

# Connection between inclusive semileptonic decays of bound and free heavy quarks

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A relativistic constituent quark model, formulated on the light front, is used to derive a new parton approximation for the inclusive semileptonic decay width of the  $B$  meson. A simple connection between the decay rate of a free heavy quark and the one of a heavy quark bound in a meson or in a baryon is established. The main features of the new approach are the treatment of the  $b$  quark as an on-mass-shell particle and the inclusion of the effects arising from the  $b$  quark transverse motion in the  $B$  meson. In a way conceptually similar to the deep-inelastic scattering case, the  $B$  meson inclusive width is expressed as the integral of the free  $b$  quark partial width multiplied by a bound-state factor related to the  $b$ -quark distribution function in the  $B$  meson. The nonperturbative meson structure is described through various quark-model wave functions, constructed via the Hamiltonian light front formalism, using as input both relativistic and nonrelativistic potential models. A link between spectroscopic quark models and  $B$ -meson decay physics is obtained in this way. Our predictions for the  $B \rightarrow X_c l \nu_l$  and  $B \rightarrow X_u l \nu_l$  decays are used to extract the CKM parameters  $|V_{cb}|$  and  $|V_{ub}|$  from available inclusive data. After averaging over the various quark models adopted and including leading-order perturbative QCD corrections, we obtain  $|V_{cb}| = (43.0 \pm 0.7_{\text{exp}} \pm 1.8_{\text{th}}) \times 10^{-3}$  and  $|V_{ub}| = (3.83 \pm 0.48_{\text{exp}} \pm 0.14_{\text{th}}) \times 10^{-3}$ , implying  $|V_{ub}/V_{cb}| = 0.089 \pm 0.011_{\text{exp}} \pm 0.005_{\text{th}}$ , in nice agreement with existing predictions. [S0556-2821(99)02621-1]

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## I. INTRODUCTION

The investigation of inclusive semileptonic  $B$ -meson decays can provide relevant information on the Cabibbo-Kobayashi-Maskawa (CKM) parameters  $|V_{cb}|$  and  $|V_{ub}|$  as well as on the internal nonperturbative structure of the  $B$  meson. In particular, a precise knowledge of  $V_{ub}$  is essential for the description of  $CP$  violation within the standard model and indeed its determination is one of the main goals of the  $b$ -flavored phenomenology.

As far as the theoretical point of view is concerned, the QCD-based operator product expansion (OPE) is widely recognized as a consistent dynamical approach for investigating inclusive heavy-flavor decays [1]. It is also well known [2,3] that an adequate description of the end-point region of the lepton spectrum requires a partial resummation of the most singular terms of the OPE. In this way, the phenomenon of the *Fermi motion* of the heavy quark inside the hadron, already introduced into phenomenological models in a *ad hoc* way a long time ago [4], emerges naturally in the OPE approach. The final result is conceptually similar to the leading-twist term of deep-inelastic lepton-nucleon scattering case. In particular, the effects of Fermi motion are encoded in a heavy-quark distribution function, whose first two moments can be expressed in terms of the matrix elements of the heavy-quark kinetic operator. However, the first few moments do not exhaust the information hidden in the heavy-quark distribution function, which cannot be calculated yet from first principles; therefore, a particular functional form is

chosen in practice (see Refs. [2,3]), introducing consequently a model dependence. In this respect, the use of phenomenological models, like the constituent quark model, could be of great interest as a complementary approach to the OPE resummation method.

Until now, two main phenomenological approaches have been applied to the description of the nonperturbative strong interaction effects in inclusive heavy-flavor decays: the Altaelli-Cabibbo-Corbò-Maiani-Martinelli (ACMM) model [4] and the *exclusive* variant based on the one-by-one summation of various final resonant channels [5–8]. The impact of the Fermi motion in the parton model has also been addressed in Refs. [9–11], where the effects due to the internal motion of the  $b$  quark inside the  $B$  meson have been encoded in a model-dependent quark distribution function. In Ref. [9] the latter has been related to the fragmentation function of heavy-quarks into heavy mesons, while in Refs. [10,11] the bound-state effects have been incorporated via the following distribution function:

$$F(x) \equiv \int d\vec{p}_\perp |\psi(x, p_\perp^2)|^2, \quad (1)$$

where  $|\psi(x, p_\perp^2)|^2$  is the probability to find the  $b$ -quark carrying a light-front (LF) fraction  $x = p_b^+/P_B^+$  of the  $B$ -meson momentum and a transverse relative momentum squared  $p_\perp^2$  ( $\equiv |\vec{p}_\perp|^2$ ). In Ref. [11] the model wave function  $\psi(x, p_\perp^2)$  has been constructed via the Hamiltonian LF for-

malism (see Refs. [12,13]) using as input the canonical wave functions corresponding to various quark potential models.

The important feature of the approach of Refs. [10,11] is the treatment of the  $b$  quark as a virtual particle with mass  $m_b^2 = x^2 M_B^2$ , where  $M_B$  is the  $B$ -meson mass, while the effects due to the transverse motion of the  $b$  quark are neglected everywhere except in the calculation of the distribution function  $F(x)$  in Eq. (1). The final expressions obtained for the semileptonic branching ratios and the lepton energy distributions clearly exhibit a close analogy with the deep-inelastic lepton-nucleon scattering case.

