Matching the heavy particle approach to relativistic theory

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On the simple model of interacting massless and heavy scalar fields it is demonstrated that the technique of heavy baryon chiral perturbation theory reproduces the results of relativistic theory. Explicit calculations are performed for diagrams including two loops. [S0556-2821(99)06621-7]

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I. INTRODUCTION

Heavy baryon chiral perturbation theory (HBCHPT), suggested in Ref. [1], is an important and effective method of calculating of different processes involving electromagnetic and strong interactions. (For review and references see Refs. [2,3].) The authors of Ref. [1] used the ideas of heavy quark effective field theory which allowed them to avoid severe complications appearing in the problem of the relativistic treatment of baryons at low energies, encountered in Ref. [4]. Jenkins and Manohar suggested taking the extremally nonrelativistic limit of the fully relativistic theory and expand in inverse powers of the baryon mass M.

In the heavy baryon approach one integrates out heavy degrees of freedom and expands the resulting nonlocal operators in inverse powers of large mass. In terms of the relativistic perturbation theory of the original field theoretical model (Feynman diagrams) heavy baryon approach corresponds to the expansion of integrands in the loop integrals in powers of $1/M$ with subsequent term by term integration of the resulting series [5]. The noncommutativity of the integration over loop momenta and the expansion in $1/M$ generates a problem of matching of heavy baryon approach to the original relativistic theory. According to Lepage's [6] argument from the uncertainty principle, one should be able to compensate the difference between the results of ''naive'' heavy baryon and relativistic approaches by including additional terms into the Lagrangian of the heavy baryon approach. While the problem of this matching has been analyzed at one loop level [5,7,8], to the best of our knowledge the matching procedure for higher order loops has not been studied.

In the present paper we consider the matching problem on a two loop level on the example of the forward scattering amplitude in a scalar theory. The consideration of the nonzero spin and the nonzero transferred momentum makes calculations more tedious and less transparent, bringing nothing new and essential in the problem considered. In our calculations we use the technique of calculation of loop integrals by dimensional counting, developed in Ref. [9].

We explicitly show that heavy baryon approach reproduces the results of the original relativistic theory at two-loop level.

II. ONE-LOOP ANALYSIS

Let us consider a field theoretical model described by the Lagrangian

$$\mathcal{L}=-\frac{1}{2}\Phi^*\partial_\mu\partial^\mu\Phi-\frac{1}{2}M^2\Phi^*\Phi -\frac{1}{2}\phi\partial_\mu\partial^\mu\phi-g\Phi^*\Phi\phi+L_1, \tag{1}$$

where Φ is a complex scalar field with mass M, ϕ is a neutral massless scalar field, g is a coupling constant, and L_1 contains all counterterms which are necessary to remove divergences (one can include also interactions with the derivatives and/or a larger number of fields and corresponding counterterms).

To avoid complications due to the infrared singularities we work in six-dimensional space-time. We use dimensional regularization and n is a dimension of space-time.

Heavy baryon approach to the processes which involve one heavy particle uses the following expansion of the heavy scalar propagator ($p_\mu=Mv_\mu+k_\mu$, $v^2=1$):

$$\frac{1}{p^2-M^2}=\frac{1}{2Mvk+k^2}=\frac{1}{2M}\frac{1}{vk+k^2/2M} =\frac{1}{2M}\left(\frac{1}{vk}-\frac{1}{2M}\frac{k^2}{(vk)^2}+\right). \tag{2}$$

This expansion corresponds to the following Lagrangian:

$$\mathcal{L}_1=-M\psi^*\left(v\partial+\frac{\partial^2}{2M}\right)\psi, \tag{3}$$

where the second term of $\mathcal{L}_1$ is treated perturbatively. This Lagrangian can be obtained from the free part of $\mathcal{L}$ corresponding to heavy scalar field, defining $\Phi=\exp\{iMvx\}\psi$.

