Probing the $WW\gamma$ vertex at hadron colliders

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(Received 15 July 1999; published 8 November 1999)

We present a new, model independent method for extracting bounds for the anomalous γWW couplings from hadron collider experiments. At the partonic level we introduce a set of three observables which are constructed from the unpolarized differential cross section for the process $d\overline{u}\to W^-\gamma$ by appropriate convolution with a set of simple polynomials depending only on the center-of-mass angle. One of these observables allows for the direct determination of the anomalous coupling usually denoted by $\Delta\kappa$, without any simplifying assumptions, and without relying on the presence of a radiation zero. The other two observables impose two sum rules on the remaining three anomalous couplings. The inclusion of the structure functions is discussed in detail for both $p\overline{p}$ and pp colliders. We show that, whilst for $p\overline{p}$ experiments this can be accomplished straightforwardly, in the pp case one has to resort to somewhat more elaborate techniques, such as the binning of events according to their longitudinal momenta. [S0556-2821(99)01523-4]

PACS number(s): 13.40.Gp, 13.85.Ni, 14.70.Bh, 14.70.Fm

I. INTRODUCTION

The importance of hadron colliders concerning the measurement of the trilinear gauge self-couplings has already been exemplified by the direct verification of the existence of such vertices through $W\gamma$ as well as WZ production at the Fermilab Tevatron [1]. The bounds on the anomalous couplings so obtained have only recently been surpassed by the CERN e^+e^- collider LEP2 measurements [2]. The advent of the Large Hadron Collider (LHC) at CERN is expected to improve the situation further, because of its high luminosity and energy reach. A significant advantage of hadron colliders in this context is the fact that the couplings of the photon and the Z boson to the W boson can be probed independently through separate processes. Taking also into account that at the sub-process level the center-of-mass energy varies, one may also study the form-factor structure of the couplings, thus furnishing important information, complementary to that obtained from the lepton colliders.

In this paper we extend to hadron collider experiments a model-independent method proposed for the extraction of bounds on the anomalous couplings at LEP2 [3]. This method is based on constructing appropriate projections of the differential cross section [4], which lead to a set of novel observables; the latter are related to the anomalous couplings by means of simple algebraic equations. The experimental determinations of these observables can in turn be used in order to impose bounds simultaneously on all anomalous couplings, without having to resort to model-dependent relations among them, or invoke any further simplifying assumptions.

Given the advantages mentioned above, the generalization of this method to the case of hadron colliders is certainly interesting, but is by no means obvious, even at the theoretical level. The crucial difference is that at hadron colliders the center-of-mass energy of the sub-processes is not fixed, and the effects of the structure functions must be included. The detailed study reveals however that, due to a very particular dependence of the differential cross-section on the center-of-mass angle, the method can in fact be applied. As a result, one obtains a set of three algebraic equations relating the four unknown couplings. In particular, one can directly extract the value for the $\Delta\kappa$ anomalous form-factor, *without* having to assume the absence of other anomalous couplings.

The paper is organized as follows: In Sec. II we present explicit results of the partonic differential cross-section for the prototype process $d\overline{u}\to W^{\pm}\gamma$, using the most general $WW\gamma$ vertex allowed from Lorentz and $U(1)_{EM}$ invariance. This process has received significant attention in the literature [5–9], mainly due to the presence of the radiation zero in its differential cross-section [10,11]. In Sec. III we define a set of observables, which depend explicitly on the various anomalous couplings, and can be experimentally extracted through the convolution of the differential cross-section with appropriately constructed polynomials of $\cos\theta$, the center-of-mass scattering angle. In Sec. IV we present an elementary discussion of the statistical properties of these observables. In Sec. V we turn our attention to the realistic cases of pp and $p\overline{p}$ collisions, and address the complications that arise due to the inclusion of the structure functions. Finally, in Sec. VI we present our conclusions.

II. THE PROCESS $d\overline{u}\to W^-\gamma$ IN THE PRESENCE OF ANOMALOUS COUPLINGS

We consider the process $d(p_d)\overline{u}(p_u)\to W^-(p_2)\gamma(q)$, shown in Fig. 1. All momenta are incoming i.e. $p_d+p_u+p_2+q=0$, and the square of center-of-mass energy, s, is given by $s=p_1^2=(p_d+p_u)^2=(p_2+q)^2$. We work in the narrow width approximation where the W is assumed to be strictly on shell, $p_2^2=M_W^2$; the inclusion of W off-shellness effects is known to give very small contributions [7].

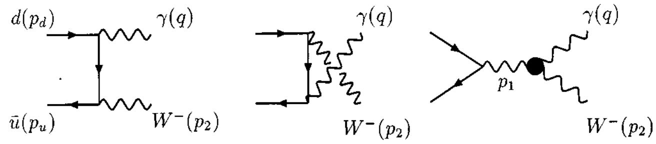

FIG. 1. Diagrams contributing to the process $d\overline{u}\rightarrow W^-\gamma$. The blob at the vertex indicates the presence of anomalous three-boson couplings.