The aim of this paper is to generalize the work of Refs. [9–11] by developing a more refined expression for the inclusive semileptonic decay width.<sup>1</sup> The new features are the treatment of the  $b$  quark as an on-mass-shell particle with mass  $m_b$  [as is required in the Hamiltonian light front (LF) formalism] and the full inclusion of the effects due to the  $b$ -quark transverse momenta. Our main result is the derivation of a new parton formula for the differential inclusive width, which is similar to the one derived by Bjorken *et al.* [15] in the case of infinitely heavy  $b$  and  $c$  quarks, viz.

$$\frac{d\Gamma_{SL}}{dq^2 dq_0} = \frac{d\Gamma_{SL}^{(\text{free})}}{dq^2} \omega(q^2, q_0), \quad (2)$$

where  $d\Gamma_{SL}^{(\text{free})}/dq^2$  is the free-quark differential decay rate and the function  $\omega(q^2, q_0)$  incorporates the effects of the Fermi motion of the heavy quark expressed in terms of the  $b$ -quark distribution function  $|\psi(x, p_\perp^2)|^2$ . In Eq. (2)  $q = p_l + p_{\nu_l}$  is the four-momentum of the lepton pair, and  $q_0$  is the dilepton energy in the  $B$ -meson rest frame. The structure of Eq. (2) suggests that in the limit of heavy quarks with infinite mass (i.e.,  $m_b \rightarrow \infty$  and  $m_c \rightarrow \infty$ ) one has

$$\int dq_0 \omega(q^2, q_0) = 1, \quad (3)$$

which means that the total inclusive width of the hadron is the same as the total inclusive width at the free quark level. The corrections to the free-quark decay picture are mainly due to the difference between the quark mass  $m_b$  and the meson mass  $M_B$  as well as to the *primordial* motion of the  $b$  quark inside the  $B$  meson. These nonperturbative corrections vanish in the heavy-quark limit  $m_b \rightarrow \infty$ , but at finite values of the  $b$ -quark mass a new parton description of inclusive semileptonic decays, based on the constituent quark model, is derived.

The plan of the paper is as follows. Section II contains a brief discussion of the kinematics relevant to inclusive semileptonic  $B$ -meson decays. In Sec. III we derive our main result, Eq. (2), for the differential inclusive semileptonic decay rate. In Sec. IV we compute the semileptonic branching ratio for the processes  $B \rightarrow X_c l \nu_l$  and  $B \rightarrow X_u \ell \nu_\ell$ , paying particular attention to the extraction of  $|V_{cb}|$  and  $|V_{ub}|$  from available inclusive data. The dependence upon the quark

model parameters will be estimated through the use of different meson wave functions, either obtained in a phenomenological way (as in Ref. [10]) or constructed via the Hamiltonian LF formalism from quark potential models (as in Ref. [11]). Finally, our conclusions are summarized in Sec. V.

## II. KINEMATICS

The inclusive semileptonic width  $\Gamma_{SL}$  for the decay process  $B \rightarrow X_{q'} l \bar{\nu}_l$ , where  $l = e, \mu$ , or  $\tau$  and  $X_{q'}$  is any possible hadronic state containing a charm quark ( $q' = c$ ) or a light quark ( $q' = u$ ), can be written in terms of the contraction among the leptonic tensor  $L^{\alpha\beta}$  and the hadronic one  $W_{\alpha\beta}$  [1]:

$$\Gamma_{SL} = \frac{1}{(2\pi)^3} \frac{G_F^2 |V_{bq'}|^2}{M_B} \int d^4q \int d\tau_l L^{\alpha\beta} W_{\alpha\beta}, \quad (4)$$

where  $d^4q = 2\pi |\vec{q}| dq^2 dq_0$ ,  $d\tau_l = |\vec{p}_l| d\Omega_l / (16\pi^2 \sqrt{q^2})$  is the leptonic phase space,  $d\Omega_l$  is the solid angle of the charged lepton  $l$ ,  $|\vec{p}_l| = \sqrt{q^2} \Phi_l / 2$  is its momentum in the dilepton center-of-mass frame and  $\Phi_l \equiv \sqrt{1 - 2\lambda_+ + \lambda_-^2}$ , with  $\lambda_\pm \equiv (m_l^2 \pm m_{\nu_l}^2) / q^2$ . The tensors  $L^{\alpha\beta}$  and  $W_{\alpha\beta}$  in Eq. (4) are explicitly given by

$$L^{\alpha\beta} = 2[p_l^\alpha p_{\nu_l}^\beta + p_l^\beta p_{\nu_l}^\alpha - g^{\alpha\beta} (p_l \cdot p_{\nu_l}) + i \epsilon^{\alpha\beta\gamma\delta} p_{l\gamma} p_{\nu_l\delta}], \quad (5)$$

$$W_{\alpha\beta} = (2\pi)^3 \sum_n \int \prod_{i=1}^n \frac{d\vec{p}_i}{(2\pi)^3 2E_i} \delta^4 \left( P_B - q - \sum_{i=1}^n p_i \right) \times \langle B | j_\alpha^\dagger(0) | n \rangle \langle n | j_\beta(0) | B \rangle, \quad (6)$$

respectively. The summation in Eq. (6) includes all possible final hadronic states,  $j_\alpha(0)$  is the weak current mediating the decay  $b \rightarrow q'$  and  $P_B$  is the  $B$ -meson four-momentum. The hadronic tensor (6) is function of the two invariants  $q^2 = q \cdot q$  and  $q_0 \equiv (q \cdot P_B) / M_B$ , where the latter is related to the invariant mass  $M_X$  of the final hadronic system by  $q_0 = (M_B^2 + q^2 - M_X^2) / 2M_B$ . In what follows we will consider the  $B$ -meson rest frame.

The integral over the leptonic phase space in Eq. (4) is given by

$$\int d\tau_l L^{\alpha\beta} = \frac{1}{4\pi} \frac{|\vec{p}_l|}{\sqrt{q^2}} \langle L^{\alpha\beta} \rangle, \quad (7)$$

with

$$\begin{aligned} \langle L^{\alpha\beta} \rangle &= \frac{1}{4\pi} \int d\Omega_l L^{\alpha\beta} \\ &= \frac{2}{3} \left\{ (1 + \lambda_1) (q^\alpha q^\beta - g^{\alpha\beta} q^2) + \frac{3}{2} \lambda_2 g^{\alpha\beta} q^2 \right\}, \quad (8) \end{aligned}$$

where  $\lambda_1 \equiv \lambda_+ - 2\lambda_-^2$  and  $\lambda_2 \equiv \lambda_+ - \lambda_-^2$ .

<sup>1</sup>A preview of our approach can be found in Ref. [14].

