are the Wilson coefficients (polarizabilities) which reflect the structure of the quarkonium; the energy of the hadrons is assumed to be small compared to the binding energy of the quarkonium, ϵ_0. The factorization scale in this formula can be chosen at ϵ_0. At small energies, the leading operator in Eq. (4) is the square of the chromo-electric field $(1/2)g^2\mathbf{E}^{a2}(0)$ [4,5]—this is the leading twist-two operator expressible in terms of gluon fields. Other twist-two operators contain covariant derivatives leading to the powers of the ratio of the energy transfer to the binding energy and are therefore suppressed at small energy; the series of these local operators can be summed up into a double dipole form [5]:

$$\begin{aligned}\frac{g^2}{N}\sum_{n=2,\text{even}}^{\infty}&\left\langle\phi\left|r^i\frac{1}{(H_a+\epsilon)^{n-1}}r^j\right|\phi\right\rangle\\&\times\mathrm{tr}[E_i(0)(-iD^0)^{n-2}E_j(0)]\\&=\frac{g^2}{N}\left\langle\mathrm{tr}\left[\mathbf{r}\cdot\mathbf{E}(0)\frac{1}{H_a+\epsilon+iD^0}\mathbf{r}\cdot\mathbf{E}(0)\right]\right\rangle,\end{aligned} \tag{5}$$

where the Wilson coefficients are explicitly given by the expectation values over the singlet state $\phi(r)$ with the binding energy ϵ; $H_a(r)$ is the effective Hamiltonian describing the intermediate, SU(N) color-adjoint quark-antiquark state; D^0 is the covariant derivative acting on $\mathbf{E}$ and the trace over the color indices of gluon operators ensures that Eq. (5) is gauge invariant. In the heavy quark limit, $\phi(r)$ can be approximated by the Coulomb wave function.

Using this multipole representation, one can write down the amplitude of the scattering of two small color dipoles at low energies (in the Born approximation) in the following form [5]:

$$V(R)=-i\int_{-\infty}^{\infty}dt\left\langle 0\left|\mathrm{T}\left(\sum_i c_iO_i(x)\right)\left(\sum_j c_jO_j(0)\right)\right|0\right\rangle. \tag{6}$$

Keeping only the leading operators, we can rewrite Eq. (6) in a simple form

$$\begin{aligned}V(R)=-i\left(\overline{d}_2\frac{a_0^2}{\epsilon_0}\right)^2\int_{-\infty}^{\infty}dt\Big\langle 0\Big|\mathrm{T}&\frac{1}{2}g^2\mathbf{E}^a\cdot\mathbf{E}^a(t,R)\\&\times\frac{1}{2}g^2\mathbf{E}^b\cdot\mathbf{E}^b(0)\Big|0\Big\rangle,\end{aligned} \tag{7}$$

where $\overline{d}_2$ is the corresponding Wilson coefficient defined by

$$\overline{d}_2\frac{a_0^2}{\epsilon_0}=\frac{1}{3N}\left\langle\phi\left|r^i\frac{1}{H_a+\epsilon}r^i\right|\phi\right\rangle, \tag{8}$$

from which we have explicitly factored out the dependence on the quarkonium Bohr radius a_0 and the Rydberg energy ϵ_0. The factors a_0 and $1/\epsilon_0$ represent the characteristic dimension and fluctuation time of the color dipole, respectively. The approximation used in deriving Eq. (7) is justified when the gluon fields change slowly compared to $1/\epsilon_0$. The Wilson coefficients (8) were computed for S [5] and P [19] states of quarkonium in the large N limit.

[2]This picture was foreseen by Bjorken [28].

In physical terms, the structure of Eq. (7) is transparent: it describes elastic scattering of two dipoles which act on each other by chromoelectric dipole fields; color neutrality permits only the square of dipole interaction. The amplitude (7) was evaluated before [5] in perturbative QCD using functional methods. For our purposes, however, it is convenient to use a spectral representation approach.[3] As a first application of this approach, we will reproduce the result of [5] by a different, and perhaps more simple, method.

First, it is convenient to express $g^2\mathbf{E}^{a2}$ in terms of the gluon field strength tensor [29]:

$$\begin{aligned} g^2\mathbf{E}^{a2} &= \frac{g^2}{2}(\mathbf{E}^{a2}-\mathbf{B}^{a2})+\frac{g^2}{2}(\mathbf{E}^{a2}+\mathbf{B}^{a2}) \\ &= -\frac{1}{4}g^2 G^a_{\alpha\beta}G^{a\alpha\beta}+g^2\left(-G^a_{0\alpha}G^{a\alpha}_0+\frac{1}{4}g_{00}G^a_{\alpha\beta}G^{a\alpha\beta}\right) \\ &= \frac{8\pi^2}{b}\theta^\mu_\mu+g^2\theta^{(G)}_{00}, \end{aligned} \tag{9}$$

where

$$\theta^\mu_\mu \equiv \frac{\beta(g)}{2g}G^{a\alpha\beta}G^a_{\alpha\beta}=-\frac{bg^2}{32\pi^2}G^{a\alpha\beta}G^a_{\alpha\beta},$$

$$\theta^{(G)}_{\mu\nu}\equiv -G^a_{\mu\alpha}G^{a\alpha}_\nu+\frac{1}{4}g_{\mu\nu}G^a_{\alpha\beta}G^{a\alpha\beta}. \tag{10}$$

Note that θ^μ_μ is the trace of the energy-momentum tensor of QCD in the chiral limit, and as a consequence of decoupling theorem [35] the β function in Eq. (10) does not contain the contribution of heavy quarks [i.e. $b=\frac{1}{3}(11N-2N_f)=9$].

Let us now write down the spectral representation for the correlator of the trace of the energy-momentum tensor

$$\begin{aligned} &\langle 0|\mathrm{T}\theta^\mu_\mu(x)\theta^\nu_\nu(0)|0\rangle \\ &\quad=\int\frac{d^4k}{(2\pi)^3}\rho_\theta(k^2)\,\theta(k_0)[e^{-ikx}\theta(x^0)+e^{ikx}\theta(-x^0)] \\ &\quad=\int d\sigma^2\rho_\theta(\sigma^2)\Delta_F(x;\sigma^2), \end{aligned} \tag{11}$$

where the spectral density is defined by

$$\rho_\theta(k^2)\,\theta(k_0)=\sum_n (2\pi)^3\delta^4(p_n-k)|\langle n|\theta^\mu_\mu|0\rangle|^2, \tag{12}$$

the phase-space integral should be understood in Eq. (12), and

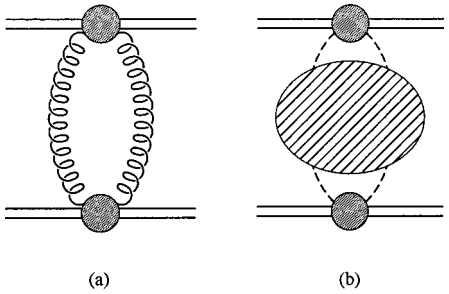

FIG. 1. Contributions to the potential between quarkonia from (a) two-gluon exchange and (b) correlated two-pion exchange.

