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Neutrino oscillations in a predictive SUSY GUT

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In this paper we present a predictive SO(10) supersymmetric grand unified theory with the family symmetry U(2)×U(1) which has several nice features. We are able to fit fermion masses and mixing angles, including recent neutrino data, with nine parameters in the charged fermion sector and four in the neutrino sector. The family symmetry plays a preeminent role. (i) The model is ''natural''—we include all terms allowed by the symmetry. It restricts the number of arbitrary parameters and enforces many zeros in the effective mass matrices. (ii) Family symmetry breaking from U(2)×U(1)→U(1)→ nothing generates the family hierarchy. It also constrains squark and slepton mass matrices, thus ameliorating flavor violation resulting from squark and slepton loop contributions. (iii) It naturally gives large angle $\nu_\mu-\nu_\tau$ mixing describing atmospheric neutrino oscillation data and small angle $\nu_e-\nu_s$ mixing, consistent with the small mixing angle Mikheyev-Smirnov-Wolfenstein (MSW) solution to solar neutrino data. (iv) Finally, in this paper we assume minimal family symmetry-breaking vacuum expectation values (VEV's). As a result we cannot obtain a three neutrino solution to both atmospheric and solar neutrino oscillations. In addition, the solution discussed here cannot fit liquid scintillation neutrino detector (LSND) data even though this solution requires a sterile neutrino ν_s. It is important to note, however, that with nonminimal family symmetry-breaking VEV's, a three neutrino solution is possible with the small mixing angle MSW solution to solar neutrino data and large angle $\nu_\mu-\nu_\tau$ mixing describing atmospheric neutrino oscillation data. In the four neutrino case, nonminimal family VEV's may also permit a solution for LSND. The results with nonminimal family breaking are still under investigation and will be reported in a future paper. [S0556-2821(99)00619-0]

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I. INTRODUCTION

Solar [1], atmospheric [2], and accelerator [3] neutrino data strongly suggest that neutrinos have small masses and nonvanishing mixing angles. This hypothesis is also constrained by reactor [4] based experiments. In the near future, many more experiments will test the hypothesis of neutrino masses. In addition, a neutrino mass necessarily implies new physics beyond the standard model. Thus there is great excitement, both experimental and theoretical, in this field.

Phenomenological neutrino mass models [5] are designed to reproduce the best fits to all or some of the neutrino data. These models are only constrained by how much of the neutrino data one wants to fit. Three neutrino models with three active neutrinos (ν_e, ν_μ and ν_τ) are consistent with solar [1] and atmospheric [2] neutrino oscillations, while four neutrino models, including a sterile (or electroweak singlet) neutrino (ν_s), are consistent with solar, atmospheric, and liquid scintillation neutrino detector (LSND) [3] neutrino experiments. There are also six neutrino models, with three active and three sterile neutrinos, motivated by complete family symmetry [6].

It is important to address the theoretical question; to what extent can this new data on neutrino masses and mixing angles constrain the physics beyond the standard model; in particular, theories of fermion masses. Since any number of sterile neutrinos may mix with the three active neutrinos, even in a grand unified theory, it may always be possible to fit neutrino data without ever constraining the charged fermion sector of the theory. This would be an unfortunate circumstance. It is the purpose of this paper, however, to show that in any ''predictive'' theory of charged fermion masses, the neutrino sector is severely constrained.

By a ''predictive'' model of fermion masses we mean the following.

The Lagrangian is ''natural'' containing all terms consistent with the symmetries and particle content of the theory.

In addition there are necessarily grand unified theory (GUT) gauge symmetries as well as family symmetries which restrict the form of the Yukawa matrices [7–9]; thereby greatly reducing the number of arbitrary parameters.

In supersymmetric (SUSY) theories, these same family symmetries can usefully constrain the form of soft SUSY breaking squark and slepton masses as well [7–9]; thus ameliorating the problem of large flavor violation in SUSY theory [10].

