

Hunting the vector hybrid

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(Received 2 February 1999; published 3 November 1999)

The current state of analysis of e^+e^- annihilation below 2.0 GeV and of the vector component of τ decay is reviewed. The evidence for and against the presence of hybrid vectors is discussed. It is concluded that the data strongly favor their inclusion, and the consequences of this are outlined. [S0556-2821(99)02021-4]

PACS number(s): 12.39.Mk, 12.39.Jh, 13.25.Jx

I. INTRODUCTION

It has been recognized for some time that the data on vector meson decays appear to be in conflict with the predictions of the 3P_0 model which has become the standard for calculating meson decays. One solution has been to suggest that the physical light vectors are mixed $q\bar{q}$ and hybrid vectors, as the latter have appropriate decay characteristics. However this has never been quantified, nor have alternative non-hybrid explanations been actively sought. Here we explore the limits of the 3P_0 model and apply a specific non- 3P_0 model to the vector decays in an attempt to avoid the introduction of hybrids. Constraints placed on these models by other decays, in particular those of the pseudoscalars $\eta(1295)$ and $\pi(1300)$, are sufficient to prevent them from providing a solution to the vector decay problem. Given that the inclusion of hybrids is unavoidable, we consider the advantages and disadvantages of the flux tube model and the constituent gluon model of hybrids in the context of the vector decays. The data prefer the constituent gluon model, and we outline briefly the consequences elsewhere of this choice.

The current information on light-quark vectors from e^+e^- annihilation and τ decay is discussed in Sec. II, and the decay problems identified. Present understanding of gluonic excitations in general and of hybrid models in particular is summarized in Sec. III. In addition to the standard 3P_0 approach, which models the string breaking, we suggest a specific hadronic ansatz for relevant light quarkonia decays in analogy to the decays of heavy quarkonia. These two approaches are evaluated in the context of pseudoscalar decays, specifically $\eta(1295)$ and $\pi(1300)$, in Sec. IV and limits put on the corresponding vector decays. These latter results are confronted with the data in Sec. V, where it is shown that the limits are too restrictive to resolve the problems identified in Sec. II. This leads naturally to a detailed consideration of the two available models for hybrids, and the data appear to favor the constituent gluon model over the flux tube model. Our conclusions and their consequences are summarized in Sec. VI.

II. e^+e^- ANNIHILATION AND τ DECAY

The existence of the isovector $\rho(1450)$ and $\rho(1700)$, and their isoscalar counterparts $\omega(1420)$ and $\omega(1600)$ is now well established [1]. The key experimental results in determining the existence of the two isovector states were $e^+e^- \rightarrow \pi^+\pi^-$ [2] and $e^+e^- \rightarrow \omega\pi$ [3]. These original data sets have subsequently been augmented by data on the corresponding charged channels in τ decay [4,5], to which they are related by conservation of vector current (CVC). These new data confirm the earlier conclusions. The data on $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ [6] and $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ [6,7] (excluding $\omega\pi$) and the corresponding charged channels in τ decay [5] are compatible with the two-resonance interpretation [8,9]. However the 4π data alone do *not* provide such good discrimination despite 4π being the major decay channel. The reason for this is straightforward. In $\omega\pi$ and $\pi\pi$ there is strong interference with the tail of the ρ , which is absent in the case of 4π . It was also found that the $e^+e^- \rightarrow \eta\pi^+\pi^-$ cross section is better fitted with two interfering resonances than with a single state [10], with parameters in fair agreement with those found in the analysis of other channels. Independent evidence for two $J^P=1^-$ states was provided in a high statistics study of the $\eta\pi\pi$ system in π^-p charge exchange [11]. Decisive evidence for both the $\rho(1450)$ and $\rho(1700)$ in their 2π and 4π decays has come from the study of $\bar{p}p$ and $\bar{p}n$ annihilation [12]. The masses and widths obtained from the analysis of the 2π channel are in reasonably good accord with those found from e^+e^- annihilation and τ decay. The significance of these results for the observation the vector mesons via their 4π decay is that they represent a combined analysis of four high-statistics channels: $\bar{p}p \rightarrow 5\pi^0$, $\bar{p}d \rightarrow \pi^-4\pi^0(p)$, $\bar{p}p \rightarrow \pi^+\pi^-3\pi^0$ and $\bar{p}d \rightarrow \pi^+2\pi^-2\pi^0(p)$. The inclusion of both the $\rho(1450)$ and the $\rho(1700)$, with masses and widths fixed at the values obtained in the 2π analysis, leads to a significant improvement in $\log(L)$.

The data initially available for the study of the corresponding isoscalar states $\omega(1420)$ and $\omega(1600)$ were $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ (which is dominated by $\rho\pi$) and $e^+e^- \rightarrow \omega\pi^+\pi^-$ [13]. The latter cross section shows a clear peak which is apparently dominated by the $\omega(1600)$. The former cross section shows little structure, but is appreciably larger than that calculated from the tails of the ω and ϕ . This im-

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TABLE I. Experimental masses and widths of the higher vector mesons.

Resonance	$\rho(1450)$	$\rho(1700)$	$\omega(1420)$	$\omega(1600)$
Mass (MeV)	1465 ± 25	1700 ± 20	1419 ± 31	1649 ± 24
Width (MeV)	310 ± 60	240 ± 60	174 ± 59	220 ± 35

plies an additional contribution and a best fit is obtained with two states [8], although a fit with only the $\omega(1600)$ cannot be excluded completely. Data on $\bar{p}N$ annihilation may help to clarify the situation, but analysis is still at a preliminary stage [14].

Although there is general consensus on the existence of the $\rho(1450)$, $\rho(1700)$, $\omega(1420)$ and $\omega(1600)$ there is considerable disparity on the parameters of these resonances. These show variation from one reaction to another and, even within one particular process, are dependent on the analysis techniques employed. Results from channels for which there is strong interference with the tail of the ρ or of the ω and ϕ are sensitive to the choice of model used to estimate this contribution. For the $\rho(1450)$ the most extreme low mass comes from an analysis of the $\pi^+\pi^-$ spectrum in the reaction $K^-p \rightarrow \pi^+\pi^-\Lambda$ [15], which gives 1266 ± 14 MeV. However such a low mass is not supported by any other analysis and does require confirmation. Most of the results of the analyses of e^+e^- annihilation, τ decay and $\bar{p}N$ annihilation are clustered round the preferred Particle Data Group (PDG) values [1], which are the ones we use here. These are given in Table I.

A natural explanation of these states is that they are the first radial, 2^3S_1 , and first orbital, 1^3D_1 , excitations of the ρ and ω as the masses are close to those predicted by the quark model [16]. This interpretation is given further credence by the observation of $\phi(1680)$ which has the appropriate mass to be a candidate for the first radial excitation of the ϕ .

Despite the reasonable agreement of the observed masses with the quark model predictions, the ratio of the e^+e^- width of the $\rho(1700)$ to that of the $\rho(1450)$ is surprisingly large. In the non-relativistic limit the e^+e^- width of the 1^3D_1 state vanishes, and although some non-zero width will be created by relativistic corrections this is expected to be small. Additionally the data on the 4π channels in e^+e^- annihilation and in τ decay do not appear at first sight to be compatible with those expected for the vector radial and orbital excitations of the $q\bar{q}$ system. This statement is of course model dependent as it assumes that we can predict the hadronic decays of the vector $q\bar{q}$ excitations. The 3P_0 model [17–21] does appear to allow this with some accuracy. A systematic study of known light $q\bar{q}$ decays shows that a 3P_0 -type amplitude dominates, and widths which are predicted to be large or small are found respectively to be so. More quantitatively, calculated widths agree with data to within 25–40%. Of course the accuracy of the 3P_0 model has only been tested on specific well-known decays and it may be that its accuracy elsewhere is less certain. However there is no hard evidence to support this latter conjecture,

TABLE II. The 3P_0 partial widths for ρ_{2S} and ρ_{1D} .

Channel	$\pi\pi$	$\pi\omega$	$\rho\eta$	πh_1	πa_1	$\rho\rho$	$\rho\sigma$	Other	Total
ρ_{2S}	68	115	18	1	3	10	1	80	295
ρ_{1D}	27	23	13	104	105	6	0	137	415

and as far as one can ascertain the 3P_0 model is reliable.

Thus the success of the 3P_0 model for well-known decays can be used to justify its application to predicting other decays, and in particular those of the radial and orbital excitations of the ρ and ω . In its simplest form the 3P_0 model contains only two parameters: an inverse length scale β which controls the meson form factors, and the pair creation strength γ . These are not known precisely, but are reasonably well constrained with $\beta \sim 0.4$ GeV, $\gamma = 0.39$. Assuming that their masses are respectively 1.45 and 1.70 GeV, the 3P_0 partial widths for ρ_{2S} and ρ_{1D} are given in Table II.