Introducing the dimensionless kinematical variables  $t \equiv q^2/m_b^2$  and  $s \equiv M_X^2/m_b^2$ , the semileptonic width (4) can be cast into the form

$$\Gamma_{SL} = \frac{G_F^2 m_b^5}{(4\pi)^3} |V_{bq'}|^2 \int_{t_{\min}}^{t_{\max}} dt \Phi_{\ell}(t) \int_{s_{\min}}^{s_{\max}} ds \frac{|\vec{q}|}{m_b} \frac{\langle L^{\alpha\beta} \rangle W_{\alpha\beta}}{M_B^2}, \quad (9)$$

where

$$\frac{2|\vec{q}|}{m_b} \equiv \alpha(t, s) = \frac{1}{x_0} \sqrt{(1+x_0^2 t - x_0^2 s)^2 - 4x_0^2 t} \quad (10)$$

and  $x_0 \equiv m_b/M_B$ . In Eq. (9) the limits of integrations in the  $t$ - $s$  plane are given by  $s_{\min} = \zeta^2/x_0^2$ ,  $s_{\max} = (1-x_0\sqrt{t})^2/x_0^2$ ,  $t_{\min} = m_l^2/m_b^2$ , and  $t_{\max} = (1-\zeta)^2/x_0^2$ , where  $\zeta \equiv M_{\text{thr}}/M_B$ , with  $M_{\text{thr}}$  being the lowest mass of the final hadronic state ( $M_{\text{thr}} = M_D$  in case of the  $b \rightarrow c$  transition and  $M_{\text{thr}} = M_{\pi}$  for the  $b \rightarrow u$  transition).

Before closing this section we note that the right-hand side of Eq. (9) is expressed in terms of an integral over physical spectral densities and phase space, both depending on meson masses. As it will be shown in the next section, in our parton picture the heavy-quark mass  $m_b$  emerges as the relevant parameter.

### III. LIGHT FRONT CONSTITUENT QUARK MODEL APPROXIMATION FOR $W_{\mu\nu}$

In this section we apply the constituent quark model to the treatment of semileptonic beauty decays,  $B \rightarrow X_q l \nu_l$ , in close analogy with the parton approximation in deep-inelastic lepton-nucleon scattering. Our approach is based on (i) the hypothesis of quark-hadron duality, which assumes that, when a sufficient number of exclusive hadronic decay modes is summed up, the decay probability into hadrons equals the one of its partons, and (ii) the dominance of the valence component in the  $B$ -meson wave function. Following this assumption, the hadronic tensor  $W_{\alpha\beta}$  is given through the optical theorem by the imaginary part of the quark box diagram describing the forward scattering amplitude [9–11]:

$$W_{\alpha\beta} = \int_0^1 \frac{dx}{x} \int d\vec{p}_{\perp} w_{\alpha\beta}^{(bq')} (p_b, p_{q'}) \delta[(p_b - q)^2 - m_q^2] \theta(\varepsilon_{q'}) |\psi(x, p_{\perp}^2)|^2, \quad (11)$$

where  $p_b \equiv (p_b^+, p_b^-, \vec{p}_{\perp}) = [xM_B, (m_b^2 + p_{\perp}^2)/xM_B, \vec{p}_{\perp}]$  with  $p_b^2 = p_b^- p_b^+ - p_{\perp}^2 = m_b^2$  and the quark tensor  $w_{\alpha\beta}^{(bq')} (p_b, p_{q'})$  is defined analogously to the lepton tensor in Eq. (5):

$$w_{\alpha\beta}^{(bq')} (p_b, p_{q'}) = 4[p_{q'\alpha} p_{b\beta} + p_{q'\beta} p_{b\alpha} - g_{\alpha\beta} (p_{q'} \cdot p_b) + i \epsilon_{\alpha\beta\gamma\delta} p_{q'}^{\gamma} p_b^{\delta}]. \quad (12)$$

Equation (11) corresponds to the average of the free-quark decay distribution over the motion of the heavy quark, described by the distribution function  $|\psi(x, p_{\perp}^2)|^2$ , whose nor-

malization is given by  $\int_0^1 dx \int d\vec{p}_{\perp} |\psi(x, p_{\perp}^2)|^2 = 1$ . In Eq. (11) the function  $\theta(\varepsilon_{q'})$ , where  $\varepsilon_{q'}$  is the  $q'$ -quark energy, is inserted for consistency with the use of the valence LF wave function  $\psi(x, p_{\perp}^2)$ , while the  $\delta$  function, expressing the decay of the  $b$ -quark to a  $q'$ -quark, can be rewritten as

$$\delta[(p_b - q)^2 - m_{q'}^2] = \frac{x}{x_0} \frac{m_b}{q^+} \delta(p_{\perp}^2 - p_{\perp}^{*2}), \quad (13)$$

where  $q^+$  is the LF *plus* component of the dilepton momentum,  $q^+ = q_0 + |\vec{q}|$ , and

$$p_{\perp}^{*2} = m_b^2 \left[ \frac{x}{x_0} \frac{m_b}{q^+} (1+t-\rho) - \left( \frac{x}{x_0} \frac{m_b}{q^+} \right)^2 t - 1 \right], \quad (14)$$

with  $\rho \equiv m_c^2/m_b^2$  being the quark mass ratio squared.

We now substitute Eq. (11) into (9) and use the fact that the contraction of the *averaged* lepton tensor  $\langle L^{\alpha\beta} \rangle$  and the quark tensor  $w_{\alpha\beta}^{bq'}$  does not depend on  $x$  and  $s$  and, therefore, can be taken out of the integral over  $x$  in Eq. (11); the explicit expression for the tensor contraction is given in the Appendix [see Eqs. (A6)–(A9)]. The differential decay rate becomes

$$\begin{aligned} \frac{d^2\Gamma_{SL}}{dt ds} &= \frac{G_F^2 m_b^5}{(4\pi)^3} |V_{bq'}|^2 \frac{|\vec{q}|}{m_b} \Phi_{\ell}(t) \\ &\times \frac{\langle L^{\alpha\beta} \rangle w_{\alpha\beta}^{(bq')}}{M_B^2} \frac{\pi}{x_0} \frac{m_b}{q^+} \int_{x_1}^{\min[1, x_2]} dx |\psi(x, p_{\perp}^{*2})|^2, \end{aligned} \quad (15)$$

where the integration limits follow from the condition  $p_{\perp}^{*2} \geq 0$ , viz.,

$$x_{1,2} = x_0 \frac{q^+}{\tilde{q}^{\pm}} = x_0 \frac{q_0 + |\vec{q}|}{\tilde{q}_0 \pm |\vec{q}|} \quad (16)$$

with  $\tilde{q}_0$  ( $\tilde{q}$ ) being the energy (three-momentum) of the lepton pair in the  $b$ -quark rest frame, viz.