The S-matrix element for the process we study is given by

$$\langle d\overline{u}|W^-\gamma\rangle = T^{\mu\beta}\epsilon^{\mu}_{\gamma}(q)\epsilon^{\beta}_{W^-}(p_2), \tag{2.1}$$

where $\epsilon^{\mu}_{\gamma}(q)$ and $\epsilon^{\beta}_{W^-}(p_2)$ are the polarization vectors of the photon and the W, respectively, and the amplitude $T^{\mu\beta}$ is the sum of three pieces,

$$T^{\mu\beta}=T^{\mu\beta}_s+T^{\mu\beta}_t+T^{\mu\beta}_u, \tag{2.2}$$

where T_s, T_t and T_u denote the s, t and u channels contributions, respectively. They are given by the following expressions:

$$T^{\mu\beta}_t=\left(\frac{iegQ_d}{\sqrt{2}}\right)\overline{v}_u\gamma_\beta P_L\frac{1}{\not p_d+\not q}\gamma_\mu u_d,$$

$$T^{\mu\beta}_u=\left(\frac{iegQ_{\overline{u}}}{\sqrt{2}}\right)\overline{v}_u\gamma_\mu\frac{1}{\not p_u+\not q}\gamma_\beta P_L u_d, \tag{2.3}$$

$$T^{\mu\beta}_s=\left(\frac{-ieg}{\sqrt{2}}\right)J^{W^-}_\alpha\left(\frac{1}{p_1^2-M_W^2}\right)\Gamma^{\mu\alpha\beta}(q,p_1,p_2),$$

where $J^{W^-}_\alpha=\overline{v}_u\gamma_\alpha P_L u_d$ $P_L=\frac{1}{2}(1-\gamma_5)$, $Q_{\overline{u}}=2Q_d=-\frac{2}{3}$, and we have neglected any quark mixing effects. $\Gamma^{\mu\alpha\beta}$ denotes the $WW\gamma$ vertex; it is written as the sum of the usual standard model piece $\Gamma^{\mu\alpha\beta}_0$, and an anomalous piece $\Gamma^{\mu\alpha\beta}_{an}$: i.e.

$$\Gamma^{\mu\alpha\beta}(q,p_1,p_2)=\Gamma^{\mu\alpha\beta}_0(q,p_1,p_2)+\Gamma^{\mu\alpha\beta}_{an}(q,p_1,p_2). \tag{2.4}$$

The canonical piece $\Gamma^{\mu\alpha\beta}_0$ is given by

$$\Gamma^{\mu\alpha\beta}_0(q,p_1,p_2)=(p_1-p_2)_\mu g^{\alpha\beta}+2q^\beta g^{\alpha\mu}-2q^\alpha g^{\mu\beta}, \tag{2.5}$$

where the relations $p_1^\alpha J^{W^-}_\alpha=0$, $q_\mu\epsilon^\mu_\gamma(q)=0$, and $p_{2\beta}\epsilon^\beta_{W^-}(p_2)=0$ have been used. The anomalous piece $\Gamma^{\mu\alpha\beta}_{an}$ reads[1]

$$\begin{aligned}\Gamma^{\mu\alpha\beta}_{an}(q,p_1,p_2)={}&\frac{\lambda}{M_W^2}(p_2-p_1)^\mu q^\alpha q^\beta+(\Delta\kappa+\lambda)q^\beta g^{\alpha\mu}\\&-\left(\Delta\kappa+\frac{1}{\rho}\lambda\right)q^\alpha g^{\mu\beta}\\&+\left[\tilde{\kappa}-\left(\frac{1+\rho}{2\rho}\right)\tilde{\lambda}\right]\varepsilon^{\mu\alpha\beta\rho}q_\rho\\&-\frac{\tilde{\lambda}}{2M_W^2}[(p_2-p_1)^\mu\epsilon^{\alpha\beta\rho\sigma}q_\rho(p_2-p_1)_\sigma\\&+s(\rho-1)\epsilon^{\mu\alpha\beta\rho}(p_2-p_1)_\rho]\end{aligned} \tag{2.6}$$

where $\rho\equiv M_W^2/s$. The anomalous couplings $\Delta\kappa$, λ, $\tilde{\kappa}$, and $\tilde{\lambda}$ parametrize the deviations from the standard model vertex; they are form-factors which must be evaluated at the kinematical point relevant for the reaction under consideration, i.e.

$$z\equiv z(p_1^2=s,p_2^2=M_W^2,q^2=0), \tag{2.7}$$

where z stands for any of the four aforementioned couplings. The anomalous vertex given above is the most general γWW vertex compatible with Lorentz and $U(1)$ gauge invariance; it has been derived from the interaction Lagrangian

$$\begin{aligned}\frac{1}{e}\mathcal{L}_{\gamma WW}={}&(ig_1W^\dagger_{\lambda\nu}W^\lambda A^\nu+\text{H.c.})\\&+i\kappa W^\dagger_\lambda W_\nu F^{\lambda\nu}+i\frac{\lambda}{M_W^2}W^\dagger_{\rho\lambda}W^\lambda_\nu F^{\nu\rho}\\&+\frac{g_4}{M_W^2}W^\dagger_\lambda W_\nu(\partial^\lambda\partial_\rho F^{\rho\nu}+\partial^\nu\partial_\rho F^{\rho\lambda})\\&-\left(\frac{g_5}{M_W^2}\epsilon^{\lambda\nu\rho\sigma}W^\dagger_\lambda\partial_\rho W_\nu\partial^\tau F_{\tau\sigma}+\text{H.c.}\right)\\&+i\tilde{\kappa}W^\dagger_\lambda W_\nu\tilde{F}^{\lambda\nu}+i\frac{\tilde{\lambda}}{M_W^2}W^\dagger_{\rho\lambda}W^\lambda_\nu\tilde{F}^{\nu\rho},\end{aligned} \tag{2.8}$$

where W^μ is the W^- field, A^μ is the photon field, the field strength tensors are all Abelian, and given by $W^{\mu\nu}=\partial^\mu W^\nu-\partial^\nu W^\mu$, $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$, $\tilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, and $\Delta\kappa\equiv\kappa-1$. The reason why the form-factors g_1, g_4, and g_5 do not appear in $\Gamma^{\mu\alpha\beta}_{an}$ is that by gauge-invariance g_1 is forced to be fixed at the value $g_1(p_1^2,p_2^2,q^2)=1$, whereas g_4 and g_5 to be proportional (in momentum space) to q^2, and thus to vanish in our case of an on-shell photon [7–9,12,13].