In this paper, we demonstrate that these same family symmetries greatly restrict the form of neutrino masses and mixing. Hence neutrino data can greatly constrain any predictive theory of fermion masses.

We show this in the context of a particular SO(10) SUSY GUT which fits charged fermion masses and mixing angles well. SUSY GUT's are very attractive. They successfully predict the unification of gauge couplings observed at the

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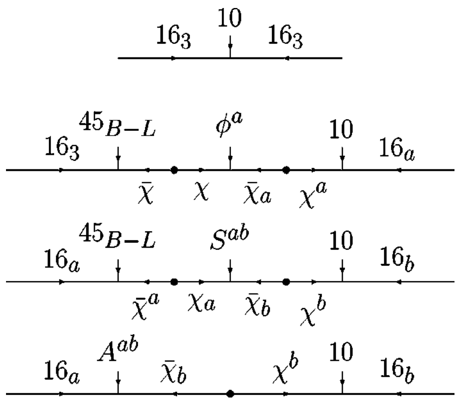

FIG. 1. Diagrams generating the Yukawa matrices.

CERN e^+e^- collider LEP [11,12]. In SO(10) one family $\{q, \overline{u}, \overline{d}, l, \overline{e}, \overline{\nu}\}$ fits into the 16-dimensional spinor representation of the group [13]. Thus up, down, charged lepton and Dirac neutrino mass matrices are related.

Of course, the last comment leads to the generic problem for any GUT description of neutrino masses. Atmospheric neutrino data [2] requires large mixing between ν_μ and ν_x, where ν_x is any neutrino species, other than ν_e [2,4]. Solar neutrino data as well can have a large mixing angle solution. Thus lepton mass matrices must give large mixing angles in sharp contrast to quark mass matrices which give small Cabibbo-Kobayashi-Maskawa mixing angles.

We consider an SO(10)×U(2)×U(1) model of fermion masses. This theory is a modification of the SO(10)×U(2) model of Barbieri, Hall, Raby, and Romanino (BHRR) [9]. The modifications only affect the results for neutrinos. Alternate descriptions of neutrinos in the context of U(2) family symmetry can be found in recent articles [14]. In Sec. II, we give the superspace potential and the resulting quark and lepton Yukawa matrices. We then give the results for charged fermion masses and mixing angles. In Sec. III, we describe the neutrino sector; giving our fits for solar and atmospheric neutrino oscillations and predictions for future experiments. A brief summary of our results follows.

Atmospheric neutrino oscillations are predominantly described by maximal $\nu_\mu - \nu_\tau$ mixing; while solar neutrino oscillations are given by small-angle Mikheyev-Smirnov-Wolfenstein (MSW) $\nu_e - \nu_s$ mixing. We are *not* able to accommodate LSND. However, this result assumes minimal family symmetry-breaking vacuum expectation values (VEV's). By allowing for more general U(2)×U(1) breaking VEV's we are able to obtain a three neutrino fit to atmospheric and solar neutrino data. In a future paper we report on this three neutrino solution in more detail. We also analyze the case of general family symmetry breaking with either one or two sterile neutrinos. It is important to determine how robust is the null LSND result. Our detailed conclusions are in Sec. IV.

II. AN SO(10)×U(2)×U(1) MODEL

The three families of fermions are contained in 16_a, $a=1,2$; and 16_3 where a is a U(2) family index. [Note U(2) = SU(2) × U(1)′ where the U(1)′ charge is +1 (−1) for each upper (lower) SU(2) index.] At the tree level, the third family of fermions couples to 10 (a Higgs field) with coupling $\lambda\, 16_3 10\, 16_3$ in the superspace potential. The Higgs field and 16_3 have zero charge under both U(1)'s, while 16_a has charge −1 and thus does not couple to the Higgs field at tree level.[1]

Three superfields ($\phi^a, S^{ab}=S^{ba}, A^{ab}=-A^{ba}$) are introduced to spontaneously break U(2)×U(1) and to generate Yukawa terms giving mass to the first and second generations. The fields (ϕ^a, S^{ab}, A^{ab}) are SO(10) singlets with U(1) charges {0,1,2}, respectively. The vacuum expectation values (VEV's) ($\phi^2 \sim S^{22} \sim \epsilon M_0^2/\langle 45\rangle$) break U(2)×U(1) to $\tilde{U}(1)$ and ($A^{12} \sim \epsilon' M_0$) completely. In this model, second generation masses are of order ϵ, while first generation masses are of order ϵ'^2/ϵ.