$$\frac{2\tilde{q}_0}{m_b} = 1 + t - \rho, \quad \frac{2|\vec{q}|}{m_b} \equiv \tilde{\alpha}(t, \rho) = \sqrt{(1+t-\rho)^2 - 4t}. \quad (17)$$

To proceed further we do not need the explicit expression for the contraction  $\langle L^{\alpha\beta} \rangle w_{\alpha\beta}^{(bq')}$ , which any way can be easily calculated using Eqs. (8) and (12) [see Eqs. (A6) and (A9) in the Appendix]. Instead we note that the same contraction appears in the differential semileptonic decay width of a free  $b$  quark into a  $q'$  quark

$$\frac{d\Gamma_{SL}^{(\text{free})}(t)}{dt} = \frac{G_F^2 m_b^5}{(4\pi)^3} |V_{bq'}|^2 \frac{|\vec{q}|}{m_b} \Phi_{\ell}(t) \frac{\langle L^{\alpha\beta} \rangle w_{\alpha\beta}^{(bq')}}{m_b^4}. \quad (18)$$

Using Eq. (18) we can express the contraction  $\langle L^{\alpha\beta} \rangle w_{\alpha\beta}^{bq'}$  through  $d\Gamma_{SL}^{(\text{free})}/dt$ . Then, from Eq. (15) we obtain our main result for the semileptonic decay width of the  $B$  meson

$$\frac{d\Gamma_{SL}}{dt} = \frac{d\Gamma_{SL}^{(\text{free})}(t)}{dt} \int_{s_{\min}}^{s_{\max}} ds \omega(t, s), \quad (19)$$

where the bound-state function  $\omega(t, s)$  is defined as

$$\omega(t, s) = m_b^2 x_0 \frac{\pi m_b}{q^+} \frac{|\vec{q}|}{|\vec{q}|} \int_{x_1}^{\min[1, x_2]} dx |\psi(x, p_{\perp}^{*2})|^2. \quad (20)$$

In Eq. (19) the region of integration over the final invariant hadron mass is characterized by a *quark* threshold  $M_{\text{thr}}^{(0)} = m_c$ , defined through the condition  $x_1 = \min[1, x_2]$  in Eq. (20), which differs from the hadronic threshold (i.e.,  $M_D$  for  $q' = c$  or  $M_{\pi}$  for  $q' = u$ ).<sup>2</sup> Explicitly one gets  $s_{\min} = \rho = (m_c/m_b)^2$  and  $s_{\max} = x_0^2(1 - x_0\sqrt{t})^2$ . Moreover, since the inequality  $t \leq (1 - \sqrt{\rho})^2$  follows from Eq. (17), the bound-state factor (20) as well as the structure functions (A8) are identically vanishing for  $q^2 \geq (m_b - m_{q'})^2$ .

To sum up, the inclusive semileptonic decay width, Eq. (19), is expressed as an integral of two factors. The first one is the *parton* differential decay rate (18) of a free heavy quark, which sets the overall scale for the decay rates and does not depend on the spectator quark. The second factor [Eq. (20)], which incorporates the nonperturbative corrections to the free-quark result (now depending on the spectator quark), is given by an integral over the distribution function of the  $b$  quark. In the heavy-quark limit,  $m_b \rightarrow \infty$ , the  $b$ -quark distribution in the  $B$ -meson becomes a delta function peaked at  $x = 1$  [more precisely  $\delta(x - 1) \cdot \delta(\vec{p}_{\perp})$ ], which implies that the function  $\omega(t, s)$  goes to  $\delta(s - \rho)$  for  $m_b \rightarrow \infty$ , leading to the sum rule (3). At finite values of the  $b$ -quark mass our result for  $\Gamma_{SL}$  exhibits an  $m_b$  dependence of the following general form:  $\Gamma_{SL} \propto m_b^5 [1 + c/m_b + O(1/m_b^2)]$ , where  $c$  is a nonvanishing coefficient depending on the particular quark model adopted. However, the mass  $m_b$  is the constituent mass of the  $b$  quark, which may differ from the pole quark mass  $\mu_b$  commonly appearing in the OPE of the  $B$ -meson decay rate (see Refs. [1] and [16]). Assuming  $m_b = \mu_b - c/5 + O(1/\mu_b)$ , the well-known result [1] of the absence of the  $1/\mu_b$  corrections to the free-quark decay may be recovered. The above argument is completely analogous to that used to eliminate the  $1/\mu_b$  corrections from the total width in the ACCMM model [17].

#### IV. NUMERICAL RESULTS

After having described the theoretical tools involved in the calculation of the inclusive rate, we now focus on the

practical way for the extraction of the CKM parameters  $|V_{cb}|$  and  $|V_{ub}|$  from available inclusive data. The nonperturbative ingredient in Eq. (20) is the meson wave function  $\psi(x, p_{\perp}^2)$ . In what follows, we will adopt both a phenomenological ansatz and the wave functions corresponding to various quark potential models. As for the phenomenological wave function, we use the exponential ansatz introduced in Ref. [10], which reads as

$$\psi(x, p_{\perp}^2) = \frac{\mathcal{N}}{\sqrt{(1-x)}} \exp \left\{ -\frac{\lambda_0}{2} \left[ \frac{1-x}{\xi_0} + \frac{\xi_0}{1-x} \left( 1 + \frac{p_{\perp}^2}{m_{sp}^2} \right) \right] \right\}, \quad (21)$$

where  $m_{sp}$  is the mass of the spectator quark in the  $B$  meson,  $\xi_0 \equiv m_{sp}/M_B$ ,  $\mathcal{N}$  is a normalization constant and  $\lambda_0 \approx 1$  is an adjustable parameter obtained in Ref. [10] from a fit of the experimental data on the differential decay rate of the exclusive process  $B \rightarrow D^* l \nu_l$ . In what follows we will refer to the phenomenological ansatz (21) as the quark model A. The other models considered ( $B$  to  $E$ ) are based on LF-type meson wave functions, which can be written in terms of the radial wave function  $w(p^2)$  corresponding to a particular quark potential model as [11]