It is straightforward to verify that the full vertex $\Gamma^{\mu\alpha\beta}$ satisfies the elementary Ward identity

$$q_\mu\Gamma^{\mu\alpha\beta}(q,p_1,p_2)=(p_2^2g^{\alpha\beta}-p_2^\alpha p_2^\beta)-(p_1^2g^{\alpha\beta}-p_1^\alpha p_1^\beta), \tag{2.9}$$

[1]Since we only consider the γWW vertex and no confusion can arise between the corresponding couplings of the ZWW vertex we suppress any super(sub)scripts γ from all form-factors.

a fact which guarantees the electromagnetic gauge-invariance of the entire amplitude, i.e. $q^{\mu}T_{\mu\beta}=0$.

The squared unpolarized amplitude for the process is given by

$$\sum_{s_{\bar{u}},s_d}\sum_{\lambda_\gamma,\lambda_W}|\langle d\bar{u}|T|W^-\gamma\rangle|^2 = \sum_{s_{\bar{u}},s_d} T_{\mu\beta}P^{\mu\mu'}(q)Q^{\beta\beta'}(p_2)T^{\dagger}_{\mu\beta}, \tag{2.10}$$

where the sum on $s_{\bar{u}}$, s_d runs over all possible helicities of the incoming quarks, $Q_{\beta\beta'}$ is the polarization sum of the W, given by

$$Q^{\beta\beta'}(p_2)=-g^{\beta\beta'}+\frac{p_2^{\beta}p_2^{\beta'}}{M_W^2}, \tag{2.11}$$

and $P^{\mu\mu'}(q)$ the polarization sum of the photon, given by

$$P^{\mu\mu'}(q)=-g^{\mu\mu'}+\frac{\eta^{\mu}q^{\mu'}+\eta^{\mu'}q^{\mu}}{\eta q}-\eta^2\frac{q^{\mu}q^{\mu'}}{(\eta q)^2}, \tag{2.12}$$

where η_μ is an arbitrary four-vector, analogous to a gauge-fixing parameter. By virtue of the Ward identity of Eq. (2.9) satisfied by $\Gamma^{\mu\alpha\beta}$ all dependence on η_μ disappears from the righ-hand side (RHS) of Eq. (2.10).

The unpolarized color averaged differential cross-section in the center-of-mass frame is given by

$$\left(\frac{d\sigma}{dx}\right)=\frac{1}{3}\left(\frac{1-\rho}{32\pi s}\right)\sum_{s_{\bar{u}},s_d}\sum_{\lambda_\gamma,\lambda_W}|\langle d\bar{u}|T|W^-\gamma\rangle|^2 \tag{2.13}$$

where $x\equiv\cos\theta$, and θ is the scattering angle of the photon relative to the incoming anti-quark in the center-of-mass frame of the two partons, or equivalently, the produced W and γ. The differential cross-section is the sum of two pieces:

$$\left(\frac{d\sigma}{dx}\right)=\left(\frac{d\sigma^0}{dx}\right)+\left(\frac{d\sigma^{an}}{dx}\right). \tag{2.14}$$

The standard contribution $(d\sigma^0/dx)$ corresponds to the case where all anomalous couplings are set equal to zero, and has been studied extensively in the literature [5–9]. In particular it exhibits a radiation zero at $x=-1/3$:

$$\frac{d\sigma^0}{dx}=\left(\frac{1}{3}\right)\frac{e^2g^2}{128\pi}\frac{1}{s(1-\rho)}\frac{(x+1/3)^2[(1+\rho)^2+(1-\rho)^2x^2]}{1-x^2}. \tag{2.15}$$

The anomalous contribution $(d\sigma^{an}/dx)$ is given by

$$\left(\frac{d\sigma^{an}}{dx}\right)=C(s)\left[\sigma_1(s)P_1(x)+\left(\frac{1}{2\rho}\right)\sigma_2(s)P_2(x)+\left(\frac{1}{4\rho^2}\right)\sigma_3(s)P_3(x)\right], \tag{2.16}$$

$$C(s)=\left(\frac{1}{3}\right)\frac{e^2g^2}{256\pi}\frac{(1-\rho)}{s}, \tag{2.17}$$

where

$$P_1(x)=x+3x^2,\quad P_2(x)=1,\quad P_3(x)=1-x^2, \tag{2.18}$$

and

$$\sigma_1(s)=-\frac{2}{3}\Delta\kappa,$$
$$\sigma_2(s)=(\Delta\kappa+\lambda)^2+(\tilde{\kappa}+\tilde{\lambda})^2, \tag{2.19}$$
$$\sigma_3(s)=2(\lambda^2+\tilde{\lambda}^2)-\rho[(\Delta\kappa-\lambda)^2+(\tilde{\kappa}-\tilde{\lambda})^2]+2\rho^2(\Delta\kappa^2+\tilde{\kappa}^2).$$

The expression given in Eq. (2.16) applies also to the process $u\bar{d}\to W^+\gamma$, with the only difference that, in that case, the scattering angle is defined between the incoming quark and the photon. Our result agrees with previous calculations which considered only $\Delta\kappa$ [5], or both the $\Delta\kappa$ and λ couplings [6].

It is well known that, due to the presence of the terms ρ^{-1} and ρ^{-2}, the cross-section of Eq. (2.16) would grossly violate unitarity at high energies, $s\to\infty$, $\rho\to 0$, if the anomalous couplings were considered to be constants, independent of s. Therefore, in analogy to nuclear form-factors, one traditionally assumes an s-dependence of the form [8]

$$z(s,M_W^2,0)=\frac{z_0}{(1+s/\Lambda^2)^n}. \tag{2.20}$$

From the explicit expression of the cross-section we see that the choices $n=1/2$ for $\Delta\kappa$, $\tilde{\kappa}$, and $n=1$ for λ, $\tilde{\lambda}$ would suffice in order for the square brackets in Eq. (2.16) to approach a constant as $\rho\to 0$; then the cross section would decrease like $1/s$ due to the flux factor in $C(s)$. The scale Λ is assumed to be of the same order of magnitude as the scale characterizing the new physics responsible for the anomalous couplings.

