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QCD condensate contributions to the effective quark potential in a covariant gauge

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We discuss QCD condensate contributions to the gluon propagator both in the fixed-point gauge and in covariant gauges for the external QCD vacuum gluon fields with the conclusion that a covariant gauge is essential to obtain a gauge-invariant QCD vacuum energy density difference and to retain the unitarity of the quark scattering amplitude. The gauge-invariant QCD condensate contributions to the effective one-gluon exchange potential are evaluated by using the effective gluon propagator, which produces a gauge-independent quark scattering amplitude. [S0556-2821(99)01321-1]

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I. INTRODUCTION

The QCD inspired one-gluon exchange potential model is successful in its description of the baryon spectrum and the static properties of hadrons [1,2]. However, nonperturbative effects in the strong interaction of quarks need further studying. On one hand, a wealth of information concerning the quarkonia spectroscopy provides an important observational window to the phenomenology of strong interactions. On the other hand, the heavy quark potential is an appropriate place to investigate QCD nonperturbative effects in quark interactions [3]. Although analytic studies of nonperturbative effects in QCD are limited by the absence of a systematic procedure to account for confinement, there have appeared some techniques to deal with nonperturbative QCD effects on quark interactions. Well known examples are QCD sum rules [4], renormalons [5–7], and lattice simulations [8–10]. The results obtained by different methods complement and check each other to a certain extent. Recently, much progress has been achieved both in lattice simulations [9,10] and in renormalons analysis [6,7]. Some attempts [11–14] have also been made to incorporate nonperturbative effects in the quark potential in the inspiration of QCD sum rules [4]. However, the results in Refs. [11–13], which were obtained in a gauge dependent way, have doubtful physical meaning. Nonperturbative effects were phenomenologically considered in Refs. [11–13] by employing the vacuum condensate to modify the free gluon propagator in the fixed-point gauge; i.e., the nonlocal two-quark and two-gluon vacuum expectation values (VEVs) were calculated in the fixed-point gauge [15]. Although this gauge is extremely simple for many lowest order expansions [16], it violates translational invariance, and could in principle conflict with the covariant gauge used to formulate QCD. In particular, an explicit ξ dependence (here and henceforth, we have specified the perturbative gauge dependence as ξ dependence) in the transverse portion of the nonperturbative gluon propagator results in a ξ-dependent quark-quark scattering amplitude. The effective quark potential obtained from a ξ-dependent amplitude is of course ξ-dependent too.

As a step towards a universal parametrization of what is known from gauge independent, first principles nonperturbative studies, the main purpose of this paper is to give a quark potential including nonperturbative effects by calculating the operator product expansion (OPE) coefficients of gluon and quark condensates in a gauge independent way. More specifically, the aim of this work is twofold: (i) to discuss QCD condensate contributions to the gluon propagator and (ii) to evaluate the nonperturbative corrections to the one-gluon exchange quark potential in covariant gauges. This is then a natural continuation of the work done in Ref. [14]. We use here the nonperturbative gluon propagator, allowing for the presence of the ghost condensate which appears in covariant gauges.

In Sec. II, we discuss the nonperturbative gluon propagator both in the fixed-point gauge and in covariant gauges. In Sec. III, we present our main results of the gauge-invariant quark potential with the nonperturbative corrections of QCD vacuum condensates. The brief summary in Sec. IV contains a partial comparison of the obtained quark potential with lattice and renormalons results, and some possible applications of the obtained quark potential.

II. NONPERTURBATIVE GLUON PROPAGATORS IN DIFFERENT GAUGES

In this section, we discuss the nonperturbative gluon propagator in the fixed-point gauge and in covariant gauges for the external QCD vacuum gluon fields (nonperturbative ones). Recently, the OPE of the gluon propagator in QCD has been extensively studied [17–22].