$$\psi^{\text{LF}}(x, p_{\perp}^2) = \sqrt{\frac{M_0}{4x(1-x)}} \left[ 1 - \left( \frac{m_b^2 - m_{sp}^2}{M_0^2} \right)^2 \right] \frac{w(p^2)}{\sqrt{4\pi}}, \quad (22)$$

where  $M_0 \equiv \sqrt{m_b^2 + p^2} + \sqrt{m_{sp}^2 + p^2} = [(p_{\perp}^2 + m_b^2)/x + (p_{\perp}^2 + m_{sp}^2)/(1-x)]^{1/2}$  is the free mass and  $p^2 \equiv p_{\perp}^2 + p_z^2$ , with  $p_z = (x - 1/2)M_0 + (m_{sp}^2 - m_b^2)/2M_0$ . In particular, models  $B - E$  correspond to the radial wave functions  $w(p^2)$  of the potential models of Refs. [18,6,19,20], respectively. The main difference among the various quark models lies in the behavior of  $w(p^2)$  at high values of the internal momentum  $p$ . Models  $B$  and  $C$  correspond to a soft Gaussian ansatz, which takes into account mainly the effects of the confinement scale, whereas the functions  $w(p^2)$  corresponding to models  $D$  and  $E$  exhibit high momentum components generated by the effective one-gluon exchange part of the interquark potential. In case of models  $A$ ,  $D$ , and  $E$  the distribution function  $F(x)$  [Eq. (1)] has been already calculated in Ref. [11]. It turns out that the models  $A - D$  yield quite similar results for  $F(x)$ , whereas model  $E$  predicts a remarkably broader  $x$  distribution. In terms of the mean value  $\langle x \rangle \equiv \int_0^1 dx x F(x)$  and the variance  $\sigma^2 \equiv \int_0^1 dx x(x - \langle x \rangle)^2 F(x)$ , one gets  $\langle x \rangle \approx 0.90$  and  $\sqrt{\sigma^2} \approx 0.06$  for models  $A - D$ , while in case of model  $E$  [the Godfrey-Isgur (GI) potential model] one has  $\langle x \rangle \approx 0.87$  and  $\sqrt{\sigma^2} \approx 0.09$ .<sup>3</sup> Such a striking difference is directly related to the larger mean value of the internal momentum characterizing the GI model with respect to the others cases  $A - D$ . The values of the constituent quark

<sup>2</sup>Note that the *quark* threshold  $M_{\text{thr}}^{(0)} = m_c$  differs also from the parameter  $M_{th}$ , which was introduced in Refs. [10,11] with the aim of separating the exclusive  $D$  and  $D^*$  channels from the hadron continuum.

<sup>3</sup>Note that  $\langle x \rangle$  does not generally coincide with the location of the maximum of  $F(x)$  as well as with the value of  $x_0$ .

TABLE I. The values of the constituent quark masses (in GeV) and of the average internal momentum squared  $\langle p^2 \rangle$  (in GeV<sup>2</sup>) for the quark models  $A-E$  (see text). The value of the  $B$  meson mass is  $m_B = 5.279$  GeV [22].

Model	A	B	C	D	E
$m_b$	4.800	4.880	5.200	5.237	4.977
$m_c$	1.400	1.550	1.820	1.835	1.628
$m_u$	0.300	0.330	0.330	0.337	0.220
$M_B - m_b$	0.479	0.399	0.079	0.042	0.302
$m_b - m_c$	3.400	3.330	3.380	3.402	3.349
$m_b - m_u$	4.500	4.550	4.870	4.900	4.757
$m_b/M_B$	0.909	0.924	0.985	0.992	0.943
$m_c/m_b$	0.292	0.318	0.350	0.350	0.327
$m_u/m_b$	0.063	0.068	0.063	0.064	0.044
$\langle p^2 \rangle$	0.234	0.252	0.277	0.262	0.553

masses as well as the mean value  $\langle p^2 \rangle \equiv \int_0^\infty dp p^4 |w(p^2)|^2$  for the different quark models considered in this paper are collected in Table I.

We have calculated Eqs. (19),(20) in case of the decay processes  $B \rightarrow X_c l \nu_l$  and  $B \rightarrow X_u l \nu_l$  adopting the five quark models  $A$  to  $E$  [in all these models the physical mass of the  $B$  meson,  $M_B = 5.279$  GeV, has been taken from the recent Particle Data Group (PDG) publication [22]]. We have also considered that perturbative QCD corrections lead approximately to an additional multiplicative factor  $J_{LO}^{\text{pert}}$  in Eq. (19), which typically reduces the semileptonic decay width. At leading order the correction associated with the running coupling constant  $\alpha_s$  is well known [23] and for the  $b \rightarrow c$  ( $b \rightarrow u$ ) transition we will consider in what follows the value  $J_{LO}^{\text{pert}} = 0.90$  (0.85). Our results for the branching ratio  $\Gamma_{SL}/\Gamma_B^{(\text{exp})}$ , where  $\Gamma_B^{(\text{exp})}$  is the experimental  $B$ -meson width ( $\Gamma_B^{(\text{exp})} = 1/\tau_B^{(\text{exp})}$  with  $\tau_B^{(\text{exp})} = 1.57 \pm 0.04$  ps [21]), are collected in Table II for the process  $B \rightarrow X_c l \nu_l$  with the branching ratio being given in units of  $10^{-2} (|V_{cb}|/0.040)^2 (\tau_B^{(\text{exp})}/1.57 \text{ ps})$ . It can be seen that the nonperturbative effects, mocked up in the  $b$ -quark distribution function  $|\psi(x, p_\perp^2)|^2$ , are mainly related both to the broadening of the  $x$  distribution, generated by the high-momentum components of the  $B$ -meson wave function (see model  $E$ ), and to the quark mass ratio  $\sqrt{\rho} = m_{q'}/m_b$  (see Table I). The values predicted for the semileptonic branching ratio  $\Gamma_{SL}/\Gamma_B^{(\text{exp})}$  exhibit a

TABLE II. The branching ratio  $\Gamma_{SL}/\Gamma_B^{(\text{exp})}$  for the inclusive process  $B \rightarrow X_c l \nu_l$  (with  $l = e, \mu$ ) in units of  $10^{-2} \times (|V_{cb}|/0.040)^2 (\tau_B^{(\text{exp})}/1.57 \text{ ps})$ , calculated within the quark models  $A-E$  and considering for the pQCD corrections an overall reduction factor equal to 0.90 [23]. The values of  $|V_{cb}|$  in units of  $10^{-3} \times \sqrt{Br_{SL}^{(\text{exp})}/10.43\%} \sqrt{1.57 \text{ ps}/\tau_B^{(\text{exp})}}$ , obtained within the various quark models considered, are also reported.

