Hadronic B decays: Supersymmetric enhancement and a simple spectator model

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Two aspects of hadronic B decays are investigated. First, the supersymmetric enhancement of hadronic b decays $b \rightarrow qg \rightarrow qq'\bar{q}'$, $q \in \{d,s\}$, by gluino penguin processes is studied through their effect on the Wilson coefficients of the effective Hamiltonian. The gluino penguin process is dominated by the magnetic dipole transition which is strongly magnified relative to the electric monopole driven standard model gluon penguin process by the renormalization-induced QCD corrections, resulting in quark decay rates for pure penguin processes which, at scales $O(m_b)$, can exceed the standard model rates. The CP asymmetries are, however, relatively unaffected. Second, hadronization of the final state quarks is studied through a simple phase space spectator model. We consider two extreme models for color flow during meson formation: one in which color flow is ignored and one of color suppression in which low mass meson formation occurs only for color singlet quark-antiquark pairs. We find that processes in which the spectator antiquark $\bar{q}_s$ combines with q' are relatively insensitive to the color flow model whereas processes in which $\bar{q}_s$ combines with q are very sensitive to color suppression. [S0556-2821(99)04423-9]

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I. INTRODUCTION

These are exciting times for B physics. Recently the CLEO Collaboration [1] reported the first measurements of a number of exclusive charmless hadronic B decays which provide conclusive evidence for hadronic penguin processes, and the next decade will see [2] intensive investigation of the B-meson system at the Cornell Electron Storage Ring (CESR), the Fermilab Tevatron, DESY ep Collider HERA, the SLAC and KEK B factories, and at the CERN Large Hadron Collider (LHC). In particular, measurements of rare flavor-changing B decays [3] will provide windows for the discovery of indirect effects of new physics beyond the standard model (SM). For example, data on CP asymmetries should indicate whether the Cabibbo-Kobayashi-Maskawa (CKM) CP violation mechanism of the SM is correct or whether new sources of CP violation are needed. The most favored candidate theory for this new physics is supersymmetry (SUSY) for which rare B decays offer possible insight because [4,5] the measured observables involve SM and SUSY processes occurring at the same order of perturbation theory. In particular, the $b \rightarrow s$ transition provides an opportunity to study CP violation from non-standard phases [6]. Probing for SUSY in $b \rightarrow s\gamma$ decays has been examined by many authors [5,7–12] and there is also significant current interest in the $b \rightarrow sg$ penguin decay [13–16] for which it has been argued [17] that enhancement for on-shell gluons is needed from non-SM physics to explain the CLEO measurement [1] of a large branching ratio for the inclusive process $B \rightarrow \eta' X_s$ and the η'-g-g gluon anomaly.

The most predictive of the SUSY models is the (constrained) minimally supersymmetric standard model (MSSM) [18,19] based on spontaneously broken $N=1$ supergravity with flat Kähler metrics [20], universal explicit soft-SUSY breaking terms at the scale $M_{\mathrm{MSSM}} \sim M_U$ and spontaneous breaking of the $SU(2) \otimes U(1)$ symmetry driven by radiative corrections. The unification scale M_U boundary conditions are a universal scalar mass m_0, universal gaugino mass $m_{1/2}$ and a universal soft SUSY-breaking trilinear scalar coupling A. After minimization of the full one-loop Higgs effective potential the MSSM is represented by a four-dimensional parameter space $\{m_0, m_{1/2}, A, \tan\beta\}$, where $\tan\beta \equiv v_2/v_1$ is the ratio of the vacuum expectation values of the two Higgs fields, together with the sign of the coupling μ between the two Higgs fields.

In an earlier paper [21] we examined the SM and MSSM predictions at the weak scale quark level for the penguin mediated decays $b \rightarrow qq'\bar{q}'$, $q \in \{d,s\}$. For the SM we presented the relative contributions of the internal u, c and t quarks to the gluon penguin processes and stressed the invalidity of the widespread assumption that the process is dominated by the t quark [22]. We also considered the relative magnitudes of the various form factors and the role of the strong and weak phases [23,24] and found, for example, that the CP violating phases for the $b \rightarrow dg$ and $b \rightarrow sg$ electric form factors, which dominate the decay amplitude, have no simple relationship with any angle of the unitarity triangle. For the MSSM we explored the allowed regions of the parameter space to locate those regions which gave the largest modifications to the SM results and found, in contrast to the SM, that the magnetic amplitude dominates the electric amplitude and that there are large regions of the MSSM parameter space for which the magnetic amplitude is greater than that of the SM. For regions of high $\tan\beta$ ($=48$) and low $(m_0, m_{1/2})$ ranging from (225,150) to (275,225) the SUSY enhancement of the gluon-mediated exclusive hadronic b decays can be at the several percent level and the SUSY penguin processes dominate the SUSY box processes. Similar

effects of the large $\tan \beta$ enhancement occur for the photon magnetic dipole operator in $b \to s \gamma$ and there is significant current interest in its implications for $B \to X_s \gamma$ and $B \to X_s \, l^+ l^-$ $(l = e, \mu, \tau)$ [5,11,12].

Calculations of weak decays of hadrons within the framework of the effective Hamiltonian involve [4,25] (i) the computation of the quark level decays $b \to q q' \bar{q}'$ at the electroweak scale M_W, (ii) determination of the Wilson coefficients $C_k(M_W)$ through matching of this full theory onto the five-quark low energy effective theory in which the $W^\pm$, t and all SUSY particles heavier than M_W are integrated out, (iii) evolution of the Wilson coefficients down to the low energy scale $\mu \sim O(m_b)$ using renormalization group improved perturbation theory [26], thereby incorporating the important short distance QCD corrections, and (iv) the calculation of hadronic matrix elements for the hadronization of the final-state quarks into particular final states, typically evaluated using the factorization assumption [27].

In this earlier paper [21], the QCD corrections arising from renormalization of the short distance results down from the electroweak scale to the scale m_b were not included. It was argued that these effects were not likely to alter the finding that the magnetic amplitude is dominant in the MSSM as the QCD induced mixing effects [26] produce an enhancement of the magnetic dipole operators in the $\Delta B = 1$ effective Hamiltonian relative to the current-current penguin operators associated with the electric monopole amplitude. Furthermore, as noted by Gérard and Hou [24], use of the q^2-dependent SM form factor $F_1^L(q^2)$ already incorporates the dominant part of the QCD corrections for the current-current penguin operators and, therefore, that the main effects of QCD corrections will be the renormalization of the strong coupling constant from $\alpha_s(M_W)$ to $\alpha_s(m_b)$. This would have the effect of increasing the penguin decay rates of the SM by the factor η^2, where $\eta \equiv \alpha_s(m_b)/\alpha_s(M_W) \approx 1.82$, and also increasing the MSSM penguin amplitudes relative to those of the SM.

In the present paper we address each of the stages of the calculation of B decay rates. In Sec. II we incorporate the MSSM penguin processes into the effective Hamiltonian and fully implement the renormalization-induced QCD corrections. We confirm that the MSSM processes are enhanced relative to those of the SM by these QCD corrections and that the dominant MSSM processes are indeed the magnetic dipole interactions. In Sec. III we compare the quark-level decay rates calculated from the QCD corrected effective Hamiltonian with $\eta^2 \Gamma^{(0)}$, where $\Gamma^{(0)}$ are our weak scale decay rates [21] calculated using q^2-dependent form factors, and find that the Gérard-Hou conjecture is generally valid to within 10% for the SM but greatly underestimates the decay rates for the SM+MSSM. The hadronization of the quark level processes is studied in Sec. IV where, as an alternative to the widely used factorization models, we adopt a simple spectator model. The results obtained from this spectator model are given in Sec. V. Section VI presents our discussions and conclusions. The input numerical values used in our calculations for the CKM parameters and quark masses are given in an appendix.

II. GLUINO PENGUIN PROCESSES AND THE EFFECTIVE HAMILTONIAN

With the inclusion of gluino-mediated penguin processes from SUSY, the total QCD penguin amplitude to lowest order in α_s for the decay process $b \to q \, g \to q \, q' \bar{q}'$ is

$$M^{\text{Peng}} = -\frac{i g_s^2}{4 \pi^2} [\bar{u}_q(p_q) T^a \hat{\gamma}_\mu u_b(p_b)]$$

$$\times [\bar{u}_{q'}(p_{q'}) \gamma^\mu T^a v_{\bar{q}'}(p_{\bar{q}'})] \tag{1}$$

where

$$\hat{\gamma}_\mu \equiv \gamma_\mu [F_1^L(q^2) P_L + F_1^R(q^2) P_R]$$

$$+ \frac{i \sigma_{\mu\nu} q^\nu}{q^2} [F_2^L(q^2) P_L + F_2^R(q^2) P_R]. \tag{2}$$

Here F_1 and F_2 are the electric (monopole) and magnetic (dipole) form factors, $q = p_b - p_q$ is the gluon momentum, $P_{L(R)} \equiv (1 \mp \gamma_5)/2$ are the chirality projection operators and T^a $(a = 1, \ldots, 8)$ are the $SU(3)_c$ generators normalized to $Tr(T^a T^b) = \frac{1}{2} \delta^{ab}$. In writing Eq. (1) we have neglected the smaller MSSM $b \to qg$ penguin processes mediated [7] by charged Higgs boson, chargino and neutralino exchanges. The gluino penguin amplitude is enhanced relative to that for these processes by both the factor α_s/α_W and the additional $\tilde{g}$-$\tilde{g}$-g coupling with its large color factor.