Let us start with the one loop self-energy correction to the scattering process in the original relativistic theory, depicted in Fig. 1(a) (the solid line corresponds to the heavy scalar

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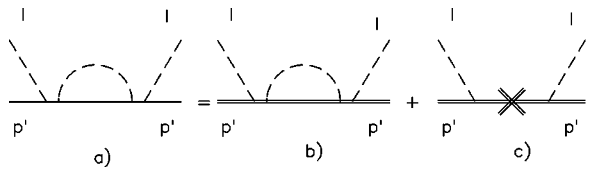

FIG. 1. One-loop corrections to the heavy scalar propagator in the heavy scalar-massless scalar scattering process.

and dashed line corresponds to massless scalar). The expression for this diagram is proportional to the following integral:

$$J_{11}=\int \frac{d^n q}{[q^2+i\epsilon][(p+q)^2-M^2+i\epsilon]}. \tag{4}$$

The straightforward integration yields ($p'=Mv,\ p=Mv+l,\ v^2=1,\ l^2=0,\ p^2=M^2+2Mvl$)

$$\begin{aligned}J_{11}=&\frac{i\pi^{n/2}}{M}\Gamma\left(\frac{n}{2}-1\right)\Gamma(3-n)(-2vl)^{n-3}{}_2F_1\\&\times\left(\frac{n}{2}-1,n-2;n-2;-\frac{2vl}{M}\right)\\&+i\pi^{n/2}(M^2)^{n/2-2}\frac{\Gamma(2-n/2)\Gamma(n-3)}{\Gamma(n-2)}\ {}_2F_1\\&\times\left(1,2-n/2;4-n;-\frac{2vl}{M}\right),\end{aligned} \tag{5}$$

where ${}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$ are the (generalized) hypergeometric functions of z [10]. Heavy baryon expansion for J_{11} is realized by expanding [according to Eq. (2)] the integrand in $1/M$ and integrating the resulting series term by term.

As it was observed in Ref. [9], expanding integrand in powers of some parameter and changing the order of integration and summation one recovers that part of the value of the integral which can be expanded in powers of given parameter with nonzero coefficients.

From the expression (5) we see that the first term can be expanded in powers of $1/M$ with nonzero coefficients, while the second one cannot—it contains $(M^2)^{n/2-2}$. Hence we expect that heavy baryon approach reproduces the first term of the expression (5).

Indeed,

$$\begin{aligned}J_{11}&=\int \frac{d^n q}{[q^2+i\epsilon][(p+q)^2-M^2+i\epsilon]}\\&=\int \frac{d^n q}{[q^2+i\epsilon][(l+q)^2+2Mv(l+q)+i\epsilon]}\\&=\frac{1}{2M}\int \frac{d^n q}{[q^2+i\epsilon][v(l+q)+(l+q)^2/2M+i\epsilon]}\end{aligned}$$

expanding the integrand and changing the order of integration and summation we obtain

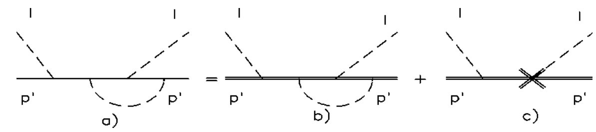

FIG. 2. One-loop corrections to the vertex in the heavy scalar-massless scalar scattering process.

$$\begin{aligned}J_{11HB}=&\frac{1}{2M}\left\{\int \frac{d^n q}{[q^2+i\epsilon][v(l+q)+i\epsilon]}\right.\\&\left.-\frac{1}{2M}\int \frac{d^n q(q+l)^2}{[q^2+i\epsilon][v(l+q)+i\epsilon]^2}\right\}+\cdots\\=&\frac{i\pi^{n/2}}{M}\Gamma\left(\frac{n}{2}-1\right)\Gamma(3-n)(-2vl)^{n-3}+\frac{i\pi^{n/2}}{2M^2}\\&\times\Gamma\left(\frac{n}{2}-1\right)\Gamma(3-n)(n-2)(-2vl)^{n-2}+\cdots.\end{aligned} \tag{6}$$

As was expected, Eq. (6) reproduces the expansion of the first term of expression (5).