Model	A	B	C	D	E
$\Gamma_{SL}/\Gamma_B^{(\text{exp})}$	10.1	9.2	9.0	9.1	8.0
$ V_{cb} $	40.7	42.6	43.1	42.8	45.6

TABLE III. The branching ratio  $\Gamma_{SL}/\Gamma_B^{(\text{exp})}$  of the inclusive process  $B \rightarrow X_u l \nu_l$  (with  $l = e, \mu$ ) in units  $10^{-2} \times (|V_{ub}|/0.0032)^2 (\tau_B^{(\text{exp})}/1.57 \text{ ps})$ , calculated within the quark models  $A-E$  and considering for the pQCD corrections an overall reduction factor equal to 0.85 [23]. The values of  $|V_{ub}|$  in units of  $10^{-3} \times \sqrt{Br_{SL}^{(\text{exp})}/0.16\%} \sqrt{1.57 \text{ ps}/\tau_B^{(\text{exp})}}$  and the values of the ratio  $|V_{ub}|/|V_{cb}|$ , obtained within the various quark models  $A-E$ , are also reported.

Model	A	B	C	D	E
$\Gamma_{SL}/\Gamma_B^{(\text{exp})}$	0.109	0.108	0.121	0.123	0.102
$ V_{ub} $	3.88	3.89	3.69	3.64	4.01
$ V_{ub} / V_{cb} $	0.095	0.091	0.086	0.085	0.088

model dependence of about  $\pm 10\%$ . Finally, assuming the experimental world average value  $Br_{SL}^{(\text{exp})}(B \rightarrow X_c e \nu_e) = (10.43 \pm 0.24)\%$  [22], the CKM matrix element  $|V_{cb}|$  can easily be obtained from the predicted values of the  $SL$  branching ratio and the corresponding results are reported in Table II. The average over the various quark-model predictions yields

$$|V_{cb}| = (43.0 \pm 0.7_{\text{exp}} \pm 1.8_{\text{th}}) \times 10^{-3} \times \sqrt{\frac{Br_{SL}^{(\text{exp})}}{10.43\%}} \sqrt{\frac{1.57 \text{ ps}}{\tau_B^{(\text{exp})}}}, \quad (23)$$

where the experimental errors of the branching ratio and the  $B$ -meson lifetime have been taken in quadrature. Our LF prediction (23) is consistent with the updated ‘‘experimental’’ determination of  $|V_{cb}|$  [24],<sup>4</sup>  $|V_{cb}|_{\text{incl}} = (39.8 \pm 0.9_{\text{exp}} \pm 4.0_{\text{th}}) \times 10^{-3}$ , as well as with the recent OPE analysis of Ref. [25],  $|V_{cb}|_{\text{incl}} = (41.3 \pm 1.6_{\text{exp}} \pm 2.0_{\text{th}}) \times 10^{-3}$ .

The other inclusive process we want to consider is the semileptonic decay  $B \rightarrow X_u l \nu_l$ . The existence of the  $b \rightarrow u l \nu_l$  transition has been demonstrated few years ago by the CLEO [26,27] and ARGUS [28] Collaborations through the observation of semileptonic  $B$ -meson decays with leptons that are too energetic to originate from the  $b \rightarrow c l \nu_l$  transition. Very recently, the CLEO Collaboration [29] has reported the first signal for exclusive semileptonic decays of the  $B$ -meson into charmless final states. In Ref. [30] the ALEPH Collaboration has announced a model-independent measurement of the inclusive  $b \rightarrow u l \nu_l$  width, viz.  $Br(b \rightarrow u l \nu_l) = (0.16 \pm 0.04)\%$ . The most important application of the analysis of the inclusive decay  $B \rightarrow X_u l \nu_l$  is the extraction of the CKM parameter  $|V_{ub}|$ . Our results obtained for the branching ratio  $\Gamma_{SL}/\Gamma_B^{(\text{exp})}$  and for the CKM parameter  $|V_{ub}|$  extracted adopting the ALEPH value for the experimental semileptonic branching ratio [i.e.  $Br_{SL}^{(\text{exp})}(B \rightarrow X_u e \nu_e) = (0.16 \pm 0.04)\%$ ], are reported in Table III. Using the val-

<sup>4</sup>The value quoted in Ref. [24] corresponds to  $Br_{SL}^{(\text{exp})} = 10.77 \pm 0.43\%$  and  $\tau_B^{(\text{exp})} = 1.60 \pm 0.03$  ps. After correcting for the values adopted in this paper, one gets  $|V_{cb}|_{\text{incl}} = (39.5 \pm 0.9_{\text{exp}} \pm 4.0_{\text{th}}) \times 10^{-3}$ .