For the SM, q^2 is not small compared to m_u^2 and m_c^2 and we must retain the q^2 dependence of the u and c contributions to $F_1^L(q^2)$. In the limit $m_q = 0$ we then have [21]

$$F_1^L(q^2) = \frac{G_F}{\sqrt{2}} \left[\sum_{i=u,c} V_{iq}^* V_{ib} f_1(x_i, q^2) + V_{tq}^* V_{tb} f_1(x_t) \right],$$

$$F_1^R(0) = 0, \tag{3}$$

$$F_2^L(0) = 0,$$

$$F_2^R(0) = \frac{G_F}{\sqrt{2}} m_b \sum_{i=u,c,t} V_{iq}^* V_{ib} f_2(x_i) \tag{4}$$

where V is the CKM matrix and $x_i \equiv m_i^2/M_W^2$. Explicit expressions for $f_1(x_i, q^2)$ are given in [21] but are not needed in the present study (see later). The $q^2 = 0$ functions are [28,29]

$$f_1(x) = \frac{1}{12(1-x)^4} [18x - 29x^2 + 10x^3 + x^4$$

$$- (8 - 32x + 18x^2) \ln x], \tag{5}$$

$$f_2(x) = \frac{-x}{4(1-x)^4} [2 + 3x - 6x^2 + x^3 + 6x \ln x]. \tag{6}$$

For small x_i, $f_2(x_i) \approx -\frac{1}{2}x_i$ whereas $f_1(x_i) \approx -\frac{2}{3}\ln x_i$.

For the MSSM form factors we have $q^2 \ll m_{\tilde{d}_j}^2$, where $m_{\tilde{d}_j}$ is the mass of the loop $\tilde{d}_j$ squark, and we can use the $q^2 = 0$ form factors [21,30,31]

$$F_1^L(0) = \sum_j \Lambda_{LLj}^{bq}[C_2(G)A(\tilde{x}_j) + C_2(R)B(\tilde{x}_j)], \quad (7)$$

$$F_1^R(0) = \sum_j \Lambda_{RRj}^{bq}[C_2(G)A(\tilde{x}_j) + C_2(R)B(\tilde{x}_j)], \quad (8)$$

$$F_2^L(0) = \sum_j \{[m_q\Lambda_{LLj}^{bq} + m_b\Lambda_{RRj}^{bq}][C_2(G)C(\tilde{x}_j)$$
$$- C_2(R)D(\tilde{x}_j)] + m_{\tilde{g}}\Lambda_{LRj}^{bq}[C_2(G)E(\tilde{x}_j)$$
$$- 4C_2(R)C(\tilde{x}_j)]\}, \quad (9)$$

$$F_2^R(0) = \sum_j \{[m_b\Lambda_{LLj}^{bq} + m_q\Lambda_{RRj}^{bq}][C_2(G)C(\tilde{x}_j)$$
$$- C_2(R)D(\tilde{x}_j)] + m_{\tilde{g}}\Lambda_{RLj}^{bq}[C_2(G)E(\tilde{x}_j)$$
$$- 4C_2(R)C(\tilde{x}_j)]\} \quad (10)$$

where $m_{\tilde{g}}$ is the gluino mass and the coefficient

$$\Lambda_{ABj}^{bq} \equiv -\frac{g_s^2}{4m_{\tilde{g}}^2} V_{\tilde{d}B}^{jq*} V_{\tilde{d}A}^{jb} \quad (11)$$

describes the rotation from the down-diagonal interaction states to the $\tilde{d}$ mass eigenstates at the $d-\tilde{d}-\tilde{g}$ vertices. Here (A,B) are chirality indices, $C_2(G) = 3$ and $C_2(R) = \sum_a T^a T^a = 4/3$ are $SU(3)$ Casimir invariants and $j = 1,\ldots,6$ labels the d squark mass eigenstates and $\tilde{x}_j \equiv m_{\tilde{d}_{L_j}}^2/m_{\tilde{g}}^2$. The matrices $V_{\tilde{d}L}$ and $V_{\tilde{d}R}$ are obtained from the (6×6) matrix $V_{\tilde{d}} = (V_{\tilde{d}L}, V_{\tilde{d}R})^T$ which diagonalizes the $\tilde{d}$ mass^2 matrix. The MSSM functions are

$$A(x) = \frac{1}{6(1-x)^4}[3 - 9x + 9x^2 - 3x^3 + (1 - 3x^2 + 2x^3)\ln x], \quad (12)$$

$$B(x) = \frac{-1}{18(1-x)^4}[11 - 18x + 9x^2 - 2x^3 + 6\ln x], \quad (13)$$

$$C(x) = \frac{-1}{4(1-x)^3}[1 - x^2 + 2x\ln x], \quad (14)$$

$$D(x) = \frac{-1}{6(1-x)^4}[2 + 3x - 6x^2 + x^3 + 6x\ln x], \quad (15)$$

$$E(x) = \frac{-1}{(1-x)^2}[1 - x + x\ln x]. \quad (16)$$

The SUSY masses and diagonalizing matrices needed to evaluate the MSSM form factors at M_W were obtained (for details see [19]) using two-loop MSSM renormalization group equations (RGEs) for the gauge and Yukawa couplings and one-loop MSSM RGEs for the other SUSY parameters. The magnitudes of the MSSM form factors satisfy $|F_2^R| > |F_1^L| \gtrsim |F_2^L| \gg |F_1^R|$ for all regions of the allowed parameter space apart from the narrow region $\tan\beta = 2$, $m_{1/2} = 150$ and $m_0 \gtrsim 1000$ where $|F_1^L|$ is slightly smaller than $|F_2^R|$. Outside this region the ratio $|F_2^R|/|F_1^L|$ exceeds unity and increases strongly with $\tan\beta$. The result that F_2^R is the largest MSSM form factor indicates that, in contrast to the SM, the magnetic dipole transition dominates the b decay process in the MSSM. To compare with the SM, we note that the ratio of the largest MSSM and SM form factors is $|F_2^R(\text{MSSM})|/|F_1^L(\text{SM})(q^2=0)| \lesssim 0.4$ GeV.

The amplitude (1) can be written

$$M^{\text{Peng}} = M^{\text{SM}} + M^{\text{MSSM}} \quad (17)$$

where

$$M^{\text{SM}} = -i\frac{G_F}{\sqrt{2}} \left\{ \frac{\alpha_s(M_W)}{\pi} \frac{1}{8} \left[\sum_{i=u,c} V_{iq}^* V_{ib} f_1(x_i, q^2) \right.\right.$$
$$\left.\left. + V_{tq}^* V_{tb} f_1(x_t) \right] O_P + \frac{1}{2} \sum_{i=u,c,t} V_{iq}^* V_{ib} f_2(x_i) O_8 \right\} \quad (18)$$

and

$$M^{\text{MSSM}} = -i\left\{ \frac{\alpha_s(M_W)}{\pi} \frac{1}{8}[F_1^L(0)O_P + F_1^R(0)\tilde{O}_P] \right.$$
$$\left. + \frac{1}{2m_b}[F_2^L(0)O_8 + F_2^R(0)\tilde{O}_8] \right\}. \quad (19)$$

Here

$$O_P = O_4 + O_6 - \frac{1}{3}(O_3 + O_5) \quad (20)$$

is a combination of the standard QCD penguin four-fermion operators [26]

$$O_{3,5} \equiv (\bar{q}_\alpha \gamma^\mu b_\alpha)_{\text{V}-\text{A}} \sum_{q'} (\bar{q}'_\beta \gamma_\mu q'_\beta)_{\text{V}\mp\text{A}} \quad (21)$$

$$O_{4,6} \equiv (\bar{q}_\alpha \gamma^\mu b_\beta)_{\text{V}-\text{A}} \sum_{q'} (\bar{q}'_\beta \gamma_\mu q'_\alpha)_{\text{V}\mp\text{A}}, \quad (22)$$

where $q' \in \{u,d,s,c,b\}$, and O_8 is the chromomagnetic dipole operator:

$$O_8 \equiv \frac{g_s^2}{4\pi^2} m_b [\bar{q}_\alpha \sigma^{\mu\nu}(1+\gamma_5) T_{\alpha\beta}^a b_\beta] \frac{q_\nu}{q^2} [\bar{q}' \gamma_\mu T^a q'] \tag{23}$$

α and β are color indices, the subscripts $V \mp A$ represent the chiral projections $1 \mp \gamma_5$.

In addition to the SM operators, the MSSM gluino penguin processes introduce operators of opposite chirality to the O_k:

$$\tilde{O}_P = \tilde{O}_4 + \tilde{O}_6 - \frac{1}{3}(\tilde{O}_3 + \tilde{O}_5), \tag{24}$$

where

$$\tilde{O}_{3,5} \equiv (\bar{q}_\alpha \gamma^\mu b_\alpha)_{V+A} \sum_{q'} (\bar{q}'_\beta \gamma_\mu q'_\beta)_{V\pm A} \tag{25}$$

$$\tilde{O}_{4,6} \equiv (\bar{q}_\alpha \gamma^\mu b_\beta)_{V+A} \sum_{q'} (\bar{q}'_\beta \gamma_\mu q'_\alpha)_{V\pm A}, \tag{26}$$

and

$$\tilde{O}_8 \equiv \frac{g_s^2}{4\pi^2} m_b [\bar{q}_\alpha \sigma^{\mu\nu}(1-\gamma_5) T_{\alpha\beta}^a b_\beta] \frac{q_\nu}{q^2} [\bar{q}' \gamma_\mu T^a q']. \tag{27}$$