$$\begin{aligned} i\Delta_F(x;\sigma^2) &= i\int\frac{d^4k}{(2\pi)^3}\delta(k^2-\sigma^2)\,\theta(k_0)[e^{-ikx}\theta(x^0) \\ &\quad +e^{ikx}\theta(-x^0)] \end{aligned} \tag{13}$$

is the Feynman propagator of a scalar field in the coordinate space. Substituting the representation (11) in Eq. (7), the potential can be expressed as a superposition of Yukawa potentials corresponding to the exchange of scalar quanta of mass σ:

$$\begin{aligned} V_\theta(R) &= -i\left(\frac{}{d_2}\frac{a_0^2}{\epsilon_0}\right)^2\left(\frac{4\pi^2}{b}\right)^2\int_{-\infty}^{\infty}dt\int d\sigma^2\rho_\theta(\sigma^2)\Delta_F(x;\sigma^2) \\ &= -\left(\frac{}{d_2}\frac{a_0^2}{\epsilon_0}\right)^2\left(\frac{4\pi^2}{b}\right)^2\int d\sigma^2\rho_\theta(\sigma^2)\frac{1}{4\pi R}e^{-\sigma R}. \end{aligned} \tag{14}$$

Our analysis so far has been completely general; the dynamics enters through the spectral density (12). Let us first evaluate this quantity in perturbation theory, where it is given by the contributions of two-gluon states [see Fig. 1(a)] defined by

$$\rho^{\mathrm{pt}}_\theta(q^2)\equiv\sum(2\pi)^3\delta^4(p_1+p_2-q)|\langle p_1\varepsilon_1 a,p_2\varepsilon_2 b|\theta^\mu_\mu|0\rangle|^2, \tag{15}$$

where the phase-space integral is understood, as well as the summations over the polarization ($\varepsilon_{1,2}$) and color indices (a,b) of the two gluons. The calculation for SU(N) color (see Appendix A) gives

$$\rho^{\mathrm{pt}}_\theta(q^2)=\left(\frac{bg^2}{32\pi^2}\right)^2\frac{N^2-1}{4\pi^2}q^4; \tag{16}$$

the appearance of q^4 dependence in Eq. (16) is of course natural from dimensional arguments. Performing the integration in Eq. (14) over the invariant mass σ^2 from zero to infinity, we get the following result ($N=3$):

$$V^{\mathrm{pt}}_\theta(R)=-g^4\left(\frac{}{d_2}\frac{a_0^2}{\epsilon_0}\right)^2\frac{15}{8\pi^3}\frac{1}{R^7}. \tag{17}$$

This result can also be derived by the functional method of Bhanot and Peskin [5] (see Appendix B).

[3]The use of dispersion theory in electrodynamics for the interaction between neutral atoms was pioneered by Feinberg and Sucher [34].

Several remarks are in order here: The $\propto R^{-7}$ dependence of the potential (17) is a classical result known from atomic physics [7]; as is apparent in our derivation, the extra R^{-1} as compared to the van der Waals potential $\propto R^{-6}$ is the consequence of the fact that the dipoles fluctuate in time, and the characteristic time of fluctuation $t \sim \epsilon_0^{-1}$ (ϵ_0 is quarkonium binding energy) is small compared to the spatial separation of the ''onia:'' $t \ll R$—note an explicit integration over time in Eq. (14). This illustrates, in a somewhat different way, the original argument of Voloshin [4] that the physical picture behind the operator product expansion (OPE) is orthogonal to the potential model—the latter is based on the assumption of instantaneous interaction, whereas the former is based on the assumption that the internal frequency of heavy quarkonium $1/\epsilon_0$ is much higher than the frequency of external soft fields. Retardation effects make questionable the possibility to describe the interactions of quarks inside a heavy quarkonium by a local potential. In our case, applying the OPE method, we first average the interactions with soft gluons over the quarkonium internal state, which corresponds to the infinite retardation. With the resulting coupling between the quarkonium and the gluons, the potential description of onium-onium scattering is adequate since at low energies the *relative* motion of heavy quarkonia is slow. The retardation effects manifest themselves in the modification of the shape of the potential.

We note that although the matrix element of the operator θ_μ^μ can in general be nonperturbative, in perturbation theory θ_μ^μ is of order g^2, and accordingly the potential (17) has the prefactor g^4. Then the second term $g^2 \theta_{00}^{(G)}$ in Eq. (9), which describes the tensor 2^{++} state of two gluons, gives the contribution in the same order in g. Adding this contribution to V_θ in Eq. (17), we recover the complete result of Ref. [5]

$$V^{\rm pt}(R) = -g^4 \left(\overline{d}_2 \frac{a_0^2}{\epsilon_0} \right)^2 \frac{23}{8\pi^3} \frac{1}{R^7}. \tag{18}$$

Note that our $\overline{d}_2$ is related to the d_2 in Ref. [5] by $d_2 a_0 \epsilon_0 = \overline{d}_2 g^2$. This perturbative expression is valid when $a_0, 1/\epsilon_0 \ll R \ll \Lambda_{\rm QCD}^{-1}$.

III. BEYOND THE PERTURBATION THEORY: THE ROLE OF GOLDSTONE BOSONS

At large distances, the perturbative description breaks down, because the potential becomes determined by the spectral density at small q^2, where the transverse momenta of the gluons become small.

A. Broken scale invariance

To see the importance of nonperturbative effects explicitly, let us consider the correlator of θ_μ^μ,

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | {\rm T} \theta_\mu^\mu(x) \theta_\nu^\nu(0) | 0 \rangle = \int d\sigma^2 \frac{\rho_\theta(\sigma^2)}{\sigma^2 - q^2 - i\epsilon}. \tag{19}$$

An important theorem [31] for this correlator states that as a consequence of the broken scale invariance of QCD,

$$\Pi(0) = -4 \langle 0 | \theta_\mu^\mu(0) | 0 \rangle. \tag{20}$$

Note that the right-hand side of Eq. (20) is divergent even in perturbation theory, and should therefore be regularized by subtracting the perturbative part. The vacuum expectation value of the θ_μ^μ operator then measures the energy density of non-perturbative fluctuations in QCD vacuum, and the low-energy theorem (20) implies a sum rule for the spectral density:

$$\int \frac{d\sigma^2}{\sigma^2} [\rho_\theta^{\rm phys}(\sigma^2) - \rho_\theta^{\rm pt}(\sigma^2)] = -4 \langle 0 | \theta_\mu^\mu(0) | 0 \rangle = -16 \epsilon_{\rm vac} \neq 0, \tag{21}$$

where the estimate for the vacuum energy density extracted from the sum rule analysis gives $\epsilon_{\rm vac} \simeq -(0.24\ {\rm GeV})^4$ [36]. Since the physical spectral density, $\rho_\theta^{\rm phys}$, should approach the perturbative one, $\rho_\theta^{\rm pt}$, at high σ^2, the integral in Eq. (21) can accumulate its value required by the RHS only in the region of relatively small σ^2. In addition, another sum rule [36–38],

$$\int d\sigma^2 \rho_\theta^{\rm phys}(\sigma^2) = \int d\sigma^2 \rho_\theta^{\rm pt}(\sigma^2) \tag{22}$$

is implied by the quark-hadron duality.

