Effects of the $K^+\to\pi^+\nu\bar{\nu}$ and of other processes on the mixing hierarchies in the four-generation model

Toshihiko Hattori*
Institute of Theoretical Physics, University of Tokushima, Tokushima 770-8502, Japan

Tsutom Hasuike†
Department of Physics, Anan College of Technology, Anan 774, Japan

Seiichi Wakaizumi‡
School of Medical Sciences, University of Tokushima, Tokushima 770-8509, Japan

(Received 25 January 1999; revised manuscript received 4 August 1999; published 12 November 1999)

We analyze in the four-generation model the first measurement of the branching ratio of the rare kaon decay $K^+\to\pi^+\nu\bar{\nu}$, along with the other processes of the K_L-K_S mass difference Δm_K, CP-violating parameter ε_K, B_d-$\bar{B}_d$ mixing, B_s-$\bar{B}_s$ mixing, $B(K_L\to\mu\bar{\mu})$, and the upper bound values of D^0-$\bar{D}^0$ mixing and $B(K_L\to\pi^0\nu\bar{\nu})$, and try to search for mixing of the fourth generation in the hierarchical mixing scheme of Wolfenstein parametrization. Using the results for the mixing of the fourth generation, we discuss predictions of D^0-$\bar{D}^0$ mixing (Δm_D) and the branching ratio of directly CP-violating decay process $K_L\to\pi^0\nu\bar{\nu}$, and the effects on the CP asymmetry in neutral B meson decays and the unitarity triangle. [S0556-2821(99)01923-2]

PACS number(s): 13.20.Eb, 12.15.−y, 12.60.−i, 13.20.−v

I. INTRODUCTION

In the physics of quarks and leptons, it has been a long time since the standard model achieved remarkable success. As we show, however, on the issue of the mass generation of quarks and leptons and physics such as supersymmetry (SUSY), physics beyond the standard model has become highly regarded. In this direction, the flavor-changing neutral current (FCNC) processes play an important role through the one-loop effects for the search of additional Higgs bosons, new gauge bosons, additional fermions, etc.

Here we focus on the new branching ratio of the FCNC process $K^+\to\pi^+\nu\bar{\nu}$, which is measured for the first time at the Brookhaven National Laboratory, $B=(4.2^{+9.7}_{-3.5})\times10^{-10}$ [1]. It should be remarked that the central value is 4–6 times larger than the standard model prediction, $B=(0.6-1.5)\times10^{-10}$ [2], though the measurement is consistent with the theory within the experimental errors.

This process $K^+\to\pi^+\nu\bar{\nu}$ was studied by Gaillard and Lee in 1974 and they obtained a branching ratio of $\sim10^{-10}$ by using the short-distance W-W box and Z^0-penguin diagrams in the ''four-quark'' model [3]. After that in 1981, Inami and Lim derived the rigorous expressions for these and other related diagrams relevant to the FCNC processes and studied the effects of superheavy quarks and leptons in $K_L\to\mu\bar{\mu}$, $K^+\to\pi^+\nu\bar{\nu}$, and K^0-$\bar{K}^0$ mixing [4], before the top quark was discovered.

In this work, we analyze the new branching ratio of $K^+\to\pi^+\nu\bar{\nu}$ in the four-generation model [5–7] under the expectation that the above-mentioned factor 4–6 of the measured value relative to the standard model predictions may imply the existence of a fourth generation with roughly the same mixing as for the third generation. We will investigate various possible mixings for the fourth generation by imposing the constraints of $K^+\to\pi^+\nu\bar{\nu}$ and other processes of K_L-K_S mass difference Δm_K, CP-violating parameter ε_K, B_d-$\bar{B}_d$ mixing, B_s-$\bar{B}_s$ mixing, D^0-$\bar{D}^0$ mixing, $B(K_L\to\pi^0\nu\bar{\nu})$ and $B(K_L\to\mu\bar{\mu})$, and we will study its effects on the D^0-$\bar{D}^0$ mixing and $B(K_L\to\pi^0\nu\bar{\nu})$, of which only the upper bounds are experimentally known, CP violation in neutral B meson decays and the unitarity triangle.

The paper is organized as follows. The four-generation model we use here is presented in Sec. II. In Sec. III we describe the phenomenological constraints on the model to search for possible mixings of the fourth generation. In Sec. IV we derive the ''maximum'' mixings allowed by the constraints. In Sec. V we discuss the consequences of the mixings on the D^0-$\bar{D}^0$ mixing and the branching ratio of another FCNC decay process $B(K_L\to\pi^0\nu\bar{\nu})$, CP asymmetry in B_d meson decays and the unitarity triangle, and finally we give conclusions.

II. THE FOUR-GENERATION MODEL

For the unitary 4×4 quark mixing matrix in the four-generation scheme, we will use the Hou-Soni-Steger parametrization [8]. The form of this parametrization is so complicated that we will not cite it here. It has, however, an advantage over the others that the third column and the fourth row have simple forms such that $(V_{ub},V_{cb},V_{tb},V_{t'b})=(s_zc_ue^{-i\phi_1},s_yc_zc_u,c_yc_zc_u,-s_u)$ and $(V_{t'd},V_{t's},V_{t'b},V_{t'b'})=(-c_uc_vs_we^{i\phi_3},-c_us_ve^{i\phi_2},-s_u,c_uc_vc_w)$, and $V_{us}=s_xc_zc_v-s_zs_us_ve^{i(\phi_2-\phi_1)}$, so that the three mixing angles $s_x(\equiv\sin\theta_x)$, s_y and s_z give the elements V_{us}, V_{cb}, and V_{ub}, respectively, as in the standard

*Email address: hattori@ias.tokushima-u.ac.jp
†Email address: hasuike@anan-nct.ac.jp
‡Email address: wakaizum@medsci.tokushima-u.ac.jp

model, and the phase ϕ_1 corresponds to the Kobayashi-Maskawa (KM) *CP*-violating phase $\delta^{\rm KM}$ [9]. The angles $s_u(\equiv \sin\theta_u)$, s_v and s_w, which give the elements $V_{t'b}$, $V_{t's}$, and $V_{t'd}$, respectively, are new mixing angles, and ϕ_2 and ϕ_3 are new phases. t' and b' are the fourth generation up and down quark, respectively.

Since the magnitude of the three elements V_{us}, V_{cb}, and V_{ub} are experimentally determined from the semileptonic decays of hyperons, B mesons to hadrons with c and u quark, respectively, and are not affected by the existence of the fourth generation, we use the same values for the three angles s_x, s_y, and s_z as in the standard model [2] as an input of our analysis,

$$s_x = 0.22, \quad s_y = 0.040\pm 0.003, \quad s_z/s_y = 0.08\pm 0.02. \tag{1}$$

We search for the mixings of the fourth generation allowed by the experimental quantities related to various FCNC processes. The mixing among the three generations in the standard model is known to be hierarchical as is well expressed by the Wolfenstein parametrization [10]

$$V^{(3)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A(\rho - i\eta)\lambda^3 \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A(1-\rho-i\eta)\lambda^3 & -A\lambda^2 & 1 \end{pmatrix}, \tag{2}$$

where $\lambda \equiv \sin\theta_C(\simeq 0.22)$ is the expansion parameter in the Wolfenstein parametrization. In the spirit of this parametrization, we will study the following cases of the fourth generation mixing to derive a ''maximum'' one allowed by the above-mentioned constraints:

$$\begin{aligned}(V_{t'd}, V_{t's}, V_{t'b}, V_{t'b'}) \simeq\ & (\lambda^5,\lambda^4,\lambda^3,1),\\ & (\lambda^4,\lambda^3,\lambda^2,1),\\ & (\lambda^3,\lambda^2,\lambda,1),\\ & (\lambda^2,\lambda^2,\lambda,1),\\ & (\lambda^3,\lambda^2,1,\lambda),\\ & (\lambda^2,\lambda,1,\lambda),\\ & (0,\lambda^3,\lambda,1),\\ & (0,\lambda^2,\lambda,1).\end{aligned} \tag{3}$$

Here we are not interested in the last two cases with $V_{t'd} = 0$ because we will focus on the factor 4–6 of the central value of the measured branching ratio of $K^+ \to \pi^+ \nu\bar{\nu}$, relative to the predicted value in the standard model.