The second term of Eq. (5) which cannot be expanded in powers of $1/M$ is analytic in momentum l and hence can be reproduced by introducing additional terms into the Lagrangian of the heavy baryon approach. Free propagators of the heavy scalar particle appearing in the expression for the considered diagram are apparently reproduced by heavy baryon approach. The same is true for all diagrams and we will not include the contributions of the free propagators in our analysis below. Fig. 1(b) schematically represents the first term on the right-hand side of the Eq. (5) [or Eq. (6)] and Fig. 1(c) corresponds to the contributions of compensating terms.

Next one-loop diagram we are considering here is drawn in Fig. 2(a) (one-loop vortex correction to the light scalar-heavy scalar vertex). The result of this diagram is proportional to the following integral:

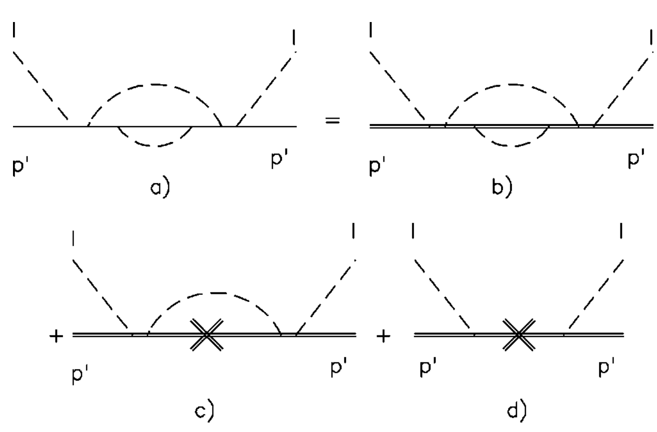

FIG. 3. Two-loop correction to the heavy scalar propagator in the heavy scalar-massless scalar scattering process.

$$J_{12}=\int\frac{d^nq}{[q^2+i\epsilon][q^2+2p'q+i\epsilon][(p+q)^2-M^2+i\epsilon]}$$
$$=-i(M^2)^{n/2-3}\pi^{n/2}\frac{\Gamma(n-4)\Gamma(3-n/2)}{\Gamma(n-3)}\;{}_3F_2\left(1,1,3-n/2;2,5-n;\frac{M^2-p^2}{M^2}\right)-i(M^2)^{n/2-3}\pi^{n/2}\left(\frac{M^2-p^2}{M^2}\right)^{n-4}$$
$$\times\frac{\Gamma(4-n)\Gamma(n/2-1)}{n-3}\;{}_3F_2\left(n/2-1,n-3;n-2;\frac{M^2-p^2}{M^2}\right)$$
$$=\frac{i\pi^{n/2}}{M^2}\Gamma(3-n)\Gamma(n/2-1)(-2vl)^{n-4}-\frac{i\pi^{n/2}}{2M^3}\Gamma(4-n)\Gamma(n/2-1)(-2vl)^{n-3}+\cdots-i(M^2)^{n/2-3}\pi^{n/2}$$
$$\times\frac{\Gamma(n-4)\Gamma(3-n/2)}{\Gamma(n-3)}\;{}_3F_2\left(1,1,3-n/2;2,5-n;\frac{-2vl}{M}\right).\tag{7}$$

On the other hand heavy baryon approach leads to

$$J_{12HB}=\frac{1}{4M^2}\int\frac{d^nq}{[q^2+i\epsilon][v(l+q)+i\epsilon][vq+i\epsilon]}$$
$$-\frac{1}{8M^3}\int\frac{d^nq(q+l)^2}{[q^2+i\epsilon][v(l+q)+i\epsilon]^2[vq+i\epsilon]}+\cdots$$
$$=\frac{i\pi^{n/2}}{M^2}\Gamma\left(\frac{n}{2}-1\right)\Gamma(3-n)(-2vl)^{n-4}$$
$$-\frac{i\pi^{n/2}}{2M^3}\Gamma\left(\frac{n}{2}-1\right)\Gamma(4-n)(-2vl)^{n-3}+\cdots.\tag{8}$$