ues of  $|V_{cb}|$  obtained in Table II, also our predictions for the ratio  $|V_{ub}/V_{cb}|$  are reported in Table III. It can be seen that the model dependence of the predicted branching ratio is about  $\pm 10\%$  as in case of the decay process  $B \rightarrow X_c l \nu_l$  (see Table II). Averaging our predictions over the various quark models, one gets

$$|V_{ub}| = (3.83 \pm 0.48_{\text{exp}} \pm 0.14_{th}) \times 10^{-3} \times \sqrt{\frac{Br_{SL}^{(\text{exp})}}{0.16\%}} \cdot \sqrt{\frac{1.57 \text{ ps}}{\tau_B^{(\text{exp})}}}, \quad (24)$$

$$|V_{ub}/V_{cb}| = 0.089 \pm 0.011_{\text{exp}} \pm 0.005_{th}. \quad (25)$$

Our result for  $|V_{ub}/V_{cb}|$  [Eq. (25)] is consistent with the model-independent value derived from the QCD-based heavy-quark expansion,  $|V_{ub}/V_{cb}| = 0.098 \pm 0.013$  [31], and with the value extracted from the measurement of the endpoint region of the lepton spectrum,  $|V_{ub}/V_{cb}| = 0.08 \pm 0.01_{\text{exp}} \pm 0.02_{th}$  [27,28], as well as with the result obtained from a LF analysis of the exclusive decays  $B \rightarrow D l \nu_l$  and  $B \rightarrow \pi l \nu_l$ ,  $|V_{ub}/V_{cb}| = 0.082 \pm 0.016$  [13]. Furthermore, our result for  $|V_{ub}|$  [Eq. (24)] is consistent within the errors with the value  $|V_{ub}| = (2.9 \pm 0.4) \times 10^{-3}$ , obtained in Ref. [13] from a LF analysis of the exclusive decay  $B \rightarrow \pi l \nu_l$ , and with the finding  $|V_{ub}| = (3.2 \pm 0.4) \times 10^{-3}$ , obtained in Ref. [32] after averaging over the exclusive  $B \rightarrow \pi l \nu_l$  and  $B \rightarrow \rho l \nu_l$  decay modes, as well as with the result  $|V_{ub}| = (3.3 \pm 0.2_{-0.4}^{+0.3} \pm 0.5) \times 10^{-3}$  quoted in a recent CLEO report [33].

Note that the larger uncertainty in our extracted value of  $|V_{ub}|$  [Eq. (24)] is the experimental one, mainly because of the large error quoted by the ALEPH Collaboration [30]. An interesting way to obtain a better determination of  $|V_{ub}|$  has been proposed recently in Ref. [34] and it is based on the investigation of the recoil mass spectrum in the  $B \rightarrow X_u l \nu_l$  decays at  $M_X \leq M_{\text{max}} < M_D$ . The suggested value for  $M_{\text{max}}$  is 1.5 GeV, chosen in order to avoid the leakage of the tail of the  $B \rightarrow X_c l \nu_l$  transitions, which can occur because of the finite resolution of the experiments. We have therefore evaluated the partial branching ratio,  $\Gamma_{SL}^*/\Gamma_B^{(\text{exp})}$ , obtained by cutting the integration over  $s$  in Eq. (19) at the value  $s_{\text{max}} = (1.5 \text{ GeV}/m_b)^2$ . The results for  $\Gamma_{SL}^*/\Gamma_B^{(\text{exp})}$  in units of  $10^{-2} \times (|V_{ub}|/0.0032)^2 (\tau_B^{(\text{exp})}/1.57 \text{ ps})$  are 0.073, 0.064, 0.074, 0.078, 0.060 in case of the quark models A–E, respectively, which correspond to a fraction of  $b \rightarrow u$  decays with  $M_X \leq 1.5 \text{ GeV}$  equal to 67, 59, 61, 63, and 59%. Their average is  $0.070 \pm 0.008$  in comparison with the total (averaged) branching ratio of  $0.113 \pm 0.009$  in the same units (see Table III), corresponding to a fraction of  $(62 \pm 4)\%$  of  $b \rightarrow u$  decays with  $M_X \leq 1.5 \text{ GeV}$ . It follows that the model dependence is only slightly increased by cutting the integration over the recoil mass  $M_X$  at  $M_{\text{max}} = 1.5 \text{ GeV}$ , at the price of reducing the number of  $B \rightarrow X_u l \nu_l$  events by  $\approx 40\%$ , in overall agreement with the findings of Ref. [34].

## V. CONCLUSIONS

In this paper we have derived a new parton formula, which establishes a simple connection between the decay

rate of a free heavy-quark and the one of a heavy-quark bound in a hadron. The main features of our approach are the treatment of the  $b$  quark as an on-mass-shell particle and the inclusion of the effects arising from the  $b$ -quark transverse motion in the  $B$  meson. Our main result is Eq. (2), or more precisely Eqs. (19), (20).

Another result of this paper is the determination of the CKM parameters  $|V_{cb}|$  and  $|V_{ub}|$  using the available experimental values for the branching ratio of the processes  $B \rightarrow X_c l \nu_l$  and  $B \rightarrow X_u l \nu_l$ . Our results exhibit a model dependence related mainly to the uncertainties associated to the nonprecise knowledge of the primordial  $b$ -quark distribution function and to the values of the constituent quark masses. These uncertainties lead to a final theoretical uncertainty in the extracted value of both  $|V_{cb}|$  and  $|V_{ub}|$  of about  $\pm 5\%$ . Including leading-order perturbative QCD corrections, we have found  $|V_{cb}| = (43.0 \pm 0.7_{\text{exp}} \pm 1.8_{th}) \times 10^{-3}$  and  $|V_{ub}| = (3.83 \pm 0.48_{\text{exp}} \pm 0.14_{th}) \times 10^{-3}$ , which imply  $|V_{ub}/V_{cb}| = 0.089 \pm 0.011_{\text{exp}} \pm 0.005_{th}$ , in nice agreement with existing predictions.

In conclusion, we point out that our LF approach can be applied to the investigation of lepton energy spectra and to inclusive processes other than semileptonic decays, such as, e.g., the nonleptonic branching ratios both for external and internal types of decays [10,11]. Another field of application is the calculation of the lifetime of the  $B_c$  meson and the theoretical estimation of the inclusive  $b \rightarrow s \gamma$  width.

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## APPENDIX

Let us remind that the dimensionless tensor  $W_{\alpha\beta}$  in Eq. (6) can be decomposed into five different Lorentz covariants, involving therefore five structure functions,  $W_1(q^2, q_0)$  to  $W_5(q^2, q_0)$ : viz,

$$\begin{aligned} W_{\alpha\beta} = & -g_{\alpha\beta} W_1(q^2, q_0) + v_\alpha v_\beta W_2(q^2, q_0) \\ & - i \epsilon_{\alpha\beta\gamma\delta} u^\gamma v^\delta W_3(q^2, q_0) + (v_\alpha u_\beta + u_\alpha v_\beta) W_4(q^2, q_0) \\ & + u_\alpha u_\beta W_5(q^2, q_0), \end{aligned} \quad (A1)$$

where  $v \equiv P_B/M_B$  is the four-velocity of the  $B$  meson and  $u \equiv q/M_B$ . Note that the terms proportional to  $u_\alpha$  or  $u_\beta$  in Eq. (8) do not contribute to the differential decay rate if the lepton masses can be neglected. Therefore, for  $l=e$  or  $\mu$  only  $W_1(q^2, q_0)$ ,  $W_2(q^2, q_0)$ , and  $W_3(q^2, q_0)$  are actually relevant. The dimensionless structure functions  $W_i(q^2, q_0)$  are functions of the two invariants  $q^2$  and  $q_0 \equiv v \cdot q$ .