Table I shows the products of the relevant mixing matrix elements of the dominant contributions to the one-loop diagrams in B_d-$\bar{B}_d$ mixing (Δm_{B_d}), $b\to s\gamma$ decay, $K^+ \to \pi^+ \nu\bar{\nu}$ decay, and short-distance (SD) contributions to $K_L \to \mu\bar{\mu}$($(K_L\to\mu\bar{\mu})_{\rm SD}$) for the standard model and the four-generation model with the first four cases of mixing of Eq. (3). As seen in Table I, the first two cases of $(V_{t'd}, V_{t's}, V_{t'b}) \simeq (\lambda^5,\lambda^4,\lambda^3)$ and $(\lambda^4,\lambda^3,\lambda^2)$ give too small contributions to affect the branching ratio of $K^+ \to \pi^+ \nu\bar{\nu}$ and they also do not give any significant contributions to Δm_{B_d} and $(K_L\to\mu\bar{\mu})_{\rm SD}$. The third case of $(V_{t'd}, V_{t's}, V_{t'b}) \simeq (\lambda^3,\lambda^2,\lambda)$ gives the same order of contributions to $K^+ \to \pi^+ \nu\bar{\nu}$ and $(K_L\to\mu\bar{\mu})_{\rm SD}$ as in the standard model. It turns out that even this favorable case of $(\lambda^3,\lambda^2,\lambda)$ does not contribute to $b\to s\gamma$ so much as in the standard model, so we will not include the process $b\to s\gamma$ in the following numerical analysis. Although the fifth and sixth cases of $(V_{t'd}, V_{t's}, V_{t'b}, V_{t'b'}) \simeq (\lambda^3,\lambda^2,1,\lambda)$ and $(\lambda^2,\lambda,1,\lambda)$ of Eq. (3) are interesting, these cases have proved not to lead to any favorable solutions in our numerical analysis.

TABLE I. Combinations of relevant mixing matrix elements for Δm_{B_d}, $b\to s\gamma$, $K^+ \to \pi^+ \nu\bar{\nu}$ and $(K_L\to\mu\bar{\mu})_{\rm SD}$ for the third generation in the standard model and the four cases of the fourth generation mixing.

Mixing	Δm_{B_d}	$b\to s\gamma$	$K^+ \to \pi^+ \nu\bar{\nu}$	$(K_L\to\mu\bar{\mu})_{\rm SD}$
(V_{td}, V_{ts}, V_{tb})	$V_{td}V_{tb}$	$V_{ts}V_{tb}$	$V_{td}V_{ts}$	$V_{td}V_{ts}$
$(\lambda^3,\lambda^2,1)$	λ^3	λ^2	λ^5	λ^5
$(V_{t'd}, V_{t's}, V_{t'b})$	$V_{t'd}V_{t'b}$	$V_{t's}V_{t'b}$	$V_{t'd}V_{t's}$	$V_{t'd}V_{t's}$
$(\lambda^5,\lambda^4,\lambda^3)$	λ^8	λ^7	λ^9	λ^9
$(\lambda^4,\lambda^3,\lambda^2)$	λ^6	λ^5	λ^7	λ^7
$(\lambda^3,\lambda^2,\lambda)$	λ^4	λ^3	λ^5	λ^5
$(\lambda^2,\lambda^2,\lambda)$	λ^3	λ^3	λ^4	λ^4

III. CONSTRAINTS ON THE MODEL

The constraints we impose on the model to search for the fourth generation mixing are the following, $K_L - K_S$ mass difference $\Delta m_K = (3.522\pm 0.016)\times 10^{-12}$ MeV [11], *CP*-violating parameter in the neutral kaon system $\varepsilon_K = (2.28 \pm 0.02)\times 10^{-3}$ [11], $\Delta m_{B_d} = (3.12\pm 0.20)\times 10^{-10}$ MeV [11] for B_d-$\bar{B}_d$ mixing, $B(K^+ \to \pi^+ \nu\bar{\nu}) = (4.2^{+9.7}_{-3.5})\times 10^{-10}$ [1], $\Delta m_{Bs} > 52.0\times 10^{-10}$ MeV [12] for B_s-$\bar{B}_s$ mixing, $\Delta m_D < 1.4\times 10^{-10}$ MeV [13] for D^0-$\bar{D}^0$ mixing, $B(K_L\to \pi^0\nu\bar{\nu}) < 5.8\times 10^{-5}$ [14] and $B(K_L\to\mu\bar{\mu})_{\rm SD} < 2.2\times 10^{-9}$, where the upper bound of the short-distance contribution to $B(K_L \to \mu\bar{\mu})$ is taken to be the value estimated by Bélanger and Geng [15]. As for the directly *CP*-violating parameter in the neutral kaon system ε'/ε, the experimental values by the two groups at CERN and Fermilab deviated from each other by more than 2.4 standard deviations and recently KTeV at Fermilab has obtained a completely consistent value of ${\rm Re}(\varepsilon'/\varepsilon) = (28.0\pm 4.1)\times 10^{-4}$ [16] with the one by *NA*31 of ${\rm Re}(\varepsilon'/\varepsilon) = (23\pm 7)\times 10^{-4}$ [17]. The formulation of ε'/ε in the four-generation model with appropriate QCD corrections is complicated and is out of the scope of our paper [18]. So, we will not include ε'/ε here.

Each of the above-mentioned eight constraints is described in the following.

(i) $K_L - K_S$ mass difference, Δm_K. The short-distance part of Δm_K comes from the well-known W-W box diagram

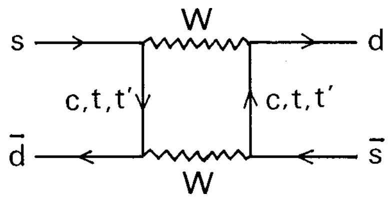

FIG. 1. W-W box diagram for K_L-K_S mass difference in the four-generation model.

with c, t, and t' as internal quarks as shown in Fig. 1 in the four-generation model and the contribution is expressed, for example, for the box diagram with two c quarks as follows:

$$\Delta m_K(c,c)=\frac{G_F^2M_W^2}{6\pi^2}f_K^2B_Km_K\,\mathrm{Re}[(V_{cs}V_{cd}^*)^2]\,\eta_{cc}^K S(x_c), \tag{4}$$

where $S(x)$ is the Inami-Lim box function [4], $x_c\equiv m_c^2/M_W^2$, m_c being the charm-quark mass, η_{cc}^K is the QCD correction factor including the next-to-leading order effects, and f_K and B_K are the decay constant and the bag parameter of the kaon, respectively. By taking for these parameters, the values of $m_c=1.3$ GeV, $\eta_{cc}^K=1.38$ [2], $f_K=0.16$ GeV and $B_K=0.75\pm0.15$ [2], we obtain from the inputs of Eq. (1) the (c,c) contribution as $\Delta m_K(c,c)=(2.6\text{–}3.9)\times10^{-12}$ MeV, which is already consistent by itself with the measured value. Numerically, (c,t) and (t,t) contributions are very small in comparison with the (c,c) contribution, so we take a constraint for the fourth-generation contributions to be

$$\left|\frac{\Delta m_K(c,t')+\Delta m_K(t,t')+\Delta m_K(t',t')}{\Delta m_K(c,c)}\right|<1 \tag{5}$$

as a loose constraint, since these are a large amount of the long-distance contributions. In Eq. (5), (t',t') contribution, $\Delta m_K(t',t')$, is given as follows,

$$\Delta m_K(t',t')=\frac{G_F^2M_W^2}{6\pi^2}f_K^2B_Km_K\,\mathrm{Re}[(V_{t's}V_{t'd}^*)^2]\,\eta_{t't'}^K S(x_{t'}), \tag{6}$$

where $x_{t'}\equiv m_{t'}^2/M_W^2$, $m_{t'}$ being the fourth-generation t' mass, and $S(x_{t'})$ can be approximated as $0.707x_{t'}^{0.82}$ for $130\leqslant m_{t'}\leqslant1200$ GeV. $\eta_{t't'}^K$ is the QCD correction factor which is taken here to the leading order as

$$\eta_{t't'}^K=[\alpha_s(m_c)]^{6/27}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{6/25}\left[\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right]^{6/23}\left[\frac{\alpha_s(m_{b'})}{\alpha_s(m_t)}\right]^{6/21}\times\left[\frac{\alpha_s(\mu_{t'})}{\alpha_s(m_{b'})}\right]^{6/19}. \tag{7}$$