The effective Hamiltonian for $\Delta B = 1$ decays at scale $\mu = O(m_b)$ has the structure

$$\begin{aligned} H_{\text{eff}}(\mu) = \frac{G_F}{\sqrt{2}} \Big\{ & V_{uq}^* V_{ub} [C_1(\mu) O_1^u + C_2(\mu) O_2^u] \\ & + V_{cq}^* V_{cb} [C_1(\mu) O_1^c + C_2(\mu) O_2^c] \\ & - V_{tq}^* V_{tb} \sum_{k=3,\dots,6,8} [C_k(\mu) O_k + \tilde{C}_k(\mu) \tilde{O}_k] \Big\} \end{aligned} \tag{28}$$

where

$$O_1^{q'} \equiv (\bar{q}_\alpha q'_\alpha)_{V-A} (\bar{q}'_\beta b_\beta)_{V-A}, \tag{29}$$

$$O_2^{q'} \equiv (\bar{q}_\alpha q'_\beta)_{V-A} (\bar{q}'_\beta b_\alpha)_{V-A} \tag{30}$$

($q' \in \{u,c\}$), are the tree current-current operators. We omit the photon magnetic dipole operator O_7 and electroweak penguin operators. The Wilson coefficients $C_k(\mu) = C_k^{\text{SM}}$ for $k=1,2$ and $C_k(\mu) = C_k^{\text{SM}} + C_k^{\text{MSSM}}$, $\tilde{C}_k(\mu) = \tilde{C}_k^{\text{MSSM}}$ for $k=3,\dots,6,8$ incorporate the physics contributions from scales higher than μ and are determined perturbatively at M_W by matching to the full SM+MSSM theory. Noting that the u and c contributions from the SM penguin processes to the coefficient of O_P in Eq. (18) cancel in this matching process [25], we obtain [in the naive dimensional regularization (NDR) scheme]

$$C_1^{\text{SM}}(M_W) = 1 - \frac{11}{6} \frac{\alpha_s(M_W)}{4\pi}, \tag{31}$$

$$C_2^{\text{SM}}(M_W) = \frac{11}{2} \frac{\alpha_s(M_W)}{4\pi}, \tag{32}$$

$$C_k^{\text{SM}}(M_W) = \frac{\alpha_s(M_W)}{24\pi} A_k \left[f_1(x_t) - \frac{2}{3} \right], \tag{33}$$

$$C_8^{\text{SM}}(M_W) = \frac{1}{2} f_2(x_t), \tag{34}$$

$$C_k^{\text{MSSM}}(M_W) = \frac{\alpha_s(M_W)}{24\pi} \left(\frac{G_F}{\sqrt{2}} V_{tq}^* V_{tb} \right)^{-1} A_k F_1^L(0), \tag{35}$$

$$C_8^{\text{MSSM}}(M_W) = \frac{1}{2} \left(\frac{G_F}{\sqrt{2}} V_{tq}^* V_{tb} m_b \right)^{-1} F_2^L(0) \tag{36}$$

where $k=3,\dots,6$ and

$$A_k \equiv (-1,3,-1,3). \tag{37}$$

The coefficients $\{\tilde{C}_k^{\text{MSSM}}, \tilde{C}_8^{\text{MSSM}}\}$ are obtained from $\{C_k^{\text{MSSM}}, C_8^{\text{MSSM}}\}$ by the replacement $F_{1,2}^L(0) \to F_{1,2}^R(0)$.

The Wilson coefficients

$$C(M_W) \equiv (C_1(M_W), \dots, C_6(M_W))^{\text{T}} \tag{38}$$

evolve under the RGEs to

$$C(\mu) = U_5(\mu, M_W) C(M_W) \tag{39}$$

where $U_5(\mu, M_W)$ is the five-flavor 6×6 evolution matrix. In next-to-leading order (NLO), $U_5(\mu, M_W)$ is given by [32,26]

$$U_5(\mu, M_W) = \left[1 + \frac{\alpha_s(\mu)}{4\pi} J \right] U_5^0(\mu, M_W) \left[1 - \frac{\alpha_s(M_W)}{4\pi} J \right] \tag{40}$$

where $U_5^0(\mu, M_W)$ is the leading order (LO) evolution matrix and J expresses the NLO corrections to the evolution. NLO Wilson coefficients in the NDR renormalization scheme were computed from Eqs. (39) and (40) using expressions given in [32,26] for J and U_5^0 and the two-loop expression for $\alpha_s(\mu)$ with five flavors and $\alpha_s^{\overline{MS}}(M_Z) = 0.118$ ($\Lambda_{\text{QCD}} = 0.225$ GeV).

The renormalization group induced mixing between O_8 and the set $\{O_1, \dots, O_6\}$ vanishes at 1-loop order and it is sufficient to use the LO evolution for $C_8(\mu)$:

$$C_8(\mu) = \eta^{-14/23} C_8(M_W) + \sum_{i=1}^{8} \bar{h}_i \eta^{-a_i} \tag{41}$$

where $\eta \equiv \alpha_s(\mu)/\alpha_s(M_W)$ and the constants $\bar{h}_i$ and a_i are tabulated in [26].

TABLE I. Wilson coefficients $C_k(\mu)$ for the SM and the combined SM+MSSM and Wilson coefficients $\tilde{C}_k$ for the MSSM at the renormalization scales $\mu=m_b$ and $\mu=2.50$ GeV. The MSSM results are for the MSSM data set (42).

	SM		SM+MSSM	
μ	m_b	2.50 GeV	m_b	2.50 GeV
C_1	1.0767	1.1266	1.0767+0.0000i	1.1266+0.0000i
C_2	−0.1811	−0.2751	−0.1811+0.0000i	−0.2751+0.0000i
C_3	0.0119	0.0181	0.0119+0.0000i	0.0182+0.0000i
C_4	−0.0331	−0.0461	−0.0332+0.0000i	−0.0462+0.0000i
C_5	0.0094	0.0113	0.0094+0.0000i	0.0113+0.0000i
C_6	−0.0398	−0.0598	−0.0399+0.0000i	−0.0599+0.0000i
C_8	−0.1449	−0.1585	−0.4768+0.0160i	−0.4556+0.0144i
$\tilde{C}_3$ [a]			−0.39(−8)+0.19(−9)i	−0.43(−8)+0.21(−9)i
$\tilde{C}_4$ [a]			0.71(−8)−0.34(−9)i	0.68(−8)−0.33(−9)i
$\tilde{C}_5$ [a]			−0.14(−8)+0.65(−10)i	−0.79(−9)+0.39(−10)i
$\tilde{C}_6$ [a]			0.11(−7)−0.55(−9)i	0.13(−7)−0.65(−9)i
$\tilde{C}_8$			−0.0768+0.0000i	−0.0976+0.0000i

[a]Numbers in parentheses denote powers of 10.

The MSSM generated QCD penguin operators $\{\tilde{O}_3(M_W),\ldots,\tilde{O}_6(M_W)\}$ and $\tilde{O}_8(M_W)$ renormalize in the same manner as $\{O_3(M_W),\ldots,O_6(M_W)\}$ and $O_8(M_W)$ [34].

The Wilson coefficients $C_k(\mu)$, $k=1,\ldots,6,8$, for the SM and the combined SM+MSSM, together with the Wilson coefficients $\tilde{C}_k(\mu)$, $k=3,\ldots,6$, for the MSSM are listed in Table I for the scales $\mu=m_b$ and $\mu=2.5$ GeV. Inclusion of the MSSM processes generates complex Wilson coefficients due to the complex mixing coefficients Λ_{ABj}^{bq} in the MSSM form factors. These mixing coefficients arise from the mismatch between the interaction and $\tilde{d}$ mass eigenstates at the d-$\tilde{d}$-$\tilde{g}$ vertices and contain non-removable RGE-generated phases in the off-diagonal components of the couplings triggered, in particular, by the complexity of the large t-quark Yukawa coupling. The numerical values of Λ_{ABj}^{bq} used in Eqs. (35) and (36) for $C_k^{\text{MSSM}}(M_W)$, $k=3,\ldots,6,8$, are generated from Eq. (11) for a given MSSM data set by numerical diagonalization of the $\tilde{d}_j$ mass^2 matrices at the scale M_W.

The MSSM results are for the data set

$$A=-300\ \text{GeV},\quad \mu>0,\quad \tan\beta=48,$$

$$m_0=275\ \text{GeV},\quad m_{1/2}=150\ \text{GeV} \tag{42}$$

which maximizes the effects of SUSY [21] as measured by the ratio $\Gamma^{\text{peng}}(\text{MSSM})/\Gamma^{\text{peng}}(\text{SM})$ of integrated decay rates at scale M_W for $b\to qq'\bar{q}'$. This ratio is largest for high $\tan\beta$ and low $(m_0,m_{1/2})$ and for the set (42) is ≈0.10 for $b\to d$ and ≈0.085 for $b\to s$. The data set (42) satisfies the constraints imposed [35] on the SUSY parameter space by recent experimental bounds on the mass of the lightest chargino ($m_{\tilde{\chi}_1^\pm}>91$ GeV) and lighter $\tilde{t}$ squark ($m_{\tilde{t}_1}>80$ GeV) and by precision electroweak data constraints on

SUSY corrections to electroweak parameters but is on the verge of being excluded by the latest measurements for $\text{Br}(B\to X_s\gamma)$.

As expected from the observations above on the relative magnitudes of the SM and MSSM form factors at scale M_W, the only significant MSSM effects occur in the magnetic dipole coefficients. For the data set (42) we find $C_8^{\text{MSSM}}(M_W)=-0.4631+0.0224i$ ($b\to d$) and $-0.4755-0.0013i$ ($b\to s$) which are much larger than $C_8^{\text{SM}}(M_W)=-0.0953$ and comparable to the largest SM coefficient $C_1^{\text{SM}}(M_W)=0.9828$. Thus $C_8(\mu)$ is greatly enhanced by the MSSM contribution at M_W. Although $\tilde{C}_8^{\text{MSSM}}(M_W)$ is only of comparable magnitude to $C_{3,\ldots,6}^{\text{SM}}$, it is also enhanced by Eq. (41) and becomes significant.