B. Matching onto the chiral theory

At small invariant masses, the physical spectral density of the correlator (19) should be saturated by the lightest state allowed in the scalar channel—two pions:

$$\rho_\theta^{\pi\pi}(q^2) = \sum (2\pi)^3 \delta^4(p_1 + p_2 - q) |\langle \pi(p_1) \pi(p_2) | \theta_\mu^\mu | 0 \rangle|^2, \tag{23}$$

where, just as in Eq. (15), the phase-space integral is understood.

Since, according to Eq. (10), θ_μ^μ is a gluonic operator, the evaluation of Eq. (23) requires the knowledge of the coupling of gluons to pions. This is a purely nonperturbative problem. Nevertheless it can be rigorously solved, as it was shown in Ref. [10] (see also Ref. [29]). The idea is the following: at small pion momenta, the energy-momentum tensor can be accurately computed using the low-energy chiral Lagrangian,

$$\mathcal{L} = \frac{f_\pi^2}{4} {\rm tr}\, \partial_\mu U \partial^\mu U^\dagger + \frac{1}{4} m_\pi^2 f_\pi^2 {\rm tr}(U + U^\dagger), \tag{24}$$

where $U = \exp(2i\pi/f_\pi)$, $\pi \equiv \pi^a T^a$ and T^a are the SU(2) generators normalized by ${\rm tr}\, T^a T^b = \frac{1}{2} \delta^{ab}$. The trace of the energy-momentum tensor for this Lagrangian is

$$\theta^\mu_\mu = -2\frac{f_\pi^2}{4}\,\mathrm{tr}\,\partial_\mu U \partial^\mu U^\dagger - m_\pi^2 f_\pi^2\,\mathrm{tr}(U+U^\dagger). \qquad (25)$$

Expanding this expression (25) in powers of the pion field, one obtains, to the lowest order,

$$\theta^\mu_\mu = -\partial_\mu \pi^a \partial^\mu \pi^a + 2m_\pi^2 \pi^a \pi^a + \cdots, \qquad (26)$$

and this leads to an elegant result [10] in the chiral limit of vanishing pion mass:

$$\langle \pi^+ \pi^- | \theta^\mu_\mu | 0\rangle = q^2. \qquad (27)$$

This result for the coupling of the operator θ^μ_μ to two pions can be immediately generalized for any (even) number of pions using Eq. (25).

Now that we know the coupling of gluons to the two-pion state, the pion-pair contribution to the spectral density (23) can be easily computed by performing the simple phase space integration with the result

$$\rho_\theta^{\pi\pi}(q^2) = \frac{3}{32\pi^2} q^4; \qquad (28)$$

in the general case of N_f light flavors, the coefficient 3 in Eq. (28) should be replaced by (N_f^2-1). Again, the q^4 dependence comes only from dimensionality. Multipion contributions can be evaluated using Eq. (25); we have found that at small invariant masses their influence is small. The dominant contribution at small invariant masses σ, in which we are primarily interested here, therefore comes from the $\pi\pi$ state.

Recalling that to the leading order in OPE the scattering amplitude is dominated by the operator $\frac{1}{2}g^2\mathbf{E}^{a2}$, we need to evaluate also the matrix element of the second term in Eq. (9), $\langle 0|g^2\theta_{00}^{(G)}|\pi\pi\rangle$ to complete our derivation of the scattering amplitude. As we mentioned in the previous section, this tensor operator contributes a substantial fraction, 8/23, to the full perturbative result. However, unlike the scalar operator, the tensor term is not coupled to the anomaly. The contribution $\langle 0|g^2\theta_{00}^{(G)}|\pi\pi\rangle$ therefore is of $O(g^2)$, and is subleading in the heavy quark limit. In this limit, we thus come to the following low-energy expression [10]:

$$\left\langle \pi\pi \left| \frac{1}{2} g^2 \mathbf{E}^{a2} \right| 0 \right\rangle = \left(\frac{4\pi^2}{b}\right) q^2 + O(\alpha_s, m_\pi^2). \qquad (29)$$

The matrix element in question is therefore known up to α_s and m_π^2 corrections.

The most important correction due to the finite pion mass is the phase space threshold; to take it into account, we modify the spectral density in the following way ($q^2 \geqslant 4m_\pi^2$):

$$\rho_\theta^{\pi\pi}(q^2) = \frac{3}{32\pi^2}\left(\frac{q^2-4m_\pi^2}{q^2}\right)^{1/2} q^4. \qquad (30)$$

This expression should be valid at small q^2. Substituting this spectral density into the general expression (14), we get the potential due to the $\pi\pi$ exchange; at large R

$$V^{\pi\pi}(R) \to -\left(\frac{\overline{d}_2 a_0^2}{\epsilon_0}\right)^2 \left(\frac{4\pi^2}{b}\right)^2 \frac{3}{2}(2m_\pi)^4 \frac{m_\pi^{1/2}}{(4\pi R)^{5/2}} e^{-2m_\pi R}. \qquad (31)$$

Note that this potential is *not* of Yukawa form. The same R dependence of $\pi\pi$ exchange at large distances was found a long time ago by Lévy [39] and Klein [40]. It has been given previously also by Bhanot and Peskin [5], but up to an unknown constant. In our approach, the strength of the potential, as well as its dependence on the numbers of colors N and flavors N_f—$\sim(N_f^2-1)/(11N_c-2N_f)^2$, is fixed by the low-energy QCD theorems.

Note also that, unlike the perturbative result (17) which is manifestly $O(g^4)$ [in addition to a factor $(\overline{d}_2 a_0^2/\epsilon_0)^2$], the amplitude (31) is $O(g^0)$—this ''anomalously'' strong interaction is the consequence of scale anomaly.[4]

C. Dynamical enhancements in the spectral density

The low-energy theorems (LET's) [31,10] not only allow us to evaluate explicitly the contribution of uncorrelated $\pi\pi$ exchange; they also tell us that this contribution alone is not the complete answer yet. Indeed, the numerical analysis shows that the $\pi\pi$ spectral density (30) alone cannot saturate the sum rule (21)—at large σ^2, the physical spectral density approaches the spectral density of perturbation theory, so the integral in Eq. (21) does not get any contribution; at small σ^2, the $\pi\pi$ spectral density (30), according to the chiral and scale symmetries is suppressed by $\sim\sigma^4$. The low energy theorems thus *require* the presence of resonant enhancement(s) [30] in the 0^{++} $\pi\pi$, and perhaps multipion $\overline{K}K$ and $\eta\eta$ channels as well. Here we will leave the complete multichannel problem for future investigations, and study only the influence of these resonances in the $\pi\pi$ channel on the potential between the color dipoles.