In Table II “other” includes $K\bar{K}$, $K^*\bar{K} + c.c.$ and 6π channels, and the σ is the broad S -wave $\pi\pi$ enhancement, considered as the 1^3P_0 $q\bar{q}$ state. Altogether 16 channels have been incorporated in the calculation in which we have not used the narrow-width approximation for final meson states, so the results differ slightly but not significantly from those that do [21].

It is not necessary to go through a detailed analysis to show that these 3P_0 model results exclude interpreting the e^+e^- and τ decay data in terms of the ρ_{2S} and ρ_{1D} if the model is strictly applied. As already implied, the key is in the 4π decays. From Table II one can see that the 4π decays of the ρ_{2S} are negligible, and so the ρ_{2S} effectively makes no contribution to the 4π channel. In contrast the 4π decays of the ρ_{1D} are large, and the two dominant ones, $h_1\pi$ and $a_1\pi$, are comparable. Now $h_1\pi$ contributes only to the $\pi^+\pi^-\pi^0\pi^0$ channel in e^+e^- annihilation, but $a_1\pi$ contributes to both this and to $\pi^+\pi^-\pi^+\pi^-$. An immediate consequence is that we would expect $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0) > \sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-)$, after subtraction of the $\omega\pi$ cross section from the total $\pi^+\pi^-\pi^0\pi^0$. This contradicts observation. Despite considerable uncertainty in the $\pi^+\pi^-\pi^0\pi^0$ cross section, enhanced by the need to subtract the $\omega\pi$ cross section, it is undeniably appreciably smaller than the $\pi^+\pi^-\pi^+\pi^-$ cross section over most of the relevant energy range.

One explanation of this has been to suggest that the $q\bar{q}$ vector states are mixed with a hybrid vector [22,23] as this decays predominantly to $a_1\pi$ in flux tube models [23], and to $a_1\pi$ and $\rho(\pi\pi)_S$ in constituent gluon models [24]. Both the $\pi^+\pi^-\pi^+\pi^-$ and the $\pi^+\pi^-\pi^0\pi^0$ channels are accessed by the $a_1\pi$ and $\rho(\pi\pi)_S$ decays so, in either case, e^+e^- annihilation and the corresponding τ decays should in principle be explicable in terms of some suitable combination of ρ , ρ_{2S} , ρ_{1D} and hybrid ρ_H , and with the implication that there must be very little ρ_{1D} to ensure the dominance of $\pi^+\pi^-\pi^+\pi^-$ over $\pi^+\pi^-\pi^0\pi^0$. The surprisingly large ratio of the e^+e^- widths is also a good indicator of mixing.

However such evidence as we have from the isoscalar states indicates that the picture might not be quite as simple

TABLE III. The 3P_0 widths for the ω_{2S} and ω_{1D} .

Channel	$\rho\pi$	$\omega\eta$	$b_1\pi$	$\omega\sigma$	Other	Total
ω_{2S}	328	12	1	8	36	385
ω_{1D}	101	13	371	0	53	561

as this. The 3P_0 widths for the ω_{2S} and ω_{1D} are given in Table III [21] assuming that their masses are respectively 1420 and 1650 MeV.

The large widths of the bare states predicted by the 3P_0 model are well in excess of the quoted experimental total widths [1]. There must be strong mixing in the isoscalar channel as the e^+e^- widths of the ω'_1 and ω'_2 are almost the same, and one would not expect either the ω_{1D} or the ω_H to have an electromagnetic coupling comparable to that of the ω_{2S} . In the flux tube model the width of the ω_H is predicted to be small, ~ 20 MeV [23], and is essentially all to $\rho\pi$. The ω_H width can be appreciably larger in constituent gluon models [24] but again the $\rho\pi$ decay dominates although some $\omega(\pi\pi)_S$ decay is allowed [24]. Thus omitting the ω_{1D} , in analogy with the isovector case, would seem difficult to reconcile with the integrated cross section for $e^+e^- \rightarrow \omega\pi\pi$ which, up to 1.8 GeV, is about 60% of the integrated $e^+e^- \rightarrow \rho\pi\pi$ cross section and could be taken to imply some significant ω_{1D} component.

The arguments relating to the $\rho(\pi\pi)_S$ and $\omega(\pi\pi)_S$ decays of ρ_{2S} and ω_{2S} presuppose that there is no mechanism which can generate these in any significant way. A possible approach is to invoke an inherent uncertainty in the 3P_0 model when applied to the decays of radial excitations to the ground state plus an S -wave $\pi\pi$ pair. In the 3P_0 model the decays of the ρ_{2S} to $\rho(\pi\pi)_S$ and of the ω_{2S} to $\omega(\pi\pi)_S$ are strongly suppressed by a cancellation between two terms, one of which is strongly dependent on the model parameters. If these decays could be sufficiently enhanced within the structure of the model then the 4π problem in the isovector sector and the comparatively large $\omega\pi\pi$ in the isoscalar sector could possibly be resolved. Note that the $\rho(\pi\pi)_S$ and $\omega(\pi\pi)_S$ decays of the ρ_{1D} and ω_{1D} respectively are strictly forbidden in the 3P_0 model.

Further, many radial excitations are known to decay preferentially to the ground state, or a lower radial excitation, plus $(\pi\pi)_S$. The most obvious ones occur in higher quarkonia. The branching fractions of these decays are $\psi(2S) \rightarrow \psi(1S)$, $50.8 \pm 3.7\%$; $Y(2S) \rightarrow Y(1S)$, $27.3 \pm 1.4\%$; $Y(3S) \rightarrow Y(1S)$, $6.5 \pm 0.4\%$, $Y(3S) \rightarrow Y(2S)$, $4.8 \pm 0.7\%$. These decays cannot proceed via string breaking and are all specifically non- 3P_0 decays. The relevance of this is the implication that the $(\pi\pi)_S$ state couples rather strongly to gluons. It has been suggested [25] on the basis of QCD sum rules that the σ -meson is a low-lying scalar glueball and an estimate [26] of its coupling to light quarks indicates that it is strong. Thus it is not surprising that a similar phenomenon is seen in light quarkonia: for example $\eta'(1295) \rightarrow \eta(\pi\pi)_S$ and $\pi(1300) \rightarrow \pi(\pi\pi)_S$, assuming for the moment that both $\eta'(1295)$ and $\pi(1300)$ are radial excitations. Both of these latter decays are essentially zero in the 3P_0 model with stan-

dard parameters. Whether a similar mechanism is operating here as for heavy quarkonia, or whether these decays arise from the sensitivity of the 3P_0 model for radial decays involving $(\pi\pi)_S$, is undetermined.

Before applying these various ideas to the vector meson decays we consider the current status of hybrid mesons and of radial decays to the corresponding ground state plus $(\pi\pi)_S$ in light quarkonia.

III. HYBRID MESONS

Evidence for the excitation of gluonic degrees of freedom has emerged in several processes. There are two independent indications of an isovector $J^{PC}=1^{-+}$ exotic resonance $\hat{\rho}(1600)$ in $\pi^-N \rightarrow \pi^+\pi^-\pi^-N$, specifically in the $\rho^0\pi^-$ channel. The E852 Collaboration [27] quote a mass of 1593 ± 8 MeV and width of 168 ± 20 MeV, which are consistent with the preliminary claim of the VES Collaboration [28] of a resonance at 1620 ± 20 MeV with a width of 240 ± 50 MeV. There is also evidence for this state in the $\eta'\pi$ channel [28,29]. It has been argued that the $\rho\pi$, $\eta'\pi$ and $\eta\pi$ couplings of this state support the hypothesis that it is indeed a hybrid meson, although other interpretations cannot be eliminated entirely [30]. A peak in the $\eta\pi$ mass spectrum at 1.4 GeV with $J^{PC}=1^{-+}$, in the reaction $\pi^-N \rightarrow \eta\pi^-N$, has also been interpreted as a resonance [31]. Additional evidence for the same state in the same mode is provided by the Crystal Barrel Collaboration [32], in an analysis of $p\bar{p} \rightarrow \eta\pi^+\pi^-$. In this case the signal is deduced from a phase variation in the $J^{PC}=1^{-+}$ amplitude seen as interference in the Dalitz plot. There is evidence from the VES Collaboration [33] for two isovector 0^{-+} states in the mass region 1.4 to 1.9 GeV. One is the well-established $\pi(1800)$ [34] with a mass of $1790 \pm 6 \pm 12$ MeV and width of $225 \pm 9 \pm 15$ MeV, and one a new state, the $\pi(1600)$, with a mass of $1580 \pm 43 \pm 75$ MeV and width of $450 \pm 60 \pm 100$ MeV [33]. The quark model predicts only one state in this mass region. Thus there is evidence for degrees of freedom beyond $q\bar{q}$, and the unusual decay pattern of the $\pi(1800)$ encourages the belief that it has a strong hybrid component [21,23].