In QCD sum rules for gauge invariant currents, the background field method is used, where the fixed-point gauge is generally employed for nonperturbative gluon fields, i.e.,

$$x_\mu B_a^\mu(x) = 0. \tag{1}$$

But, for perturbative gluon fields, a covariant gauge is usually adopted, i.e.,

$$iD^{ab}_{\mu\nu}(q)=i\,\delta_{ab}\left[-\frac{g_{\mu\nu}}{q^2}+(1-\xi)\frac{q_\mu q_\nu}{(q^2)^2}\right]. \tag{2}$$

In the fixed-point gauge, the nonlocal two-gluon VEV can be written as [4,23]

$$\begin{aligned}\langle 0|B^a_\mu(x)B^b_\nu(y)|0\rangle &= \frac{1}{4}x^\rho y^\sigma\langle 0|G^a_{\rho\mu}G^b_{\sigma\nu}|0\rangle+\cdots\\ &=\frac{\delta_{ab}}{48(N_c^2-1)}x^\rho y^\sigma(g_{\rho\sigma}g_{\mu\nu}-g_{\rho\nu}g_{\sigma\mu})\\ &\quad\times\langle 0|G^2|0\rangle+\cdots,\end{aligned} \tag{3}$$

where

$$\langle 0|G^2|0\rangle=\langle 0|G^a_{\rho\mu}G^{\rho\mu}_a|0\rangle. \tag{4}$$

Obviously, the expansion (3) violates translational invariance since the right hand side (RHS) of Eq. (3) is a function of xy instead of $(x-y)$.

To obtain the correct nonperturbative gluon propagator, it is essential to obtain the expansion of $\langle 0|B^a_\mu(x)B^b_\nu(y)|0\rangle$ with translational invariance. The basic requirements for translational invariance were studied before [24]. According to these requirements and the Lorentz gauge condition

$$\partial^\mu B^a_\mu(x)=0, \tag{5}$$

the nonlocal two-gluon VEV can be expressed as

$$\begin{aligned}\langle 0|B^a_\mu(x)B^b_\nu(y)|0\rangle &=\langle 0|B^a_\mu(0)B^b_\nu(0)|0\rangle\\ &\quad-\frac{\delta_{ab}}{2(N_c^2-1)}(x-y)^\rho(x-y)^\sigma\\ &\quad\times\langle 0|\partial_\rho B^d_\mu(0)\partial_\sigma B^d_\nu(0)|0\rangle+\cdots,\end{aligned} \tag{6}$$

where

$$\begin{aligned}\langle 0|B^a_\mu(0)B^b_\nu(0)|0\rangle &=\frac{g_{\mu\nu}}{4}\frac{\delta_{ab}}{(N_c^2-1)}\langle 0|B^d_\rho(0)B^\rho_d(0)|0\rangle\\ &=\frac{g_{\mu\nu}}{4}\frac{\delta_{ab}}{(N_c^2-1)}\langle 0|B^2|0\rangle\end{aligned} \tag{7}$$

and

$$\begin{aligned}&\frac{1}{2}\langle 0|\partial_\rho B^a_\mu(0)\partial_\sigma B^a_\nu(0)|0\rangle\\ &\quad=\left[Sg_{\mu\nu}g_{\rho\sigma}+\frac{R}{2}(g_{\rho\nu}g_{\sigma\mu}+g_{\rho\mu}g_{\nu\sigma})\right].\end{aligned} \tag{8}$$

Contracting Eq. (8) with $g^{\rho\sigma}g^{\mu\nu}$, $g^{\rho\mu}g^{\sigma\nu}$, and $g^{\rho\nu}g^{\sigma\mu}$ leads to

$$\frac{1}{2}\langle 0|\partial^\sigma B^\nu_a(0)\partial_\sigma B^a_\nu(0)|0\rangle=16S+4R, \tag{9}$$

$$\frac{1}{2}\langle 0|\partial^\mu B^a_\mu(0)\partial^\nu B^a_\nu(0)|0\rangle=4S+10R, \tag{10}$$

and

$$\frac{1}{2}\langle 0|\partial^\nu B^\mu_a(0)\partial_\mu B^a_\nu(0)|0\rangle=4S+10R, \tag{11}$$

respectively. According to the Lorentz gauge condition (5), (10) means

$$R=-\frac{2}{5}S. \tag{12}$$

Furthermore, using the definition of the gluon field strength and by only retaining the contribution of the vacuum intermediate state [25], one can easily find that

$$\begin{aligned}&\langle 0|G^a_{\rho\mu}(0)G^b_{\sigma\nu}(0)|0\rangle\\ &\approx[\langle 0|\partial_\rho B^a_\mu(0)\partial_\sigma B^b_\nu(0)|0\rangle+\langle 0|\partial_\mu B^a_\rho(0)\partial_\nu B^b_\sigma(0)|0\rangle]\\ &\quad-[\langle 0|\partial_\mu B^a_\rho(0)\partial_\sigma B^b_\nu(0)|0\rangle+\langle 0|\partial_\rho B^a_\mu(0)\partial_\nu B^b_\sigma(0)|0\rangle]\\ &\quad+\frac{\pi\alpha_s N_c}{3(N_c^2-1)^2}\delta_{ab}[g_{\rho\sigma}g_{\mu\nu}-g_{\mu\sigma}g_{\rho\nu}]\langle 0|B^2|0\rangle^2\end{aligned} \tag{13}$$