The superspace potential for the charged fermion sector of this theory, including the heavy Froggatt-Nielsen states [15], is given by

$$
\begin{aligned}
W \supset\; & 16_3 10\, 16_3 + 16_a 10\, \chi^a \\
& + \overline{\chi}_a (M\chi^a + \phi^a \chi + S^{ab}\chi_b + A^{ab} 16_b) \\
& + \overline{\chi}^a (M' \chi_a + 45\, 16_a) + \overline{\chi}(M''\chi + 45\, 16_3), \quad (1)
\end{aligned}
$$

where $M = M_0(1 + \alpha X + \beta Y)$. X, Y are SO(10) breaking VEV's in the adjoint representation with X corresponding to the U(1) in SO(10) which preserves SU(5), Y is standard weak hypercharge and α, β are arbitrary parameters. The field 45 is assumed to obtain a vacuum expectation value (VEV) in the $B-L$ direction. Note, this theory differs from BHRR [9] in that the fields ϕ^a and S^{ab} are now SO(10) singlets [rather than SO(10) adjoints] and the SO(10) adjoint quantum numbers of these fields, necessary for acceptable masses and mixing angles, has been made explicit in the field 45 with U(1) charge 1.[2] This theory thus requires much fewer SO(10) adjoint representations. Moreover our neutrino mass solution depends heavily on this change.

The effective mass parameters M_0, M', M'' are SO(10) invariants. The scales are assumed to satisfy $M_0 \sim M' \sim M'' \gg \langle \phi^2 \rangle \sim \langle S^{22} \rangle \gg \langle A^{12} \rangle$ where M_0 may be of order of the GUT scale. In the effective theory below M_0, the Froggatt-Nielsen states $\{\chi, \overline{\chi}, \chi^a, \overline{\chi}_a, \chi_a, \overline{\chi}^a\}$ may be integrated out, resulting in the effective Yukawa matrices for up quarks,

[1]There are in fact three additional U(1)'s implicit in the superspace potential [Eq. (1)]. These are a Peccei-Quinn symmetry in which all 16s have charge +1, all $\overline{16}$'s have charge −1, and 10 has charge −2; a flavon symmetry in which (ϕ^a, S^{ab}, A^{ab}) and M have charge +1 and $\overline{\chi}_b$ has charge −1; and an R symmetry in which all chiral superfields have charge +1. The family symmetries of the theory may be realized as either global or local symmetries. For the purposes of this paper, it is not necessary to specify which one. However, if it is realized locally, as might be expected from string theory, then not all of the U(1)'s are anomaly free. We would then need to specify the complete set of anomaly free U(1)'s.

[2]This change (see BHRR [9]) is the reason for the additional U(1).

down quarks, charged leptons, and the Dirac neutrino Yukawa matrix given by (see Fig. 1)

$$Y_u = \begin{pmatrix} 0 & \epsilon'\rho & 0 \\ -\epsilon'\rho & \epsilon\rho & r\epsilon T_{\bar{u}} \\ 0 & r\epsilon T_Q & 1 \end{pmatrix} \lambda, \qquad (2)$$

$$Y_d = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & r\sigma\epsilon T_{\bar{d}} \\ 0 & r\epsilon T_Q & 1 \end{pmatrix} \xi,$$

$$Y_e = \begin{pmatrix} 0 & -\epsilon' & 0 \\ \epsilon' & 3\epsilon & r\epsilon T_{\bar{e}} \\ 0 & r\sigma\epsilon T_L & 1 \end{pmatrix} \xi,$$