The interpretation of the peak in the $\eta\pi$ mass spectrum at 1.4 GeV as a resonance [31] has been challenged [35]. It was shown that the E852 $\eta\pi$ peak and phase can be obtained without the need to invoke the presence of an exotic resonance. The two key ingredients are the presence of a strongly coupled threshold in this mass region (taken to be $b_1\pi$) with rescattering to produce the $\eta\pi$ signal. A Deck-type background interfering with a hybrid resonance of higher mass, for which the $\hat{\rho}$ at 1.6 GeV is an obvious candidate, was considered as the production mechanism. The Deck mechanism also provides the predominant natural parity exchange for the 1.4 GeV peak which is observed experimentally, in contrast to the 1.6 GeV state which has a significant contribution from unnatural parity exchange. Of course the Deck mechanism is not applicable to the $p\bar{p}$ annihilation experiment [32], but the strongly-coupled threshold with rescattering can generate sufficient phase variation without requiring a resonance at 1.4 GeV [36].

If the above is the correct interpretation of the $\eta\pi$ data

then the $\hat{\rho}(1600)$ is the lowest mass exotic hybrid. An alternative viewpoint is to accept that the 1.4 GeV $\eta\pi$ signal really is an exotic resonance [24] and to explore the consequences. One of these, which will be relevant for our subsequent discussion, is that the $J^{PC}=0^{-+}$ $\pi(1300)$ is predominantly a hybrid meson, although there may be some admixture of the $q\bar{q} 2^1S_0$ state which has the same quantum numbers. One argument usually given in favor of the hybrid interpretation is a large $\pi(\pi\pi)_S$ branching fraction, much larger than $\pi\rho$, although as we shall see in Sec. IV B there is considerable experimental disagreement on this point. A large $\pi(\pi\pi)_S$ width would be in complete disagreement with the predictions for the $q\bar{q} 2S$ level in the 3P_0 model for which the $\pi(\pi\pi)_S$ mode is strongly suppressed. However if the $\pi(1300)$ is a $q\bar{q}$ radial excitation then, as noted in Sec. II, the decay is very sensitive to the 3P_0 parameters. The decay may also be generated by a non- 3P_0 mechanism. Thus the $\pi(\pi\pi)_S$ decay may be possible without invoking a hybrid. This possibility will be explored fully in Sec. IV.

Ideally we would have mass predictions for hybrids comparable to those for the $q\bar{q}$ states. Unfortunately the absolute mass scale for light-quark hybrids is not precisely determined, with predictions for the lightest hybrids lying between 1.3 and 1.9 GeV. Bag models [37,38] tend towards the lower end of this range, but it is not clear just how reliable their results are. Parameters are tuned to fit the $q\bar{q}$ spectrum and it is questionable whether the same parameters should be used for the $q\bar{q}g$ states. Attempts to accommodate this lead to considerable variation in the predictions, giving a mass value for the lightest hybrid in the range 1.4 to 1.7 GeV. However it is perfectly possible to accommodate an 0^{-+} at ~ 1.3 GeV and an 1^{-+} at ~ 1.4 GeV with the lightest 1^{--} hybrid at ~ 1.65 GeV. Flux tube models [39] predict hybrid masses to be considerably higher than these, at about 1.9 GeV. The constituent gluon model [40] gives the light-quark hybrid mass at 1.7–1.8 GeV. In principle QCD sum rules could resolve the issue of mass scale as they are a powerful tool for the understanding of hadron properties in terms of the vacuum condensates of QCD. However even here there is a major divergence of view, either giving an upper limit of 1.5 GeV on the mass of the 1^{-+} hybrid [41], with a preference for a somewhat lower value, or putting it somewhere in the range 1.6 to 2.1 GeV [42], with a preference for the upper end. The principal difference between the two calculations is the application of a low-energy theorem in [41] which in turn gives an important role to the $g^3\langle G^3\rangle$ term with the effect of lowering the mass. This is acknowledged in [42]. Applications of lattice gauge theories to the light-quark hybrid sector [43–46] give essentially consistent results for the mass of the 1^{-+} hybrid, predicting it to be around 2 GeV, although admittedly with quite large errors. Indeed they are still sufficiently large that this mass is considered not to be inconsistent with the hybrid interpretation of the $\hat{\rho}(1600)$ [44].

However it does seem to be generally agreed that the mass ordering is $0^{-+} < 1^{-+} < 1^{--} < 2^{-+}$. This is certainly the case for bag models [37,38], and also appears to hold in the heavy-quark sector [47]. The same mass ordering emerges if one assumes that the splittings are due to the

spin-spin contact interaction [24]. The heavy-quark expansion of QCD in Coulomb gauge [48] demonstrates that spin-orbit splitting of low-lying hybrids with $J^{PC}=J^{-+}$ and J^{+-} is such that $J=1$ lies between $J=0$ and $J=2$, and the ordering is the same for both sets. However either of $J=0$ or $J=2$ can be the lowest-lying. In lattice QCD calculations of heavy-quark hybrid states it is found that $0^{+-} < 2^{+-}$ [49] so that combining the two results gives $0^{-+} < 1^{-+} < 2^{-+}$ (and $0^{+-} < 1^{+-} < 2^{+-}$). Whether it makes sense to extrapolate these heavy-quark results to the light quark sector is debatable, but nonetheless the qualitative agreement with the bag and constituent model results is encouraging. It has to be noted that in the light-quark sector it is found that nominally $2^{+-} < 0^{+-}$ [44]. However the errors on the masses, which are much greater relative to the central values than in the heavy-quark sector, are still sufficiently large for this result not to be considered definitive.

The lack of precision in mass estimates is matched by uncertainty on decay modes. Again the two standard approaches are the constituent gluon model [24,50] and the flux tube model [23,39]. In the former the hybrids are considered specifically as having three components: quark, antiquark and gluon. Decays proceed by dissociation of the constituent gluon [50]. In the latter it is assumed that the hybrids are quark-antiquark states moving on an adiabatic surface generated by an excited flux tube of gluons, with the standard $q\bar{q}$ mesons corresponding to the unexcited flux tube. Decays of hybrids and $q\bar{q}$ mesons then proceed by the same phenomenological pair-creation mechanism, for example the 3P_0 model, coupled with a flux tube overlap [39]. While there are many common features in the decay modes predicted by these approaches there are some substantial differences which become rather crucial in interpreting data, and which are caused partly by the different level of flexibility allowed within the models. In the constituent model the decay strength is proportional to the strong coupling constant $\alpha_s(q^2)$ given at some characteristic scale q^2 . As the present level of modeling does not permit definition of this scale, the decay strength was treated as a model parameter in [24,50]. The analysis in [24] was based on the assumption of $\hat{\rho}(1400)$ being a hybrid and some upper limits of its $\rho\pi$ mode, which were used as an input to define the decay strength. In the original version [39] of the flux tube model the decay strength was defined from the data on $q\bar{q}$ decays. In principle, in the flux tube model the quark pair creation vertex is uncorrelated with the gluonic modes of the hybrid. This permits the inclusion of different decay vertices within the same overall structure. One has recently been proposed [51], motivated by the heavy-quark limit of the QCD Hamiltonian, and its predictions compared with those of the standard 3P_0 vertex [48]. Once again there are many similarities but some major differences which should be amenable to experimental test although present data cannot distinguish.

The hybrid decays of particular interest to us here are those of the isovector 0^{-+} and 1^{-+} , and both the isovector and isoscalar 1^{--} . There are some substantial differences between the flux tube model [39,51] and the constituent gluon model [24,50] for these.

0^{-+} : In the flux tube model, the principal decay modes

are $\rho\pi$ and, if the hybrid is sufficiently massive, $f_0(1370)\pi$. The $\rho\pi$ width in the constituent gluon model is comparable to that of the flux tube model, but it also has a very large $\pi(\pi\pi)_S$ width which dominates the decay.

1^{-+} : In this case the flux tube model and the constituent gluon model are in reasonable accord. The principal decay modes are $\rho\pi$ and $b_1\pi$, with the latter the larger of the two.

1^{--} : The flux tube model predicts a rather narrow isovector state, with $a_1\pi$ as the dominant mode. In contrast the constituent gluon model predicts a much larger width, still with $a_1\pi$ dominant, but with significant $\rho(\pi\pi)_S$ and $\omega\pi$ components. For the corresponding isoscalar, the flux tube model predicts a very narrow state decaying almost entirely to $\rho\pi$. Again the constituent gluon model predicts a much larger width, having $\rho\pi$ as the largest decay mode but also with a significant $\omega(\pi\pi)$ fraction.

We would like to comment here on an important point concerning decays which include $(\pi\pi)_S$ in the final state. The flux tube calculations do not consider such decays at all, on the grounds that the complicated dynamics of the $(\pi\pi)_S$ final state is incompatible with the simple decay chain of $q\bar{q}$ in the 3P_0 state going into $\pi\pi$. It has even been suggested that an effective Lagrangian approach may provide a better simulation of dynamics when the $(\pi\pi)_S$ system is involved [52]. Nevertheless, as the ${}^3P_0 q\bar{q}$ couples strongly to $\pi\pi$, it should participate in the $\pi\pi$ dynamics even if there exists a non- $q\bar{q}$ mechanism which generates this dynamics. So there is no reason to neglect the decay channels with $q\bar{q}$ in the 3P_0 state, unless the corresponding amplitude is very small *per se*.