In Eq. (7), $\alpha_s(m)$ is the running coupling constant in QCD and is expressed as

$$\alpha_s(m)=\frac{4\pi}{\beta_0\ln(m^2/\Lambda^2)}, \tag{8}$$

where Λ is the QCD scale of 0.10 GeV and $\beta_0=11-\frac{2}{3}N_f$, N_f being the number of active quark favors at the relevant energy scale, and $\mu_{t'}\simeq O(m_{t'})$. $\eta_{t't'}^K$ turns out to be 0.61 for $m_c=1.3$ GeV, $m_b=4.4$ GeV, $m_t=180$ GeV, $m_{b'}=370$ GeV, and $m_{t'}=400$ GeV, the constraint on the fourth-generation quark masses being described at the end of this section. Similarly, $\Delta m_K(t,t')$ and $\Delta m_K(c,t')$ are expressed as

$$\Delta m_K(t,t')=2\frac{G_F^2M_W^2}{6\pi^2}f_K^2B_Km_K\,\mathrm{Re}[V_{ts}V_{td}^*V_{t's}V_{t'd}^*]\times\eta_{tt'}^K S(x_t,x_{t'}), \tag{9}$$

$$\Delta m_K(c,t')=2\frac{G_F^2M_W^2}{6\pi^2}f_K^2B_Km_K\,\mathrm{Re}[V_{cs}V_{cd}^*V_{t's}V_{t'd}^*]\times\eta_{ct'}^K S(x_c,x_{t'}), \tag{10}$$

where $S(x_t,x_{t'})$ is the Inami-Lim function for the W-W box diagram with t and t' quark in the internal line [4] and the QCD correction factors $\eta_{tt'}^K$ and $\eta_{ct'}^K$ are taken as 0.5 and 0.6, respectively.

(ii) CP-violating parameter in neutral kaon system, ε_K. The quantity ε_K is expressed by the imaginary part of hadronic matrix element of the effective Hamiltonian with $\Delta S=2$ between K^0 and $\bar{K}^0$, to which the short-distance contribution comes from the W-W box diagram as in Δm_K. The box contributions with c and t quark and with two t quarks give the expressions of

$$\varepsilon_K(c,t)=\frac{1}{\sqrt{2}\Delta m_K}\frac{G_F^2M_W^2}{6\pi^2}f_K^2B_Km_K\,\mathrm{Im}[V_{cs}V_{cd}^*V_{ts}V_{td}^*]\times\eta_{ct}^K S(x_c,x_t), \tag{11}$$

$$\varepsilon_K(t,t)=\frac{1}{\sqrt{2}\Delta m_K}\frac{G_F^2M_W^2}{12\pi^2}f_K^2B_Km_K\,\mathrm{Im}[(V_{ts}V_{td}^*)^2]\times\eta_{tt}^K S(x_t). \tag{12}$$

If we take the QCD correction factors including the next-to-leading order as $\eta_{ct}^K=0.47$ and $\eta_{tt}^K=0.57$ [2], the dominant terms in the (c,t)- and (t,t)-box contributions lead to $\varepsilon_K(c,t)\simeq2.83\times10^{-3}B_K\sin\phi_1$ and $\varepsilon_K(t,t)\simeq2.41\times10^{-3}B_K\sin(2\phi_1)$ in the standard model, where ϕ_1 is the CP-violating phase δ^{KM}. Since the magnitude of these two contributions is already close to the measured value $\varepsilon_K=(2.28\pm0.02)\times10^{-3}$ by taking into consideration the theoretical uncertainty in the bag parameter value, $B_K=0.75\pm0.15$, we take the constraint from ε_K on the model that the sum of the contributions from c, t, and t' quarks should be within the 1σ error of the measured value,

$$\sum_{i,j=c,t,t',i\leqslant j}\varepsilon_K(i,j)=(2.28\pm0.02)\times10^{-3}. \tag{13}$$

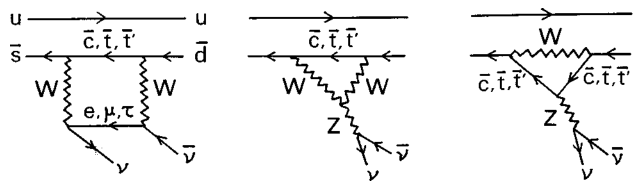

FIG. 2. W-W box and Z^0-penguin diagrams for $K^+ \to \pi^+ \nu \overline{\nu}$.

(iii) B_d-$\overline{B}_d$ mixing, Δm_{B_d}. The mass difference between the two mass eigenstates of B_d-$\overline{B}_d$ system is given by the W-W box diagram, and the (t,t)-box contribution is expressed by

$$\Delta m_{B_d}(t,t) = \frac{G_F^2 M_W^2}{6\pi^2} f_B^2 B_B m_{B_d} |V_{tb} V_{td}^*|^2 \eta_{tt}^B S(x_t), \quad (14)$$

where f_B and B_B are the decay constant and the bag parameter for B_d meson, respectively, and η_{tt}^B is the QCD correction factor including the next-to-leading order effects. By taking for these parameters the values of $\sqrt{B_B} f_B = (0.20 \pm 0.04)$ GeV and $\eta_{tt}^B = 0.55$ [2] and by using the inputs of Eq. (1), we obtain the (t,t) contribution $\Delta m_{B_d}(t,t) = (1.75\text{–}3.95) \times 10^{-10}$ MeV in the standard model. This value is consistent with the measured value, $\Delta m_{B_d} = (3.12 \pm 0.20) \times 10^{-10}$ MeV [11]. Since (c,c) and (c,t) contributions are numerically very small in comparison with the (t,t) contribution, we take the constraint from Δm_{B_d} on the model that the sum of the contributions from t and t' should be within the 1σ error of the measured value as follows:

$$\begin{aligned} &\frac{G_F^2 M_W^2}{6\pi^2} f_B^2 B_B m_{B_d} \big| (V_{tb} V_{td}^*)^2 \eta_{tt}^B S(x_t) + (V_{t'b} V_{t'd}^*)^2 \eta_{t't'}^B S(x_{t'}) \\ &\quad + 2 V_{tb} V_{td}^* V_{t'b} V_{t'd}^* \eta_{tt'}^B S(x_t, x_{t'}) \big| \\ &\quad = (3.12 \pm 0.20) \times 10^{-10} \ \text{MeV}, \end{aligned} \quad (15)$$

where we take for the QCD correction factor $\eta_{t't'}^B$ the following expression to the leading order,

$$\eta_{t't'}^B = [\alpha_s(m_t)]^{6/23} \left[\frac{\alpha_s(m_{b'})}{\alpha_s(m_t)} \right]^{6/21} \left[\frac{\alpha_s(\mu_{t'})}{\alpha_s(m_{b'})} \right]^{6/19}, \quad (16)$$

which turns out to be 0.58 for the same set of parameter values as for $\eta_{t't'}^K$. Another QCD correction factor $\eta_{tt'}^B$ in Eq. (15) is taken as 0.5.

(iv) $B(K^+ \to \pi^+ \nu \overline{\nu})$. The short-distance contributions to the FCNC decay $K^+ \to \pi^+ \nu \overline{\nu}$ come from the W-W box diagram and Z^0-penguin diagrams as shown in Fig. 2 in the four-generation model. The expression for the contributions including the next-to-leading order QCD effects is given by Buchalla and Buras [19,20] in the standard model and are summarized in Ref. [2]. We add to their expression of the branching ratio the contribution from t'-quark exchange as follows:

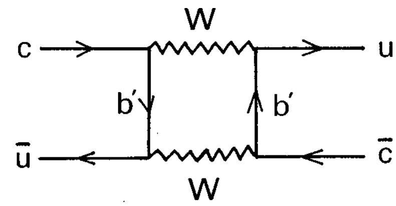

FIG. 3. The dominant W-W box diagram for D^0-$\overline{D}^0$ mixing in the four-generation model.

$$\begin{aligned} B(K^+ \to \pi^+ \nu \overline{\nu}) = \kappa_+ &\left| \frac{V_{cd} V_{cs}^*}{\lambda} P_0 + \frac{V_{td} V_{ts}^*}{\lambda^5} \eta_t X_0(x_t) \right. \\ &\left. + \frac{V_{t'd} V_{t's}^*}{\lambda^5} \eta_{t'} X_0(x_{t'}) \right|^2, \end{aligned} \quad (17)$$

where $\kappa_+ = 4.57 \times 10^{-11}$, P_0 is the sum of charm contributions to the two diagrams including the next-to-leading order QCD corrections [20] and X_0 is the sum of the W-W box and Z^0-penguin functions without QCD corrections calculated by Inami and Lim [4], the expressions of P_0 and X_0 being summarized in Ref. [2]. In Eq. (17), $\eta_t (=0.985)$ is the next-to-leading order QCD correction factor to the t-quark exchange [2,19], and we take $\eta_{t'} = 1.0$ for t' exchange, since η_t is almost 1.0 and the running distance for the QCD corrections for t' exchange is shorter for $m_{t'} > m_t$ than that for the t exchange. The constraint is that the branching ratio of Eq. (17) should be consistent with the measured value of branching ratio $B = (4.2^{+9.7}_{-3.5}) \times 10^{-10}$ [1], since the long-distance contribution is estimated to be very small ($B \sim 10^{-13}$) [21]. We do not take into consideration the mixing effect in the leptonic sector.