To do this, we define the pion scalar form factor by $\langle \pi^+\pi^-|\theta^\mu_\mu|0\rangle = q^2 F(q^2)$ (in the chiral limit) and write down the spectral density as

$$\rho_\theta^{\pi\pi}(s) = \frac{3}{32\pi^2}\left(\frac{s-4m_\pi^2}{s}\right)^{1/2} s^2 |F(s)|^2. \qquad (32)$$

It may be illustrative to consider first the idealized case of a sharp σ resonance. For simplicity, let us assume that the difference between the physical and perturbative spectral densities is due to this σ resonance alone, and write the spectral density as $\rho_\theta^{\rm phys}(s) - \rho_\theta^{\rm pt}(s) = c\,\delta(s-m_\sigma^2)$. The LET (21) then fixes the contribution of the narrow σ state of mass m_σ as

$$\int \frac{ds}{s}[\rho_\theta^{\rm phys}(s) - \rho_\theta^{\rm pt}(s)] = \frac{c}{m_\sigma^2} = -16\epsilon_{\rm vac}. \qquad (33)$$

[4]Of course, in the heavy quark limit the amplitude (31) will nevertheless vanish, since $a_0 \to 0$ and $\epsilon_0 \to \infty$.

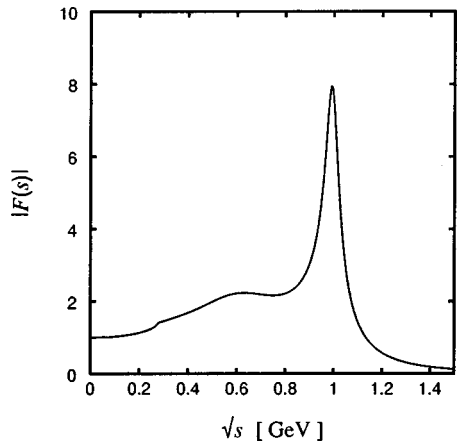

FIG. 2. Scalar-isoscalar form factor of the pion (35).

The corresponding potential is of Yukawa type,

$$V(R) = -\left(\frac{a_0^2}{d_2 \epsilon_0}\right)^2 \left(\frac{4\pi^2}{b}\right)^2 c \frac{1}{4\pi R} e^{-m_\sigma R}. \tag{34}$$

In this idealized situation, the strength of the potential is directly related to the energy density of nonperturbative QCD vacuum. Note, however, that this simplified model of the sharp σ resonance is inconsistent with the asymptotics derived from the broken chiral symmetry [Cf. Eq. (31)].

The form factor $F(s)$ is directly related to the experimental $\pi\pi$ phase shifts by the Omnès-Muskhelishvili equation [41,42]. Within the single-channel treatment $F(s)$ has a solution

$$F(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_1} ds' \frac{\delta_0^0(s')}{s'(s'-s-i\epsilon)}\right], \tag{35}$$

where $\delta_0^0(s)$ is the phase shift of the $\pi\pi$ scattering in the scalar-isoscalar channel, and formally $s_1 \to \infty$. With this formula we can make a full use of the experimental information on the $\pi\pi$ correlations.

In our calculation we use a simple analytic form [43] for the phase shift $\delta_0^0(s)$ which has been shown to fit the experimental data up to $s_{\pi\pi} \simeq 1$ GeV^2. Beyond this energy, one should take into account the contributions of other channels, such as $\bar{K}K$. We performed the integral in Eq. (35) numerically up to $s_1 = (5\ \text{GeV})^2$ by extrapolating the low-energy fit of the phase shift. When we change s_1 to $(20\ \text{GeV})^2$, the change in $F(s)$ at 1 GeV^2 is a few percent. In Fig. 2 we show the resulting scalar formfactor of the pion, $F(s)$. The structure of $F(s)$ may be interpreted as due to a broad σ and narrow f_0 resonances. For a more realistic evaluation of the formfactor, the multi-channel calculation has to be done; the results will be reported elsewhere. In this paper, as a simple model for the $\rho_\theta^{\text{phys}}$, we will take the form

$$\rho_\theta^{\text{phys}}(s) = \begin{cases} \rho_\theta^{\pi\pi}(s) & (4m_\pi^2 < s < s_0), \\ \rho_\theta^{\text{pt}}(s) & (s_0 < s), \end{cases} \tag{36}$$

where s_0 is a matching scale.

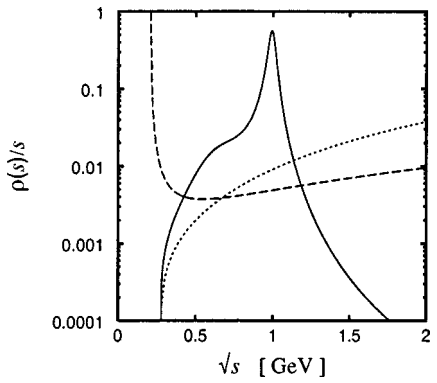

FIG. 3. Spectral density of the correlator $\langle 0|\mathrm{T}\theta_\mu^\mu(x)\theta_\nu^\nu(0)|0\rangle$ at low energy (solid line). The uncorrelated two-pion contribution is shown in dotted line, and the perturbative one with one-loop running coupling constant ($\Lambda_{\text{QCD}} = 200$ MeV) in dashed line.

D. The analysis of the sum rule

Let us consider the sum rule (21) within our simple model for the spectral density. When the model (36) for $\rho_\theta^{\text{phys}}(s)$ is used, the upper limit of the integral in Eq. (21) can be replaced by s_0. In Fig. 3 we show the physical and perturbative parts of the integrand in the sum rule (21) with solid and dashed lines, respectively. Since for the spectral density in the perturbation theory there is no scale other than s, the coupling constant should be taken running with this scale:

$$\rho_\theta^{\text{pt}}(s) = \left(\frac{9\alpha_s(s)}{8\pi}\right)^2 \frac{2}{\pi^2} s^2, \tag{37}$$

where $\alpha_s(s) = 4\pi/[b \ln(s/\Lambda_{\text{QCD}}^2)]$ with $\Lambda_{\text{QCD}} = 200$ MeV.

We note that the spectral density for uncorrelated pions $\rho_\theta^{\pi\pi}$ (30), which is shown in dotted line in Fig. 3, has the same functional form as the ρ_θ^{pt}, up to the logarithm and the threshold factor. As a consequence of this, we find that the uncorrelated $\pi\pi$ spectral density (30) cannot obey both the sum rule (21) and the duality constraint (22). On the other hand, the spectral density obtained with the Omnès-Muskhelishvili solution has a nontrivial structure (see Fig. 3); one can clearly see a narrow peak at the f_0 resonance region with a shoulder coming from the broad ''σ'' around 0.6 GeV.