The interaction in the $(\pi\pi)_S$ channel is very strong and requires the unitarized coupled channel analysis, but the $\pi\pi$ phase shift can be described with the ${}^3P_0 q\bar{q}$ as an intermediate state (see the detailed analysis of [53] and a simple model in Appendix A). It is not surprising that a naive quark model, such as the 3P_0 one, fails to describe the low-mass part of the $\pi\pi S$ -wave phase shift, where constituent quarks are not the proper degrees of freedom and chiral physics enters the game instead. It still remains an open question of how to incorporate the chiral symmetry constraints into the quark model unless it is done in a purely phenomenological way. In the simple model described in Appendix A the 3P_0 amplitude is modified to interpolate smoothly between the chiral perturbation theory regime with Adler zeros and the confinement regime with string-breaking modeled by the 3P_0 mechanism.

In summary there are two main lines which can be followed.

(i) Hybrids are comparatively light, the $\pi(1300)$ and $\hat{\rho}(1400)$ are hybrid states (or, in the former case, predominantly hybrid) and the mass of the hybrid ρ_H , $\omega_H \sim 1.6$ GeV. This would allow strong mixing of the vector hybrids with the radial and orbital excitations of the ρ and ω , but is not compatible with flux tube models.

(ii) Hybrids are comparatively heavy, the $\hat{\rho}(1600)$ is the lightest 1^{-+} state and the $\pi(1600)$ presumably the corresponding 0^{-+} hybrid (or at least contains a significant hybrid component). This scenario is compatible with flux tube

models, but puts the 1^{--} vector hybrid mass at ~ 2.0 GeV, making strong mixing with the radial and orbital excitations unlikely.

IV. PSEUDOSCALAR DECAYS TO GROUND STATE PLUS $(\pi\pi)_S$

We have already commented that in the constituent gluon model of hybrids the decays of the 0^{-+} state is very different from those predicted by the flux tube model. It has been suggested [24] that if the $\hat{\rho}(1405)$ does exist then the $\pi(1300)$ should have a large hybrid admixture and the $\pi(1300)$ decays would then allow a test of the two models. We explore both this hypothesis and the hypothesis that the $\pi(1300)$ is a $q\bar{q}$ state, specifically the first radial 2^1S_0 excitation of the π . For the latter assumption we can use the decays of the $\eta(1295)$ as a control. To begin with we summarize the current experimental situation with respect to the decays of both these mesons.

A. $\eta(1275)$

The most detailed study of the decay $\eta(1295) \rightarrow \eta(\pi\pi)_S$ comes from the E852 charge-exchange reaction $\pi^- p \rightarrow \eta\pi^- \pi^+ n$ [54]. The data are sufficiently precise to allow a separation of the $a_0\pi$ and $\eta(\pi\pi)_S$ decays despite the similarity of these two channels. The $a_0\pi/\eta(\pi\pi)_S$ branching ratio is estimated to be 0.48 ± 0.22 , although this may contain a large systematic error due to the difficulty of distinguishing unambiguously between the $a_0\pi$ and $\eta(\pi\pi)_S$ decays. This ratio disagrees with the GAMS result of 1.86 ± 0.60 [55], although it also is possibly subject to similar systematic errors. The total width of the $\eta(1295)$ is rather well defined: 66 ± 13 MeV from E852, 53 ± 6 MeV from Fukui *et al.* [56]. Assuming that the two results for the $\Gamma(a_0\pi)/\Gamma(\eta(\pi\pi)_S)$ branching ratios give reasonable upper and lower limits, we can conclude that the partial width for $\eta(1295) \rightarrow \eta(\pi\pi)_S$ is in the approximate range 20 to 40 MeV.

B. $\pi(1300)$

Until recently there has been little information on the $\pi(\pi\pi)_S$ branching fraction of the $\pi(1300)$, and there is still considerable uncertainty in the total width, which can lie somewhere in the range 200 to 600 MeV [1]. The recent VES data [33,34] show a clear $\pi(1300)$ peak in 3π , with a width of $\Gamma \sim 400$ to 500 MeV in both $\pi(\pi\pi)_S$ and $\pi\rho$. The latter appears particularly strong and it has been suggested [21] that as the size of the Deck background in $\pi(\pi\pi)_S$ is uncertain it could provide the totality of the $\pi(\pi\pi)_S$ signal. If this is correct then the dominant decay would be $\rho\pi$. In contrast the E852 experiment [57] claims three decay modes of the $\pi(1300)$: $\pi\rho$, $\pi f_2(1270)$ and $\pi(\pi\pi)_S$. No comment is made on the mass or width of the $\pi(1300)$ other than it is broad. As for the VES experiment, the genuine resonance signal could be confused by interference with the Deck background.

An uncertain Deck background is not a problem for $p\bar{p}$ annihilation experiments. In their study of $f_0(1500)$ decays

in $4\pi^0$ in $p\bar{p} \rightarrow 5\pi^0$ at rest, the Crystal Barrel experiment [58] found a very substantial improvement in their fit when the $\pi\pi(1300)$ decay of the $f_0(1500)$ was included. As the final state in this case is all π^0 the decay of the $\pi(1300)$ cannot be to $\pi\rho$ and must be to $\pi(\pi\pi)_S$. However because of the restriction to the purely neutral channel nothing can be said about the $\pi(\pi\pi)_S$ branching fraction. The parameters of the $\pi(1300)$, if left free in the fit, are determined to be $M = 1.114$ GeV, $\Gamma = 340$ MeV. Errors on these are not given. In a preliminary analysis [59] of $p\bar{p}$ and $p\bar{n}$ annihilation to 5π , the ratio $[\pi(1300) \rightarrow \pi(\pi\pi)_S]/[\pi(1300) \rightarrow \pi\rho]$ is quoted as being not more than 10%. The mass and width of the $\pi(1300)$ are found to be $M = 1.400 \pm 0.04$ GeV and $\Gamma = 275 \pm 50$ MeV respectively. The Obelix experiment [60], in their analysis of $p\bar{p} \rightarrow 2\pi^+ 2\pi^-$ also find a significant improvement in their fit if the $\pi(1300)$ is included in the decay chain. The $\pi(1300)$ parameters are found to be $M = 1.275 \pm 0.015$ GeV, $\Gamma = 218 \pm 100$ MeV. Both the $\pi(\pi\pi)_S$ and $\pi\rho$ modes were required by the fit, and the ratio between them was found to be large: $[\pi(1300) \rightarrow \pi(\pi\pi)_S]/[\pi(1300) \rightarrow \pi\rho] = 5.25 \pm 0.7$. This is appreciably larger than the result ~ 2.12 obtained from much earlier data [54]. Thus the situation is very confused with gross discrepancies among the experiments. However it is reasonable to believe that the decay $\pi(1300) \rightarrow \pi(\pi\pi)_S$ does exist, but with a completely undetermined branching fraction. It is perhaps significant that the $p\bar{p}$ annihilation experiments find a smaller width than the $\pi p \rightarrow (3\pi)p$ production experiments, which could be due to the effect of the Deck background on the latter. If this is the case, then a total width of ~ 300 MeV would seem reasonable, with a partial width for $\pi(\pi\pi)_S$ somewhere between 30 and 200 MeV.

C. Pseudoscalar decays in the 3P_0 model

For this part of the discussion we assume that the $\pi(1300)$ is a $q\bar{q}$ state, specifically the first radial 2^1S_0 excitation of the π . For calculating decays in the 3P_0 model we also assume that the $(\pi\pi)_S$ is contained in that model and is the 1^3P_0 state. A two-channel model which reproduces the experimental S -wave $\pi\pi$ amplitude is described in Appendix A. Both versions of the $(\pi\pi)_S$ amplitude described there were used in these calculations i.e. without and with the Adler zero. The results are very much the same in both cases.