We note that the coupling λ contributes only quadratically in the cross section. We have no simple explanation of this fact, which seems to be characteristic to the specific process. Of course the *C*, *P* violating couplings $\tilde{\kappa},\tilde{\lambda}$ can only contribute quadratically. Thus, if the anomalous couplings are small enough ($\leqslant 10^{-3}$) so that for certain center of mass energies all quadratic terms could be neglected as unobservable, then any deviation would be a direct measurement of $\Delta\kappa$. This is however not the case for typical Tevatron and LHC energies; therefore, quadratic terms must be kept in general, and are typically of the same order, or even larger than the linear one.

It is interesting to notice that the term linear in $\Delta\kappa$ does *not* distort the appearance of the radiation zero at $x=-1/3$, for arbitrary values of $\Delta\kappa$. The radiation zero is washed out only due to the quadratic anomalous terms, which reduce it to a dip. Since the quadratic coefficients of λ are much larger than those of $\Delta\kappa$, due to the pre-factor $(1/4\rho^2)$ in front of σ_3 in Eq. (2.16), the cross section is extremely sensitive to nonstandard λ values in the neighborhood of the radiation zero.

We also notice that the quadratic terms are completely symmetric under $(\Delta\kappa,\lambda)\leftrightarrow(\tilde{\kappa},\tilde{\lambda})$. Furthermore the coefficient in front of $\Delta\kappa$ is independent of the energy. Thus in experimental analyses where the fitting is carried out by allowing one of the anomalous couplings to differ from zero at a time, the λ and $\tilde{\lambda}$ distributions as well as other related bounds would be identical. The same would be true for $\Delta\kappa$ and $\tilde{\kappa}$, if the term linear in $\Delta\kappa$ were negligible; this could be the case at high energies, if $\Delta\kappa$ is not very small ($\Delta\kappa > 10^{-2}$).

III. PROJECTING OUT THE ANOMALOUS COUPLINGS

In this section we show how one can extract experimental values for the quantities σ_1, σ_2, and σ_3, defined in Eq. (2.19). This will furnish a system of three independent algebraic equations involving the four unknown gauge couplings.

To accomplish this, one first notices that the polynomials $P_i(x)$ constitute a linearly-independent set; their Wronskian $W(P_i)$ is simply $W(P_i)=2$. One may then construct a set of three other polynomials, $\tilde{P}_i(x)$, which are orthonormal to the $P_i(x)$, i.e. they satisfy

$$\int_{-1}^{1}\tilde{P}_i(x)P_j(x)dx=\delta_{ij}. \tag{3.1}$$

These polynomials are[2]

$$\tilde{P}_1(x)=\frac{3}{2}x,$$

$$\tilde{P}_2(x)=-\frac{3}{4}-\frac{9}{2}x+\frac{15}{4}x^2, \tag{3.2}$$

$$\tilde{P}_3(x)=\frac{15}{8}+\frac{9}{2}x-\frac{45}{8}x^2.$$

It is important to observe that the polynomials $P_i(x)$ and $\widetilde{P_i}(x)$ are *independent* of the (sub)-process energy *s*. This is to be contrasted to the equivalent set of polynomials obtained in the context of the process $e^+e^-\rightarrow W^+W^-$ [3], which depend explicitly on the (fixed) *s*. As we will explain in Sec. V the *s*-independence of the polynomials $\widetilde{P_i}(x)$ is crucial for the applicability of the proposed method to the realistic case of $p\bar{p}$ and pp scattering, where the structure functions must be included.

By means of the polynomials $\tilde{P}_i(x)$ one may then invert Eq. (2.19), and project out the individual σ_i as follows:

$$\sigma_1(s)=C^{-1}(s)\int_{-1}^{1}dx\left(\frac{d\sigma^{an}}{dx}\right)\tilde{P}_1(x),$$

$$\sigma_2(s)=2\rho C^{-1}(s)\int_{-1}^{1}dx\left(\frac{d\sigma^{an}}{dx}\right)\tilde{P}_2(x), \tag{3.3}$$

$$\sigma_3(s)=4\rho^2 C^{-1}(s)\int_{-1}^{1}dx\left(\frac{d\sigma^{an}}{dx}\right)\tilde{P}_3(x).$$

In order to extract the experimental values σ_i^{exp} for the observables σ_i we simply substitute in the left-hand side of Eq. (3.3) the experimental value $(d\sigma_{exp}^{an}/dx)$ given by

$$\left(\frac{d\sigma_{exp}^{an}}{dx}\right)=\left(\frac{d\sigma_{exp}}{dx}\right)-\left(\frac{d\sigma^0}{dx}\right). \tag{3.4}$$

We notice from Eq. (2.19) that σ_1^{exp} would determine *directly* the experimental value for $\Delta\kappa$, *without* any assumptions on the size of the other three couplings. The remaining two equations involving σ_2^{exp} and σ_3^{exp} can then be used as sum rules, in order to impose experimental constraints on the three remaining couplings λ, $\tilde{\kappa}$, and $\tilde{\lambda}$.