The comparison of Eqs. (7) and (8) shows that heavy baryon approach reproduces that part of relativistic answer which can be expanded in inverse powers of M. The second part is analytic in l and can be reproduced by adding appropriate terms into the Lagrangian of the heavy baryon approach. Figure 2(b) and 2(c) correspond to Eq. (8) and the contributions of compensating terms, respectively. The analysis of the rest of one-loop diagrams lead to the same result: heavy baryon approach reproduces those parts of diagrams which are nonanalytic in the momenta and the remaining parts, analytic in momenta can be reproduced by adding terms into the effective Lagrangian of the heavy baryon approach.

III. TWO-LOOP ANALYSIS

Two-loop diagrams have more complicated structure. Let us consider two-loop correction to the propagator of the heavy scalar in original relativistic theory depicted in Fig. 3(a). The result of this diagram is proportional to the following integral:

$$J_{21}=\int\frac{d^nq_1d^nq_2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][(p+q_1)^2-M^2+i\epsilon]^2[(p+q_1+q_2)^2-M^2+i\epsilon]}.\tag{9}$$

From the method of dimensional counting [9] it follows that

$$J_{21}=\delta^{2n-7}(p^2)^{n-5}\sum_{k=0}^{\infty}f_{1k}\delta^k+\delta^{n-4}(p^2)^{n-5}\sum_{k=0}^{\infty}f_{2k}\delta^k$$
$$+(p^2)^{n-5}\sum_{k=0}^{\infty}f_{3k}\delta^k,\tag{10}$$

where

$$\delta=\frac{M^2-p^2}{p^2}=\frac{-2vl}{M}\frac{1}{1+2vl/M}\tag{11}$$

and the coefficients f_{ik} are determined by the original integral (9) [9]. Substituting Eq. (11) into Eq. (10) we obtain

$$J_{21}=(-2vl)^{2n-7}M^{-3}\sum_{k=0}^{\infty}D_{1k}\left(\frac{2vl}{M}\right)^k$$
$$+(-2vl)^{n-4}M^{n-6}\sum_{k=0}^{\infty}D_{2k}\left(\frac{2vl}{M}\right)^k$$
$$+M^{2n-10}\sum_{k=0}^{\infty}D_{3k}\left(\frac{2vl}{M}\right)^k,\tag{12}$$

where D_{ik} do not depend on M, l, or v.

Actual calculations of J_{21} can be performed using the methods of Ref. [9] as follows. Let us rewrite

$$J_{21}=\int \frac{d^n q_1 d^n q_2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][2Mv(l+q_1)+(l+q_1)^2+i\epsilon]^2[2Mv(l+q_1+q_2)+(l+q_1+q_2)^2+i\epsilon]}. \tag{13}$$

First we expand the integrand in inverse powers of M, change the order of integration and summation and obtain

$$\begin{aligned}J_{21}^{(1)}=&\frac{1}{4M^3}\int \frac{d^n q_1 d^n q_2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][v(l+q_1)+i\epsilon]^2[v(l+q_1+q_2)+i\epsilon]}\\&-\frac{1}{4M^4}\left(\int \frac{d^n q_1 d^n q_2 (q_1+l)^2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][v(l+q_1)+i\epsilon]^3[v(l+q_1+q_2)+i\epsilon]}\right.\\&\left.+\frac{1}{2}\int \frac{d^n q_1 d^n q_2 (q_1+q_2+l)^2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][v(l+q_1)+i\epsilon]^2[v(l+q_1+q_2)+i\epsilon]^2}\right)+\cdots.\end{aligned} \tag{14}$$