We plot the integral in the LET for the physical (solid) and perturbative (dashed) parts separately as a function of the upper limit, s_0. One can see that the value of the integral for the physical spectral density is mainly accumulated in 1 GeV region, and the ''σ'' contributes to it about 20% (see Fig. 4). The perturbative part behaves as s^2 up to the logarithm, weighting the higher energy region. The LET tells us that the difference of these two contributions should be equal to the energy density of the QCD vacuum. In our model for the spectral density, the LET (21) is satisfied when we choose $s_0 = (2 \sim 2.5\ \text{GeV})^2$. As for the duality relation (22), the equality of the integrals of the physical and perturbative spectral densities is achieved when we choose $s_0 \sim (2\ \text{GeV})^2$ (Fig. 5)—this value of the matching scale therefore provides a consistent solution to both the LET and the duality relation.

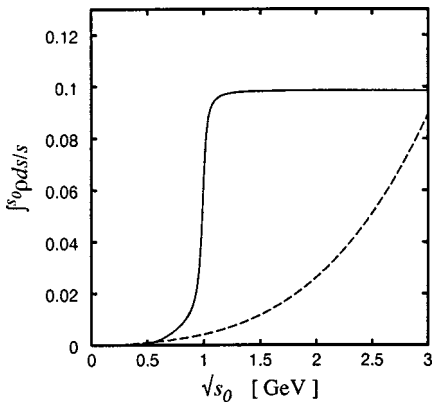

FIG. 4. Physical (solid) and perturbative (dashed) parts of the integral (21) as a function of the upper limit s_0. The LET states that the difference of the two should be equal to the QCD vacuum energy density, $16|\epsilon_{\rm vac}| \simeq 0.053$ GeV^4.

Even though our spectral density cannot be taken seriously in the high mass region beyond ~ 1 GeV, our calculation nevertheless shows the following: Nonperturbative dynamics of QCD generates enhancements in the intermediate mass region in the form of hadronic resonances, which make the physical spectral density consistent with the LET (21). The narrow $f_0(980)$ is more important for the LET than the low mass, broad σ resonance. Therefore, to discuss the influence of heavier resonances [like $f_0(1500)$] we need to perform a coupled-channel analysis including the $\overline{K}K$ and other states. In the rest of this paper we put the matching scale $\sqrt{s_0}=2$ GeV.

IV. THE POTENTIAL BETWEEN COLOR DIPOLES

As a concrete example, let us consider the potential between two J/ψ's at rest. Although the charm quark is perhaps not heavy enough to justify the heavy quark limit, we try to extrapolate our result to J/ψ and discuss its implications.

For the pure Coulombic bound state, $\overline{d_2}=7/36$, and the Bohr radius and Rydberg energy are given by $a_0=4/(3\alpha_s m)$ and $\epsilon_0=(3\alpha_s/4)^2 m=1/(a_0^2 m)$, respectively

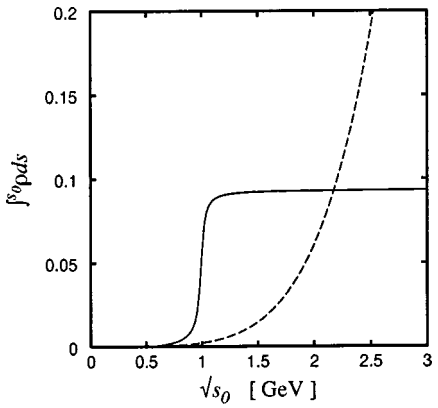

FIG. 5. Integral of the duality relation. The notations are the same as in Fig. 4.

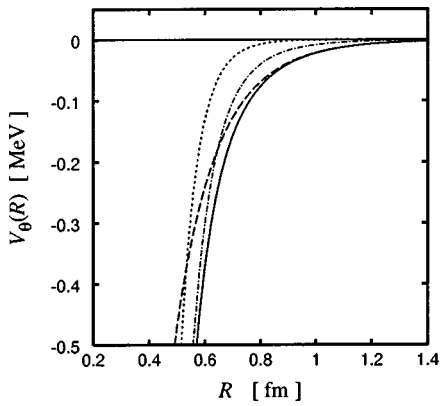

FIG. 6. Potential (14) between two J/ψ's (solid line). Contributions of the spectral densities of $s>s_0$ and $s<s_0$ with $\sqrt{s_0}=2$ GeV, respectively, are plotted in dotted and dashed lines. The perturbative result (17) is shown in dashed-dotted line, for reference.

($\alpha_s=g^2/4\pi$). We have $\alpha_s(J/\psi)=0.87$ and $a_0=0.20$ fm for the J/ψ with the phenomenological inputs, $\epsilon_0=2M_D-M(J/\psi)=642$ MeV and $m=1.5$ GeV. These values show the application to J/ψ will be qualitative at most, because of the large α_s value and because $a_0>s_0^{-1/2}$; the latter means that nonperturbative effects penetrate inside the radius of J/ψ.

In Fig. 6 the resulting potential (14) between two J/ψ's is shown as a solid line. In our model for the spectral density, the potential consists of two components, high q^2 (dotted) and low q^2 (dashed), separated by s_0, which we set (2 GeV)^2. (As in the heavy quark limit, we omit the contribution from the tensor exchange $\theta_{00}^{(G)}$.)

First, we see that the potential at large distances is naturally determined by the spectral density of the low q^2 region. Moreover the total strength of the potential at large distances is enhanced by the nonperturbative spectrum of QCD, compared to the formal perturbative result (17) denoted by the dashed-dotted line. The region where the two components compete is $R\simeq 0.5\sim 0.6$ fm, which is much larger than the scale determined by $s_0^{-1/2}\sim 0.1$ fm. This is in contrast with naive expectation that beyond the scale $s_0^{-1/2}$, the potential should be dominated by the nonperturbative spectral density. The reason for this lies in the large value of $\alpha_s(J/\psi)$, reflecting the fact that charmonium is still far from the heavy quark limit.

In the discussion of the LET (21), we used the running coupling constant, while the coupling constant used here is frozen at the J/ψ scale. This is because in the heavy quark limit, it is natural to renormalize the coupling constant at the scale of quarkonium $g(\epsilon_0)$ with $\epsilon_0 \gg s_0^{1/2}\Lambda_{\rm QCD}$. The matrix element of G^2 should then contain the effects of quantum fluctuations below this energy scale. Within the perturbative approach the renormalization group ensures independence of the final result on the choice of renormalization point, at least in the leading-log approximation, which we used in Eq. (37). In the case of J/ψ, the renormalization scale (chosen at the binding energy ϵ_0) is still *lower* than s_0, and the spectral density (16) with fixed $\alpha_s(J/\psi)$ is significantly larger than

the one with the running coupling constant (37). Again, this reflects the fact that nonperturbative effects penetrate ''inside'' the J/ψ. The most important feature seen in Fig. 6 is the dominance of low-q^2 enhancements in the spectral density in the behavior of potential at large distances.