In the 3P_0 model, with standard wave-function parameters, the decay of a radial excitation to the corresponding ground state plus $(\pi\pi)_S$ is small, a few MeV at most. This is true for ρ_S , ω_S , $\eta(1295)$, $\pi(1300)$. It is caused by a node in the wave function, with the consequence that the decay is very sensitive to the parameter β . Of course there are limits within which β can vary. A good guide to these limits is provided by [20]. Of the standard decays used to specify β and γ , $a_1 \rightarrow \rho\pi$ provides no constraint because of the very large experimental uncertainty on its width; $h_1 \rightarrow \rho\pi$ provides only a weak constraint ($0.3 \leq \beta \leq 0.5$ GeV), again because of experimental uncertainty coupled with a rather weak theoretical dependence on β ; and $b_1 \rightarrow \omega\pi$ also provides

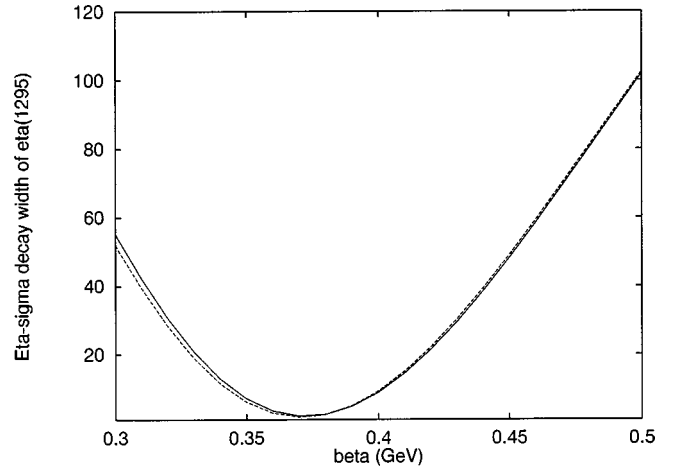


FIG. 1. Width of the decay $\eta'(1295) \rightarrow \eta(\pi\pi)_S$ as a function of β . The solid line is calculated using the 3P_0 $\pi\pi$ phase shift and the dashed line using the 3P_0 $\pi\pi$ phase shift modified to include the Adler zero.

only a weak constraint, in this case because the theoretical width is almost independent of β over quite a wide range. In contrast the theoretical widths of $f_2 \rightarrow \pi\pi$, $a_2 \rightarrow \rho\pi$ and $\rho \rightarrow \pi\pi$ vary strongly with β . The two former favor a value of β close to the mean of 0.4 GeV, but the latter prefers a much smaller value, ~ 0.3 GeV. In contrast the D/S ratios for the decays $b_1 \rightarrow \omega\pi$ and $a_1 \rightarrow \rho\pi$, which are sensitive tests of the 3P_0 model, prefer a larger value, particularly the latter, which could go as high as $\beta \sim 0.5$ GeV. Taking everything into account, 0.3 GeV and 0.5 GeV do seem to provide extreme lower and upper limits on β , with 0.35 GeV and 0.45 GeV being more reasonable.

The variation of the $\eta(\pi\pi)_S$ width of the $\eta(1295)$ is given in Fig. 1 as a function of the value of β for $(\pi\pi)_S$. Clearly the experimental limits on the $\eta(1295)$ width provide a strong constraint on the allowed values of β , independently of other decays. In the 3P_0 model with standard values of the parameters β and γ the width $\Gamma(\pi' \rightarrow \pi\rho) \sim 200$ MeV and the width $\Gamma[\pi' \rightarrow \pi(\pi\pi)_S] \sim 0$ MeV [21]. The $\pi\rho$ partial width does have quite a strong dependence on β , decreasing from ~ 300 MeV at $\beta = 0.3$ GeV to ~ 100 MeV at $\beta \sim 0.5$ GeV [21]. The variation of the $\pi(\pi\pi)_S$ width is much stronger, due to the effect of the node in the wave function. This variation is shown in Fig. 2, where we have taken into account the effect of symmetrization of like pions. The procedure for this is outlined in Appendix B. The constraints imposed by the $\eta(1295)$ decay width clearly restrict the maximum partial width for the $\pi(1300) \rightarrow \pi(\pi\pi)_S$ to about 60 MeV. This is consistent with the latest result from Crystal Barrel [59], but is appreciably smaller than the result from Obelix [60].

We can conclude the the 3P_0 model cannot be used to explain a very large $\pi(\pi\pi)_S$ width of the $\pi(1300)$, should that prove to be required. However a large $\pi(\pi\pi)_S$ width of the $\pi(1300)$ does not necessarily provide an argument in favor of a hybrid interpretation and we now consider an alternative non- 3P_0 mechanism.

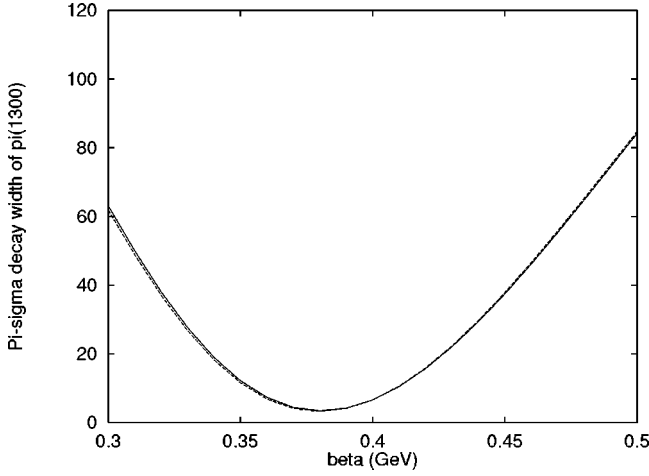


FIG. 2. Width of the decay $\pi(1300) \rightarrow \pi(\pi\pi)_S$ as a function of β . The curves are as in Fig. 1.

D. Non- 3P_0 decays of the pseudoscalars

The decays of heavy-quarkonia radial excitations to the ground state, or to a lower radial, plus $(\pi\pi)_S$ raise the possibility of analogous non- 3P_0 decays in the light-quark sector. The detailed mechanism is presumably rather different from that of the models applied to the decays of heavy quarkonia as they rely on a multipole expansion of the gluon field [61,62]. The interactions of gluons with a wavelength λ much larger than the radius $r_{Q\bar{Q}}$ of quarkonia are suppressed in the multipole expansion by powers of $r_{Q\bar{Q}}/\lambda$. So although it is reasonable to consider only the leading operator for heavy quarkonia it is not so for light quark states.

An alternative mechanism which does not rely on a multipole expansion, “vacuum excitation,” has been suggested [63]. This is applicable in principle to both light and heavy quarkonia. The essential idea is that a radial excitation decays to a lower radial excitation or to the ground state by exciting a virtual state from the vacuum into reality. This naturally has the quantum numbers of the vacuum, i.e. $I=0, J=0$ which of course are precisely those of $(\pi\pi)_S$. For our present purposes we simplify the calculation by considering relative phase-space as the spatial wave functions of the $\eta(1295)$ and $\pi(1300)$ are identical, and so overlap integrals will be the same. We consider both unweighted and weighted phase space, using for the latter the same S -wave $\pi\pi$ amplitude of Appendix A. It is irrelevant whether this is or is not a genuine $q\bar{q}$ state. All that is required is an accurate representation of the amplitude. We find that the ratio of $[\pi(1300) \rightarrow \pi(\pi\pi)_S]/[\eta(1295) \rightarrow \eta(\pi\pi)_S]$ is 3.6 for unweighted phase space and 7.7 for weighted phase space. Thus given a width of 20 to 40 MeV for the $\eta(1295)$ decay it is not difficult to generate the required large width for the $\pi(1300)$ decay.

So we conclude that this particular non- 3P_0 mechanism can correlate the $\eta(1295)$ and $\pi(1300)$ decays and provide a large $\pi(\pi\pi)_S$ width for the latter without the need to invoke a hybrid. Of course this remains a hypothesis as we do not have a specific model with which to calculate these decays for light quarks.

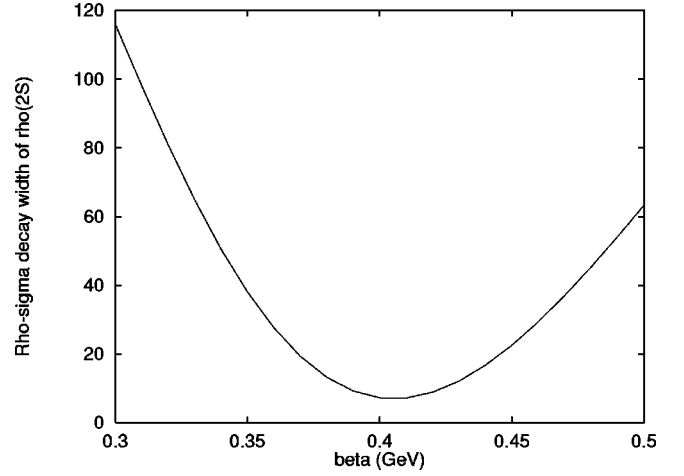


FIG. 3. Width of the decay $\rho(2S) \rightarrow \rho(\pi\pi)_S$ as a function of β . In contrast to Figs. 1 and 2 only one curve is shown as the results of using the different phase shifts are indistinguishable.

V. VECTOR STATES

We now consider the implications of the results of the previous sections for the interpretation of the data from e^+e^- annihilation and τ decay. We consider two extremes.

(1) The vector hybrids are too heavy to permit significant mixing with the $q\bar{q}$ states in the relevant kinematical region. The ρ_S and ω_S decays are some combination of 3P_0 and direct hadronic decay to $\rho(\pi\pi)_S$ and $\omega(\pi\pi)_S$ respectively. The ρ_D and ω_D decays are purely 3P_0 . Some mixing between the $2S$ and $1D$ states can be allowed.

(2) The vector hybrids are sufficiently light to allow strong mixing with the $q\bar{q}$ states. For completeness we consider the predictions of both the flux tube model and the constituent gluon model for the hybrid decays. However it must be remembered that the flux tube model prefers a higher mass for the vector hybrid.