The Wilson coefficients in NLO precision depend upon the renormalization scheme [32,36,33] and this unphysical dependence is compensated by the perturbative one-loop QCD corrections to the matrix elements of the four-quark operators $O_1,\ldots,O_6$ (and $\tilde{O}_3,\ldots,\tilde{O}_6$) at the scale μ. This is equivalent to using the effective Hamiltonian (28) with the Wilson coefficients $C_k(\mu)$, $k=1,\ldots,6$, replaced by the effective q^2-dependent Wilson coefficients $C_k^{\text{eff}}(\mu,q^2)$ such that

$$C_k(\mu)\langle O_k(\mu)\rangle=C_k^{\text{eff}}(\mu,q^2)\langle O_k\rangle^{\text{tree}}. \tag{43}$$

The two sets of Wilson coefficients are related by [33] (in the NDR scheme)

$$C_k^{\text{eff}}(\mu,q^2)=\left[1+\frac{\alpha_s(\mu)}{4\pi}\left(r_V^{\text{T}}+\gamma_V^{\text{T}}\log\frac{m_b}{\mu}\right)\right]_{kj}C_j(\mu)$$

$$+\frac{\alpha_s(\mu)}{24\pi}A_k'(c_t+c_p+c_g) \tag{44}$$

TABLE II. Effective Wilson coefficients $C_k^{\text{eff}}(\mu, q^2)$ for the SM and the combined SM+MSSM and effective Wilson coefficients $\tilde{C}_k^{\text{eff}}$ for the MSSM at the renormalization scale $\mu = 2.50$ GeV and momentum $q^2 = m_b^2/2$ for $b \rightarrow d$ and $\bar{b} \rightarrow \bar{d}$ transitions. Results are given for the MSSM data set (42).

	SM		SM+MSSM	
	$b \rightarrow d$	$\bar{b} \rightarrow \bar{d}$	$b \rightarrow d$	$\bar{b} \rightarrow \bar{d}$
C_1^{eff}	$1.1679 + 0.0000i$	$1.1679 + 0.0000i$	$1.1679 + 0.0000i$	$1.1679 + 0.0000i$
C_2^{eff}	$-0.3525 + 0.0000i$	$-0.3525 + 0.0000i$	$-0.3525 + 0.0000i$	$-0.3525 + 0.0000i$
C_3^{eff}	$0.0233 + 0.0018i$	$0.0250 + 0.0047i$	$0.0188 + 0.0019i$	$0.0204 + 0.0046i$
C_4^{eff}	$-0.0540 - 0.0053i$	$-0.0588 - 0.0142i$	$-0.0403 - 0.0057i$	$-0.0451 - 0.0137i$
C_5^{eff}	$0.0172 + 0.0018i$	$0.0189 + 0.0047i$	$0.0127 + 0.0019i$	$0.0143 + 0.0046i$
C_6^{eff}	$-0.0675 - 0.0053i$	$-0.0723 - 0.0142i$	$-0.0539 - 0.0057i$	$-0.0587 - 0.0137i$
$\tilde{C}_3^{\text{eff}}$			$-0.0010 + 0.0000i$	$-0.0010 + 0.0000i$
$\tilde{C}_4^{\text{eff}}$			$0.0029 + 0.0000i$	$0.0029 + 0.0000i$
$\tilde{C}_5^{\text{eff}}$			$-0.0010 + 0.0000i$	$-0.0010 + 0.0000i$
$\tilde{C}_6^{\text{eff}}$			$0.0029 + 0.0000i$	$0.0029 + 0.0000i$

where $A_k' \equiv (0,0,-1,3,-1,3)^T$, the 6×6 constant matrices r_V and γ_V (given in [33]) represent the process-independent parts of the vertex corrections and c_t, c_p and c_g are the contributions arising from the penguin-type corrections to the current-current operators $O_{1,2}$, penguin-type corrections to $O_3, \ldots, O_6$, and the tree level diagram of O_8 respectively. The quantities c_t, c_p and c_g are given by [33]

$$c_t = -\frac{C_1}{V_{tq}^* V_{tb}} \left[V_{uq}^* V_{ub} \left(\frac{2}{3} + \mathcal{F}_1(m_u, q^2) \right) \right.$$
$$\left. + V_{cq}^* V_{cb} \left(\frac{2}{3} + \mathcal{F}_1(m_c, q^2) \right) \right], \tag{45}$$

$$c_p = C_3 \left[\frac{4}{3} + \mathcal{F}_1(m_q, q^2) + \mathcal{F}_1(m_b, q^2) \right]$$
$$+ (C_4 + C_6) \sum_{j = u,d,s,c,b} \mathcal{F}_1(m_j, q^2), \tag{46}$$

$$c_g = -\frac{2 m_b}{\sqrt{q^2}} C_8 \tag{47}$$

where

$$\mathcal{F}_1(m, q^2) \equiv \frac{2}{3} \log \left(\frac{m^2}{\mu^2} \right) - \Delta F_1 \left(\frac{q^2}{m^2} \right) \tag{48}$$

and

$$\Delta F_1(z) = -4 \int_0^1 dx \, x(1-x) \log[1 - zx(1-x) - i\epsilon]. \tag{49}$$

Note that $\mathcal{F}_1(m, q^2)$ is just $-f_1(x, q^2)$ of [21] with M_W^2 replaced by μ^2 and is complex for $q^2 > 4m^2$. These strong phases in $\mathcal{F}_1$, generated at the $u\bar{u}$ and $c\bar{c}$ thresholds, combine with the weak CKM phases to produce effective Wilson

coefficients C_k^{eff}, $k = 3, \ldots, 6$, which differ in both magnitude and phase for b and $\bar{b}$ decays and hence CP violation is manifest.

Some comments are appropriate about whether or not to include the term c_g in Eq. (44). This term is absent in most applications of the effective Hamiltonian method [37,15,39] and was first included by [33]. The O_8 contribution to $\langle qq'\bar{q}'|H_{\text{eff}}|b\rangle$ can be written

$$M_8 = i \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{2\pi} F_2^R(0) \frac{m_b}{q^2} [\bar{u}_q \gamma_\mu \gamma_\nu q^\nu (1 + \gamma_5) T^a u_b]$$
$$\times [\bar{u}_{q'} \gamma^\mu T^a v_{\bar{q}'}]. \tag{50}$$

In the factorization model used by [33] color considerations preclude q' and $\bar{q}'$ combining into the same meson and it is reasonable to assume that $\boldsymbol{p}_{q'} = -\boldsymbol{p}_{\bar{q}'}$ in the b rest frame. The gluon momentum is then

$$q^\mu = \sqrt{q^2} p_b^\mu / m_b. \tag{51}$$

This allows M_8 to be expressed in terms of the penguin operator combination O_P given in Eq. (20), and yields the contribution c_g to C_k^{eff}, $k = 3, \ldots, 6$. q^2 is conventionally replaced by an averaged value $\langle q^2 \rangle$ taken to lie in the range $m_b^2/4 \leq \langle q^2 \rangle \leq m_b^2/2$. If the assumption (51) is not valid, then c_g should be omitted and a proper treatment of the magnetic dipole contribution to decay rates must be based upon Eq. (50). This then involves an integration over a range of q^2 determined by kinematical considerations (see later and Refs. [40,21,14]).

We have computed effective Wilson coefficients with and without the term c_g. Although the assumption $\boldsymbol{p}_{q'} = -\boldsymbol{p}_{\bar{q}'}$ is valid for many of the processes (so-called A processes) considered in our spectator model (see later), we choose not to make this assumption in our effective Wilson coefficients and hence do not include the c_g term in our calculations.

We list in Table II the effective Wilson coefficients

TABLE III. Effective Wilson coefficients $C_k^{\text{eff}}(\mu,q^2)$ for the SM and the combined SM+MSSM and effective Wilson coefficients $\tilde{C}_k^{\text{eff}}$ for the MSSM at the renormalization scale $\mu=2.50$ GeV and momentum $q^2=m_b^2/2$ for $b\to s$ and $\bar{b}\to\bar{s}$ transitions. Results are given for the MSSM data set (42).

| | SM | | SM+MSSM | |
	$b\to s$	$\bar{b}\to\bar{s}$	$b\to s$	$\bar{b}\to\bar{s}$
C_1^{eff}	$1.1679+0.0000i$	$1.1679+0.0000i$	$1.1679+0.0000i$	$1.1679+0.0000i$
C_2^{eff}	$-0.3525+0.0000i$	$-0.3525+0.0000i$	$-0.3525+0.0000i$	$-0.3525+0.0000i$
C_3^{eff}	$0.0249+0.0032i$	$0.0248+0.0030i$	$0.0202+0.0032i$	$0.0201+0.0030i$
C_4^{eff}	$-0.0585-0.0095i$	$-0.0583-0.0090i$	$-0.0445-0.0095i$	$-0.0442-0.0090i$
C_5^{eff}	$0.0188+0.0032i$	$0.0187+0.0030i$	$0.0141+0.0032i$	$0.0140+0.0030i$
C_6^{eff}	$-0.0721-0.0095i$	$-0.0718-0.0090i$	$-0.0581-0.0095i$	$-0.0578-0.0090i$
$\tilde{C}_3^{\text{eff}}$			$-0.0010+0.0000i$	$-0.0010+0.0000i$
$\tilde{C}_4^{\text{eff}}$			$0.0031+0.0000i$	$0.0031+0.0000i$
$\tilde{C}_5^{\text{eff}}$			$-0.0010+0.0000i$	$-0.0010+0.0000i$
$\tilde{C}_6^{\text{eff}}$			$0.0031+0.0000i$	$0.0031+0.0000i$

$C_k^{\text{eff}}(\mu,q^2)$, $k=1,\ldots,6$ and $\tilde{C}_k^{\text{eff}}(\mu,q^2)$, $k=3,\ldots,6$ evaluated from Eq. (44) at $q^2=m_b^2/2$ and $\mu=2.50$ GeV for the processes $b\to dq\bar{q}'$ and $\bar{b}\to\bar{d}q\bar{q}'$. Table III lists the same quantities for the processes $b\to sq\bar{q}'$ and $\bar{b}\to\bar{s}q\bar{q}'$. The values of the $\tilde{C}_k^{\text{eff}}$ coefficients are due nearly entirely to the A_k' term in Eq. (44). If the input CKM matrix is changed from the Particle Data Group parametrization to the Wolfenstein form with the parameter values $A=0.81$, $\rho=0.05$ and $\eta=0.36$ used by [33], and we include c_g, we obtain values for the SM C_k^{eff} which agree very closely with those given in [33].