V. THE SUM RULE FOR THE POTENTIAL

We can derive an interesting sum rule for the strength of the potential. According to Eqs. (14) and (17), we have

$$\int_a^\infty d^3\mathbf{R}[V_\theta(R)-V_\theta^{\rm pt}(R)] = -\left(\overline{d}_2\frac{a_0^2}{\epsilon_0}\right)^2\left(\frac{4\pi^2}{b}\right)^2\int\frac{d\sigma^2}{\sigma^2}[\rho_\theta^{\rm phys}(\sigma^2)-\rho_\theta^{\rm pt}(\sigma^2)] \times\Gamma(2,\sigma a), \tag{38}$$

where a should be chosen to be of the order of the onium radius, and $\Gamma(z,p)=\int_p^\infty dt t^{z-1}e^{-t}$. As we discussed previously, the physical spectral density $\rho_\theta^{\rm phys}(\sigma^2)$ differs from the perturbative one, $\rho_\theta^{\rm pt}(\sigma^2)$, in the region $\sigma^2\lesssim s_0$. In the heavy quark limit, $a\propto 1/(\alpha_s m)$ and $a\sqrt{s_0}\ll 1$. Therefore we can rewrite the sum rule (38) in a more suggestive form:

$$\int_a^\infty d^3\mathbf{R}(V_\theta(R)-V_\theta^{\rm pt}(R)) = -\left(\overline{d}_2\frac{a_0^2}{\epsilon_0}\right)^2\left(\frac{4\pi^2}{b}\right)^2\int\frac{d\sigma^2}{\sigma^2}[\rho_\theta^{\rm phys}(\sigma^2)-\rho_\theta^{\rm pt}(\sigma^2)] = -\left(\overline{d}_2\frac{a_0^2}{\epsilon_0}\right)^2\left(\frac{4\pi^2}{b}\right)^2 16|\epsilon_{\rm vac}|, \tag{39}$$

which relates the overall strength of the interaction between small color dipoles to the energy density of the nonperturbative QCD vacuum.

VI. QUARKONIUM INTERACTIONS WITH PIONS

As another application of our formalism, we evaluate the cross sections of elastic scattering $\pi\Phi\to\pi\Phi$ and of excitation process $\pi\Phi\to\pi\Phi'$; the latter cross section was previously computed in Refs. [20,21]. These cross sections are important for the analyses of quarkonium production in heavy ion collisions [32,33]. The fact that soft pions effectively decouple from heavy quarkonia was previously noted in Ref. [19].

A. Elastic $\pi\Phi$ scattering

Within the OPE formalism (4), it is straightforward to write down the amplitude of pion-quarkonium elastic scattering at small energies: to the leading order in OPE,

$$\mathcal{M}^{kl}(P',p';P,p)=-\overline{d}_2\frac{a_0^2}{\epsilon_0}\left\langle\pi^k(p')\left|\frac{1}{2}g^2\mathbf{E}^{a2}(0)\right|\pi^l(p)\right\rangle. \tag{40}$$

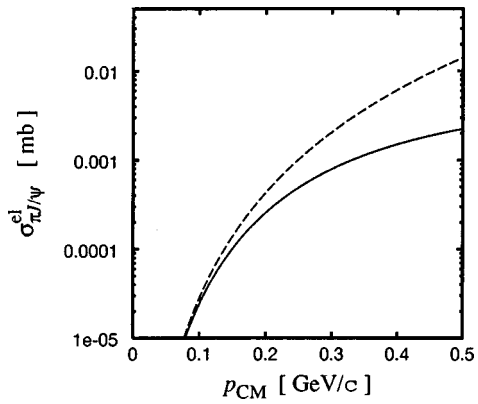

FIG. 7. $\pi J/\psi$ elastic cross section (solid) as a function of the c.m. momentum. Dashed line is the case of $F(t)=1$.

The matrix element $\langle\pi^k|g^2\mathbf{E}^{a2}(0)|\pi^l\rangle$ can be found from $\langle\pi^k\pi^l|g^2\mathbf{E}^{a2}(0)|0\rangle$, Eq. (29), by crossing; the LET (29) tells us that up to α_s and m_π^2 corrections

$$\left\langle\pi^k(p')\left|\frac{1}{2}g^2\mathbf{E}^{a2}(0)\right|\pi^l(p)\right\rangle=\frac{4\pi^2}{b}\langle\pi^k(p')|\theta^\mu_\mu(0)|\pi^l(p)\rangle = \delta^{kl}\frac{4\pi^2}{b}tF(t), \tag{41}$$

where $t=(p-p')^2$, and we have introduced, as before, the pion scalar form factor, $F(t)$. Taking into account the nonrelativistic normalization of the Φ state, we have the expression for the total elastic cross section in the c.m. frame.

$$\sigma(s)=\frac{1}{2p_0\overline{v}_{rel}}\int\frac{d^3P'}{(2\pi)^3}\int\frac{d^3p'}{(2\pi)^3 2p_0'}|\mathcal{M}|^2(2\pi)^4 \times\delta^4(P'+p'-P-p) =\frac{1}{16\pi s}\frac{M^2}{\mathbf{p}^2}\left(\overline{d}_2\frac{a_0^2}{\epsilon_0}\right)^2\left(\frac{4\pi^2}{b}\right)^2\int_0^{4\mathbf{p}^2}d(-t)t^2|F(t)|^2, \tag{42}$$

where $\overline{v}_{rel}=\sqrt{(Pp)^2-M^2m^2}/P^0p^0$ is the relative velocity of the incoming J/ψ and pion, and $\mathbf{p}^2=[s-(M-m_\pi)^2][s-(M+m_\pi)^2]/4s$ is the c.m. momentum.

The result for the elastic $\pi J/\psi$ cross section is shown in Fig. 7. The pion scalar form factor $F(q^2)$ and other parameters are the same as in the previous section. Note that the chiral symmetry requires a strong momentum dependence, t^2 in Eq. (42); therefore at low energies the J/ψ interaction with pions is very weak. Extrapolation of the scalar form factor $F(t)$ to the scattering region, $t<0$, induces additional suppression of the $\pi J/\psi$ interaction. At small energies (see Fig. 7) the cross section is on the order of 0.01 mb, which is much smaller than the geometrical cross section of the J/ψ. For quarkonium production in heavy ion collisions, this implies that the interactions with secondary pions do not contribute to the broadening of the quarkonium transverse momentum spectra.

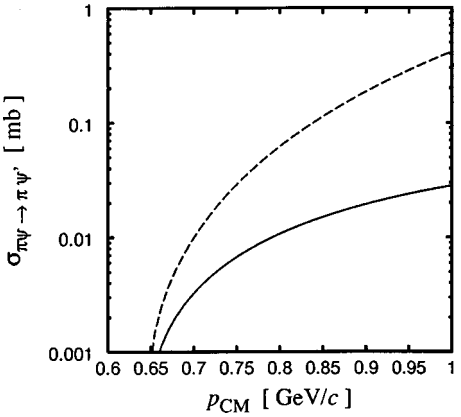

FIG. 8. $\pi J/\psi \to \pi\psi'$ cross section. The notations are the same as in Fig. 7.