which results in

$$S\approx\frac{5\langle 0|G^2|0\rangle}{288}-\frac{5N_c\pi\alpha_s}{72(N_c^2-1)}\langle 0|B^2|0\rangle^2. \tag{14}$$

In addition, combining Eq. (11) with Eqs. (9) and (12), one can get

$$S=\frac{5\langle 0|(\partial_\nu B^a_\sigma-\partial_\sigma B^a_\nu)^2|0\rangle}{288}. \tag{15}$$

Comparing Eqs. (14) and (15) leads to

$$\alpha_s\langle 0|B^2|0\rangle^2\approx 0 \tag{16}$$

provided that the approximation of the vacuum dominance in intermediate states is accepted, and that the equality of $\langle 0|(\partial_\nu B^a_\sigma-\partial_\sigma B^a_\nu)^2|0\rangle$ and the Abelian part of $\langle 0|G^2|0\rangle$ has been used [20]. The dimension-two condensate $\langle 0|B^2|0\rangle$ is not gauge invariant. According to our estimate of its value, this term (maybe due to a spontaneous gauge symmetry breaking) is very small and can be omitted as compared with the gauge-invariant gluon condensate $\langle 0|G^2|0\rangle$. We will show in the following discussion that the vanish of the condensate $\langle 0|B^2|0\rangle$ is essential to retain the gauge invariance of the vacuum energy density difference.

Therefore, by considering Eq. (12), Eq. (6) can also be rewritten as

$$\langle 0|B^a_\mu(x)B^b_\nu(y)|0\rangle = -\frac{\delta_{ab}}{(N_c^2-1)}S \times\left[(x-y)^2 g_{\mu\nu} - \frac{2}{5}(x-y)_\mu (x-y)_\nu\right] + \cdots \tag{17}$$

with manifest translational invariance. This result was used before by Bagan *et al.* [22] without looking at the condensate $\langle 0|B^2|0\rangle$. Now let us focus our attention on discussing the vacuum energy density difference and the quark scattering amplitude, which are related to the nonperturbative gluon propagator.

A. ξ dependence of the vacuum energy density difference

The vacuum energy density difference (the effective potential of Coleman and Weinberg [26]) is defined as the perturbative contribution to the difference between the energy densities of the physical and bare vacua. The condensate contribution to the vacuum energy difference [17] is of interest because it can tell us something about the energy dependence on the condensates of the QCD vacuum. In order to calculate QCD condensate contributions to the vacuum energy density difference, we need quark condensate contributions to the quark self-energy and the gluon vacuum polarization. They can be expressed as [17]

$$\Sigma^{\langle\bar{q}q\rangle} = \sum_f \frac{(N_c^2-1)\pi\alpha_s\langle 0|\overline{q_f}q_f|0\rangle}{2N_c^2 q^2}\left[3+\xi-\xi m_f \frac{\not{q}}{q^2}\right] \tag{18}$$

and

$$\Pi^{\langle\bar{q}q\rangle}_{\mu\nu} = \sum_f \frac{4\pi\alpha_s m_f\langle 0|\overline{q_f}q_f|0\rangle}{N_c q^2} g^\perp_{\mu\nu}(q), \tag{19}$$

respectively, with $g^\perp_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu/q^2$. The form of the gluon condensate contribution depends on the choice of the gauge for the external QCD gluon fields. In the fixed-point gauge, the contribution to the quark self-energy is [23]

$$\Sigma^{\langle G^2\rangle} = \frac{\pi\alpha_s\langle 0|G^2|0\rangle m_f(q^2 - m_f\not{q})}{N_c(q^2-m_f^2)^3} \tag{20}$$

and, by employing Eq. (3), the gluon condensate contribution to the gluon polarization can be obtained as

$$\Pi^{\langle G^2\rangle}_{\mu\nu} = \frac{N_c\pi\alpha_s\langle 0|G^2|0\rangle}{12(N_c^2-1)q^2} \times\left[(36+\xi)g^\perp_{\mu\nu}(q) - \left(12+\frac{18}{\xi}\right)\frac{q_\mu q_\nu}{q^2}\right]. \tag{21}$$