A. No vector hybrids

The width of the decay $\rho_{2S} \rightarrow \rho(\pi\pi)_S$ in the 3P_0 model is shown in Fig. 3. Not surprisingly it is too small to account for the observed large 4π width of the $\rho(1450)$, even at the maximum acceptable value of β . To extend the non- 3P_0 decay process suggested in Sec. IV D to the decay of the ρ_{2S} requires the assumption that only S -waves are relevant. This is certainly in accord with the decay of heavy quarkonia. The matrix element for the decay of $\eta(1295)$ or $\pi(1300)$ is simply a constant, f_S say. The general form of the matrix element for the decay of ρ_{2S} is

$$A_{\mu\nu} = f \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} + \frac{q_\mu q_\nu}{q^2} - \frac{(kq)k_\mu q_\nu}{k^2 q^2} \right) + g \left(\frac{(kq)k_\mu}{k^2} - q_\mu \right) \left(k_\nu - \frac{(kq)q_\nu}{q^2} \right)$$

where k and q are respectively the four-momenta of the ρ_{2S} and the ρ . If only S -wave is present then

$$f = f_S, \quad g = -f_S/m_0(m_1 + E_1)$$

$$|\rho'_1\rangle \sim \cos\phi|\rho_S\rangle + \sin\phi|\rho_H\rangle.$$

where m_0 and m_1 are the masses of the ρ_{2S} and ρ , and E_1 is the energy of the ρ in the ρ_{2S} rest frame. One has then to calculate $\frac{1}{3}A_{\mu\nu}^*A_{\mu\nu}$ which is simply f_S^2 . So the only difference between the decays of the vector radial excitations and the pseudoscalar radial excitations is due to phase space. As the available phase space for the decay $\rho_{2S} \rightarrow \rho(\pi\pi)_S$ is not very different from that for the decay $\eta(1295) \rightarrow \eta(\pi\pi)_S$, the partial width will be comparable.

Thus neither mechanism by itself can explain the 4π decays of the $\rho(1450)$. Nor can one add them to increase the width, as the combined strength is controlled by the $\eta(1295)$ decay. It should be recalled that mixing of the ρ_{2S} and the ρ_{1D} is unlikely to resolve the difficulty as, even with an increase in the $\rho(\pi\pi)_S$ width of the former within the limits allowed, the $\pi^+\pi^-\pi^0\pi^0$ cross section will still exceed the $\pi^+\pi^-\pi^+\pi^-$ cross section. This conclusion is not affected by adjusting the 3P_0 parameters for the ρ_{1D} decays, as the $a_1\pi$ and $h_1\pi$ widths move in unison and the equality remains essentially unchanged. Thus we are forced to conclude that the $e^+e^- \rightarrow 4\pi$ data cannot be explained in terms of conventional $q\bar{q}$ dynamics.

The partial width for the decay $\omega_{2S} \rightarrow \omega(\pi\pi)_S$ in either of these models is comparable to that of the corresponding ρ_{2S} decay. This does not pose any particular problem for the isoscalar sector and does not provide any further insight into the likely mechanisms as it is not nearly sufficient in itself to provide the requisite integrated $\omega\pi\pi$ fraction. Thus we remain with the earlier conclusion that within the isoscalar $q\bar{q}$ structure there must be a significant ω_{1D} component.

B. Vector hybrids

It is now apparent that the inclusion of a isovector vector hybrid is essential to explain the $e^+e^- \rightarrow 4\pi$ data, and consequently the corresponding isoscalar vector hybrid must be included in any discussion of the $e^+e^- \rightarrow \rho\pi, \omega\pi\pi$ data. It is reasonable to assume that the hidden-strange vector hybrid is sufficiently massive not to affect the discussion.

The real question is, what can be inferred about the nature of these hybrids? The flux tube model with its dominant $a_1\pi$ decay would appear to resolve the problem in the isovector sector. To achieve strong mixing with the ρ_{2S} and ω_{2S} requires a comparatively low mass. We have seen in Sec. III that the mass scale for the flux tube model is high, making strong mixing unlikely. However spin-dependent forces may lower the mass of the hybrid ρ and ω , which are spin $S=0$ in contrast to the conventional $q\bar{q}$ components which are $S=1$, sufficiently to allow strong mixing between hybrid and conventional quarkonia. There is perhaps less of a problem with the constituent gluon model, but it is still a concern. However accepting the low-mass hybrid scenario, one could then have

$$|V\rangle = \cos\phi\{\cos\theta|2{}^3S_1\rangle + \sin\theta|1{}^3D_1\rangle\} + \sin\phi|V_H\rangle.$$

To explain the predominance of the $\pi^+\pi^-\pi^+\pi^-$ channel in this model it is necessary to take $\theta \sim 0$, so that ρ'_1 is given by

This simple scheme would make the $\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^0\pi^0$ widths of the $\rho(1450)$ the same, with only the rather small $\rho(\pi\pi)_S$ decay of the ρ_H contributing to the observed difference. Further this suppression of the ρ_{1D} contribution is not compatible with the isoscalar data. Without the ω_{1D} there is no source for the strong $\omega\pi\pi$ channel observed experimentally. The $\omega(\pi\pi)_S$ decay of ω_H is not sufficient to redress the imbalance. The dynamics would be complex indeed if the 3D_1 state were absent, or nearly so, in the isovector channel but gave a significant contribution in the isoscalar channel. This is perhaps not entirely implausible as the isoscalar channel is complicated by the presence of the ϕ_{2S} right in the middle of the relevant mass range. Hadronic mixing with the nearby ω_{2S} , ω_{1D} and ω_H will certainly occur at some level, modifying the isoscalar mixing pattern. However this mixing would have to be remarkably strong to produce the differences observed.

The opposite view, that the hybrid mass is high, also does not permit a simple mixing scheme. Mixing essentially between the hybrid and the 3D_1 state is immediately ruled out by the $\rho(1450)$ decays, although it would appear a sustainable option for the isoscalar hadronic decays. Further the e^+e^- widths of the $\rho(1700)$ and $\omega(1650)$ demand mixing with the ρ_{2S} .

The necessity to consider a complicated mixing scheme brings us back to the problem of the hybrid mass scale. The simplest way to achieve strong mixing between ρ_{2S} and the hybrid is to have them nearly degenerate, which means that the hybrid is very light, about 1.4 GeV. Setting the mass scale in such a way opens up exciting possibilities for the spectroscopy of states with nonexotic quantum numbers, along the lines discussed in [24]. It has the additional advantage of being compatible with the sum rules result [41], which we consider as the most reliable of the sum rules analyses. Nevertheless, the exotic ρ - π signals at 1.6 GeV from BNL and VES appear sound, and an alternative explanation of the BNL signal at 1.4 GeV has been proposed. Moreover, it is very improbable to have two exotic hybrids so close to each other. Another argument against such a light hybrid comes from the pseudoscalar sector: if there is no need to invoke a hybrid interpretation to explain the decay pattern of $\pi(1300)$, the new pseudoscalars $\pi(1600)$ and $\pi(1800)$ from VES can be economically considered as admixtures of $3S$ $q\bar{q}$ and hybrid states. It should be remembered that the hybrid interpretation of the $\pi(1800)$ [21,23] emerges from its decay properties.

An alternative scenario is to let the hybrid mix strongly with the $1D$ state through their near-degeneracy, and then let the lower of the two mixed states mix with the $2S$. This would mean that the 4π decays of the two observed physical states would be quasi-identical. This also suggests that the third state is put rather high and, in addition, does not have much electromagnetic coupling. This scenario assumes the vector hybrid at ~ 1.7 GeV, which makes the mass problem easier to swallow and is compatible with the exotic and pseudoscalar hybrid sector, including the splitting ordering.

However this scenario is no longer the straightforward two-level mixing scheme initially proposed for the isovector channel. All three states must be included, and in the isoscalar channel one cannot ignore the possible additional complications arising from the ϕ_{2S} . The results of such mixing depend on the fine details of the positions of the bare states and the mixing strengths. It is worth mentioning here that one could expect rather strong mixing between quarkonia and hybrids in the constituent model via constituent gluon emission and/or absorption. On the other hand, there are no distinguishable gluons in the flux tube model, and, consequently, no obvious mechanism to provide such mixing.

We have not attempted to construct a detailed mixing scheme here as it requires consideration of all channels and a theoretically-constrained fit to the data [65]. As a first step we present below a simple three-level mixing model which describes qualitatively the isovector data.