B. $\pi\Phi \to \pi\Phi'$ transition amplitude

Our next example is the transition process $\pi\Phi \to \pi\Phi'$. In this case, however, the transferred momentum is on the order of the binding energy, $\Delta = M' - M = O(\epsilon_0)$, which may invalidate our assumption on the factorization between the short and long distances. Fortunately, since the size of quarkonium $a_0 \sim 1/(g^2 m) \ll 1/\epsilon_0 \sim 1/(g^4 m)$ in the heavy quark limit, the double-dipole form

$$\mathcal{M} = \left\langle \phi'\pi \left| \mathrm{tr}\left[\mathbf{r}\cdot\mathbf{E} \frac{1}{H_a + \epsilon + iD^0} \mathbf{r}\cdot\mathbf{E} \right] \right| \phi\pi \right\rangle \qquad (43)$$

is still valid [5]. The structure of this formula is transparent: the initial quarkonium Φ absorbs or emits a gluon, then propagates with the internal energy, $-\epsilon + Q$, and emits or absorbs another gluon to form a color-singlet, excited quarkonium state Φ'; these gluons originate from pions.

To apply our formalism, let us approximate $-iD^0$ in Eq. (43) by the typical value of the gluon momentum, Δ. Within this (rather crude) approximation, the quarkonium part and the pion part can be factorized in the matrix element (43) and the relevant Wilson coefficient, which for this process reads

$$\overline{d}_2' \frac{a_0^2}{\epsilon_0} = \frac{1}{3N} \left\langle \phi' \left| r^i \frac{1}{H_a + \epsilon - \Delta} r^i \right| \phi \right\rangle. \qquad (44)$$

The transition matrix element then reduces to the same form as in the elastic case:

$$\mathcal{M} = \left(\overline{d}_2' \frac{a_0^2}{\epsilon_0} \right) \left(\frac{4\pi^2}{b} \right) tF(t), \qquad (45)$$

and the total transition cross section can be written as

$$\sigma(s) = \frac{1}{16\pi s} \frac{MM'}{\mathbf{p}^2} \left(\overline{d}_2' \frac{a_0^2}{\epsilon_0} \right)^2 \left(\frac{4\pi^2}{b} \right)^2 \int_{t_{\min}}^{t_{\max}} d(-t) t^2 |F(t)|^2. \qquad (46)$$

In Fig. 8 we show the result for the $\pi J/\psi \to \pi\psi'$ cross section, evaluated assuming the $1s$ and $2s$ Coulomb wave functions, and $\Delta = (3/4)\epsilon_0$. It shows that the cross section is on the order of 0.01–0.1 mb.

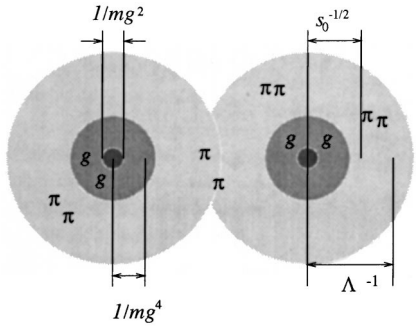

FIG. 9. Schematic picture of the potential between heavy onia.

We also evaluated the partial width of the ψ' due to $\psi' \to \psi\pi\pi$ decay within the same formalism, and obtained $\Gamma = 260$ (70) keV with (without) using the form factor $F(s)$. This should be compared with the experimental value 135 ± 20 keV [44]. We conclude that our calculations, due to the assumption of the heavy quark limit, hold to within a factor of 2 only. Additional confirmation of the s dependence of the matrix element comes from the dipion invariant mass distribution in the $\psi' \to \psi\pi\pi$ decay [45].

VII. SUMMARY AND DISCUSSION

We have shown that at large distances the interaction of small color dipoles becomes totally nonperturbative. This result has a deep physical origin: indeed, one can trace it back to the sum rule (21) for the correlator of the energy-momentum tensor, which reflects the fact that the nonperturbative vacuum of QCD is characterized by nonzero energy density.

For QCD practitioners, ''nonperturbative'' is often a substitute for ''incalculable.'' Nonetheless, in our case, we were able to evaluate explicitly this, nonperturbative, scattering amplitude in a model-independent way. The key ingredients in our approach were (i) the use of spectral representation in the t-channel and (ii) the low-energy theorem arising from the (broken) scale and chiral invariances of QCD.

What are the implications of our results? First, we find that the long-distance interactions of small color dipoles are dominated by pion clouds; the qualitative picture of this interaction is illustrated in Fig. 9. The size occupied by the heavy quark-antiquark pair in the quarkonium (see Fig. 9) is $a_0 \sim 1/(g^2 m)$; the gluon cloud spreads up to the distances on the order of the inverse binding energy $1/\epsilon_0 \sim 1/(g^4 m)$, since the typical momentum K of gluons is $K \sim \epsilon_0$. (This picture of quarkonium structure emerges also from the NRQCD approach of Bodwin, Braaten, and Lepage [46].) The pion cloud begins to dominate at the distances $\sim s_0^{-1/2}$, and spreads up to the distances $(2\mu)^{-1}$, where μ is the pion mass (s_0 is the mass scale at which the nonperturbative effects begin to dominate in the spectral density, see Sec. III). This pion cloud may as well be important in high-energy scattering. One may even speculate (see Bjorken [28]) that

pions are responsible for the so-called ''soft Pomeron,'' even though it is not yet clear how to extend our calculations to high-energy scattering—this would require evaluation of higher orders in the multipole expansion. Nevertheless the possibility that the diffusion of partons toward small k_t makes pionic degrees of freedom important looks very plausible to us. Second, the fact that pions (and therefore light quarks) dominate the long-distance interactions of heavy quark systems is important for lattice QCD simulations. Even though naively one may think that light quarks are relatively unimportant for the studies of heavy quarkonia on the lattice, our findings show that the opposite is true. This suggests that to extract the properties of heavy quarkonia from the lattice QCD one has to use ''unquenched'' theory with light quarks. The importance of pionic degrees of freedom in determination of the mass splittings of heavy quarkonia was investigated in Ref. [22].

Third, we find that both inelastic and elastic $\pi J/\psi$ scattering cross sections are very small, less than 0.1 mb. The smallness of inelastic cross section suggests that pions are very ineffective in dissociating J/ψ's, lending support to the idea to use quarkonia as a signal of deconfinement [32]. The smallness of the elastic cross section explains why the transverse momentum distributions of J/ψ's seem to be unaffected by the final state interactions with secondary pions, whereas much bigger ψ''s, to which our multipole expansion analysis does not apply, can show significantly larger mean transverse momenta.