By using the above self-energies, we obtain the condensate contributions to the vacuum energy density in the fixed-point gauge as

$$V(\langle\bar{q}q\rangle,\langle G^2\rangle) = \left[\frac{9\pi\alpha_s N_c\langle 0|G^2|0\rangle}{48}(10-\xi) + \sum_f \frac{6\pi\alpha_s(N_c^2-1)m_f\langle 0|\overline{q_f}q_f|0\rangle}{N_c}\right] \times\int_{q^2<-\mu^2}(-i)\frac{d^4q}{(2\pi)^4}\frac{1}{(q^2+i\epsilon)^2}, \tag{22}$$

which is ξ dependent. The renormalization point μ^2 is used in QCD to divide the momentum range into a perturbative and a nonperturbative regions. As pointed out by Jackiw [27], any gauge dependence of the effective potential for a particular operator makes a physical interpretation questionable. This problem can be avoided in covariant gauges. Actually, Lavelle and Schaden [17] got a gauge invariant vacuum energy density difference by taking into account the ghost condensate contribution in covariant gauges. With our estimate of $\langle 0|B^2|0\rangle$ given in Eq. (16), the vacuum energy density difference obtained in Ref. [17] is gauge invariant even if there exists a spontaneous gauge symmetry breaking.

B. Unitarity of the quark scattering amplitude

It is well known that the ξ dependence of the perturbative gluon propagator does not carry through to the quark scattering amplitude. However, it is quite another story to prove this for the nonperturbative gluon propagator. In the fixed-point gauge, the QCD condensate contribution to leading order in α_s to the gluon propagator can easily be obtained as

$$iD_{\mu\nu}(q) = i\left\{-\frac{1}{q^2}A_T g^\perp_{\mu\nu}(q) + A_L\frac{q_\mu q_\nu}{q^4}\right\}, \tag{23}$$

where

$$A_T = \frac{N_c\pi\alpha_s\langle 0|G^2|0\rangle}{12(N_c^2-1)q^4}(36+\xi) + \sum_f \frac{4\pi\alpha_s m_f\langle 0|\overline{q_f}q_f|0\rangle}{N_c q^2(q^2-m_f^2)}, \tag{24}$$

$$A_L = \frac{N_c\pi\alpha_s\langle 0|G^2|0\rangle}{12(N_c^2-1)q^4}(18\xi+12\xi^2). \tag{25}$$

The gluon vacuum polarization used in deriving Eq. (23) is given in Eq. (21). Notice that the longitudinal term in Eq. (21) is both nonzero and ξ dependent. Hence, the unitarity in the non-Abelian coupling amplitude cannot be satisfied, which stimulates us to try to use a covariant gauge for the external QCD vacuum gluon fields. At least, by introducing the covariant gauge for the gluon VEV, the difficulty of two potentially conflicting gauge conditions for the gluon VEV and the perturbative gluon fields in QCD can be avoided.

In covariant gauges, by using the nonlocal two-gluon VEV given in Eq. (17), the lowest-dimension gluon condensate contribution to the gluon propagator can be written as

$$iG_{\mu\nu}^{\langle G^2\rangle}(q)=i\left\{\frac{(25-3\xi)\pi\alpha_s\langle 0|G^2|0\rangle}{24q^6}g_{\mu\nu}^{\perp}(q)-\frac{3\xi^2\pi\alpha_s\langle 0|G^2|0\rangle}{8q^8}q_\mu q_\nu\right\}. \tag{26}$$

There is also a longitudinal term in the gluon polarization of Eq. (26), i.e., the Slavnov-Taylor identities (STI) [28] is not fulfilled here. This problem can be fixed by allowing for the presence of the ghost condensate. In Ref. [19], Lavelle and Schaden got a transverse gluon vacuum polarization by taking into account mixing with equation of motion condensates. Their result for the gluon vacuum polarization in covariant gauges with the corrections of gluon and ghost condensates as depicted in Figs. 1(a)–1(c) is

$$\Pi_{\mu\nu}(q^2)=-\frac{N_c\pi\alpha_s(68+3\xi)}{18(N_c^2-1)q^2}g_{\mu\nu}^{\perp}\langle 0|(\partial_\nu B_\sigma^a-\partial_\sigma B_\nu^a)^2|0\rangle, \tag{27}$$