C. A simple mixing scheme

We consider the mixing of the $2S$, the $1D$ and the hybrid H_0 . For the 3×3 mixing matrix use the standard PDG one without the phase:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}. \quad (1)$$

Then

$$\begin{aligned} |\psi_1\rangle &= c_{12}c_{13}|2S\rangle + s_{12}c_{13}|H_0\rangle + s_{13}|1D\rangle \\ |H\rangle &= -[s_{12}c_{23} + c_{12}s_{23}s_{13}]|2S\rangle + [c_{12}c_{23} \\ &\quad - s_{12}s_{23}s_{13}]|H_0\rangle + s_{23}c_{13}|1D\rangle \\ |\psi_2\rangle &= [s_{12}s_{23} - c_{12}c_{23}s_{13}]|2S\rangle - [c_{12}s_{23} \\ &\quad + s_{12}c_{23}s_{13}]|H_0\rangle + c_{23}c_{13}|1D\rangle. \end{aligned} \quad (2)$$

We hypothesize that we want no direct mixing between the $|2S\rangle$ and the $|1D\rangle$ so set $s_{13} \sim 0$ and $c_{13} \sim 1$. Thus we have

$$\begin{aligned} |\psi_1\rangle &= c_{12}|2S\rangle + s_{12}|H_0\rangle \\ |H\rangle &= -s_{12}c_{23}|2S\rangle + c_{12}c_{23}|H_0\rangle \\ &\quad + s_{23}|1D\rangle \\ |\psi_2\rangle &= s_{12}s_{23}|2S\rangle - c_{12}s_{23}|H_0\rangle + c_{23}|1D\rangle. \end{aligned} \quad (3)$$

Obviously we identify $|\psi_1\rangle$ with the $\rho(1450)$. In the absence of a proper dynamical model of the mixing we are free to identify $\rho(17000)$ either with $|H\rangle$ or with $|\psi_2\rangle$. For definiteness we choose the former, but the subsequent discussion follows analogously for the latter.

It is reasonable to assume that the bare hybrid $|H_0\rangle$ has no direct electromagnetic coupling. We make no specific assumption on the electromagnetic coupling of the bare $|1D\rangle$

other than that it should be small compared to the $|2S\rangle$. The e^+e^- amplitudes for the physical states are

$$\begin{aligned} \langle e^+e^-|\psi_1\rangle &= c_{12}\langle e^+e^-|2S\rangle \\ \langle e^+e^-|H\rangle &= -s_{12}c_{23}\langle e^+e^-|2S\rangle \\ &\quad + s_{23}\langle e^+e^-|1D\rangle \\ \langle e^+e^-|\psi_2\rangle &= s_{12}s_{23}\langle e^+e^-|2S\rangle \\ &\quad + c_{23}\langle e^+e^-|1D\rangle. \end{aligned} \quad (4)$$

We see immediately that provided $s_{12}c_{23} > 0$ (and $s_{23} > 0$ as the electromagnetic coupling of $|1D\rangle$ has the opposite sign to that of $|2S\rangle$) we will get the correct relative signs of the electromagnetic couplings of the observed states [8]. In the limit of a vanishing electromagnetic width for the $|1D\rangle$ then we require $s_{12}c_{23}$ to be $\sim \frac{1}{2}$ to agree with the data analyses.

We know that the $\rho(1700)$ has a small $\omega\pi$ width, which puts another constraint on the mixing. The $\omega\pi$ amplitudes are

$$\begin{aligned} \langle \omega\pi|\psi_1\rangle &= c_{12}\langle \omega\pi|2S\rangle \\ \langle \omega\pi|H\rangle &= -s_{12}c_{23}\langle \omega\pi|2S\rangle \\ &\quad + s_{23}\langle \omega\pi|1D\rangle \\ \langle \omega\pi|\psi_2\rangle &= s_{12}s_{23}\langle \omega\pi|2S\rangle \\ &\quad + c_{23}\langle \omega\pi|1D\rangle. \end{aligned} \quad (5)$$

We have already established that $s_{12}c_{23} > 0$ and, in principle, that $s_{23} > 0$. The latter is important as it allows here some cancellation between the two terms for the $\omega\pi$ decay of the $|H\rangle$. A very small width is thus not ruled out in principle, although in practice it may not be quite so simple as in the 3P_0 model the $|2S\rangle$ width is very much larger than the $|1D\rangle$ width.

Finally we look at the 4π decays. We can ignore the $a_1\pi$ and $h_1\pi$ decays of the $|2S\rangle$ but we can let it have some $\rho(\pi\pi)_S$ decay generated by other means. We should also let the bare hybrid have some $\rho(\pi\pi)_S$ decay as well. The amplitudes are then

$$\begin{aligned} \langle a_1\pi|\psi_1\rangle &= s_{12}\langle a_1\pi|H_0\rangle \\ \langle a_1\pi|H\rangle &= c_{12}c_{23}\langle a_1\pi|H_0\rangle \\ &\quad + s_{23}\langle a_1\pi|1D\rangle \\ \langle a_1\pi|\psi_2\rangle &= -c_{12}s_{23}\langle a_1\pi|H_0\rangle \\ &\quad + c_{23}\langle a_1\pi|1D\rangle \end{aligned} \quad (6)$$

and

$$\begin{aligned} \langle h_1\pi|\psi_1\rangle &= 0 \\ \langle h_1\pi|H\rangle &= s_{23}\langle h_1\pi|1D\rangle \\ \langle h_1\pi|\psi_2\rangle &= c_{23}\langle h_1\pi|1D\rangle \end{aligned} \quad (7)$$

and

$$\begin{aligned}
\langle \rho(\pi\pi)_S | \psi_1 \rangle &= \langle \rho(\pi\pi)_S c_{12} | 2S \rangle + s_{12} \langle \rho(\pi\pi)_S | H_0 \rangle \\
\langle \rho(\pi\pi)_S | H \rangle &= -s_{12} c_{23} \langle \rho(\pi\pi)_S | 2S \rangle \\
&\quad + c_{12} c_{23} \langle \rho(\pi\pi)_S | H_0 \rangle \\
\langle \rho(\pi\pi)_S | \psi_2 \rangle &= s_{12} s_{23} \langle \rho(\pi\pi)_S | 2S \rangle \\
&\quad - c_{12} s_{23} \langle \rho(\pi\pi)_S | H_0 \rangle. \tag{8}
\end{aligned}$$

None of these are qualitatively inconsistent with observation. They all seem reasonable, particularly the lack of any $h_1\pi$ decay of $|\psi_1\rangle$. Note that $|\psi_2\rangle$ has a non-zero e^+e^- width, and is presumably somewhere ‘‘off-stage.’’ There is some evidence for isovector states in the vicinity of 2.0 GeV which decay strongly into 6π [64]. One of these could be the missing member of the trio.

VI. CONCLUSIONS

Our general conclusion is that the e^+e^- annihilation and τ decay data require the existence of a ‘‘hidden’’ vector hybrid in both the isovector and isoscalar channels. The argument is based strongly on the pattern of the observed decays to $\rho(\pi\pi)_S$ for the isovectors and to $\omega(\pi\pi)_S$ for the isoscalars. The strong mixing evident from the electromagnetic widths is also a key feature. The mixing required is non-trivial, involving the first radial and the first orbital $q\bar{q}$ excitations and the ground-state vector hybrid. Before coming to our general conclusion we explored the limits of the 3P_0 model and proposed a specific non- 3P_0 model in an unsuccessful attempt to explain these data without going beyond the $q\bar{q}$ sector.

More specifically we are inclined towards the constituent gluon model rather than the flux tube model to describe the characteristics of the hybrids. This is based on two aspects: the constituent gluon model can more readily encompass a hybrid mass in the relevant range; and the hybrid decays in the constituent gluon model are the more compatible with the data. It is interesting to note that there are indications in lattice gauge theories in the heavy-quark sector [67] that at $Q\bar{Q}$ separations of less than 2 fm treating glue as string fluctuations is doubtful i.e. the flux tube is not an appropriate picture. Whether these results are straightforwardly applicable to the light quark sector is of course an open question.

The suggestion that the ground state vector hybrid is comparatively light, i.e. about 1.6 GeV, has major consequences. Given the hybrid spin-parity sequence discussed in Sec. III, namely $0^{-+} < 1^{-+} < 1^{--}$, it reopens the question of a hybrid component in the $\pi(1300)$ [24] and emphasizes the urgency of clarifying the status of the $\hat{\rho}(1405)$. For the former it is essential to resolve the current major experimental discrepancies on the $\pi(\pi\pi)_S$ width, and for the latter to disprove the possibility that it may be an artifact of a strongly coupled threshold [35,36].

For the vector mesons there are three key requirements. The first is to identify the missing partner of the $\rho(1450)$ and $\rho(1700)$, and equally that of the $\omega(1420)$ and $\omega(1600)$, which

are predicated to exist on the basis of hybrid mixing. The second is to establish the decay modes of these vector states, particularly the specific content of the 4π channel for the $\rho(1450)$ and $\rho(1700)$ and of the 5π channel for the $\omega(1420)$ and $\omega(1600)$. The third is to obtain a better understanding of the corresponding ϕ states, of which rather little is known at present.

The obvious place to study the vector mesons is in e^+e^- annihilation, and DAΦNE would be an ideal facility in its higher energy mode. Photoproduction and/or electroproduction provide another possibility, and the higher energies proposed for TJNAF would make this an excellent source. Diffractive production is unlikely to be the dominant mechanism at TJNAF energies, so the vector mesons will not be produced in a clean environment. However this is not necessarily negative, as interference with known states can provide important phase information and other states of interest in the hybrid story e.g. the $\pi(1300)$ and the $\hat{\rho}(1405)$, if it is genuine, will also be produced at these intermediate energies [68,69]. The expected continuation of the VES program will provide information on a wide range of mesonic states and could resolve the problem of the $\pi(1300)$ decays and of the existence of the $\hat{\rho}(1405)$.