(v) B_s-$\overline{B}_s$ mixing. The dominant contributions to B_s-$\overline{B}_s$ mixing are the W-W box diagrams with t and t' exchanges as in B_d-$\overline{B}_d$ mixing. We take the constraint that the sum of (t,t), (t,t'), and (t',t') contributions to Δm_{B_s} should be larger than the present experimental lower bound; $\Delta m_{B_s} > 52.0 \times 10^{-10}$ MeV [12], where Δm_{B_s} is the mass difference of the two mass eigenstates of B_s-$\overline{B}_s$ system. The constraint is expressed as follows:

$$\begin{aligned} &\frac{G_F^2 M_W^2}{6\pi^2} f_{B_s}^2 B_{B_s} m_{B_s} \big| (V_{tb} V_{ts}^*)^2 \eta_{tt}^B S(x_t) \\ &\quad + (V_{t'b} V_{t's}^*)^2 \eta_{t't'}^B S(x_{t'}) \\ &\quad + 2 V_{tb} V_{ts}^* V_{t'b} V_{t's}^* \eta_{tt'}^B S(x_t, x_{t'}) \big| \\ &\quad > 52.0 \times 10^{-10} \ \text{MeV}. \end{aligned} \quad (18)$$

We take the quantity $\sqrt{B_{B_s}} f_{B_s}$ to be equal to that for B_d-$\overline{B}_d$ mixing, and the QCD correction factors η_{tt}^B, $\eta_{t't'}^B$, and $\eta_{tt'}^B$ are equal to the ones for B_d-$\overline{B}_d$ mixing.

(vi) D^0-$\overline{D}^0$ mixing. The dominant contribution to D^0-$\overline{D}^0$ mixing in the four-generation model is the W-W box diagram with fourth-generation down-quark b' exchange [22] as

TABLE II. Typical solutions for the fourth-generation mixing $(s_w, s_v, s_u)=(0.8\lambda^3, 0.8\lambda^2, \lambda)$ in case of $(m_{t'}, m_{b'})=(400, 370)$ GeV for the three phases of (ϕ_1,ϕ_2,ϕ_3). Predictions of $B(K^+\to\pi^+\nu\bar{\nu})\times(10^{-10})$, Δm_D (in 10^{-12} MeV), $B(K_L\to\pi^0\nu\bar{\nu})(10^{-10})$, and the CP asymmetry for $B_d\to J/\psi K_S$ are added.

ϕ_1	ϕ_2	ϕ_3	$K^+\to\pi^+\nu\bar{\nu}$	Δm_D	$K_L\to\pi^0\nu\bar{\nu}$	$C_f(B_d\to J/\psi K_S)$
$3\pi/2$	$\pi/6$	$\pi/2$	2.8	0.8	11.7	−0.37
$\pi/4$	$\pi/3$	0	2.2	1.8	8.7	0.24
$3\pi/2$	$\pi/2$	$\pi/4$	2.3	1.6	8.6	−0.34
$7\pi/12$	$\pi/2$	$3\pi/4$	4.1	0.6	16.7	0.26
$\pi/2$	$5\pi/6$	$\pi/2$	1.7	0.7	7.0	0.25
$3\pi/4$	π	$3\pi/4$	1.5	0.8	5.8	0.17
$13\pi/8$	π	$3\pi/4$	2.7	0.7	9.9	−0.35
$\pi/2$	$7\pi/6$	$11\pi/12$	1.7	0.4	6.6	0.30
$\pi/3$	$4\pi/3$	$13\pi/12$	2.1	0.4	8.0	0.32
$\pi/2$	2π	$7\pi/4$	1.6	2.0	6.2	0.34

shown in Fig. 3. We take the constraint that this contribution to the mass difference between the two mass eigenstates of the D^0-$\bar{D}^0$ system should be smaller than the present experimental upper bound [13], $\Delta m_D(b',b')<1.4\times10^{-10}$ MeV, since the standard model box contribution of two s-quarks exchange [23] and the long-distance contributions [24] are estimated to be three to four orders of magnitude smaller than the upper bound. The constraint is expressed as

$$\Delta m_D(b',b')=\frac{G_F^2M_W^2}{6\pi^2}f_D^2B_Dm_D\,\mathrm{Re}[(V_{cb'}^*V_{ub'})^2]\times\eta_{b'b'}^D S(x_{b'})<1.4\times10^{-10}\ \text{MeV}, \tag{19}$$

where $x_{b'}\equiv m_{b'}^2/M_W^2$. We take for the QCD correction factor $\eta_{b'b'}^D$ the following expression to the leading order,

$$\eta_{b'b'}^D=[\alpha_s(m_b)]^{6/25}\left[\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right]^{6/23}\left[\frac{\alpha_s(\mu_{b'})}{\alpha_s(m_t)}\right]^{6/21}, \tag{20}$$

which is about 0.58 for $\mu_{b'}\simeq m_{b'}=370$ GeV, $m_t=180$ GeV, and $m_b=4.4$ GeV. We tentatively take $f_D\sqrt{B_D}=0.2$ GeV in the following numerical analyses, since the numerical result of $\Delta m_D(b',b')$ is of the order of 10^{-12} MeV for the range of $f_D\sqrt{B_D}=(0.1$–$0.3)$ GeV. Incidentally, the standard model prediction of Δm_D is around 10^{-14} MeV [22].

(vii) $B(K_L\to\pi^0\nu\bar{\nu})$. The process $K_L\to\pi^0\nu\bar{\nu}$ is the "direct" CP-violating decay [25] and the rate is expressed by the imaginary part of sum of the same W-W box and Z^0-penguin diagram amplitudes as in $K^+\to\pi^+\nu\bar{\nu}$ [2], since the CP-conserving contribution is known to be very strongly suppressed [26]. Therefore, we take the constraint that the sum of t and t' contributions to the branching ratio should be smaller than the experimental upper bound [14], $B(K_L\to\pi^0\nu\bar{\nu})<5.8\times10^{-5}$. The constraint is expressed as

$$\kappa_L\left(\frac{\mathrm{Im}(V_{td}V_{ts}^*)}{\lambda^5}\eta_tX_0(x_t)+\frac{\mathrm{Im}(V_{t'd}V_{t's}^*)}{\lambda^5}\eta_{t'}X_0(x_{t'})\right)^2<5.8\times10^{-5}, \tag{21}$$

where $\kappa_L=1.91\times10^{-10}$, X_0 is the same function and η_t and $\eta_{t'}$ are the same QCD correction factors as appeared in Eq. (17) for $K^+\to\pi^+\nu\bar{\nu}$.

TABLE III. Typical solutions for the fourth-generation mixing $(s_w, s_v, s_u)=(0.5\lambda^3, 0.5\lambda^2, \lambda)$ in case of $(m_{t'}, m_{b'})=(800, 770)$ GeV for the three phases of (ϕ_1,ϕ_2,ϕ_3). Predictions of $B(K^+\to\pi^+\nu\bar{\nu})\times(10^{-10})$, Δm_D (in 10^{-12} MeV), $B(K_L\to\pi^0\nu\bar{\nu})(10^{-10})$, and the CP asymmetry for $B_d\to J/\psi K_S$ are added.