III. DECAY RATES FOR $b\to qq'\bar{q}''$

In calculating the *b* quark decay rates we allow for the more general case where the antiquark produced is not necessarily $\bar{q}'$. We firstly consider the SM only and, in accordance with most studies of the SM, omit the small magnetic dipole term C_8^{SM} from the effective Hamiltonian (28) in the calculation of decay rates.

The partial decay rate, *b* spin averaged and summed over final spin states, has overall spherical symmetry. Apart from its overall orientation, a final state is specified by only two parameters, say $p_q=|\boldsymbol{p}_q|$ and $p_{q'}=|\boldsymbol{p}_{q'}|$. The partial decay rate in the *b* rest frame is

$$\frac{d^2\Gamma}{dp_q dp_{q'}} = \frac{G_F^2}{\pi^3} p_q p_{q'} E_{\bar{q}''} \left[\alpha_1 \frac{p_q \cdot p_{q'}}{E_q E_{q'}} + \alpha_2 \frac{p_q \cdot p_{\bar{q}''}}{E_q E_{\bar{q}''}} \right.$$
$$\left. + \alpha_3 \frac{m_{q'} m_{\bar{q}''}}{E_{q'} E_{\bar{q}''}} \right] \quad (52)$$

where

$$\alpha_1 = |d_1+d_2+d_3+d_4|^2 + 2|d_1+d_4|^2 + 2|d_2+d_3|^2,$$

$$\alpha_2 = |d_5+d_6|^2 + 2|d_5|^2 + 2|d_6|^2,$$

$$\alpha_3 = \Re[(3d_1+d_2+d_3+3d_4)d_6^*$$
$$+ (d_1+3d_2+3d_3+d_4)d_5^*] \quad (53)$$

and

$$d_{1,2}(\mu,q^2) \equiv V_{q''q}^* V_{q'b} C_{1,2}^{\text{eff}}(\mu,q^2),$$

$$d_{3,\ldots,6}(\mu,q^2) \equiv -V_{tq}^* V_{tb} C_{3,\ldots,6}^{\text{eff}}(\mu,q^2). \quad (54)$$

Here $p_q \cdot p_{q'} = [m_b^2 + m_{q''}^2 - m_q^2 - m_{q'}^2 - 2m_b E_{\bar{q}''}]/2$, $p_q \cdot p_{\bar{q}''} = [m_b^2 + m_{q'}^2 - m_q^2 - m_{q''}^2 - 2m_b E_{q'}]/2$ and the angles between the particle velocities must be physical, for example $|\cos(\theta_{qq'})| \leq 1$. Note that, for $q \in \{d,s\}$, only the tree current-current terms $d_{1,2}$ contribute when $q' \neq q'' \in \{u,c\}$ whereas, for $\Delta C = \Delta U = 0$ transitions, only the penguin terms $d_{3,\ldots,6}$ contribute for $q' = q'' \in \{d,s\}$ and both tree and penguin terms contribute for $q' = q'' \in \{u,c\}$.

For massless final state quarks, the decay rate is [37]

$$\Gamma = 12\Gamma_0 \int_0^1 d\xi [\alpha_1(\xi) + \alpha_2(\xi)] \left[\frac{1}{6} - \frac{\xi^2}{2} + \frac{\xi^3}{3} \right] \quad (55)$$

where $\Gamma_0 \equiv G_F^2 m_b^5/192\pi^3$ and $\xi \equiv q^2/m_b^2$. For SM pure penguin decays, the QCD corrected decay rates at scale μ calculated from Eq. (55) generally agree to better than 30% (see Table IV) with $\eta^2\Gamma^{(0)}$ where $\Gamma^{(0)}$ are the weak scale decay rates calculated using q^2-dependent form factors [21] and the factor η^2 accounts for the running of α_s from M_W to μ. To this extent the results support the conjecture of Gérard and Hou [24] mentioned in the Introduction.

To study the effects of the MSSM on the quark decay rates, we note that the only significant effects of the MSSM are confined to the magnetic dipole coefficients $C_8(\mu)$ and

TABLE IV. Comparison of QCD corrected integrated decay rates $\Gamma(\mu)$ at scale μ calculated from Eqs. (55) and (56) for pure penguin $b \rightarrow q$ and $\bar{b} \rightarrow \bar{q}$, $q \in \{d,s\}$, transitions with the scaled decay rates $\Gamma^{(0)}(\mu) = \eta^2 \Gamma^{(0)}(M_W)$ where $\Gamma^{(0)}(M_W)$ are the weak scale decay rates calculated using q^2-dependent form factors and $\eta^2 \equiv [\alpha_s(\mu)/\alpha_s(M_W)]^2$ accounts for the renormalized α_s. Results are calculated for the cutoff $q^2_{\min} = 1.0 \text{ GeV}^2$ and for the MSSM data set (42). The numbers in parentheses denote powers of 10.

Γ (GeV)	SM				SM+MSSM			
	$b \rightarrow d$	$\bar{b} \rightarrow \bar{d}$	$b \rightarrow s$	$\bar{b} \rightarrow \bar{s}$	$b \rightarrow d$	$\bar{b} \rightarrow \bar{d}$	$b \rightarrow s$	$\bar{b} \rightarrow \bar{s}$
$\Gamma^{(0)}(M_W)$	1.480($-$17)	2.210($-$17)	2.991($-$16)	2.921($-$16)	2.062($-$17)	2.926($-$17)	4.080($-$16)	3.997($-$16)
$\Gamma^{(0)}(m_b)$	4.898($-$17)	7.311($-$17)	9.897($-$16)	9.667($-$16)	6.823($-$17)	9.681($-$17)	1.350($-$15)	1.323($-$15)
$\Gamma(m_b)$	6.304($-$17)	9.076($-$17)	1.303($-$15)	1.276($-$15)	1.192($-$16)	1.560($-$16)	2.353($-$15)	2.317($-$15)
$\Gamma^{(0)}(2.50)$	7.594($-$17)	1.134($-$16)	1.534($-$15)	1.499($-$15)	1.058($-$16)	1.501($-$16)	2.093($-$15)	2.051($-$15)
$\Gamma(2.50)$	8.103($-$17)	1.222($-$16)	1.714($-$15)	1.674($-$15)	1.578($-$16)	2.130($-$16)	3.157($-$15)	3.103($-$15)

$\tilde{C}_8(\mu)$ (see Table I) and the terms $\tilde{C}_k$, $k=3,\ldots,6$ of the effective Hamiltonian (28) can be neglected in any calculation of decay rates. Since $|\tilde{C}_8(\mu)|/|C_8(\mu)| \lesssim 0.2$ for both $b \rightarrow d$ and $b \rightarrow s$ transitions, it would be reasonable to also neglect the $\tilde{C}_8$ term so that, if the assumption (51) about the momenta of q' and $\bar{q}''$ is applicable, the major effects of the MSSM could be incorporated in a modified value of c_g. However, for the general case, the c_g term should be omitted from C_k^{eff}, $k=3,\ldots,6$, and the magnetic dipole terms directly included. For massless final state quarks the dipole contributions add the terms [21]

$$\frac{1}{3}\left(\frac{\alpha_s}{\pi}\right)^2 (|d_8|^2 + |\tilde{d}_8|^2)\left(\frac{1}{3\xi} - \frac{1}{2} + \frac{\xi^2}{6}\right)$$

$$-\frac{2}{3}\frac{\alpha_s}{\pi}\Re[(d_1 + d_4 + d_6)d_8^*](1-\xi)^2 \quad (56)$$

to the integrand of Eq. (55), where

$$d_8(\mu) \equiv -V_{tq}^* V_{tb} C_8(\mu), \quad \tilde{d}_8(\mu) \equiv -V_{tq}^* V_{tb} \tilde{C}_8(\mu), \quad (57)$$

thus requiring the imposition of a lower cutoff on q^2 (taken to be 1.0 GeV^2). The MSSM gluino penguin processes so included are greatly enhanced relative to the SM penguin amplitudes by the renormalization-induced QCD corrections (see Table IV) and the resulting decay rates Γ for the SM +MSSM exceed the η^2-scaled weak scale decay rates Γ^0. We also note from Table IV that, whereas the QCD corrections only mildly affects the magnitudes of the CP asymmetries $a_{CP} \equiv (\Gamma - \bar{\Gamma})/(\Gamma + \bar{\Gamma})$ for the SM, they decrease the magnitude of a_{CP} for the SM+MSSM.