All the inclusive observables can be expressed in terms of the structure functions  $W_i(q^2, q_0)$  ( $i=1,5$ ). Thus, the contraction  $\langle L^{\alpha\beta} \rangle W_{\alpha\beta}/M_B^2$  in Eq. (9) is given by

$$\frac{\langle L^{\alpha\beta} \rangle W_{\alpha\beta}}{M_B^2} = \frac{4}{3} x_0^2 F(t, s), \quad (\text{A2})$$

where

$$F(t, s) \equiv 3t(1 + \lambda_1 - 2\lambda_2)W_1(t, s) + \left[ (1 + \lambda_1) \frac{|\vec{q}|^2}{m_b^2} + \frac{3}{2} \lambda_2 t \right] W_2(t, s) + \frac{3}{2} \lambda_2 t [(1 + x_0^2 t - x_0^2 s)W_4(t, s) + x_0^2 t W_5(t, s)]. \quad (\text{A3})$$

We have derived our basic result (19),(20) without calculating explicitly the structure functions  $W_i(t, s)$  in our LF parton approximation (11). However, since we have in mind also more general applications, such as, e.g., the calculation of the lepton energy spectrum, we now present our LF formulas for the functions  $W_i(t, s)$ . To this end, after averaging over the azimuthal angle  $\varphi$  of the transverse momentum  $p_\perp$ , the quark tensor  $w_{\alpha\beta}^{bq'}(p_b, p_{q'})$  can be cast into the form

$$\int \frac{d\varphi}{2\pi} w_{\alpha\beta}^{bq'}(p_b, p_{q'}) = m_b^2 \{ -g_{\alpha\beta} w_1 + v_\alpha v_\beta w_2 - i \epsilon_{\alpha\beta\gamma\delta} v^\gamma u^\delta w_3 + (v_\alpha u_\beta + v_\beta u_\alpha) w_4 + u_\alpha u_\beta w_5 \}, \quad (\text{A4})$$

which implies ( $i=1,5$ )

$$W_i(t, s) = m_b^2 \frac{\pi}{r} \int_{x_1}^{\min[1, x_2]} dx w_i(t, s; x) |\psi(x, p_\perp^2)|^2, \quad (\text{A5})$$

where  $r \equiv x_0 q^+ / m_b$ . Note that the contraction  $\langle L^{\alpha\beta} \rangle w_{\alpha\beta}^{bq'}$  of the lepton and quark tensors is given in terms of the functions  $w_i$  by

$$\frac{\langle L^{\alpha\beta} \rangle w_{\alpha\beta}^{bq'}}{m_b^4} = \frac{4}{3} \tilde{F}(t, \rho), \quad (\text{A6})$$

where

$$\tilde{F}(t, \rho) \equiv 3t(1 + \lambda_1 - 2\lambda_2)w_1 + \left[ (1 + \lambda_1) \frac{|\vec{q}|^2}{m_b^2} + \frac{3}{2} \lambda_2 t \right] w_2 + \frac{3}{2} \lambda_2 t [(1 + x_0^2 t - x_0^2 s)w_4 + x_0^2 t w_5]. \quad (\text{A7})$$

The calculation of the functions  $w_i(t, s; x)$  is straightforward and leads to

$$\begin{aligned} w_1(t, s; x) &= 2[(\xi - 1/2)z_0 - \xi^2 t], \\ w_2(t, s; x) &= \frac{8}{\alpha^2(t, s)} \left[ 3\xi^2 t^2 - 3\xi t z_0 + t + \frac{1}{2} z_0^2 \right], \\ w_3(t, s; x) &= -\frac{2}{x_0 \alpha(t, s)} [z_0 - 2\xi t], \\ w_4(t, s; x) &= \frac{4}{x_0^2 \alpha^2(t, s)} \left\{ \left[ x - \frac{x_0^2}{r} (z_0 - \xi t) - \frac{x_0 \alpha(t, s)}{2} \right] \right. \\ &\quad \left. \times (z_0 - 2\xi t) + [\xi z_0 - \xi^2 t - 1](1 + x_0^2 t - x_0^2 s) \right\}, \\ w_5(t, s; x) &= \frac{4}{x_0^4 \alpha^2(t, s)} \left\{ \left[ x - \frac{x_0^2}{r} (z_0 - \xi t) \right] \left[ x - x_0^2 \frac{\xi}{x} (z_0 - \xi t) - x_0 \alpha(t, s) \right] \right. \\ &\quad \left. - 2x_0^2 [\xi z_0 - \xi^2 t - 1] \right\}, \quad (\text{A8}) \end{aligned}$$

where  $\xi \equiv x/r$ ,  $z_0 \equiv 1 + t - \rho$  and the quantity  $\alpha(t, s)$  is defined by Eq. (10). Using Eq. (A8) it can be easily verified that

$$\tilde{F}(t, \rho) = (1 + \lambda_1) [(1 - \rho)^2 + (1 + \rho)t - 2t^2] - 3\lambda_2 t (1 + \rho - t). \quad (\text{A9})$$

Using Eqs. (18), (A6), and (A9) one obtains the well-known result

$$\frac{d\Gamma_{SL}^{(\text{free})}(t)}{dt} = \frac{G_F^2 m_b^5}{48\pi^3} |V_{bq'}|^2 \frac{|\vec{q}|}{m_b} \Phi_l(t) \{ (1 + \lambda_1) [(1 - \rho)^2 + (1 + \rho)t - 2t^2] - 3\lambda_2 t (1 + \rho - t) \}. \quad (\text{A10})$$

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