where condensate terms which vanish due to the equation of motion are not shown. It is noteworthy that leading mixed condensate contributions to the gluon polarization (27) have also been taken into account (the diagrams for the mixed condensate contributions are not shown in Fig. 1, see Ref. [19]). Although the vacuum polarization (27) is transverse, it is also explicitly ξ dependent. To obtain a gauge invariant effective gluon propagator, Lavelle [20] investigated the effects of the $\langle 0|G^2|0\rangle$ condensate on the effective gluon propagator in quark interactions by using the pinch technique (PT) in the context of QCD [29,30] (see diagrams in Ref. [20] for the PT). The effective gluon propagator was finally obtained as

$$iG_{\mu\nu}^{T}(q)=i\left\{-\frac{1}{q^2}+\left[\frac{34N_c\pi\alpha_s\langle 0|G^2|0\rangle}{9(N_c^2-1)q^6}-\frac{4\pi\alpha_s}{N_cq^4}\sum_f m_f\langle 0|\overline{q}_fq_f|0\rangle\times\left(\frac{1}{q^2-m_f^2}+\frac{1}{2}\frac{m_f^2}{(q^2-m_f^2)^2}\right)\right]\right\}g_{\mu\nu}^{\perp}(q)-\xi\frac{q_\mu q_\nu}{q^4}. \tag{28}$$

Here, following Ref. [20], we have identified $\langle 0|(\partial_\nu B_\sigma^a-\partial_\sigma B_\nu^a)^2|0\rangle$ with the Abelian part of $\langle 0|G^2|0\rangle$. The ghost condensate is necessary in order to achieve a gauge invariant form of the gluon propagator. The fact that the ghost condensate contribution to the gluon propagator (28) is not explicit is due to the following two reasons: (i) gluon and ghost equations of motion and (ii) pinch technique (PT) (see Ref. [20]). The quark condensate contribution term differs from that in Eq. (23) due to the fact that the next-to-leading-order term in the full coefficient of the $\langle\overline{q}q\rangle$ component of the nonperturbative two-quark VEV [23] is retained.

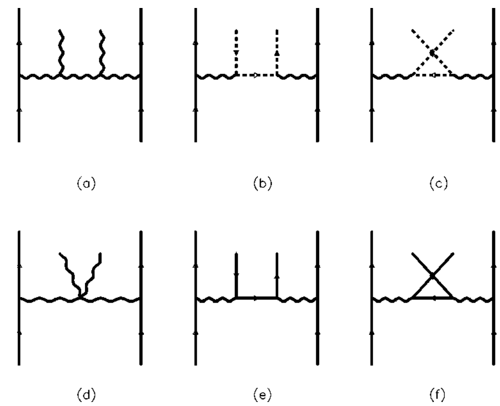

FIG. 1. The Feynman diagrams for the contributions of the nonperturbative corrections to perturbative quark-quark potential in the one-gluon exchange approximation with the lowest dimensional gluon, ghost, and quark condensates.

After the above detailed discussion on the nonperturbative gluon propagator, we can now stress our main reason for deriving the quark interaction potential from the quark scattering amplitude in covariant gauges for the external QCD vacuum gluon fields. First, one can obtain a gauge invariant vacuum energy density difference in covariant gauges. However, the ξ dependence of Eq. (21) in the fixed-point gauge brings about the explicit ξ dependence in the vacuum energy density difference as shown in Eq. (22), which raises doubts about its physical validity. Second, the transverse portion of the nonperturbative gluon propagators in covariant gauges is ξ independent. In contrast to this, the explicit ξ dependence in the transverse portion of the nonperturbative gluon propagator in the fixed-point gauge results in a quark scattering amplitude with doubtful physical meaning. All of this motivates us to employ the nonperturbative gluon propagator in covariant gauges to derive the effective quark interaction potential.