The impact on $b \rightarrow sg$ of new physics exhibited by an enhanced chromomagnetic coupling C_8 involving an unconstrained new CP phase has recently been studied by Hou [14] within an effective Hamiltonian framework. q^2-dependent decay rates and CP asymmetries are given for phase differences of $\pi/4$, $\pi/2$ and $3\pi/4$ between the C_8 amplitude and the complex SM $C_{3,\ldots,6}$ penguin amplitudes.

In all cases Hou finds large rate asymmetries above the $c\bar{c}$ threshold, a feature not present in our present calculations or in our earlier weak scale calculations incorporating a q^2-dependent SM $F_1^L(q^2)$ form factor [21], because of the much smaller MSSM phases (for $b \rightarrow s$ the phases of the dominant MSSM form factors F_2^R and F_1^L are nearly constant [21] at ≈ -0.016 over the allowed MSSM parameter space).

Although the QCD corrected SM+MSSM decay rates are quite sensitive to the choice of lower cutoff for q^2, it is clear that the values obtained for the MSSM data set (42) are unacceptably large. For example, assuming a maximum branching ratio of 1% for $b \rightarrow sd\bar{d}$ gives an experimental upper bound on the $b \rightarrow s$ decay rate of $\lesssim 3 \times 10^{-15}$ GeV, comparable to our calculated rate. Even though the MSSM data set used in the present calculations is not typical of the MSSM parameter space, having been chosen to maximize the hadronic b decay rate at M_W, our findings here suggest that experimental data on hadronic b decays will exclude a similar low $(m_0, m_{1/2})$ region of the MSSM parameter space to that excluded by the $b \rightarrow s\gamma$ constraints [35].

From here on we focus on decay rates in the SM for b decays which are not pure penguin processes. We therefore neglect the magnetic dipole interaction and retain final state quark masses. If we make the common assumption that the coefficients α_i, $i=1,2,3$, are constant, evaluated at $q^2 = m_b^2/2$ for example, then the decay rate is

$$\Gamma = \Gamma_0[\alpha_1(\xi = \tfrac{1}{2})I_1 + \alpha_2(\xi = \tfrac{1}{2})I_2 + \alpha_3(\xi = \tfrac{1}{2})I_3] \quad (58)$$

where the phase space integrals I_i, $i=1,2,3$ have to be computed numerically. Decay rates computed from Eq. (58) are shown in Table V, together with branching ratios for various b decays. Branching ratios are less sensitive to the value chosen for m_b and have been obtained by dividing the decay rates by the total decay rate Γ^{total} computed using the $O(\alpha_s)$ QCD-corrected semileptonic decay rate

$$\Gamma(b \rightarrow cl^-\bar{\nu}_l) = \Gamma_0|V_{cb}|^2 I_1\left[1 - \frac{2\alpha_s}{3\pi}f(\rho_c)\right], \quad (59)$$

where $\rho_c \equiv (m_c/m_b)^2$, with $f(\rho_c) = 2.51$ [38].

TABLE V. SM phase space integrals I_i, $i=1,2,3$, decay rates and branching ratios B for various b decays calculated using effective Wilson coefficients $C_k^{\text{eff}}(\mu,q^2)$ evaluated at the renormalization scale $\mu=2.50$ GeV. The hadronic decay rates Γ are calculated from Eq. (58) using q^2 fixed at $m_b^2/2$. The numbers in parentheses denote powers of 10.

Decay process	I_1	I_2	I_3	Γ (GeV)	B
$b \to c\ d\ \bar{u}$	0.5011			1.846(-13)	5.007(-1)
$b \to c\ s\ \bar{u}$	0.4911			9.280(-15)	2.517(-2)
$b \to c\ d\ \bar{c}$	0.1765	0.1766	8.69(-2)	2.921(-15)	7.706(-3)
$b \to c\ s\ \bar{c}$	0.1702	0.1706	8.49(-2)	5.507(-14)	1.494(-1)
$b \to u\ d\ \bar{u}$	1.0000	0.9952	8.38(-6)	2.354(-15)	6.386(-3)
$\bar{b} \to \bar{u}\ \bar{d}\ u$				2.586(-15)	7.014(-3)
$b \to u\ s\ \bar{u}$	0.9884	0.9823	8.34(-6)	2.462(-15)	6.677(-3)
$\bar{b} \to \bar{u}\ \bar{s}\ u$				2.228(-15)	6.043(-3)
$b \to u\ d\ \bar{c}$	0.4982			6.362(-17)	1.726(-4)
$\bar{b} \to \bar{u}\ \bar{d}\ c$				6.362(-17)	1.726(-4)
$b \to u\ s\ \bar{c}$	0.4882			1.235(-15)	3.351(-3)
$b \to c\ e^-\ \bar{\nu}_e$	0.5012			4.991(-14)	1.354(-1)
$b \to c\ \mu^-\ \bar{\nu}_\mu$	0.4982			4.961(-14)	1.346(-1)
$b \to c\ \tau^-\ \bar{\nu}_\tau$	0.1121			1.117(-14)	3.030(-2)

IV. SPECTATOR QUARK MODEL FOR $B \to h_1 h_2$

We now wish to make the transition from decay rates at the quark level to decay rates for two body hadronic decays $B \to h_1 h_2$. Calculation of the required hadronic matrix elements $\langle h_1 h_2 | O_k | B \rangle$ from first principles is currently not possible and several approximate schemes are available in the literature. Much work has been done on the factorization model [27] and the more recent generalized factorization model [33,39,41] of this process in which final state interactions are neglected and the hadronic matrix elements are factorized into a product of two hadronic currents: $\langle h_1 | J_{1\mu} | B \rangle \langle h_2 | J_2^\mu | 0 \rangle$. The operators O_2^q and $O_{4,6}$ are Fierz transformed into a combination of color singlet-singlet and octet-octet terms and the octet-octet terms then discarded. The singlet-singlet current matrix elements are then expressed in terms of known decay rates and form factors. Consequently, the hadronic matrix elements are expressed in terms of the combinations

$$a_{2k-1} = C_{2k-1}^{\text{eff}} + \frac{1}{N_c} C_{2k}^{\text{eff}}, \quad a_{2k} = C_{2k}^{\text{eff}} + \frac{1}{N_c} C_{2k-1}^{\text{eff}} \quad (60)$$

where $k=1,2,3$ and the number of colors N_c is usually treated as a free phenomenological parameter in order to compensate for the discarded octet-octet terms. Factorization works reasonably well for B decays to heavy hadrons [42], in that a_1 and a_2 seem to assume universal values, and it is argued [33] to also account for B decays to light hadrons such as $B \to K\pi$ and $B \to \pi\pi$. The generalized factorization approach has been criticized [43,25] because the effective Wilson coefficients (44) are gauge dependent, implying that the value of N_c extracted from comparisons of factorization model predictions with data cannot have any physical

meaning.[1] These authors [43] present an alternative model based upon Wick contractions in the matrix elements of the NLO effective Hamiltonian. Furthermore, the applicability of factorization to the hadronization of outgoing light quark pairs moving in opposite directions has been questioned [45] and it has been emphasized there that hadronic B decays, especially as they are originated by three partons, are phase space driven.

In the spectator model [46] alternative to factorization, the b quark and spectator quark are treated as quasifree particles with a distribution of momenta due to their Fermi motion relative to the B meson. In these models the b quark is treated as a virtual particle with an invariant mass $m_b(p_s)$ satisfying $m_b^2(p_s) = m_B^2 + m_s^2 - 2m_B\sqrt{p_s^2 + m_s^2}$. We, however, consider here a tentative spectator model based upon the idea of duality between quark and hadron physics at the high energies of b quark and B meson decays. The decays at the quark level, even including the penguin processes, are basically short distance processes. In our proposed spectator quark model the long distance hadronization is largely a matter of incoherently assigning regions of the final quark phase space to the different mesonic systems.

For example, we consider a B meson $b\bar{u}$ or $b\bar{d}$ to be a heavy stationary b quark accompanied by a light spectator constituent antiquark which has a spherically symmetric normalized momentum distribution $P(|\boldsymbol{p}_s|^2)d^3p_s$. The total meson decay rate through a particular mode is then assumed to be

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[1] Gauge invariant and infrared finite effective Wilson coefficients have recently become available [44].

$$\Gamma = \int \frac{d^2\Gamma}{dp_q dp_{q'}} P(|\boldsymbol{p}_s|^2) d^3 p_s dp_q dp_{q'}, \tag{61}$$

equal to the initiating decay rate. Neglecting, for the moment, any constraints due to quark color, we suppose the spectator antiquark to combine with the quark i ($i = q'$ or q) to form the meson system. For example, if $i = q'$, we assign a mass $M_{q's}$ to the system such that

$$M_{q's}^2 = (p_{q'} + p_s) \cdot (p_{q'} + p_s)$$
$$= m_{q'}^2 + m_s^2 + 2(E_{q'}E_s - p_{q'}p_s \cos\theta_{q's}). \tag{62}$$

Constraining $p_{q'}$ and p_s to have mass $M_{q's}$, we can infer from Eq. (61) that

$$\frac{d\Gamma}{dM_{q's}} = 2M_{q's} \int \frac{d^2\Gamma}{dp_q dp_{q'}} P(|\boldsymbol{p}_s|^2) \delta(M_{q's}^2 - m_{q'}^2 - m_s^2$$
$$- 2(E_{q'}E_s - p_{q'}p_s \cos\theta_{q's})) d^3 p_s dp_q dp_{q'}. \tag{63}$$

Hence

$$\frac{d\Gamma}{dM_{q's}} = 2\pi M_{q's} \int \frac{p_s dp_s}{p_{q'}} \frac{d^2\Gamma}{dp_q dp_{q'}} P(|\boldsymbol{p}_s|^2) dp_q dp_{q'} \tag{64}$$

where the integration region is restricted by the condition $|\cos\theta_{q's}| \leq 1$.