$$Y_\nu = \begin{pmatrix} 0 & -\omega\epsilon' & 0 \\ \omega\epsilon' & 3\omega\epsilon & \frac{1}{2}\omega r\epsilon T_{\bar{\nu}} \\ 0 & r\sigma\epsilon T_L & 1 \end{pmatrix} \lambda,$$

with

$$\omega = \frac{2\,\sigma}{2\,\sigma - 1} \qquad (3)$$

and

$$T_f = (\text{Baryon\#} - \text{Lepton\#}) \quad \text{for} \quad f = \{Q, \bar{u}, \bar{d}, L, \bar{e}, \bar{\nu}\}. \qquad (4)$$

In our notation, fermion doublets are on the left and singlets are on the right. Note, we have assumed that the Higgs doublets of the minimal supersymmetric standard model (MSSM) are contained in the 10 such that $\lambda\, 10 \supset \lambda\, H_u + \xi\, H_d$. We can then consider two important limits — case (1) $\lambda = \xi$ (no Higgs mixing) with large $\tan\beta$, and case (2) $\lambda \gg \xi$ or small $\tan\beta$.

Results for charged fermion masses and mixing angles

We have performed a global χ^2 analysis, incorporating two- (one-) loop renormalization group (RG) running of dimensionless (dimensionful) parameters from M_G to M_Z in the MSSM, one-loop radiative threshold corrections at M_Z, and three-loop QCD (one-loop QED) RG running below M_Z.[3] Electroweak symmetry breaking is obtained self-consistently from the effective potential at one loop, with all one-loop threshold corrections included. This analysis is performed using the code of Blazek *et al.* [16].[4] In this paper, we just present the results for one set of soft SUSY breaking parameters m_0, $M_{1/2}$ with all other parameters varied to obtain the best fit solution. In Table I we give the 20 observables which enter the χ^2 function, their experimental values and the uncertainty σ (in parentheses).[5] In most cases σ is determined by the one standard deviation experimental un-

TABLE I. Charged fermion masses and mixing angles. Initial parameters: large $\tan\beta$ case ($\lambda = \xi$), $(1/\alpha_G, M_G, \epsilon_3) = (24.52,\ 3.05 \times 10^{16}$ GeV, $-4.08\%)$, $(\lambda, r, \sigma, \epsilon, \rho, \epsilon') = (0.79,\ 12.4,\ 0.84,\ 0.011,\ 0.043,\ 0.0031)$, $(\Phi_\sigma, \Phi_\epsilon, \Phi_\rho) = (0.73,\ -1.21,\ 3.72)$rad, $[m_0,\ M_{1/2},\ A_0,\ \mu(M_Z)] = (1000,\ 300,\ -1437,\ 110)$ GeV, $[(m_{H_d}/m_0)^2,\ (m_{H_u}/m_0)^2, \tan\beta] = (2.22,\ 1.65,\ 53.7)$.

Observable	Data(σ) (masses)	Theory (GeV)
M_Z	91.187 (0.091)	91.17
M_W	80.388 (0.080)	80.40
$G_\mu \times 10^5$	1.1664 (0.0012)	1.166
α_{EM}^{-1}	137.04 (0.14)	137.0
$\alpha_s(M_Z)$	0.1190 (0.003)	0.1174
$\rho_{new} \times 10^3$	−1.20 (1.3)	+0.320
M_t	173.8 (5.0)	175.0
$m_b(M_b)$	4.260 (0.11)	4.328
$M_b - M_c$	3.400 (0.2)	3.421
m_s	0.180 (0.050)	0.148
m_d/m_s	0.050 (0.015)	0.0589
Q^{-2}	0.00203 (0.00020)	0.00201
M_τ	1.777 (0.0018)	1.776
M_μ	0.10566 (0.00011)	0.1057
$M_e \times 10^3$	0.5110 (0.00051)	0.5110
V_{us}	0.2205 (0.0026)	0.2205
V_{cb}	0.03920 (0.0030)	0.0403
V_{ub}/V_{cb}	0.0800 (0.02)	0.0691
$\hat{B}_K$	0.860 (0.08)	0.8703
$B(b \to s\gamma) \times 10^4$	3.000 (0.47)	2.995
TOTAL χ^2	3.39	