In the case when experimental cuts restrict the angular region from $[a,b]$ instead of $[-1,1]$ appropriate orthonormal polynomials $\tilde{P}_i(x)$ can still be easily constructed, by requiring

$$\int_{a}^{b}\tilde{P}_i(x)P_j(x)dx=\delta_{ij}. \tag{3.5}$$

Their closed form reads

$$\tilde{P}_i(x)=c_{i0}+c_{i1}x+c_{i2}x^2, \quad i=1,2,3 \tag{3.6}$$

with

$$c_{10}=36D(a^3+4a^2b+4ab^2+b^3),$$

$$c_{11}=-48D(4a^2+7ab+4b^2),$$

[2]Of course this set is not uniquely determined; here we derive the set with the lowest possible degree in *x*.

$$
\begin{aligned}
c_{12}&=180D(a+b),\\
c_{20}&=-3D[3a^4+12a^3(3+b)\\
&\quad+(10+36b+3b^2)(b^2+4ab)\\
&\quad+2a^2(5+72b+15b^2)],\\
c_{21}&=36D[a^3+4a^2(4+b)+b(5+16b+b^2)\\
&\quad+a(5+28b+4b^2)],\\
c_{22}&=-30D[6+a^2+18b+b^2+2a(9+2b)],\\
c_{30}&=6D[18a^3+(5+18b)(4a+b)b+a^2(5+72b)],\\
c_{31}&=-36D[16a^2+b(5+16b)+a(5+28b)],\\
c_{32}&=180D(1+3a+3b),
\end{aligned}
\tag{3.7}
$$

where

$$D\equiv(a-b)^{-5}. \tag{3.8}$$

As a check one may verify that the choice $a=-1$ and $b=1$ in Eq. (3.7) reproduces the set given in Eq. (3.2).

IV. STATISTICAL PROPERTIES OF THE σ_i OBSERVABLES

In this section we will present an elementary study of the basic statistical properties of the σ_i observables introduced in the previous section. In particular we will compute their correlations, using simple assumptions about the distribution of the anomalous couplings. For convenience, in this section we introduce the following uniform notation:

$$z_1\equiv\Delta\kappa,\quad z_2\equiv\lambda,\quad z_3\equiv\tilde{\kappa},\quad z_4\equiv\tilde{\lambda}. \tag{4.1}$$

To study the correlations of the σ_i observables we will that assume each of the couplings z_i obeys independently a normal (Gaussian) probability distribution, with mean μ_i and variance δ_i^2, i.e.

$$p_i(z_i,\mu_i,\delta_i^2)=\frac{1}{\delta_i(2\pi)^{1/2}}\exp\left[-\frac{(z_i-\mu_i)^2}{2\delta_i^2}\right]. \tag{4.2}$$

Then, the expectation value $\langle\sigma_i\rangle$ of the observable σ_i is given by

$$\langle\sigma_i\rangle=\prod_{j=1}^{4}\int_{-\infty}^{+\infty}[dz_j]p_j\sigma_i, \tag{4.3}$$

the corresponding covariance matrix by

$$V_{ij}=\langle\sigma_i\sigma_j\rangle-\langle\sigma_i\rangle\langle\sigma_j\rangle, \tag{4.4}$$

and the correlations r_{ij} by

$$r_{ij}=\frac{V_{ij}}{V_{ii}^{1/2}V_{jj}^{1/2}}. \tag{4.5}$$

We will next assume that the Gaussian distribution is peaked around the standard model values of the couplings, i.e. $\mu_i=0$, and will use the elementary results

$$
\begin{aligned}
&\int_{-\infty}^{+\infty}[dz_i]p_i^{(0)}z_i=0,\\
&\int_{-\infty}^{+\infty}[dz_i]z_i^2p_i^{(0)}=\delta_i^2,\qquad \int_{-\infty}^{+\infty}[dz_i]z_i^4p_i^{(0)}=\frac{3}{4}\delta_i^4,
\end{aligned}
\tag{4.6}
$$

where $p_i^{(0)}\equiv p_i(z_i,0,\delta_i^2)$. After a straightforward calculation we obtain the following expressions for the various V_{ij}:

$$
\begin{aligned}
V_{11}&=\frac{4}{9}\delta_1^2,\\
V_{12}&=0,\\
V_{13}&=0,\\
V_{22}&=4\delta_1^2\delta_2^2+\frac{1}{2}\delta_1^4+\frac{1}{2}\delta_2^4+4\delta_3^2\delta_4^2+\frac{1}{2}\delta_3^4+\frac{1}{2}\delta_4^4,\\
V_{33}&=-2\rho(\delta_2^4+\delta_4^4)+\frac{1}{2}\rho^2(8\delta_1^2\delta_2^2+\delta_1^4+\delta_2^4+8\delta_3^2\delta_4^2+\delta_3^4\\
&\quad+\delta_4^4)-2\rho^3(\delta_1^4+\delta_3^4)+2\rho^4(\delta_1^4+\delta_3^4)+2(\delta_2^4+\delta_4^4),\\
V_{23}&=\frac{1}{2}\rho(8\delta_1^2\delta_2^2-\delta_1^4-\delta_2^4+8\delta_3^2\delta_4^2-\delta_3^4-\delta_4^4)+\rho^2(\delta_1^4+\delta_3^4)\\
&\quad+\delta_2^4+\delta_4^4.
\end{aligned}
\tag{4.7}
$$

Evidently, the only non-zero correlation is r_{23}. By varying the value of the corresponding variances δ_i one can study the limits where some of the couplings are excluded on the grounds of certain discrete symmetries. For example, by choosing δ_3 and δ_4 very small compared to δ_1 and δ_2 we approach the limit where the C and P symmetries are individually respected by the anomalous couplings. The other interesting parameter to vary is of course the center-of-mass energy s. In Table I we display some characteristic cases.

We see that, with the exception of relatively low energies, the values of r_{23} are rather reasonable, and they tend to decrease as the sub-process energy s increases.

V. INCLUSION OF THE STRUCTURE FUNCTIONS

The analysis presented so far is valid at the partonic level, and the quarks appearing in the initial state were assumed to be in their center-of-mass frame. In reality the initial states are protons and anti-protons, a fact which introduces two

TABLE I. The correlation r_{23} as a function of the center-of-mass energy s, for various choices of the parameters δ_i.