We also assign a mass $M_{q\bar{q}''}$ to the second quark antiquark system such that

$$M_{q\bar{q}''}^2 = (p_q + p_{\bar{q}''}) \cdot (p_q + p_{\bar{q}''})$$
$$= (p_b - p_{q'}) \cdot (p_b - p_{q'})$$
$$= m_b^2 + m_{q'}^2 - 2m_b E_{q'}. \tag{65}$$

The variable $E_{q'}$, and hence $p_{q'}$, determines the mass $M_{q\bar{q}''}$. Taking this mass to be the independent variable, we have

$$\frac{d^2\Gamma}{dM_{q's} dM_{q\bar{q}''}} = \frac{2\pi M_{q's} M_{q\bar{q}''}}{m_b}$$
$$\times \int \frac{E_{q'} p_s}{p_{q'}^2} \frac{d^2\Gamma}{dp_q dp_{q'}} P(|\boldsymbol{p}_s|^2) dp_s dp_q. \tag{66}$$

We call this mode of quark and antiquark combination *process A*. Finally, by integration, we compute the partial decay rates $\Gamma(M_{q's}, M_{q\bar{q}''})$ into quark systems with masses less than $M_{q's}$ and $M_{q\bar{q}''}$. With suitable binning we equate these partial decay rates with corresponding rates into mesons. To calculate an exclusive decay into a particular two meson system would require one to follow the flow of quark spins. Although the model could be generalized to do so, we at-

tempt to take large enough mass bins to enclose all the spin combinations. When searching for CP asymmetries we would advocate looking at more inclusive decay rates which will be larger and hence more readily measurable. Of particular interest are the quark antiquark systems forming the lowest mass 0^- and 1^- meson states as data exist for charmed quark systems which can be used to test the spectator model. Also, data which should exhibit matter-antimatter asymmetry are eagerly awaited for the rare decays into light quark systems.

It is also possible that the spectator antiquark combines with the quark q, for which we get

$$\frac{d^2\Gamma}{dM_{qs} dM_{q'\bar{q}''}} = \frac{2\pi M_{qs} M_{q'\bar{q}''}}{m_b}$$
$$\times \int \frac{E_q p_s}{p_q^2} \frac{d^2\Gamma}{dp_q dp_{q'}} P(|\boldsymbol{p}_s|^2) dp_s dp_{q'}. \tag{67}$$

We call this *process B*.

In some meson decays, for example $B^- \to (\bar{u}d) + (\bar{u}u)$ which is initiated by the quark decay $b \to ud\bar{u}$, the spectator $\bar{u}$ could have combined with the u or the d. However, for light meson systems such as $\pi^- \rho^0$, we find that the different combinations come from very different regions of phase space of the initiating b decay and we conclude that we are not double counting.

Turning now to the flow of color, we examine what may be regarded as two extreme possibilities. In the first, called here *model I*, we take the formulas (52), (66) and (67) at face value; that is, we make no attempt to follow the flow of color and assume that all color flow is taken care of by the gluon fields in the meson system. In the second, called here *model II*, we consider the possibility that the lowest mass meson states are only formed if the quark-antiquark pair are in a color singlet state. That is, we assume that the color disruption caused by a quark-antiquark pair in a color octet state will result in more complex meson systems than the lowest mass 0^- or 1^- states.

Projecting out the color singlet states results only in a modification of the coefficients $\alpha_{1,2}$ of Eq. (52). For *process A* represented by Eq. (66) they become

$$\alpha_1 = 3\left| d_1 + \frac{1}{3}d_2 + \frac{1}{3}d_3 + d_4 \right|^2$$

$$\alpha_2 = 3\left| \frac{1}{3}d_5 + d_6 \right|^2, \tag{68}$$

whereas for *process B* represented by Eq. (67) they are

$$\alpha_1 = 3\left| \frac{1}{3}d_1 + d_2 + d_3 + \frac{1}{3}d_4 \right|^2$$

$$\alpha_2 = 3\left| d_5 + \frac{1}{3}d_6 \right|^2. \tag{69}$$

V. SPECTATOR MODEL RESULTS

Calculations have been performed for two spectator quark momentum distributions. One is a fixed spherical bag model [47] in which the spectator is confined in a spherical cavity of radius R and has a wave function of the form

$$\phi(\boldsymbol{r}) = \frac{1}{\pi} \sqrt{\frac{\Lambda}{2}} \frac{\sin(\Lambda r)}{r}. \tag{70}$$

This implies a momentum distribution

$$P(|\boldsymbol{p}_s|^2) = \left(\frac{\Lambda}{\pi}\right)^3 \frac{\sin^2(\pi p_s/\Lambda)}{|\boldsymbol{p}_s|^2(\Lambda^2 - |\boldsymbol{p}_s|^2)^2} \tag{71}$$

and a mean square momentum $\overline{|\boldsymbol{p}_s|^2} = \Lambda^2$. The parameter Λ is determined from the mean square B meson radius

$$\bar{r}^2 = \frac{1}{\Lambda^2}\left[\frac{\pi^2}{3} - \frac{1}{2}\right]. \tag{72}$$

Taking $\sqrt{\bar{r}^2} = 0.55$ fm yields the value $\Lambda = 0.6$ GeV which we have used in our calculations. As a check on the sensitivity of our results to the model, we have also used a Gaussian spectator quark distribution with $\overline{|\boldsymbol{p}_s|^2} = \Lambda^2$:

$$P(|\boldsymbol{p}_s|^2) = \left(\frac{3}{2\pi\Lambda}\right)^{3/2} e^{-3|\boldsymbol{p}_s|^2/2\Lambda^2}. \tag{73}$$

For the maximum mass of the quark-antiquark systems we take a value midway between the lowest mass 1^- state and the next most massive meson. Thus we take, for $(c\bar{u})$ or $(c\bar{d})$, $M_{c\bar{u}} = 2.214$ GeV between the $D^*(2.007)$ and the $D^*(2.420)$; for $(c\bar{s})$, $M_{c\bar{s}} = 2.323$ GeV between $D_s^*(2.112)$ and $D_s^*(2.533)$; for $(s\bar{u})$ or $(s\bar{d})$, $M_{s\bar{u}} = 1.081$ GeV between $K^*(0.892)$ and $K^*(1.270)$; for $(u\bar{d})$, $M_{u\bar{d}} = 0.877$ GeV between $\rho(0.770)$ and $a_0(0.984)$; and for $(u\bar{u})$ and $(d\bar{d})$, $M_{u\bar{u}} = M_{d\bar{d}} = 0.870$ GeV between $\omega(0.782)$ and $\eta'(0.958)$. For the $(c\bar{c})$ system we make an exception and take the maximum $M_{c\bar{c}} = 3.552$ GeV between $\chi(3.417)$ and $\psi(2s)(3.686)$.

The results are presented in Table VI and compared, where data are available, with the sum of the branching ratios into mesons with masses less than the above cutoff masses. All calculations are based upon partial decay rates (52) with the α_i coefficients evaluated at $q^2 = m_b^2/2$. The semileptonic decay rates are also shown in Table VI. For these decays the lepton momenta cover all the allowed phase space.

We have repeated the calculations using the SM + MSSM effective Wilson coefficients of Tables II and III but neglecting the very small $\widetilde{C}^{\text{eff}}$ coefficients. The MSSM effects are insignificant for the $b \to u d\bar{u}$ transitions. For type A processes all branching ratios are slightly increased ($\sim 1\%$). For $b \to u s\bar{u}$ transitions the MSSM effects are more significant and result in branching ratio decreases for type A

decays of 23% for $b \to u s\bar{u}$ and 28% for $\bar{b} \to \bar{u}\bar{s}u$. The $b \to d s\bar{d}$ and $\bar{b} \to \bar{d}\bar{s}d$ branching ratios are also decreased by about 30%. Inclusion into our spectator model of the MSSM-enhanced magnetic dipole term is a matter for future investigation.

VI. DISCUSSION AND CONCLUSIONS

We have investigated two aspects of hadronic B decays. The first is the effects of renormalization-induced QCD corrections on the inclusion of SUSY gluino penguin contributions to the hadronic b decays $b \to qg \to qq'\bar{q}'$, $q' \in \{d,s\}$, which have been studied by calculating the Wilson coefficients of the effective SM + MSSM Hamiltonian. The SUSY enhancement of these gluon mediated processes at M_W by the MSSM magnetic dipole transition, which is especially significant for [21] large $\tan\beta$ and low $(m_0, m_{1/2})$, is further amplified by the QCD corrections, resulting in quark decay rates at scales $\mu \sim O(m_b)$ for pure SM + MSSM penguin processes which exceed those obtained by simply scaling the rates at M_W by the factor η^2 to allow for the renormalization of the strong coupling constant from M_W to μ. The rates are also significantly larger than estimated experimental bounds, suggesting that experimental data on hadronic b decays will exclude a similar low $(m_0, m_{1/2})$ region of the MSSM parameter space to that excluded by the $b \to s\gamma$ constraints [35]. We find that the QCD corrections only mildly affect the magnitudes of the CP asymmetries a_{CP}.