Second we rescale $q_2\to q_2 M$, extract a noninteger power of mass, expand the integrand in inverse powers and change the order of integration and summation. The result is

$$\begin{aligned}J_{21}^{(2)}=&\frac{M^{n-6}}{4}\int \frac{d^n q_1 d^n q_2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][v(l+q_1)+i\epsilon]^2[q_2^2+2vq_2+i\epsilon]}\\&-\frac{M^{n-7}}{4}\left(\int \frac{d^n q_1 d^n q_2 (q_1+l)^2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][v(l+q_1)+i\epsilon]^3[q_2^2+2vq_2+i\epsilon]}\right.\\&\left.+\int \frac{d^n q_1 d^n q_2 \{v(q_1+l)-2q_2(q_1+l)\}}{[q_1^2+i\epsilon][q_2^2+i\epsilon][v(l+q_1)+i\epsilon]^2[q_2^2+2vq_2+i\epsilon]^2}\right)+\cdots,\end{aligned} \tag{15}$$

and third we rescale $q_1\to Mq_1$, $q_2\to Mq_2$, extract noninteger power of the mass, change the order of integration and summation and obtain

$$\begin{aligned}J_{21}^{(3)}=&M^{2n-10}\int \frac{d^n q_1 d^n q_2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][q_1^2+2vq_1+i\epsilon]^2[(q_1+q_2)^2+2v(q_1+q_2)+i\epsilon]}\\&-M^{2n-11}\left(\int \frac{d^n q_1 d^n q_2 (4lq_1+4vl)}{[q_1^2+i\epsilon][q_2^2+i\epsilon][q_1^2+2vq_1+i\epsilon]^3[(q_1+q_2)^2+2v(q_1+q_2)+i\epsilon]}\right.\\&\left.+\int \frac{d^n q_1 d^n q_2 \{2l(q_1+q_2)+2vl\}}{[q_1^2+i\epsilon][q_2^2+i\epsilon][q_1^2+2vq_1+i\epsilon]^2[(q_1+q_2)^2+2v(q_1+q_2)+i\epsilon]^2}\right)+\cdots.\end{aligned} \tag{16}$$

Integral J_{21} is nothing but a sum of $J_{21}^{(1)}$, $J_{21}^{(2)}$, and $J_{21}^{(3)}$ [9].

Evidently, $J_{21}^{(1)}$ is expandable in inverse powers of M and hence heavy baryon approach reproduces this part of J_{21}. $J_{21}^{(3)}$ cannot be reproduced (because it contains M^{2n-10}) but it is analytic in l and hence it can be taken into account by adding compensating terms into the Lagrangian of the heavy baryon approach. It is $J_{21}^{(2)}$, corresponding to the second term in Eq. (12) which is nontrivial and might cause problems: terms which are not expandable in powers of M and have nonanalytic dependence on l can appear.

This feature does not appear on a one loop level. Let us consider this problem in details.

The above given representation for J_{21} can be obtained also as follows: One loop subintegral over q_2 can be represented as a sum of two parts: the first part is a result of expanding integrand of this subintegral in inverse powers of M and changing the order of integration and summation. The second part is obtained by rescaling $q_2\to q_2 M$, extracting noninteger factor of M, expanding the integrand in powers of $1/M$ and changing the order of integration and summation:

$$J_{21}=\int \frac{d^n q_1}{[q_1^2+i\epsilon][2Mv(l+q_1)+(l+q_1)^2+i\epsilon]^2}\times\{F_1(M,l+q_1)+M^{n-4}F_2(M,l+q_1)\}, \tag{17}$$

where F_1 and F_2 represent series in $1/M$. As we concluded from one-loop analysis heavy baryon approach reproduces

F_1 at one loop level and $M^{n-4}F_2$ is reproduced by adding compensating terms into the Lagrangian of the heavy baryon approach.