ϕ_1	ϕ_2	ϕ_3	$K^+\to\pi^+\nu\bar{\nu}$	Δm_D	$K_L\to\pi^0\nu\bar{\nu}$	$C_f(B_d\to J/\psi K_S)$
$5\pi/12$	$\pi/3$	0	2.8	1.1	11.5	0.26
$\pi/2$	$\pi/2$	$\pi/6$	2.6	0.9	10.6	0.19
$5\pi/12$	$2\pi/3$	$\pi/3$	2.7	0.6	10.8	0.17
$\pi/2$	$5\pi/6$	$\pi/2$	2.7	0.3	11.0	0.22
$\pi/2$	π	$4\pi/3$	4.8	0.1	19.8	0.38
$7\pi/12$	$7\pi/6$	$3\pi/2$	4.5	0.3	18.5	0.37
$15\pi/8$	$3\pi/2$	0	5.1	0.4	19.5	−0.13
$\pi/2$	$3\pi/2$	$11\pi/6$	4.6	0.7	19.2	0.32
$7\pi/4$	$5\pi/3$	$\pi/6$	5.0	0.5	18.6	−0.32
$19\pi/12$	$11\pi/6$	$\pi/3$	5.2	0.4	18.0	−0.40

TABLE IV. Typical solutions for the fourth-generation mixing $(s_w, s_v, s_u) = (0.3\lambda^3, 0.3\lambda^2, \lambda)$ in case of $(m_{t'}, m_{b'}) = (1200, 1170)$ GeV for the three phases of (ϕ_1, ϕ_2, ϕ_3). Predictions of $B(K^+ \to \pi^+ \nu\bar{\nu}) \times (10^{-10})$, Δm_D (in 10^{-12} MeV), $B(K_L \to \pi^0 \nu\bar{\nu})(10^{-10})$, and the *CP* asymmetry for $B_d \to J/\psi K_S$ are added.

ϕ_1	ϕ_2	ϕ_3	$K^+ \to \pi^+ \nu\bar{\nu}$	Δm_D	$K_L \to \pi^0 \nu\bar{\nu}$	$C_f(B_d \to J/\psi K_S)$
$5\pi/12$	$\pi/6$	$23\pi/12$	1.0	0.7	3.8	0.30
$\pi/2$	$\pi/3$	0	1.5	0.5	6.1	0.27
$7\pi/12$	$\pi/2$	$\pi/6$	1.4	0.3	5.7	0.18
$2\pi/3$	$5\pi/6$	$7\pi/12$	0.9	0.1	3.7	0.18
$\pi/2$	π	$3\pi/4$	1.0	0.01	3.8	0.29
$5\pi/12$	$4\pi/3$	$5\pi/3$	2.9	0.1	12.0	0.39
$5\pi/12$	$3\pi/2$	$7\pi/4$	2.1	0.3	8.5	0.36
$5\pi/12$	$5\pi/3$	$23\pi/12$	2.3	0.4	9.0	0.30
$\pi/4$	$11\pi/6$	$\pi/6$	3.1	0.4	12.8	0.14
$\pi/2$	2π	$7\pi/4$	1.1	0.5	4.2	0.35

(viii) $B(K_L \to \mu\bar{\mu})_{\rm SD}$. The process $K_L \to \mu\bar{\mu}$ is the *CP*-conserving decay. The short-distance (SD) contribution is given by the *W*-*W* box and Z^0-penguin diagrams and the branching ratio for this part is expressed as [2]

$$B(K_L \to \mu\bar{\mu})_{\rm SD} = \kappa_\mu \left[\frac{{\rm Re}(V_{cd}V_{cs}^*)}{\lambda} P_0' + \frac{{\rm Re}(V_{td}V_{ts}^*)}{\lambda^5} \eta_t^Y Y_0(x_t) + \frac{{\rm Re}(V_{t'd}V_{t's}^*)}{\lambda^5} \eta_{t'}^Y Y_0(x_{t'}) \right]^2, \qquad (22)$$

where $\kappa_\mu = 1.68 \times 10^{-9}$, P_0' is the sum of charm contributions to the two diagrams including the next-to-leading order QCD corrections [20] and Y_0 the sum of the *W*-*W* box and Z^0-penguin functions without QCD corrections calculated by Inami and Lim [4], the expressions of P_0' and Y_0 being summarized in Ref. [2]. In Eq. (22), $\eta_t^Y (=1.026)$ is the next-to-leading order QCD correction factor to the *t*-quark exchange [2,19] and we take $\eta_{t'}^Y = 1.0$ for t' exchange for the same reason as stated for $K^+ \to \pi^+ \nu\bar{\nu}$. We take the constraint that the branching ratio of Eq. (22) should be smaller than the upper bound of the short-distance contribution [15] as stated before at the beginning of this section, $B(K_L \to \mu\bar{\mu})_{\rm SD} < 2.2 \times 10^{-9}$. We do not take into consideration the mixing effect in the leptonic sector.

For the masses of t' and b', there is a constraint from ρ parameter. If we denote the parameter ρ_0 as

$$\rho_0 = \frac{M_W^2}{M_Z^2 \cos^2\theta_W \hat{\rho}}, \qquad (23)$$

where $\sin^2\theta_W$ is the Weinberg angle and $\hat{\rho}$ is the quantity $M_W^2/(M_Z^2\cos^2\theta_W)$, which involves the radiative correction effects from Higgs doublets and top-quark mass, then $\rho_0 - 1$ describes new sources of SU(2) breaking. The fourth generation makes ρ_0 deviate from 1 as [27]

$$\rho_0 = 1 + \frac{3G_F}{8\sqrt{2}\pi^2} \left(m_{t'}^2 + m_{b'}^2 - \frac{4m_{t'}^2 m_{b'}^2}{m_{t'}^2 - m_{b'}^2} \ln \frac{m_{t'}}{m_{b'}} \right). \qquad (24)$$

The value of ρ_0 is now $\rho_0 = 0.9998 \pm 0.0008$ [27], and this constrains the masses of t' and b'.

IV. POSSIBLE MIXINGS OF FOURTH GENERATION

We search for possible mixings of the fourth generation allowed by the eight constraints in the previous section by testing the typical hierarchical mixings of Eq. (3) with the intention to obtain the "maximum" mixing compatible with the considerably large branching ratio of the rare decay $K^+ \to \pi^+ \nu\bar{\nu}$ with a factor of 4–6 as compared with the predic-

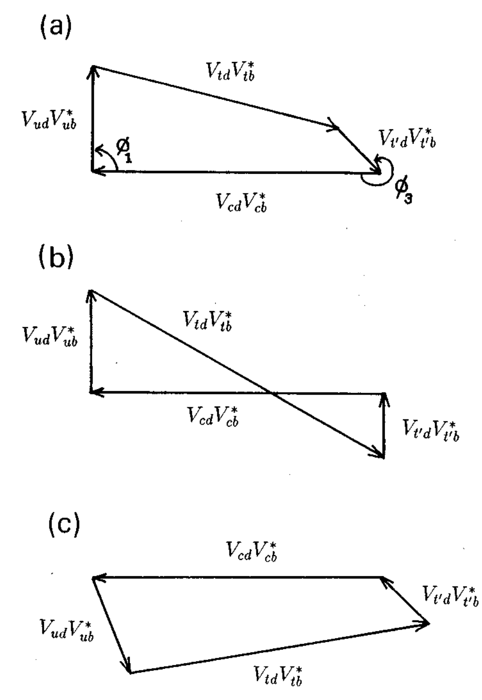

FIG. 4. Typical examples of the unitarity quadrangle for $(s_w, s_v, s_u) = (0.8\lambda^3, 0.8\lambda^2, \lambda)$ in case of $(m_{t'}, m_{b'}) = (400, 370)$ GeV. (a) $\phi_1 = \pi/2$, $\phi_2 = 2\pi$, $\phi_3 = \frac{7}{4}\pi$; $B(K^+ \to \pi^+ \nu\bar{\nu}) = 1.6 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = 0.34$. (b) $\phi_1 = \pi/2$, $\phi_2 = \frac{5}{6}\pi$, $\phi_3 = \pi/2$; $B(K^+ \to \pi^+ \nu\bar{\nu}) = 1.7 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = 0.25$. (c) $\phi_1 = \frac{13}{8}\pi$, $\phi_2 = \pi$, $\phi_3 = \frac{3}{4}\pi$; $B(K^+ \to \pi^+ \nu\bar{\nu}) = 2.7 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = -0.35$.