III. QCD CONDENSATE CONTRIBUTIONS TO THE QUARK INTERACTION POTENTIALS

To derive the quark-quark interaction potential from the nonperturbative gluon propagator, we write down a proper scattering amplitude between two quarks as

$$M=(-ig)^2\overline{\psi}(p_1)\gamma^\mu\frac{\lambda^a}{2}\psi(p_1')G_{\mu\nu}^T(q)\overline{\psi}(p_2)\gamma^\nu\frac{\lambda^a}{2}\psi(p_2') \tag{29}$$

with $q=p_1-p_1'=p_2'-p_2$ and the spinors $\psi(p_i)$ being the solution of free quarks. We then obtain the effective potential in momentum space by carrying out the Breit-Fermi expansion with the approximation $q_0=0$. Applying the three-dimensional Fourier transformation to convert the potential in momentum space to coordinate space, we finally obtain the total effective potential between quarks as

$$U_{qq}(x)=U_{qq}^{\mathrm{OGEP}}(x)+U_{qq}^{\mathrm{NP}}(x), \tag{30}$$

where $U_{qq}^{\rm OGEP}(x)$ is the perturbative one-gluon-exchange quark potential

$$U_{qq}^{\rm OGEP}(x)=\delta(t)\frac{\lambda_1^a\lambda_2^a}{4}\alpha_s\Bigg\{\frac{1}{|\vec{x}|}-\frac{\pi}{m_1m_2}\left(\frac{(m_1+m_2)^2}{2m_1m_2}+\frac{2}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\delta(\vec{x})+\frac{|\vec{p}|^2}{m_1m_2|\vec{x}|}-\frac{1}{4m_1m_2|\vec{x}|^3}\times\left[\frac{3}{|\vec{x}|^2}(\vec{\sigma}_1\cdot\vec{x})(\vec{\sigma}_2\cdot\vec{x})-(\vec{\sigma}_1\cdot\vec{\sigma}_2)\right]-\frac{1}{4m_1m_2|\vec{x}|^3}\times\left[\left(2+\frac{m_2}{m_1}\right)\vec{\sigma}_1+\left(2+\frac{m_1}{m_2}\right)\vec{\sigma}_2\right]\cdot(\vec{x}\times\vec{p})\Bigg\},\quad(31)$$

and $U_{qq}^{\rm NP}(x)$, the nonperturbative correction to the perturbative quark-quark interaction due to the quark, gluon, and ghost condensates, can be expressed as

$$U_{qq}^{\rm NP}(x)=\delta(t)\frac{\lambda_1^a\lambda_2^a}{4}\pi\alpha_s^2\Bigg[A_3|\vec{x}|^3+(A_1+2C_1)|\vec{x}|+2C_{-1}|\vec{x}|^{-1}+2\sum_f\,(\tilde{C}_0^{(f)}+\tilde{C}_{-1}^{(f)}|\vec{x}|^{-1})e^{-m_f|\vec{x}|}\Bigg],\quad(32)$$

where

$$A_3=\frac{17N_c\langle 0|G^2|0\rangle}{108(N_c^2-1)}\left(1+\frac{|\vec{p}|^2}{m_1m_2}\right),\quad(33)$$

$$A_1=\frac{17N_c\langle 0|G^2|0\rangle}{72(N_c^2-1)}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2+\frac{17N_c\langle 0|G^2|0\rangle}{432m_1m_2(N_c^2-1)}(8\vec{\sigma}_1\cdot\vec{\sigma}_2-S_{12})+\frac{17N_c\langle 0|G^2|0\rangle}{144m_1m_2(N_c^2-1)}\left[\left(2+\frac{m_2}{m_1}\right)\vec{\sigma}_1+\left(2+\frac{m_1}{m_2}\right)\vec{\sigma}_2\right]\cdot(\vec{x}\times\vec{p}),\quad(34)$$

$$C_1=\left(1+\frac{|\vec{p}|^2}{m_1m_2}\right)\sum_f\frac{\langle 0|\bar{q}_fq_f|0\rangle}{N_cm_f},\quad(35)$$

$$C_{-1}=\frac{1}{4N_cm_1m_2}\sum_f\frac{\langle 0|\bar{q}_fq_f|0\rangle}{m_f}\left\{\frac{(m_1+m_2)^2}{m_1m_2}+\frac{S_{12}}{3}+\frac{4}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2+\left[\left(2+\frac{m_2}{m_1}\right)\vec{\sigma}_1+\left(2+\frac{m_1}{m_2}\right)\vec{\sigma}_2\right]\cdot(\vec{x}\times\vec{p})\right\},\quad(36)$$