Now, expanding the denominator appearing in Eq. (17)

$$\frac{1}{[q_1^2+i\epsilon][2Mv(l+q_1)+(l+q_1)^2+i\epsilon]^2}$$

in inverse powers of M and changing the order of integration and summation in Eq. (17) we get the result which is equal to $J_{21}^{(1)}+J_{21}^{(2)}$. This makes clear that heavy baryon approach reproduces $J_{21}^{(2)}$ which was addressed as a possible source of the trouble. As for $J_{21}^{(3)}$ it is analytic in l and can be reproduced by compensating terms. Note that $J_{21}^{(3)}$ is obtained from Eq. (17) by rescaling $q_1 \to q_1 M$: one extracts noninteger power of M, expands the integrand in powers of $1/M$ and changes the order of integration and summation. Doing so one gets $M^{n-6}f_1+M^{2n-10}f_2$, where $M^{2n-10}f_2$ is equal to $J_{21}^{(3)}$ and f_1 turns out to be a sum of trivial terms (zeros). Figures 3(b), 3(c), and 3(d) correspond to $J_{21}^{(1)}$, $J_{21}^{(2)}$, and $J_{21}^{(3)}$, respectively.

Next we consider two-loop correction to the heavy scalar propagator in original relativistic theory depicted in Fig. 4. The result of this diagram is proportional to the following integral:

$$J_{22}=\int \frac{d^nq_1d^nq_2}{[q_1^2+i\epsilon][q_2^2+i\epsilon][(p+q_1)^2-M^2+i\epsilon][(p+q_1+q_2)^2-M^2+i\epsilon][(p+q_2)^2-M^2+i\epsilon]}. \tag{18}$$

For later use before analyzing Eq. (18) let us consider J_v—an off-mass shell integral of the one loop correction to the vertex:

$$J_v=\int \frac{d^nq}{[q^2+i\epsilon][(p+q)^2-M^2+i\epsilon][(k'+q)^2-M^2+i\epsilon]}, \tag{19}$$

where $p=Mv+l$ and $k'=Mv+l'$:

$$J_v=\int \frac{d^nq}{[2Mv(l+q)+(l+q)^2+i\epsilon][2Mv(l'+q)+(l'+q)^2+i\epsilon]}=J_v^1(l,l')+J_v^2(l,l'), \tag{20}$$

where J_v^1 is obtained by expanding the integrand of Eq. (20) in inverse powers of M and changing the order of integration and summation:

$$J_v^1(l,l')=\frac{1}{4M^2}\int \frac{d^nq}{[q^2+i\epsilon][vl+vq+i\epsilon][vl'+vq+i\epsilon]} - \frac{1}{8M^3}\left(\int \frac{d^nq}{[q^2+i\epsilon][vl+vq+i\epsilon]^2[vl'+vq+i\epsilon]}+\int \frac{d^nq}{[q^2+i\epsilon][vl+vq+i\epsilon][vl'+vq+i\epsilon]^2}\right)+\cdots. \tag{21}$$

J_v^2 is obtained by rescaling $q \to qM$, extracting noninteger power of M, expanding the integrand in negative powers of the mass and changing the order of integration and summation:

$$J_v^2(l,l')=M^{n-6}\int \frac{d^nq}{[q^2+i\epsilon][2vq+q^2+i\epsilon]^2} - M^{n-7}\left(\int \frac{d^nq2(vl+lq)}{[q^2+i\epsilon][2vq+q^2+i\epsilon]^3} + \int \frac{d^nq2(vl'+l'q)}{[q^2+i\epsilon][2vq+q^2+i\epsilon]^3}\right)+\cdots. \tag{22}$$

The heavy baryon approach reproduces J_v^1. J_v^2 is analytic in l and l' and can be reproduced by adding compensating terms into the Lagrangian of the heavy baryon approach.