The second aspect of hadronic B decays studied is the hadronization of the final state quarks. We have adopted a spectator model as an alternative to the widely used factorization models. The semileptonic decay rates given in Table VI indicate semi-quantitative agreement between our model and data. Where there are data, the same is true for the hadronic decays that proceed by *process A*, suggesting that color suppression does not have a large influence on these processes. This is not so for *process B* hadronic decays which we find to be very sensitive to color suppression and there is clear evidence for color suppression in the data. In fact, for decay modes which can proceed only through *process B*, the data are for the most part upper bounds. The decays $B^0 \to \psi(1s) + (K^0, K^{*0})$ and $B^+ \to \psi(1s) + (K^+, K^{*+})$, for which the experimental branching ratios are $(2.24 \pm 0.3) \times 10^{-3}$ and $(2.46 \pm 0.37) \times 10^{-3}$ respectively, are *process B* decays in the context of our model. Taking a cutoff mass $M_{c\bar{c}} = 3.257$ GeV to include $\psi(1s)$ but exclude the $\chi(1P)$ we find, with no color suppression, a branching ratio of 9.22×10^{-3}. This suggests a color suppression factor of about 4, not the orders of magnitude obtained from insisting that the $c\bar{c}$ pair are produced as a color singlet (*model II*).

The decays into light quark systems exhibit matter-antimatter asymmetry. The present calculations indicate that, averaged over a few low mass π mesonic states, the B^+ and B^0 branching ratios are larger than those for B^- and $\bar{B}^0$; the opposite is true if a K meson is involved. The calculations suggest the asymmetry is not large. In the effective Hamiltonian used here, this asymmetry comes only from the

TABLE VI. Experimental and spectator model branching ratios for B meson decays. The notation, for example, $B^0 \rightarrow (D^-, D^{*-})$ $+(\pi^+, \rho^+)$ indicates the branching ratio $B^0 \rightarrow [D^-(1.869)$ or $D^{*-}(2.010)] + (\pi^+$ or $\rho^+)$. The process classification A or B refers to whether the spectator antiquark combines with the quark q', Eq. (66) or q, Eq. (67). The model classification I or II refers to no color suppression and total color suppression respectively in the formation of the decay mesons (see the text). For the decays into light quark systems, the data are for the most part upper bounds which are not shown. The model predictions are in accordance with these bounds. Only $\bar{b}$ quark decays are shown, as in the PDG tables.

Decay	Experiment	Process	Model I	Model II
$\bar{b} \rightarrow \bar{c}\, l^+ \nu_l$				
$B^+ \rightarrow (\bar{D}^0, \bar{D}^{*0}) + l^+ \nu_l$	$(7.16 \pm 1.3) \times 10^{-2}$		8.6×10^{-2}	
$B^0 \rightarrow (D^-, D^{*-}) + l^+ \nu_l$	$(6.6 \pm 0.5) \times 10^{-2}$		8.6×10^{-2}	
$\bar{b} \rightarrow \bar{u} l^+ \nu_l$				
$B^0 \rightarrow (\pi^-, \rho^-) + l^+ \nu_l$	$(4.3 \pm 1.6) \times 10^{-4}$		5.2×10^{-4}	
$\bar{b} \rightarrow \bar{c} u \bar{d}$				
$B^0 \rightarrow (D^-, D^{*-}) + (\pi^+, \rho^+)$	$(2.04 \pm 0.53) \times 10^{-2}$	A	3.00×10^{-2}	2.73×10^{-2}
$B^+ \rightarrow (\bar{D}^0, \bar{D}^{*0}) + (\pi^+, \rho^+)$	$(3.88 \pm 0.58) \times 10^{-2}$	A	3.00×10^{-2}	2.78×10^{-2}
		B	1.39×10^{-2}	1.55×10^{-5}
$B^0 \rightarrow (\bar{D}^0, \bar{D}^{*0}) + (\pi^0, \eta, \rho^0, \omega)$	$< 3.2 \times 10^{-3}$	B	1.39×10^{-2}	1.55×10^{-5}
$\bar{b} \rightarrow \bar{c} s \bar{c}$				
$B^+ \rightarrow (\bar{D}^0, \bar{D}^{*0}) + (D_s^+, D_s^{*+})$	$(6.1 \pm 2.3) \times 10^{-2}$	A	3.2×10^{-2}	2.9×10^{-2}
$B^0 \rightarrow (D^-, D^{*-}) + (D_s^+, D_s^{*+})$	$(4.8 \pm 1.8) \times 10^{-2}$	A	3.2×10^{-2}	2.9×10^{-2}
$B^0 \rightarrow (\psi(1s), \chi(1P)) + (K^0, K^{*0})$	$< 6 \times 10^{-3}$	B	3.7×10^{-2}	4.1×10^{-5}
$B^+ \rightarrow (\psi(1s), \chi(1P)) + (K^+, K^{*+})$		B	3.7×10^{-2}	4.1×10^{-5}
$\bar{b} \rightarrow \bar{u} d \bar{u}$				
$B^0 \rightarrow (\pi^+, \rho^+) + (\pi^-, \rho^-)$		A	1.07×10^{-4}	0.95×10^{-4}
$B^+ \rightarrow (\pi^+, \rho^+) + (\pi^0, \eta, \rho^0, \omega)$		A	1.07×10^{-4}	0.95×10^{-4}
		B	1.09×10^{-4}	1.9×10^{-7}
$B^0 \rightarrow (\pi^0, \eta, \rho^0, \omega) + (\pi^0, \eta, \rho^0, \omega)$		B	1.09×10^{-4}	1.9×10^{-7}
$b \rightarrow u d \bar{u}$				
$\bar{B}^0 \rightarrow (\pi^+, \rho^+) + (\pi^-, \rho^-)$		A	0.95×10^{-4}	0.87×10^{-4}
$B^- \rightarrow (\pi^-, \rho^-) + (\pi^0, \eta, \rho^0, \omega)$		A	0.95×10^{-4}	0.87×10^{-4}
		B	0.97×10^{-4}	1.9×10^{-7}
$\bar{B}^0 \rightarrow (\pi^0, \eta, \rho^0, \omega) + (\pi^0, \eta, \rho^0, \omega)$		B	0.97×10^{-4}	1.9×10^{-7}
$\bar{b} \rightarrow \bar{u} s \bar{u}$				
$B^0 \rightarrow (K^+, K^{*+}) + (\pi^-, \rho^-)$		A	0.503×10^{-4}	0.422×10^{-4}
$B^+ \rightarrow (K^+, K^{*+}) + (\pi^0, \eta, \rho^0, \omega)$		A	0.503×10^{-4}	0.422×10^{-4}
		B	1.09×10^{-4}	0.93×10^{-6}
$B^0 \rightarrow (K^0, K^{*0}) + (\pi^0, \eta, \rho^0, \omega)$		B	1.09×10^{-4}	0.93×10^{-6}
$b \rightarrow u s \bar{u}$				
$\bar{B}^0 \rightarrow (K^-, K^{*-}) + (\pi^+, \rho^+)$		A	0.624×10^{-4}	0.531×10^{-4}
$B^- \rightarrow (K^-, K^{*-}) + (\pi^0, \eta, \rho^0, \omega)$		A	0.624×10^{-4}	0.531×10^{-4}
		B	1.21×10^{-4}	0.93×10^{-6}
$\bar{B}^0 \rightarrow (\bar{K}^0, \bar{K}^{*0}) + (\pi^0, \eta, \rho^0, \omega)$		B	1.21×10^{-4}	0.93×10^{-6}
$\bar{b} \rightarrow \bar{d} s \bar{d}$				
$B^+ \rightarrow (K^0, K^{*0}) + (\pi^+, \rho^+)$		A	0.491×10^{-4}	0.410×10^{-4}
$B^0 \rightarrow (K^0, K^{*0}) + (\pi^0, \eta, \rho^0, \omega)$		A	0.491×10^{-4}	0.410×10^{-4}
		B	1.082×10^{-4}	0.920×10^{-6}
$b \rightarrow d s \bar{d}$				
$B^- \rightarrow (\bar{K}^0, \bar{K}^{*0}) + (\pi^-, \rho^-)$		A	0.499×10^{-4}	0.417×10^{-4}
$\bar{B}^0 \rightarrow (\bar{K}^0, \bar{K}^{*0}) + (\pi^0, \eta, \rho^0, \omega)$		A	0.499×10^{-4}	0.417×10^{-4}
		B	1.096×10^{-4}	0.914×10^{-6}

''strong'' phases of the penguin diagrams. The renormalization group construction of the effective Hamiltonian, although incorporating important QCD improvements, does not include the ''strong'' phases which must be present even at the first order of α_s corrections. The penguin diagram includes just one $O(\alpha_s)$ correction to the ''strong'' phase; it would be of interest to include all $O(\alpha_s)$ corrections to the ''strong'' phase.

We have compared our results for the light quark systems with the factorization model calculations of [36] which presents results for pseudoscalar and vector meson final states. Because of the constraints of spin and isospin, which are averaged over in our model, we are reluctant to use cuts which bracket individual particles. However, the results of [36] when summed over the pseudoscalar and vector final particles are generally smaller than our calculations, but by less than factors of 2. Such factors can easily be accommodated within the uncertainties of just the CKM parameters for the light quark couplings.

APPENDIX: INPUT PARAMETERS FOR NUMERICAL CALCULATIONS

We use the standard Particle Data Group [48] parametrization of the CKM matrix with the central values

$$\theta_{12} = 0.221, \quad \theta_{13} = 0.0035, \quad \theta_{23} = 0.041 \quad \text{(A1)}$$

and choose the CKM phase δ_{13} to be $\pi/2$.

Following Ali and Greub [33], we treat internal quark masses in penguin loops as constituent masses with the values

$$m_d = m_u = 0.2 \text{ GeV}, \quad m_s = 0.5 \text{ GeV},$$

$$m_c = 1.5 \text{ GeV}, \quad m_b = 4.88 \text{ GeV}. \quad \text{(A2)}$$

For the light quarks in the spectator model we take

$$m_u = 0.005 \text{ GeV}, \quad m_d = 0.01 \text{ GeV}, \quad m_s = 0.2 \text{ GeV} \quad \text{(A3)}$$

and a spectator quark mass of 0.01 GeV.

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