[3]The predicted values of the low-energy observables are highly correlated. Thus a global χ^2 analysis is necessary in order to test the accuracy of the fit. We note that fermion masses and mixing angles are the precision electroweak data which constrain any theory beyond the standard model. It is important to know how well a theory beyond the standard model fits this data, even though in some cases this data still has large theoretical uncertainties.

[4]We assume universal scalar mass m_0 for squarks and sleptons at M_G. We have not considered the flavor violating effects of U(2) breaking scalar masses in this paper.

[5]The Jarlskog parameter $J = \mathrm{Im}(V_{ud}V_{ub}^* V_{cb} V_{cd}^*)$ is a measure of *CP* violation. We test J by a comparison to the experimental value extracted from the well-known K^0-$\bar{K}^0$ mixing observable $\epsilon_K = (2.26 \pm 0.02) \times 10^{-3}$. The largest uncertainty in such a comparison, however, comes in the value of the QCD bag constant $\hat{B}_K$. We thus exchange the Jarlskog parameter J for $\hat{B}_K$ in the list of low-energy data we are fitting. Our theoretical value of $\hat{B}_K$ is defined as that value needed to agree with ϵ_K for a set of fermion masses and mixing angles derived from the GUT scale. We test this theoretical value against the ''experimental'' value of $\hat{B}_K$. This value, together with its error estimate, is obtained from recent lattice calculations [17].

certainty, however in some cases the theoretical uncertainty ($\sim$0.1%) inherent in our renormalization-group running and one-loop threshold corrections dominates.

For large $\tan\beta$[6] there are six real Yukawa parameters and three complex phases. We take the complex phases to be Φ_ρ, Φ_ϵ, and Φ_σ. With 13 fermion mass observables (charged fermion masses and mixing angles [$\hat{B}_K$ replacing ϵ_K as a ''measure of *CP* violation'']) we have four predictions. For low $\tan\beta$, $\lambda \neq \xi$, we have one less prediction. From Table I it is clear that this theory fits the low-energy data quite well.[7] Note, fits with $\lambda \gg \xi$ and small $\tan\beta$ fit just as well.

Finally, the squark, slepton, Higgs, and gaugino spectrum of our theory is consistent with all available data. The lightest chargino and neutralino are higgsino-like with the masses close to their respective experimental limits. As an example of the additional predictions of this theory consider the *CP*-violating mixing angles which may soon be observed at *B* factories. For the selected fit we find

$$(\sin 2\alpha, \sin 2\beta, \sin\gamma) = (0.74,\ 0.54,\ 0.99) \tag{5}$$

or equivalently the Wolfenstein parameters

$$(\rho, \eta) = (-0.04,\ 0.31). \tag{6}$$

III. NEUTRINO MASSES AND MIXING ANGLES

The parameters in the Dirac Yukawa matrix for neutrinos [Eq. (2)] mixing $\nu - \bar{\nu}$ are now fixed. Of course, neutrino masses are much too large and we need to invoke the Gell-Mann–Ramond–Slansky–Yanagida [18] seesaw mechanism.