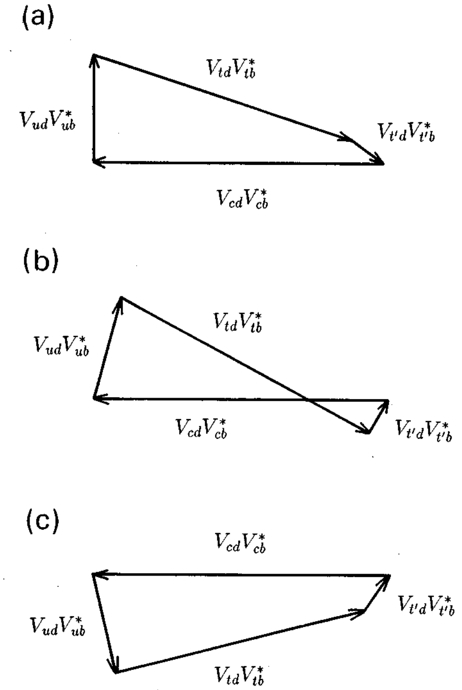

FIG. 5. Typical examples of the unitarity quadrangle for $(s_w, s_v, s_u) = (0.5\lambda^3, 0.5\lambda^2, \lambda)$ in case of $(m_{t'},\ m_{b'}) = (800,\ 770)$ GeV. (a) $\phi_1 = \pi/2$, $\phi_2 = \frac{3}{2}\pi$, $\phi_3 = \frac{11}{6}\pi$; $B(K^+ \to \pi^+ \nu \bar{\nu}) = 4.6 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = 0.32$. (b) $\phi_1 = \frac{5}{12}\pi$, $\phi_2 = \frac{2}{3}\pi$, $\phi_3 = \pi/3$; $B(K^+ \to \pi^+ \nu \bar{\nu}) = 2.7 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = 0.17$. (c) $\phi_1 = \frac{19}{12}\pi$, $\phi_2 = \frac{11}{6}\pi$, $\phi_3 = \pi/3$; $B(K^+ \to \pi^+ \nu \bar{\nu}) = 5.2 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = -0.40$.

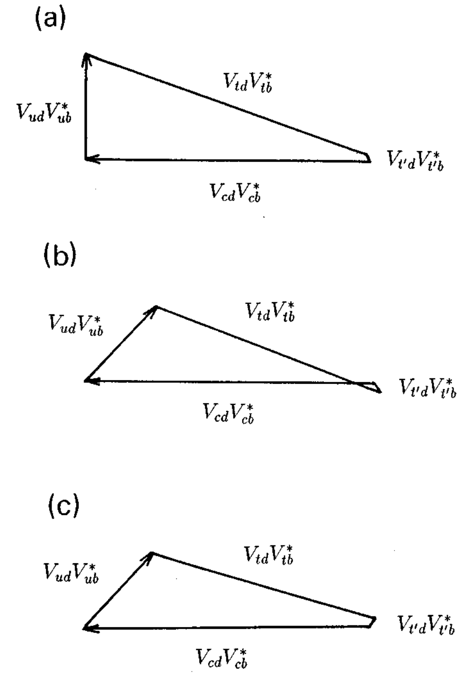

FIG. 6. Typical examples of the unitarity quadrangle for $(s_w, s_v, s_u) = (\lambda^4, \lambda^3, \lambda^2)$ in case of $(m_{t'},\ m_{b'}) = (400,\ 370)$ GeV. (a) $\phi_1 = \pi/2$, $\phi_2 = \pi/6$, $\phi_3 = \frac{7}{4}\pi$; $B(K^+ \to \pi^+ \nu \bar{\nu}) = 0.94 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = 0.30$. (b) $\phi_1 = \pi/4$, $\phi_2 = \pi/6$, $\phi_3 = \frac{3}{4}\pi$; $B(K^+ \to \pi^+ \nu \bar{\nu}) = 0.89 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = 0.28$. (c) $\phi_1 = \pi/4$, $\phi_2 = \pi/3$, $\phi_3 = \frac{5}{4}\pi$; $B(K^+ \to \pi^+ \nu \bar{\nu}) = 1.0 \times 10^{-10}$, $C_f(B_d \to J/\psi K_S) = 0.29$.

tions in the standard model. From this point of view, the last two cases with $V_{t'd} = 0$ of Eq. (3) are not interesting here.

Free parameters are the three phases ϕ_1, ϕ_2, and ϕ_3 of the 4×4 mixing matrix. As for the masses of the fourth generation quarks, we choose tentatively $(m_{t'},\ m_{b'}) = (400,\ 370)$, $(800,\ 770)$, and $(1200,\ 1170)$ GeV as typical ones, which are compatible with the constraint of Eq. (24). We vary the three phases in the range of $0 \leq \phi_1, \phi_2, \phi_3 \leq 2\pi$. We found no solutions compatible with the eight constraints for the exotic fifth and sixth cases of $(V_{t'd}, V_{t's}, V_{t'b}, V_{t'b'}) \simeq (\lambda^3, \lambda^2, 1, \lambda)$ and $(\lambda^2, \lambda, 1, \lambda)$ of Eq. (3). So, we focus on the first four cases of Eq. (3) here.

Strong constraints come from Δm_K, ε_K, B_d-$\bar{B}_d$ mixing, $K^+ \to \pi^+ \nu \bar{\nu}$ and $(K_L \to \mu \bar{\mu})_{\rm SD}$. In the standard model, the largest contribution comes from the top quarks for B_d-$\bar{B}_d$ mixing, $K^+ \to \pi^+ \nu \bar{\nu}$ and $(K_L \to \mu \bar{\mu})_{\rm SD}$, and there the combination of the relevant Cabibbo-KM (CKM) matrix elements is $V_{td}V_{tb} \sim \lambda^3$ for B_d-$\bar{B}_d$ mixing and $V_{td}V_{ts} \sim \lambda^5$ for $K^+ \to \pi^+ \nu \bar{\nu}$ and $(K_L \to \mu \bar{\mu})_{\rm SD}$. On the other hand, the combinations of the corresponding matrix elements for t' quark are shown in Table I for each of the above four cases. By comparing these combinations between the standard model and the four-generation model, the numerical analyses give the following results: the cases of (s_w, s_v, s_u) $(\simeq (|V_{t'd}|, |V_{t's}|, |V_{t'b}|)) = (\lambda^5, \lambda^4, \lambda^3)$ and $(\lambda^4, \lambda^3, \lambda^2)$ give almost the same predictions to the above-mentioned eight quantities as in the standard mode, since the contributions of the fourth generation are very small, as seen from Table I. For the case of $(\lambda^3, \lambda^2, \lambda)$, almost all the quantities satisfy the constraints with only one exception of $B(K_L \to \mu \bar{\mu})_{\rm SD}$, for which this mixing gives a value several times larger than the upper bound. The last case of $(\lambda^2, \lambda^2, \lambda)$ predicts too large values for $B(K^+ \to \pi^+ \nu \bar{\nu})$ and $B(K_L \to \mu \bar{\mu})_{\rm SD}$. These results imply that the mixing of $(\lambda^3, \lambda^2, \lambda)$ is a little large for the fourth generation and it turns out that a mixing with s_w and s_v reduced by 20%, that is, $(s_w, s_v, s_u) = (0.8\lambda^3, 0.8\lambda^2, \lambda)$ satisfies all of the eight constraints for $(m_{t'},\ m_{b'}) = (400,\ 370)$ GeV, the one with s_w and s_v reduced by 50%, that is, $(s_w, s_v, s_u) = (0.5\lambda^3, 0.5\lambda^2, \lambda)$ satisfies them for $(m_{t'},\ m_{b'}) = (800,\ 770)$ GeV and the one with $(s_w, s_v, s_u) = (0.3\lambda^3, 0.3\lambda^2, \lambda)$ does for $(m_{t'},\ m_{b'}) = (1200,\ 1170)$ GeV as a maximum mixing. This strong energy-dependence of the reduction factors $(s_w/\lambda^3, s_v/\lambda^2, s_u/\lambda)$ is valid and reasonable, because the contribution of the t'-quark exchange to the decay amplitudes of both $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ is proportional to $V_{t'd}V^*_{t's}X_0(x_{t'}) \simeq \frac{1}{8} s_w s_v (m_{t'}/M_W)^2 e^{i(\phi_3 - \phi_2)}$, the one to the amplitude of $(K_L \to \mu \bar{\mu})_{\rm SD}$ is $\mathrm{Re}(V_{t'd}V^*_{t's})Y_0(x_{t'}) \simeq \frac{1}{8} s_w s_v (m_{t'}/M_W)^2 \cos(\phi_3 - \phi_2)$, the contribution to Δm_K is $\mathrm{Re}[(V^*_{t'd}V_{t's})^2] S(x_{t'}) \simeq 0.707 s_w^2 s_v^2 (m_{t'}/M_W)^{1.64} \cos 2(\phi_2 - \phi_3)$, and the one to Δm_{B_d} is $|V^*_{t'd}V_{t'b}|^2 S(x_{t'}) \simeq 0.707 s_w^2 s_u^2 (m_{t'}/M_W)^{1.64}$.