ACKNOWLEDGMENTS

This research is supported in part by PPARC, by grant RFBR 96-15-96740 and by the EEC-TMR Program (contract NCT98-0169). We are grateful to Frank Close for helpful discussions.

APPENDIX A: THE $(\pi\pi)_S$ AMPLITUDE

The simple 3P_0 model for the $\pi\pi$ S -wave amplitude assumes that the scattering takes place via 0^{++} intermediate states. There are two such states in this case, one is $(1/\sqrt{2})(u\bar{u} + d\bar{d})$ and the other is $s\bar{s}$.

The $\pi\pi$ phase shift δ and the inelasticity η are given by

$$\begin{aligned}
\tan(2\delta) &= \frac{ab_1 - a_1b}{aa_1 + bb_1}, \\
\eta^2 &= \frac{(aa_1 + bb_1)^2 + (ab_1 - a_1b)^2}{a^2 + b^2},
\end{aligned}$$

where

$$\begin{aligned}
a = a(s) &= 1 + \frac{1}{(s-s_1)(s-s_2)} [(\text{Im } \Delta_{12})^2 - \text{Im } \Delta_{11} \text{Im } \Delta_{22}], \\
b = b(s) &= \frac{1}{s-s_1} \text{Im } \Delta_{11} + \frac{1}{s-s_2} \text{Im } \Delta_{22}, \\
a_1 = a_1(s) &= 1 + \frac{1}{(s-s_1)(s-s_2)} [(\text{Im } \tilde{\Delta}_{12})^2 \\
&\quad - \text{Im } \tilde{\Delta}_{11} \text{Im } \tilde{\Delta}_{22}],
\end{aligned}$$

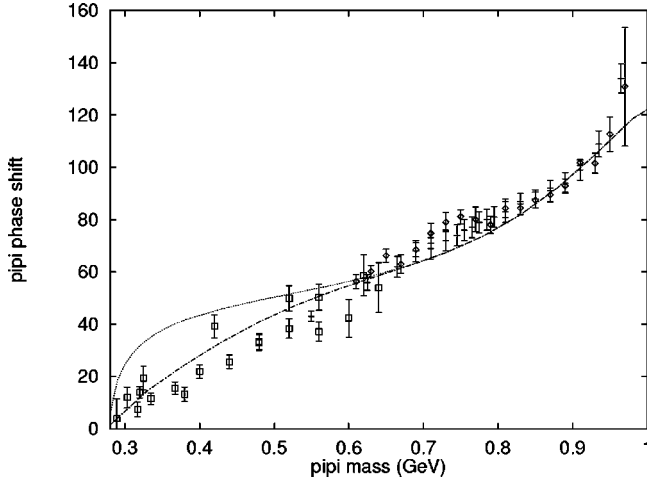


FIG. 4. The S -wave $\pi\pi$ phase shift. The solid curve is the result of the 3P_0 calculation and the dashed curve is the result of the low-energy modification to include the Adler zero. The data are from [66].

$$b_1 = b_1(s) = \frac{1}{s-s_1} \text{Im} \tilde{\Delta}_{11} + \frac{1}{s-s_2} \text{Im} \tilde{\Delta}_{22}.$$

In these formulas $\sqrt{s_1}$ and $\sqrt{s_2}$ are the masses of intermediate states, and

$$\text{Im} \Delta_{ab} = \sum_j \text{Im} \Delta_{ab}(j),$$

$$\text{Im} \tilde{\Delta}_{ab} = \text{Im} \Delta_{ab} - 2 \text{Im} \Delta_{ab}(\pi\pi),$$

where $a, b = 1, 2$ label the intermediate states, and $j = \pi\pi, K\bar{K}$ are the coupled channels taken into account.

The quantities $\text{Im} \Delta_{ab}(j)$ are calculated in the framework of 3P_0 model as

$$\text{Im} \Delta_{ab}(j) = \frac{P_j}{8\pi\sqrt{s}} f_a(j) f_b(j)$$

with the 3P_0 form factors $f_a(j)$ defined as in [18].

This model is an oversimplified version of a coupled channel method and preserves unitarity but not analyticity; in contrast to the much more sophisticated model [53] the physical resonance positions $\sqrt{s_a}$ are taken to be constants and are not defined via dispersion relations.

The $\pi\pi$ S -wave phase shift given by the 3P_0 model is shown as the solid line in Fig. 4. To improve the agreement with the data close to the $\pi\pi$ threshold the Adler zero is now introduced. We make it pragmatically by substituting

$$f_a(\pi\pi) \rightarrow \tilde{f}_a(\pi\pi) = f_a(\pi\pi) F(s)$$

with $F(s)$ being a smooth function of s . $F(m_\pi^2/2) = 0$, $F(s) \rightarrow 1$ when s increases. In such a way we have the Adler

zero at $s_0 = m_\pi^2/2$ in the $\pi\pi$ amplitude as required by chiral symmetry. The $\pi\pi$ S -wave phase shift with this modification is shown as the dashed line in Fig. 4 for the simple choice

$$F(s) = \theta(s_1 - s) \left[- \left(\frac{\sqrt{s} - \sqrt{s_0}}{\sqrt{s_1} - \sqrt{s_0}} \right)^2 + 2 \left(\frac{\sqrt{s} - \sqrt{s_0}}{\sqrt{s_1} - \sqrt{s_0}} \right) \right]$$

with $\sqrt{s_1} = 0.7$ GeV so that chiral dynamics and quark dynamics are matched at 0.7 GeV.

The idea to insert Adler zeroes into the quark model form factors was suggested in [53]. Our procedure differs from that one. In contrast to [53] we believe that the soft pion physics governs only the lower end of the $\pi\pi$ mass spectrum and does not affect the whole mass range available, leaving room for the strong interaction of quarks at higher energies. Moreover it is our belief that with the more QCD-motivated model for hadronic decays [48], which takes into account the Goldstone nature of pions, the quark model description is valid at lower $\pi\pi$ masses as well. This makes the Adler zero constraints responsible for only a rather small energy range just above the $\pi\pi$ threshold.

APPENDIX B: THE PIPRIME DECAY WIDTH

The width of the decay of π'^0 into $\pi^+\pi^-\pi^0$ is given by

$$d\Gamma = \frac{2}{3} \frac{1}{(2\pi)^3} \frac{1}{32m_{\pi'}^3} |M_c|^2 ds_{12} ds_{13}$$

$$|M_c|^2 = D$$

and the width into three neutral pions is

$$d\Gamma = \frac{1}{3} \frac{1}{(2\pi)^3} \frac{1}{32m_{\pi'}^3} |M_0|^2 ds_{12} ds_{13}$$

$$|M_0|^2 = D + 2I$$

$$D = f_{\pi'}^2(s_{12}) f_{\sigma'}^2(s_{12}) P(s_{12})$$

$$I = f_{\pi'}(s_{12}) f_{\pi'}(s_{13}) f_{\sigma}(s_{12}) f_{\sigma}(s_{13}) P(s_{12}) P(s_{13}) \\ \times [(s_{12} - m_\sigma^2)(s_{13} - m_\sigma^2) + \text{Im}(s_{12})\text{Im}(s_{13})]$$

$$f_{\pi'}(s) = f_{\pi'\sigma\pi}(m_{\pi'}^2, s, m_\pi^2)$$

$$f_{\sigma}(s) = f_{\sigma\pi\pi}(s, m_\pi^2, m_\pi^2)$$

$$P(s) = [(s - m_\sigma^2)^2 + \text{Im}^2(s)]^{-1}$$

$$\text{Im}(s) = \frac{1}{8\pi\sqrt{s}} p(s, m_{\pi'}^2, m_{\pi}^2) f_{\sigma}^2(s).$$

The total widths of these decays are given by

$$\Gamma = \int_{s_{12min}}^{s_{12max}} \int_{s_{13min}}^{s_{13max}} d\Gamma$$

$$s_{13max} = \frac{m_{\pi'}^2 + 3m_{\pi}^2 - s_{12}}{2} + 2\sqrt{\frac{(s_{12} - 4m_{\pi}^2)(m_{\pi'}^2 - m_{\pi}^2 - s_{12})^2 - 4m_{\pi}^2 s_{12}}{4 \cdot 4s_{12}}}$$

$$s_{13min} = \frac{m_{\pi'}^2 + 3m_{\pi}^2 - s_{12}}{2} - 2\sqrt{\frac{(s_{12} - 4m_{\pi}^2)(m_{\pi'}^2 - m_{\pi}^2 - s_{12})^2 - 4m_{\pi}^2 s_{12}}{4 \cdot 4s_{12}}}$$

$$s_{12max} = (m_{\pi'} - m_{\pi})^2$$

$$s_{12min} = 4m_{\pi}^2.$$

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