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APPENDIX A: DERIVATION OF EQS. (14) AND (16)

The Feynman propagator of a scalar field φ of mass σ is defined by

$$\begin{aligned} i\Delta_F(x;\sigma^2) &= i\langle \mathrm{T}\varphi(x)\varphi(0)\rangle \\ &= i\int \frac{d^4k}{(2\pi)^3}\delta(k^2-\sigma^2)\theta(k_0) \\ &\quad\times[e^{-ikx}\theta(x^0)+e^{ikx}\theta(-x^0)] \\ &= \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ikx}}{\sigma^2-k^2-i\epsilon} \\ &= \frac{i}{4\pi^2}\frac{\sigma^2}{\sqrt{(-x^2+i\epsilon)\sigma^2}}K_1[\sqrt{(-x^2+i\epsilon)\sigma^2}], \end{aligned} \tag{A1}$$

where K_1 is the modified Bessel function. The Born amplitude of one-boson exchange with coupling g is

$$\begin{aligned} i\mathcal{M}_B(\mathbf{q}) &= \int d^4x e^{i\mathbf{q}\cdot\mathbf{x}}\langle \mathrm{T} ig\varphi(x) ig\varphi(0)\rangle \\ &= ig^2\int d^4x e^{i\mathbf{q}\cdot\mathbf{x}} i\Delta_F(x;\sigma^2), \end{aligned} \tag{A2}$$

which may be related to an interaction potential by $i\mathcal{M}(\mathbf{q}) = -iV(\mathbf{q})$. Going to Euclidean space, where $x_E^2=\tau^2+\mathbf{x}^2 = \tau^2+R^2$, we can show that

$$\begin{aligned} V(R) &= -g^2\int_{-\infty}^{\infty} dt i\Delta_F(x;\sigma^2) \\ &= -g^2\int_{-\infty}^{\infty} d\tau \frac{1}{4\pi^2}\frac{\sigma^2}{\sqrt{x_E^2\sigma^2}}K_1(\sqrt{x_E^2\sigma^2}) \\ &= -\frac{g^2}{4\pi R}e^{-\sigma R}. \end{aligned} \tag{A3}$$

To the leading in the OPE, the potential between color dipoles (14) is a superposition of this Yukawa potential with the spectral function, $\rho_\theta(\sigma^2)$. In the perturbative calculation, the matrix element of G^2 between the vacuum and the two-gluon state is

$$\begin{aligned} &\langle p_1\varepsilon_1 a, p_2\varepsilon_2 b|G^{\alpha\beta c}G^c_{\alpha\beta}|0\rangle \\ &\quad = 4(-p_1\cdot p_2\varepsilon_1^*\cdot\varepsilon_2^* + p_1\cdot\varepsilon_2^* p_2\cdot\varepsilon_1^*)\delta^{ab}+O(g^2), \end{aligned} \tag{A4}$$

where p_i, ε_i and a are the momentum, polarization and SU(N) color index of the gluon, respectively. Noting that the sum over physical polarizations yields the projection, $\Sigma_{\mathrm{pol}}\varepsilon_\mu\varepsilon_\nu^* = \delta_{ij}-p_ip_j/\mathbf{p}^2$, we have a compact expression,

$$\begin{aligned} \sum_{\mathrm{pol,col}}|\langle p_1\varepsilon_1 a, p_2\varepsilon_2 b|G^2|0\rangle|^2 &= 4^2(N^2-1)2(p_1\cdot p_2)^2 \\ &= 8(N^2-1)q^4, \end{aligned} \tag{A5}$$

which depends only on $q^2=(p_1+p_2)^2$. With the phase space factor of two identical particles,

$$\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2\omega_1}\int\frac{d^3p_2}{(2\pi)^3 2\omega_2}(2\pi)^3\delta^4(p_1+p_2-q) = \frac{1}{32\pi^2}, \tag{A6}$$

we find Eq. (16) as the spectral density of the correlator of $\theta^\mu_\mu = -(bg^2/32\pi^2)G^2$, to the leading in g.

Similarly the spectral density of the two-pion states (28) can be calculated, but with $\langle \pi^k\pi^l|\theta^\mu_\mu|0\rangle = q^2\delta^{kl}$ in the chiral limit $(k,l=1,2,3)$.

APPENDIX B: ALTERNATIVE DERIVATION OF EQ. (17)

To confirm our result, we derive here Eq. (17) applying the functional method of Ref. [5] to the scalar part of the interaction (7) (in Euclidean space):

$$V_\theta(R) = -\left(\overline{d}_2 \frac{a_0^2}{\epsilon_0}\right)^2 \int_{-\infty}^{\infty} d\tau \left\langle 0 \left| \frac{g^2}{8} G^2(x) \frac{g^2}{8} G^2(0) \right| 0 \right\rangle$$
$$= -\left(\overline{d}_2 \frac{a_0^2}{\epsilon_0}\right)^2 \frac{g^4}{32} \int_{-\infty}^{\infty} d\tau \langle 0 | G^a_{\alpha\beta}(x) G^b_{\alpha'\beta'}(0) | 0 \rangle^2, \tag{B1}$$

where all indices are summed over. Using the expression for the two-point function of the gluon in Feynman gauge,

$$\langle A^a_\mu(x) A^b_\nu(y) \rangle = \frac{1}{4\pi^2} \frac{1}{(x-y)^2} \delta_{\mu\nu} \delta^{ab}, \tag{B2}$$

we have

$$\langle G^a_{\mu\nu}(x) G^b_{\mu'\nu'}(0) \rangle = \frac{2}{4\pi^2} \frac{\delta^{ab}}{x^6} \{ \delta_{\nu\nu'} (\delta_{\mu\mu'} x^2 - 4x_\mu x_{\mu'})$$
$$+ \delta_{\mu\mu'} (\delta_{\nu\nu'} x^2 - 4x_\nu x_{\nu'})$$
$$- \delta_{\nu\mu'} (\delta_{\mu\nu'} x^2 - 4x_\mu x_{\nu'})$$
$$- \delta_{\mu\nu'} (\delta_{\nu\mu'} x^2 - 4x_\nu x_{\mu'}) \} + O(g^2), \tag{B3}$$

and

$$\langle G^a_{\mu\nu}(x) G^b_{\mu'\nu'}(0) \rangle^2 = (N^2-1) \frac{24}{\pi^4} \frac{1}{x^8} + O(g^2). \tag{B4}$$

Substituting Eq. (B4) into Eq. (B1), we obtain the leading expression (17) for the potential of the scalar part ($x^2 = \tau^2 + R^2$),

$$V_\theta(R) = -\left(\overline{d}_2 \frac{a_0^2}{\epsilon_0}\right)^2 \frac{g^4}{32} \int_{-\infty}^{\infty} d\tau (N^2-1) \frac{24}{\pi^4} \frac{1}{x^8},$$
$$= -g^4 \left(\overline{d}_2 \frac{a_0^2}{\epsilon_0}\right)^2 \frac{15}{8\pi^3} \frac{1}{R^7}. \tag{B5}$$

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