We show several typical solutions with respect to the three phases (ϕ_1, ϕ_2, ϕ_3) for the maximum mixing

TABLE V. Comparison of $B(K^+\to\pi^+\nu\bar{\nu})$, x_s (B_s-$\bar{B}_s$ mixing), Δm_D, and $B(K_L\to\pi^0\nu\bar{\nu})$ among the experimental values, standard model (SM) predictions, and four-generation model predictions.

	$B(K^+\to\pi^+\nu\bar{\nu})$	x_s	Δm_D (MeV)	$B(K_L\to\pi^0\nu\bar{\nu})$
Experiments	$(4.2^{+9.7}_{-3.5})\times10^{-10}$	>12.8	$<1.4\times10^{-10}$	$<5.8\times10^{-5}$
SM	$(0.6-1.5)\times10^{-10}$	19–27	$\sim10^{-14}$	$(1.1-5.0)\times10^{-11}$
Four-generation	$(0.6-5.2)\times10^{-10}$	19–29	$(0.01-2.1)\times10^{-12}$	$(0.05-22)\times10^{-10}$

$(s_w,s_v,s_u)=(0.8\lambda^3,0.8\lambda^2,\lambda)$ in the case of $(m_{t'},\ m_{b'})=(400,\ 370)$ GeV in Table II, the ones for $(s_w,s_v,s_u)=(0.5\lambda^3,0.5\lambda^2,\lambda)$ in the case of $(m_{t'},\ m_{b'})=(800,\ 770)$ GeV in Table III and the ones for $(s_w,s_v,s_u)=(0.3\lambda^3,0.3\lambda^2,\lambda)$ in the case of $(m_{t'},\ m_{b'})=(1200,\ 1170)$ GeV in Table IV. The values of (ϕ_1,ϕ_2,ϕ_3) allowed by the constraints constitute a certain region in the plane, surrounding each of the solutions in Tables II, III, and IV. In the tables, we also give the predictions of $B(K^+\to\pi^+\nu\bar{\nu})$, Δm_D, $B(K_L\to\pi^0\nu\bar{\nu})$, and the *CP* asymmetry for $B_d\to J/\psi K_S$, which is explained in the following section, for each of the solutions.

As can be seen from Tables II–IV for the ''maximum'' mixing of the fourth generation, the constraints from all the seven quantities considered here except $B(K^+\to\pi^+\nu\bar{\nu})$ could predict the values $(0.6-5.2)\times10^{-10}$ for $B(K^+\to\pi^+\nu\bar{\nu})$, including the values just outside the predictions of the standard model, $(0.6-1.5)\times10^{-10}$, and not so large as the upper part of the measured value of $(0.7-13.9)\times10^{-10}$. This means that all the seven quantities except the present measurement of $B(K^+\to\pi^+\nu\bar{\nu})$ have already implied the fourth generation with the mixing as large as $(s_w,s_v,s_u)=(0.8\lambda^3,0.8\lambda^2,\lambda)$ for $m_{t'}=400$ GeV and so on and that they could predict the quantities of x_s, Δm_D, and $B(K_L\to\pi^0\nu\bar{\nu})$ in the range of values shown in Table V, which is explained in detail in the next section.

V. DISCUSSIONS AND CONCLUSIONS

We can obtain the following predictions from these maximum mixings: the branching ratio of $K^+\to\pi^+\nu\bar{\nu}$ takes a range from the standard model (SM) values to the central value of the new measurement as $B=(0.6-5.2)\times10^{-10}$, the strength of B_s-$\bar{B}_s$ mixing is $19\leqslant x_s\leqslant29$, where $x_s\equiv\Delta m_{B_s}/\Gamma_{B_s}$, Γ_{B_s} being the total decay rate of B_s meson, Δm_D of D^0-$\bar{D}^0$ mixing could have a value $(0.01-2.1)\times10^{-12}$ MeV, extending to about two orders of magnitude larger than the SM prediction ($\sim10^{-14}$ MeV [23]), and the branching ratio of $K_L\to\pi^0\nu\bar{\nu}$ takes a range of $(0.05-22)\times10^{-10}$, ranging from the SM values to the values of two orders of magnitude larger than the SM prediction [$(1.1-5.0)\times10^{-11}$ [2]]. These results are summarized in Table V.

The maximum mixing gives an interesting effect on the *CP* asymmetry of the decay rates of the ''gold-plate'' mode of B_d meson, $B_d\to J/\psi K_S$. The asymmetry is given by

$$C_f=\frac{\Gamma(B_d\to J/\psi K_S)-\Gamma(\bar{B}_d\to J/\psi K_S)}{\Gamma(B_d\to J/\psi K_S)+\Gamma(\bar{B}_d\to J/\psi K_S)},\tag{25}$$

and it is expressed as [28]

$$C_f=-\frac{x_d}{1+x_d^2}\,\mathrm{Im}\,\Lambda=\frac{x_d}{1+x_d^2}\sin2\beta,\tag{26}$$

$$\Lambda\equiv\sqrt{\frac{M_{12}^*}{M_{12}}}\frac{A(\bar{B}_d\to J/\psi K_S)}{A(B_d\to J/\psi K_S)},\tag{27}$$

where x_d is the mixing strength of B_d-$\bar{B}_d$ mixing, M_{12} the off-diagonal element of the mass matrix in B_d-$\bar{B}_d$ system, A the decay amplitude, and β is one of the angles of the unitarity triangle. In the standard model [29], the quantity C_f takes a positive sign as $0.18\leqslant C_f\leqslant0.37$ for $B_d\to J/\psi K_S$, resulting from the phase range of $0<\phi_1<\pi$, which is constrained from the positive sign of the *CP*-violating parameter ε_K. However, in the four-generation model [7], C_f can take a negative sign also as $-0.38\leqslant C_f\leqslant0.40$, since the phase ϕ_1 takes the whole range of $0<\phi_1<2\pi$ due to the occurrence of the two more new phases ϕ_2 and ϕ_3. For the moment, $\sin2\beta$ of Eq. (26) has recently been measured to be positive as $\sin2\beta=(3.2^{+1.8}_{-2.0}\pm0.5)$ by OPAL Collaboration [30] and $\sin2\beta=(1.8\pm1.1\pm0.3)$ by Collider Detector at Fermilab (CDF) Collaboration [31], which means that C_f is positive as in the standard model. We should add that although the penguin diagrams could affect the decay amplitude in the four-generation model, they would bring at most several percent change of C_f.

The unitarity triangle in the standard model transforms into unitarity quadrangle in the four-generation model [32]. For the ''maximum'' mixing obtained here, some of the typical quadrangles are shown in Fig. 4 for $m_{t'}=400$ GeV and in

TABLE VI. The same as in Table II except that $B_K=0.75\pm0.05$ and $f_B^2B_B=(0.20\pm0.01)^2$ GeV^2.

ϕ_1	ϕ_2	ϕ_3	$K^+\to\pi^+\nu\bar{\nu}$	Δm_D	$K_L\to\pi^0\nu\bar{\nu}$	$C_f(B_d\to J/\psi K_S)$
$\pi/4$	$\pi/3$	0	2.2	1.8	8.7	0.24
$\pi/2$	$5\pi/6$	$\pi/2$	1.7	0.7	7.0	0.25
$\pi/2$	$7\pi/6$	$11\pi/12$	1.7	0.4	6.6	0.30

TABLE VII. The same as in Table III except that $B_K = 0.75 \pm 0.05$ and $f_B^2 B_B = (0.20 \pm 0.01)^2$ GeV^2.