$\sqrt{s}$(GeV)	100	200	300	500	1000
r_{23}	0.93	0.50	0.40	0.35	0.32
		$\delta_1^2=\delta_2^2 \geqslant \delta_3^2=\delta_4^2$			
$\sqrt{s}$(GeV)	100	200	300	500	1000
r_{23}	0.92	0.44	0.32	0.25	0.23
		$\delta_1^2=2\delta_2^2=10\delta_3^2=10\delta_4^2$			
$\sqrt{s}$(GeV)	100	200	300	500	1000
r_{23}	0.91	0.40	0.27	0.21	0.18
		$\delta_1^2=3\delta_2^2=50\delta_3^2=50\delta_4^2$			

additional complications. First, the final state can be reached by different combinations of partons, which are not in their center-of-mass any more, but carry momentum fractions x_a and x_b of the corresponding parent hadrons. This is taken into account by introducing structure functions. Second, the partonic center-of-mass frame has to be reconstructed from the data on an event by event basis. However, not all kinematical information on the final state particles is available, since the final W boson decays to a lepton and an (unobserved) neutrino. Although the transverse momentum of the neutrino is identified with the missing transverse momentum of the event, its longitudinal momentum can only be determined with a twofold ambiguity by constraining the lepton-neutrino pair invariant mass to equal the W mass [8,9]. Using the fact that, at least at the Tevatron, the W boson is highly polarized, one can arrive at the correct choice with a success-rate of 73% [14].

For a $p\bar{p}$ collider the total cross-section for the process $p\bar{p}\rightarrow W^-\gamma X$, reads [5]

$$\frac{d\sigma_{p\bar{p}}(S,x)}{dx}=\sum_{u,d}\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a,s) f_{\bar{u}/\bar{p}}(x_b,s) \times\frac{d\sigma_{d\bar{u}}(s,\cos\theta_1)}{d\cos\theta_1}+\sum_{u,d}\int_0^1\int_0^1 dx_a dx_b \times f_{\bar{u}/p}(x_a,s) f_{d/\bar{p}}(x_b,s)\frac{d\sigma_{\bar{u}d}(s,\cos\theta_2)}{d\cos\theta_2}, \tag{5.1}$$

where S is the square of the $p\bar{p}$ center-of-mass energy, $S=(p_p+p_{\bar{p}})^2$, x_a and x_b are the fractional longitudinal momenta of the quarks inside the proton, $0\leqslant x_{a,b}\leqslant 1$, and $s=x_a x_b S$ is the squared center-of-mass energy of the subprocess. In the above equation u denotes a generic *up*-type quark (u,c,t), while d a generic *down*-type quark (d,s,b). The double sum runs over all possible combinations of *up* and *down* quarks $(\bar{u}d,\ \bar{u}s,\ \bar{c}d,\ \dots)$ which give rise to the desired final state $W^-\gamma$, and for which the corresponding structure functions are not negligible. $d\sigma_{d\bar{u}}$ denotes the situation where the d quark originates from the proton and the $\bar{u}$ from the anti-proton beam; in that case valence-quark structure functions are involved for both quarks. $d\sigma_{\bar{u}d}$ denotes the reverse situation; now both structure functions involve sea-quarks and are equal. We define the $W\gamma$ center-of-mass angle $x=\cos\theta_{c.m.}$, to be the angle between the photon and the anti-proton beam. Thus, the angle involved in $d\sigma_{d\bar{u}}$ is $\theta_1=\theta_{c.m.}$, while in $d\sigma_{\bar{u}d}$ it is $\theta_2=\pi-\theta_{c.m.}$, or $\cos\theta_1=x$ and $\cos\theta_2=-x$. Clearly, we have that $d\sigma_{\bar{u}d}(s,x)=d\sigma_{d\bar{u}}(s,-x)$.

Notice that if the structure function weights $f(x_a,s)f(x_b,s)$ multiplying each term were the same, then all terms linear in x would cancel in Eq. (5.1); in such a case the polynomial $P_1(x)\rightarrow\bar{P}_1(x)=3x^2$, a fact which would render the set of polynomials $\bar{P}_1(x)$, $P_2(x)$, $P_3(x)$ linearly dependent, thus reducing the usefulness of the proposed method. This is however not the case, since the first term on the RHS of Eq. (5.1) originates from the valence quarks inside the proton and anti-proton, whereas the second term comes from the sea-quarks. The sea-quark contribution is significantly smaller than that of the valence-quarks, a fact which is appropriately encoded in the form of the corresponding structure functions, convoluted with the above elementary processes. One could therefore, to a good approximation, omit this term. If, nonetheless, such a term were to be kept, the necessary procedure would be as follows: Since $d\sigma_{\bar{u}d}(s,x)=d\sigma_{d\bar{u}}(s,-x)$, it follows from Eq. (2.16) and Eq. (2.18) that $d\sigma_{\bar{u}d}(s,x)$ is a linear combination of $P_2(x)$, $P_3(x)$, and $\hat{P}_1(x)=P_1(-x)=3x^2-x$. The next step is to write $\hat{P}_1(x)$ as a linear combination of P_1(x), $P_2(x)$, and $P_3(x)$, i.e.

$$\hat{P}_1(x)=-P_1(x)+6P_2(x)-6P_3(x), \tag{5.2}$$

after which the quantities σ_1, σ_2, and σ_3 will be simply modified by small corrections.