Applying the method of dimensional counting [9] to J_{22} we obtain the following expression:

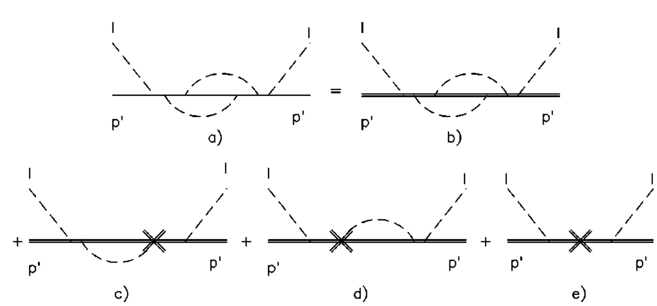

FIG. 4. Two-loop correction to the heavy scalar propagator in the heavy scalar-massless scalar scattering process.

$$
\begin{aligned}
J_{22}=&(-2vl)^{2n-7}M^{-3}\sum_{k=0}^{\infty}A_{1k}\left(\frac{2vl}{M}\right)^k\\
&+(-2vl)^{n-3}M^{n-7}\sum_{k=0}^{\infty}A_{2k}\left(\frac{2vl}{M}\right)^k\\
&+(-2vl)^{n-3}M^{n-7}\sum_{k=0}^{\infty}A_{3k}\left(\frac{2vl}{M}\right)^k\\
&+M^{2n-10}\sum_{k=0}^{\infty}A_{4k}\left(\frac{2vl}{M}\right)^k\\
=&J_{22}^{(1)}+J_{22}^{(2)}+J_{22}^{(3)}+J_{22}^{(4)},
\end{aligned}
\tag{23}
$$

where A_{ik} do not depend on M, l, or v. In Eq. (23) $J_{22}^{(1)}$ is the result of expanding integrand in inverse powers of M and integrating the series, $J_{22}^{(2)}$ and $J_{22}^{(3)}$ are obtained by rescaling $q_1\to q_1M$ and $q_2\to q_2M$ correspondingly with subsequent expansion of the integrand and change of the order of integration and summation, and $J_{22}^{(4)}$ is the result of the simultaneous rescaling $q_1\to q_1M$ and $q_2\to q_2M$ and integration of the resulting expansion.

The heavy baryon approach reproduces straightforwardly $J_{22}^{(1)}$; $J_{22}^{(4)}$ is analytic in momenta and hence can be reproduced by compensating terms in the Lagrangian of the heavy baryon approach. The terms $J_{22}^{(2)}$ and $J_{22}^{(3)}$ are not expandable in $1/M$ and they are not analytic in momenta. In a full analogy with the previous analysis for J_{21}^2 these terms are reproduced by taking into account the contributions of the compensating terms which have to be introduced into the Lagrangian of the heavy baryon approach in order to reproduce the expression for the one loop subdiagrams of this two-loop diagram.

To see that this is actually the case let us represent J_{22} in the following way:

$$
\begin{aligned}
J_{22}=&\int\frac{d^nq_1}{[q_1^2+i\epsilon][(p+q_1)^2-M^2+i\epsilon]}\int\frac{d^nq_2}{[q_2^2+i\epsilon][(p+q_1+q_2)^2-M^2+i\epsilon][(p+q_2)^2-M^2+i\epsilon]}\\
=&\int\frac{d^nq_1}{[q_1^2+i\epsilon][2Mv(l+q_1)+(l+q_1)^2+i\epsilon]}\{J_v^1(l+q_1,l)+J_v^2(l+q_1,l)\}.
\end{aligned}
\tag{24}
$$

Note that $J_v^2(l+q_1,l)$ corresponds to compensating terms included into the Lagrangian of the heavy baryon approach.