$$\tilde{C}_0^{(f)}=\frac{2}{N_c}\frac{\langle 0|\bar{q}_fq_f|0\rangle}{m_f}\left[\frac{1}{2m_f}\left(1+\frac{|\vec{p}|^2}{m_1m_2}\right)+\frac{m_f(m_1+m_2)^2}{16m_1^2m_2^2}-\frac{m_f}{24m_1m_2}S_{12}+\frac{m_f}{12m_1m_2}\vec{\sigma}_1\cdot\vec{\sigma}_2\right],\quad(37)$$

and

$$\tilde{C}_{-1}^{(f)}=-\frac{2}{N_c}\frac{\langle 0|\bar{q}_fq_f|0\rangle}{m_f}\left\{\frac{(m_1+m_2)^2}{8m_1^2m_2^2}+\frac{S_{12}}{6m_1m_2}+\frac{1}{6m_1m_2}\vec{\sigma}_1\cdot\vec{\sigma}_2-\frac{3}{24m_1m_2}\left[\left(2+\frac{m_2}{m_1}\right)\vec{\sigma}_1+\left(2+\frac{m_1}{m_2}\right)\vec{\sigma}_2\right]\cdot(\vec{x}\times\vec{p})\right\},\quad(38)$$

with $S_{12}=3(\vec{\sigma}_1\cdot\vec{n})(\vec{\sigma}_2\cdot\vec{n})-\vec{\sigma}_1\cdot\vec{\sigma}_2$ and $\vec{n}=\vec{x}/|\vec{x}|$.

For heavy quarkonium systems, where the potential concept is applicable, the quark-antiquark interaction can be obtained by taking the color generators for an antiquark as $-\lambda^T$, i.e.,

$$U_{q\bar{q}}^{\rm Direct}(x)=U_{qq}(x)|_{\lambda_1^a\lambda_2^a\to-\lambda_1^a(\lambda_2^a)^T}.\quad(39)$$

However, in this case, if the quark and antiquark have the same flavor, the annihilation mechanism should also be taken into account. The condensate contributions to this mechanism can be sketched in Fig. 2 (as for leading mixed condensate contributions and digrams for the PT, see Refs. [19,20]). The total $q\bar{q}$-pair annihilation potential can be obtained by summing up the contributions of all diagrams including nonperturbative ones in Fig. 2 and the corresponding perturbative one

$$U_{q\bar{q}}^{\rm ann(total)}(x)=U_{q\bar{q}}^{\rm ann}(x)+U_{q\bar{q}}^{\rm ann(NP)}(x)\quad(40)$$

where $U_{q\bar{q}}^{\rm ann}(x)$, the perturbative $q\bar{q}$ pair-annihilation potential in coordinate representation, is

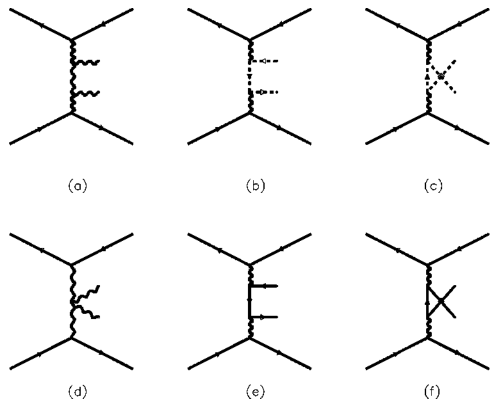

FIG. 2. The Feynman diagrams for the contributions of the nonperturbative corrections to perturbative $q\overline{q}$-pair annihilation potential in the one-gluon exchange approximation with the lowest dimensional gluon, ghost, and quark condensates.

$$U_{q\overline{q}}^{\mathrm{ann}}(x)=\delta(t)\frac{\alpha_s}{4}\frac{\pi}{16N_cm^2}(\lambda_1-\lambda_2^{\mathrm{T}})^2(1-\vec{\tau}_1\cdot\vec{\tau}_2)\times\left\{(\vec{\sigma}_1+\vec{\sigma}_2)^2\left(1-\frac{1}{3m^2}\vec{\nabla}^2\right)\delta(\vec{x})-\frac{4}{m^2}\left[(\vec{\sigma}_1\cdot\vec{\nabla})\times(\vec{\sigma}_2\cdot\vec{\nabla})-\frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\nabla}^2\right]\delta(\vec{x})\right\} \tag{41}$$

and

$$U_{q\overline{q}}^{\mathrm{ann(NP)}}(x)=\frac{\pi\alpha_s}{m^2}\left\{\frac{N_c}{(N_c^2-1)}\frac{17\langle 0|G^2|0\rangle}{72m^2}+\frac{1}{N_c}\sum_f\frac{m_f\langle 0|\overline{q}_fq_f|0\rangle}{(4m^2-m_f^2)^2}(8m^2-m_f^2)\right\}U_{q\overline{q}}^{\mathrm{ann}}(x). \tag{42}$$

IV. SUMMARY AND DISCUSSION

In this paper, we discussed the condensate contributions to the gluon propagator which is then used to derive the nonperturbative contributions to the quark potentials. To help the reader to understand what is new in this paper, we summarize some important points.