ϕ_1	ϕ_2	ϕ_3	$K^+ \to \pi^+ \nu\bar{\nu}$	Δm_D	$K_L \to \pi^0 \nu\bar{\nu}$	$C_f(B_d \to J/\psi K_S)$
$\pi/2$	$5\pi/6$	$\pi/2$	2.7	0.3	11.0	0.22
$\pi/2$	π	$4\pi/3$	4.8	0.1	19.8	0.38
$15\pi/8$	$3\pi/2$	0	5.1	0.4	19.5	−0.13

Fig. 5 for $m_{t'} = 800$ GeV. The fourth side of the quadrangle, $V_{t'd} V_{t'b}^*$, is of order λ^4, while the other three sides are of order λ^3. The first two quadrangles of Figs. 4 and 5 are for positive sign of C_f. The third ones of Figs. 4 and 5 are for the negative sign of C_f and are reversed with respect to the base line $V_{cd} V_{cb}^*$, since $\phi_1 > \pi$, where ϕ_1 corresponds to the anticlockwise angle measured from $V_{cd} V_{cb}^*$ to $V_{ud} V_{ub}^*$ and ϕ_3 to the anticlockwise angle from $V_{cd} V_{cb}^*$ to $V_{t'd} V_{t'b}^*$. Incidentally, the quadrangles for the solutions with smaller mixing of $(s_w, s_v, s_u) = (\lambda^4, \lambda^3, \lambda^2)$ for $m_{t'} = 400$ GeV are given in Fig. 6. In this case, the size of the fourth side, $V_{t'd} V_{t'b}^*$, is of order λ^6 and is about 1/100 that of the side $V_{cd} V_{cb}^*$ and the quadrangle could not be distinguished from the triangle, and the branching ratio of $K^+ \to \pi^+ \nu\bar{\nu}$ is predicted to be in the range of $(0.6–1.2) \times 10^{-10}$, which agrees with the predictions of the standard model. So, if the future measurements of $K^+ \to \pi^+ \nu\bar{\nu}$ show its branching ratio to be in the range of the standard mode values, the large mixing of the fourth generation obtained here as the ''maximum'' one will not be allowed.

We should remark that this large mixing of the fourth generation we found here is not due to the fairly large theoretical uncertainties in $B_K = 0.75 \pm 0.15$ and $f_B \sqrt{B_B} = (0.20 \pm 0.04)$ GeV. Even if we prescribe to reduce the uncertainties of these quantities by 1/3 to 1/4 keeping the central values as $B_K = 0.75 \pm 0.05$ and $f_B \sqrt{B_B} = (0.20 \pm 0.01)$ GeV, we can still find some of the solutions such as listed in Tables VI and VII for $m_{t'} = 400$ GeV and 800 GeV, respectively.

Summarizing, we find ''maximum'' mixings of the fourth generation $(V_{t'd}, V_{t's}, V_{t'b}) \simeq (0.8\lambda^3, 0.8\lambda^2, \lambda)$ for $(m_{t'},\ m_{b'}) = (400,\ 370)$ GeV, $(V_{t'd}, V_{t's}, V_{t'b}) \simeq (0.5\lambda^3, 0.5\lambda^2, \lambda)$ for $(m_{t'},\ m_{b'}) = (800,\ 770)$ GeV and $(V_{t'd}, V_{t's}, V_{t'b}) \simeq (0.3\lambda^3, 0.3\lambda^2, \lambda)$ for $(m_{t'},\ m_{b'}) = (1200,\ 1170)$ GeV, which are consistent with the eight constraints of Δm_K, ε_K, B_d-$\bar{B}_d$ mixing, $K^+ \to \pi^+ \nu\bar{\nu}$, B_s-$\bar{B}_s$ mixing, D^0-$\bar{D}^0$ mixing, $K_L \to \pi^0 \nu\bar{\nu}$, and $(K_L \to \mu\bar{\mu})_{\mathrm{SD}}$. The mass difference Δm_D from D^0-$\bar{D}^0$ mixing and the branching ratio of $K_L \to \pi^0 \nu\bar{\nu}$ could reach the values one to two orders of magnitude larger than the standard model predictions, and the *CP* asymmetry of the decay rates of $B_d \to J/\psi K_S$ could take a value of the opposite sign to the SM one. Measurements of Δm_D and $B(K_L \to \pi^0 \nu\bar{\nu})$ are expected to be done and further data of $B(K^+ \to \pi^+ \nu\bar{\nu})$ with more statistics are required.

ACKNOWLEDGMENTS

We are grateful to Takeshi Komatsubara, Minoru Tanaka, Takeshi Kurimoto, Xing Zhi-Zhong, Masako Bando, C. S. Lim, and Morimitsu Tanimoto for helpful discussions.

[1] S. Adler *et al.*, Phys. Rev. Lett. **79**, 2204 (1997).
[2] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996), and references therein; G. Buchalla and A. J. Buras, Nucl. Phys. **B548**, 309 (1999).
[3] M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).
[4] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981).
[5] M. Gronau and J. Schechter, Phys. Rev. D **31**, 1668 (1974); I. I. Bigi, Z. Phys. C **27**, 303 (1985); A. A. Anselm, J. L. Chkareuli, N. G. Utraltsev, and T. A. Zhukovskaya, Phys. Lett. **156B**, 102 (1985); X. G. He and S. Pakvasa, *ibid.* **156B**, 236 (1985); U. Türke, E. A. Paschos, H. Usler, and R. Decker, Nucl. Phys. **B285**, 313 (1985); T. Hayashi, M. Tanimoto, and S. Wakaizumi, Prog. Theor. Phys. **75**, 353 (1986).
[6] U. Türke, Phys. Lett. **168B**, 296 (1986); I. I. Bigi and S. Wakaizumi, Phys. Lett. B **188**, 501 (1987); G. Eilam, J. L. Hewett, and T. G. Rizzo, *ibid.* **193**, 533 (1987).
[7] T. Hasuike, T. Hattori, T. Hayashi, and S. Wakaizumi, Mod. Phys. Lett. A **4**, 2465 (1989); Prog. Theor. Phys. **83**, 265 (1990); Phys. Rev. D **41**, 1691 (1990); **44**, 3582 (1991).
[8] W.-S. Hou, A. Soni, and H. Steger, Phys. Lett. B **192**, 441 (1987).
[9] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[10] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[11] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[12] W. Adam *et al.*, Phys. Lett. B **414**, 382 (1997); R. Barate *et al.*, Eur. Phys. J. C **4**, 367 (1998).
[13] E. M. Aitala *et al.*, Phys. Rev. Lett. **77**, 2384 (1996).
[14] M. Weaver *et al.*, Phys. Rev. Lett. **72**, 3758 (1994).
[15] G. Bélanger and C. G. Geng, Phys. Rev. D **43**, 140 (1991).
[16] KTeV Collaboration, A. Alavi-Harati *et al.*, Phys. Rev. Lett. **83**, 22 (1999).
[17] G. D. Barr *et al.*, Phys. Lett. B **317**, 233 (1993).
[18] A. J. Buras, TUM-HEP-349/99, hep-ph/9905437.
[19] G. Buchalla and A. J. Buras, Nucl. Phys. **B398**, 285 (1993); **B400**, 225 (1993).
[20] G. Buchalla and A. J. Buras, Nucl. Phys. **B412**, 106 (1994).
[21] D. Rein and L. M. Sehgal, Phys. Rev. D **39**, 3325 (1989).
[22] K. S. Babu, X-G. He, X.-Q. Li, and S. Pakvasa, Phys. Lett. B **205**, 540 (1988).
[23] A. Datta and D. Kumbhakar, Z. Phys. C **27**, 515 (1985).

[24] L. Wolfenstein, Phys. Lett. **164B**, 170 (1985); J. Donoghue, E. Golowich, B. R. Holstein, and J. Trampetić, Phys. Rev. D **33**, 179 (1986).
[25] L. S. Littenberg, Phys. Rev. D **39**, 3322 (1989).
[26] G. Buchalla and G. Isidori, Phys. Lett. B **440**, 170 (1998).
[27] Particle Data Group [11], pp. 97–98.
[28] A. B. Carter and A. I. Sanda, Phys. Rev. D **23**, 1567 (1981); I. I. Bigi and A. I. Sanda, Nucl. Phys. **B193**, 85 (1981).
[29] I. Dunietz and J. L. Rosner, Phys. Rev. D **34**, 1404 (1986).
[30] OPAL Collaboration, K. Akerstaff *et al.*, Eur. Phys. J. C **5**, 379 (1998).
[31] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **81**, 5513 (1998).
[32] Y. Nir and H. R. Quinn, in *B Decays*, edited by S. Stone (World Scientific, Singapore, 1992).