Expanding denominator

$$
\frac{1}{[q_1^2+i\epsilon][2Mv(l+q_1)+(l+q_1)^2+i\epsilon]}
\tag{25}
$$

in $1/M$ and changing the order of integration and summation we reproduce $J_{22}^{(1)}+J_{22}^{(2)}$. So, heavy baryon approach reproduces these two terms (here $J_{22}^{(2)}$ occurred because we included contributions of compensating terms corresponding to one-loop subdiagrams). $J_{22}^{(3)}$ and $J_{22}^{(4)}$ are reproduced by rescaling $q_1\to q_1M$, extracting noninteger factors of M, expanding integrand in $1/M$, and changing the order of integration and summation.

$J_{22}^{(4)}$ is analytic in momenta and hence can be reproduced by compensating terms included into the Lagrangian of the heavy baryon approach. As for $J_{22}^{(3)}$ it is equal to $J_{22}^{(2)}$ and comes from one-loop compensating terms as well. This fact should be clear from Fig. 4 where Figs. 4(b), 4(c), 4(d), and 4(e) correspond to $J_{22}^{(1)}$, $J_{22}^{(2)}$, $J_{22}^{(3)}$, and $J_{22}^{(4)}$, respectively.

Analogous results are obtained for all the remaining two-loop diagrams. From the above analysis it follows that heavy baryon approach reproduces the results of the original relativistic theory at a two-loop order.

IV. CONCLUSIONS

In this work we have addressed the problem of matching of heavy baryon approach to the original relativistic theory. The heavy baryon approach corresponds to the expansion of the integrand in inverse powers of the large mass with subsequent change of the order of integration and summation. As this two procedures are not commutative, the difference has to be compensated by adding terms into the Lagrangian of the heavy baryon approach. As the addressed problem does not actually depend on the details of the given model we considered a simple example of heavy and massless interacting scalar fields. Using the method of calculation of loop integrals by dimensional counting outlined in Ref. [9] we analyzed one and two loop diagrams and demonstrated how the difference between relativistic and heavy baryon calculations is compensated by adding terms to the Lagrangian of the heavy baryon approach. At two-loop level the difference can be compensated only after one includes the contributions of compensating terms for one loop subdiagrams. While we included only selected diagrams in this paper, the very same conclusions are valid for one and two loop diagrams which were not included in here. We believe that the iterative procedure of considering contributions of compensating terms for one-loop diagrams in two-loop calculations which is crucial to resolve the matching problem leads to analogous results for higher loops.

While we considered a simple model of scalar fields the problems of interchange of integration and expansion in inverse powers of heavy particle mass are the same for more realistic models with included fermionic and vector fields. In heavy baryon chiral perturbation theory the compensating terms with similar structure are actually summed up and included as redefinitions of already existing coupling constants. This redefinition is crucial, it actually leads to the consistent power counting of the heavy baryon chiral perturbation theory. In heavy baryon approach the coupling constants which correspond to redefined relativistic coupling constants are introduced as initially free parameters which are to be fixed from experimental data. Working up to some given order in heavy baryon approach one actually resums low order contributions of an infinite number of relativistic high order loop diagrams.

It was shown in Ref. [2] that, within HBCHPT, an infinite number of internal line insertions must be summed to describe the scalar form-factor of the nucleon near threshold. As we demonstrated above the heavy baryon expansion reproduces the expansion of the relativistic result. This conclusion is formally still correct for the scalar form-factor of the nucleon, but the problem is that the expansion of the relativistic result is not convergent near threshold. This problem has been successfully resolved recently by Becher and Leutwyler using ''infrared regularization'' [11].

As was demonstrated above relativistic diagrams contain parts which can not be altered by adding local terms into the Lagrangian. These parts are directly reproduced by heavy baryon approach and they respect power counting. Other parts which are responsible for violation of the power counting in relativistic theory can be changed by adding counterterms. Hence it should be more or less clear that the problems of the relativistic approach, in particular that multiloop diagrams contribute into low order calculations encountered in Ref. [4], can be solved within relativistic approach using appropriately chosen normalization condition. Hence one could from the very beginning work within original relativistic approach and never encounter the problems of near threshold behavior of the scalar form factor of the nucleon. These problems will be addressed in a future paper.

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