(1) We estimated the value of $\langle 0|B^2|0\rangle^2$ in the approximation of the vacuum dominance in intermediate states and found that this non-gauge-invariant condensate can be omitted as compared with the gauge-invariant gluon condensate contribution, which is of crucial importance for having gauge invariant vacuum energy density differences [17].

(2) We gave a detailed discussion about the nonperturbative gluon propagator and showed that it is essential to adopt covariant gauges in order to obtain a gauge invariant vacuum energy density difference and to retain the unitarity of the quark scattering amplitude.

(3) The gauge-invariant nonperturbative contributions to the one-gluon exchange quark potentials were obtained by employing the nonperturbative gluon propagator, with the validity of the unitarity of the quark scattering amplitude from which the quark potential is derived.

In the fixed-point gauge, some uncertainties in the nonperturbative calculation such as three-point fermionic Green function $G^{\mu}_{\alpha\beta}(p',p,q)$ are unavoidable [31]. To overcome this, one can complete the calculation in covariant gauges [32]. Of course, it is not enough to judge whether the result in any gauge for the external QCD vacuum gluon fields is better than the one in others only by calculating gauge-dependent objects such as quark-gluon n-point Green functions. However, we should accept the fact that the explicit ξ dependence of the energy density difference in the fixed-point gauge is incompatible with the physical meaning of a gauge invariant object. In the context of the OPE, the fixed-point gauge is very convenient. But, as we have seen here, one should be very careful in using this gauge.

Lattice QCD, which is becoming more and more successful, can include nonperturbative effects by means of the Wilson loop. An OPE based parameterization of the lattice results would be highly welcome phenomenologically. Since a lot of lattice results on the interquark potential have been obtained recently [9], and in order to make the gauge-invariant quark potential achieved here more convincing, we make a partial comparison with what has been obtained from lattice calculations. Let us look into the spin dependent piece of the nonperturbative corrections. The spin dependent (SD) correction due to the quark condensate provides a piece of the spin dependent term proportional to $1/|x|$ in lattice simulations, a result given by Bali, Schilling, and Wachter in Ref. [9]. With the phenomenological value of the quark condensates in QCD sum rules [33], the momentum-dependent (MD) result proportional to the $1/|x|$ piece both in our potential and in that of lattice simulations [9] not only have the same sign, but also are compatible in magnitude. Although there is not a formal relationship between the results of the present work and those of lattice simulations, they reflect the same nonperturbative effects of QCD to some degree. Very recently, evidence of an unexpected Λ^2/q^2 power correction to the gluon condensate has been obtained in the lattice study of Ref. [34]. In the case of the static potential in position space, such a Λ^2/q^2 correction results in a term proportional to the quark separation, $|x|$. Akhoury and Zakharov [7] discuss nonperturbative corrections to the Coulomb-like potential of heavy quarks at short distances by considering the standard framework provided by infrared renormalons. They found that the leading correction at short distances is linear in $|x|$. In our modified potential, the linear correction which appears due to the quark and gluon condensates is consistent with both the lattice simulations results and renormalons calculations.

There are many possible applications of the obtained quark potentials, for instance, in the study of the nonperturbative effect in the spectra of J/ψ and Υ, especially to improve the spin splitting for these systems. As an extension of this work, we will verify whether the potentials obtained here can be used to improve the hadronic spectra and hadronic properties of J/ψ and Υ families by including perturbative closed-loop contributions in the same order of α_s as shown by Gupta *et al.* [35], Fulcher [36], and Pantaleone *et al.* [37]. In addition, this work is intended to serve as a step forward in the direction of solving long-standing problems in light baryon spectroscopy such as the energy level order between the positive- and negative-parity partner states, in particular, Roper resonance puzzle, and the baryon spin-orbit structure puzzle. The nonperturbative correction to the quark potential is bound to enrich our understanding of the quark interaction.

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