Omitting for simplicity such corrections from sea-quarks, and keeping only the $(\bar{u}d)$ term in the sum on the RHS, Eq. (5.1) becomes

$$\frac{d\sigma_{p\bar{p}}(S,x)}{dx}=\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\bar{u}/\bar{p}}(x_b)\frac{d\sigma_{\bar{u}d}(s,x)}{dx}, \tag{5.3}$$

and its deviation from the canonical value due to the anomalous gauge boson couplings is given by

$$\frac{d\sigma_{pp}^{(an)}(S,x)}{dx}=\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\overline{u}/\overline{p}}(x_b)\frac{d\sigma_{ud}^{(an)}(s,x)}{dx}$$
$$=\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\overline{u}/\overline{p}}(x_b) C(s)\left[\sigma_1(s)P_1(x)+\left(\frac{1}{2\rho(s)}\right)\sigma_2(s)P_2(x)+\left(\frac{1}{4\rho^2(s)}\right)\sigma_3(s)P_3(x)\right]. \quad (5.4)$$

At this point we can repeat the same procedure for projecting out the anomalous couplings which has been presented in Sec. III. This is possible because the polynomials $P_i(x)$ and the corresponding projective polynomials $\widetilde{P}_i(x)$ do *not* depend on the sub-process energy s. Writing Eq. (5.4) in the form

$$\frac{d\sigma_{p\overline{p}}(S,x)}{dx}=\sum_{i=1}^{3}\Sigma_i(S)P_i(x) \quad (5.5)$$

with

$$\Sigma_1(S)\equiv\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\overline{u}/\overline{p}}(x_b) C(s)\sigma_1(s),$$

$$\Sigma_2(S)\equiv\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\overline{u}/\overline{p}}(x_b)\left(\frac{C(s)}{2\rho(s)}\right)\sigma_2(s), \quad (5.6)$$

$$\Sigma_3(S)\equiv\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\overline{u}/\overline{p}}(x_b)\left(\frac{C(s)}{4\rho^2(s)}\right)\sigma_3(s),$$

we see that the quantities Σ_i may be extracted from the differential cross-section by means of the projective polynomials $\widetilde{P}_i(x)$. Their experimental value $\Sigma_i^{exp}(S)$ are obtained from

$$\Sigma_i^{exp}(S)=\int_{-1}^{1}\left[\frac{d\sigma_{pp}^{(exp)}(S,x)}{dx}-\frac{d\sigma_{pp}^{(0)}(S,x)}{dx}\right]\widetilde{P}_i(x)dx. \quad (5.7)$$

In order to place bounds on the unknown couplings from the values of $\Sigma_i^{exp}(S)$ one will have to take into account the fact that the couplings depend on the sub-process energy s, i.e. they are functions of the integration variables x_a and x_b. For example, assuming an energy dependence such as the one given in Eq. (2.20), we have for $\Delta\kappa$ (with $n=1/2$)

$$\Sigma_1^{exp}(S)=-\frac{2}{3}\Delta\kappa_0\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\overline{u}/\overline{p}}(x_b)\times\frac{C(x_a x_b S)}{\sqrt{1+x_a x_b S/\Lambda^2}}. \quad (5.8)$$

Thus, measuring Σ_1 will determine $\Delta\kappa_0$ as a function of the arbitrary scale Λ.

In the case of a pp collider such as the LHC, the process $pp\to W^-\gamma X$ proceeds only through sea-quark interactions. The corresponding differential cross-section reads

$$\frac{d\sigma_{pp}(S,\cos\theta_{c.m.})}{d\cos\theta_{c.m.}}=\sum_{u,d}\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\overline{u}/p}(x_b)\times\frac{d\sigma_{d\overline{u}}}{d\cos\theta_1}+\sum_{u,d}\int_0^1\int_0^1 dx_a dx_b\times f_{\overline{u}/p}(x_a) f_{d/p}(x_b)\frac{d\sigma_{\overline{u}d}}{d\cos\theta_2}. \quad (5.9)$$

Both terms in the above sum contain the product of a valence-quark and a sea-quark distribution. This makes the rates of pp cross sections lower than the relative $p\overline{p}$ ones, a fact which is compensated by the higher luminosities of the pp machines. Therefore, one important difference between Eq. (5.1) and Eq. (5.9) is that, after setting again $\cos\theta_1=x$ and $\cos\theta_2=-x$, the terms linear in x cancel in the latter. Indeed, since the cross-section is symmetric in $x_a\leftrightarrow x_b$ both terms contribute with equal weight under the integral and the cross-section assumes the form

$$\frac{d\sigma_{pp}(S,x)}{dx}=\sum_{u,d}\int_0^1\int_0^1 dx_a dx_b f_{d/p}(x_a) f_{\overline{u}/p}(x_b)\times\left[\frac{d\sigma_{d\overline{u}}(s,x)}{dx}+\frac{d\sigma_{d\overline{u}}(s,-x)}{dx}\right]. \quad (5.10)$$

As mentioned before, the absence of the linear x term would render the set of polynomials P_i linearly dependent. To avoid this we propose to follow the method of binning the events according to their longitudinal momentum, which was introduced in [15]. The basic point is to break the symmetricity of the $\int_0^1\int_0^1 dx_a dx_b$ integration by imposing an asymmetric constraint on the values of x_a and x_b. Let us for example impose the linear constraint $x_a-x_b\geq\delta$, with $\delta<1$. Then the ''binned'' differential cross-section $d\sigma_{pp}^{(b)}/d\cos\theta_{c.m.}$ reads

$$\frac{d\sigma_{pp}^{(b)}(S,\cos\theta_{c.m.})}{d\cos\theta_{c.m.}} = \sum_{u,d} \int_{\delta}^{1}\int_{0}^{1-\delta} dx_a dx_b f_{d/p}(x_a) f_{\bar{u}/p}(x_b) \frac{d\sigma_{d\bar{u}}}{d\cos\theta_1} + \sum_{u,d} \int_{\delta}^{1}\int_{0}^{1-\delta} dx_a dx_b f_{\bar{u}/p}(x_a) f_{d/p}(x_b) \frac{d\sigma_{\bar{u}d}}{d\cos\theta_2}. \tag{5.11}$$

or, equivalently,

$$\frac{d\sigma_{pp}^{(b)}(S,x)}{dx} = \sum_{u,d} \int_{\delta}^{1}\int_{0}^{1-\delta} dx_a dx_b \left[f_{d/p}(x_a) f_{\bar{u}/p}(x_b) \frac{d\sigma_{d\bar{u}}(s,x)}{dx} + f_{\bar{u}/p}(x_a) f_{d/p}(x_b) \frac{d\sigma_{d\bar{u}}(s,-x)}{dx} \right] = \sum_{i=1}^{3} \Sigma_i^{(b)}(S) P_i(x), \tag{5.12}$$

with

$$\Sigma_1^{(b)}(S) \equiv \int_{\delta}^{1}\int_{0}^{1-\delta} dx_a dx_b [f_{d/p}(x_a) f_{\overline{u/p}}(x_b) - f_{d/p}(x_b) f_{\overline{u/p}}(x_a)] C(s)\sigma_1 ,$$

$$\Sigma_2^{(b)}(S) \equiv \int_{\delta}^{1}\int_{0}^{1-\delta} dx_a dx_b \left[f_{d/p}(x_a) f_{\overline{u/p}}(x_b) \frac{\sigma_2}{2\rho} + f_{d/p}(x_b) f_{\overline{u/p}}(x_a) \left(\frac{\sigma_2}{2\rho} + 6\sigma_1 \right) \right] C(s), \tag{5.13}$$

$$\Sigma_3^{(b)}(S) \equiv \int_{\delta}^{1}\int_{0}^{1-\delta} dx_a dx_b \left[f_{d/p}(x_a) f_{\overline{u/p}}(x_b) \frac{\sigma_3}{4\rho^2} + f_{d/p}(x_b) f_{\overline{u/p}}(x_a) \left(\frac{\sigma_3}{4\rho^2} - 6\sigma_1 \right) \right] C(s),$$

where we have used Eq. (5.2). Similarly, the experimental values for the $\Sigma_i^{(b)}(S)$ will be given by

$$\Sigma_i^{(b,exp)}(S) = \int_{-1}^{1} \left[\frac{d\sigma_{pp}^{(b,exp)}(S,x)}{dx} - \frac{d\sigma_{pp}^{(b,0)}(S,x)}{dx} \right] \tilde{P}_i(x) dx. \tag{5.14}$$

In practice one should verify that the bounds for the anomalous couplings obtained by applying the above procedure do not depend heavily on how the binning is carried out, by choosing, for example, different values for δ, or different functional forms for the asymmetric constraint imposed.

VI. CONCLUSIONS

In this paper we have presented a model-independent method for extracting bounds on the anomalous γWW couplings from pp and $p\bar{p}$ experiments, using the process $d\bar{u} \to W^- \gamma$ as a prototype. At the partonic level this method gives rise to three observables, which depend explicitly on the various anomalous couplings through simple algebraic relations. These observables may be extracted from the experimentally measured unpolarized differential cross-section for this process by means of a convolution with appropriately constructed polynomials. These polynomials are quadratic functions of the center-of-mass angle only; most notably, they do not depend on the center-of-mass energy of the (sub)-process. One of these observables is linearly related to $\Delta\kappa$ only; therefore, its measurement can furnish the experimental value of this quantity, without further assumptions on the values of the remaining couplings. The other two observable constitute a system of two equations for the remaining three anomalous couplings; thus they can be used as sum rules, in conjunction with other possible observables, physically motivated constraints, or model-inspired relations.

The generalization of the method to the realistic case of hadron colliders, where the initial particles are not partons but protons or anti-protons, presents experimental and theoretical complications, which, however, can be overcome. From the experimental point of view, it is clear that in the case of hadron colliders the center-of-mass frame for the $W\gamma$ must be reconstructed; the ability to achieve this is of course crucial for the applicability of proposed method, given that the latter relies heavily on the use of the center-of-mass scattering angle. One way to accomplish this seems to be the following: The produced W is in general closely on-shell, and highly polarized; using the first fact, one can impose the constraint $M_W^2 = (p_l + p_\nu)^2$ to determine the longitudinal momentum of the neutrino with a twofold ambiguity [9,7] (the transverse one is the missing transverse momentum of the event), whereas the second fact guarantees that one can select the correct solution 73% of the time [14]. Thus, the W momentum can be reconstructed, and from it the center-of-mass angle of the event may be deduced.

At the theoretical level the method carries over straightforwardly from the partonic level to the case of $p\bar{p}$ colliders, since the inclusion of the structure functions does not interfere with any of the underlying assumptions. After the inclusion of the structure functions one needs to assume a certain functional dependence of the unknown form-factors on the sub-process energies, over which one integrates; this is of course a general limitation in this type of analysis, and is not particular to this method. On the other hand, in the case of pp colliders the structure functions conspire to eliminate the linear terms in x, a fact which invalidates one of the main assumptions, namely that the polynomials $P_i(x)$ are linearly independent. This may be circumvented if one considers the ''binned'' instead of the usual differential cross-section, which may be obtained by introducing an asymmetric constraint among the longitudinal momenta appearing in the arguments of the structure functions [15].

It would be interesting to study whether the analysis presented here carries over to the process $d\bar{u}\rightarrow W^{-}Z$, which would probe directly and separately the possible anomalous couplings appearing in the ZWW vertex. Finally, we note that these results can be easily translated to the $e^{-}\gamma \rightarrow W^{-}\nu_e$ process, which would be of interest for linear colliders, if the $e\gamma$ option is realised.

ACKNOWLEDGMENTS

The work of J.P. was supported by a EC grant TMR-ERBFMBICT 972024 and that of K.P. by a EC grant FM-BICT 961033. The authors thank W. J. Stirling and M. Stratmann for useful discussions.

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