Since the **16** of SO(10) contains the ''right-handed'' neutrinos $\bar{\nu}$, one possibility is to obtain $\bar{\nu} - \bar{\nu}$ Majorana masses via higher dimension operators of the form[8]

$$\begin{aligned}
&\frac{1}{M}\,\overline{16}\ 16_3\,\overline{16}\ 16_3,\\
&\frac{1}{M^2}\overline{16}\ \ 16_3\overline{16}\ \ 16_a\ \phi^a, \\
&\frac{1}{M^2}\ \overline{16}\ 16_a\ \overline{16}\ 16_b\ S^{a\,b}.
\end{aligned} \tag{7}$$

The second possibility, which we follow, is to introduce SO(10) singlet fields N and obtain effective mass terms $\bar{\nu} - N$ and $N - N$ using only dimension-four operators in the superspace potential. To do this, we add three new SO(10) singlets $\{N_a,\ a=1,2;\ N_3\}$ with U(1) charges $\{-1/2,+1/2\}$. These then contribute to the superspace potential

$$W \supset \overline{16}\,(N_a\chi^a + N_3\,16_3) + \tfrac{1}{2}\,N_a\,N_b\,S^{a\,b} + N_a\,N_3\,\phi^a, \tag{8}$$

where the field $\overline{16}$ with U(1) charge $-1/2$ is assumed to get a VEV in the ''right-handed'' neutrino direction. Note, this VEV is also needed to break the rank of SO(10).

Finally, we allow for the possibility of adding a U(2) doublet of SO(10) singlets $\bar{N}^a$ or a U(2) singlet $\bar{N}^3$. They enter the superspace potential as follows:

$$W \supset \mu'\,N_a\,\bar{N}^a + \mu_3\,N_3\bar{N}^3 \tag{9}$$

The dimensionful parameters μ', μ_3 are assumed to be of the order of the weak scale. The notation is suggestive of the similarity between these terms and the μ term in the Higgs sector. In both cases, we are adding supersymmetric mass terms and in both cases, we need some mechanism to keep these dimensionful parameters small compared to the Planck scale.

We define the 3×3 matrix

$$\tilde{\mu} = \begin{pmatrix} \mu' & 0 & 0 \\ 0 & \mu' & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \tag{10}$$

The matrix $\tilde{\mu}$ determines the number of *coupled* sterile neutrinos, i.e., there are four cases labeled by the number of neutrinos ($N_\nu = 3,4,5,6$): ($N_\nu = 3$) 3 active ($\mu' = \mu_3 = 0$); ($N_\nu = 4$) 3 active+1 sterile ($\mu' = 0;\ \mu_3 \neq 0$); ($N_\nu = 5$) 3 active+2 sterile ($\mu' \neq 0;\ \mu_3 = 0$); ($N_\nu = 6$) 3 active+3 sterile ($\mu' \neq 0;\ \mu_3 \neq 0$). In this paper we consider the cases $N_\nu = 3$ and 4 [19].

The generalized neutrino mass matrix is then given by[9]

$$(\nu \quad \bar{N} \quad \bar{\nu} \quad N),$$

$$\begin{pmatrix} 0 & 0 & m & 0 \\ 0 & 0 & 0 & \tilde{\mu}^T \\ m^T & 0 & 0 & V \\ 0 & \tilde{\mu} & V^T & M_N \end{pmatrix}, \tag{11}$$

where

$$m = Y_\nu \langle H_u^0 \rangle = Y_\nu \frac{v}{\sqrt{2}} \sin\beta \tag{12}$$

and

[6]We warn the reader that according to quite standard conventions the angle β is used in two inequivalent ways. $\tan\beta$ is the ratio of Higgs VEV's in the minimal supersymmetric standard model; while $\sin 2\beta$ refers to the *CP*-violating angle in the unitarity triangle, measured in *B* decays. We hope that the reader can easily distinguish the two from the context.

[7]In a future paper we intend to explore the dependence of the fits on the SUSY breaking parameters and also U(2) flavor violating effects. Note also the strange quark mass $m_s(1\ \text{GeV}) \sim 150$ MeV is small, consistent with recent lattice results.

[8]This possibility has been considered in the paper by Carone and Hall [14].

[9]This is similar to the double seesaw mechanism suggested by Mohapatra and Valle [20].

$$V=\begin{pmatrix} 0 & \epsilon' V_{16} & 0 \\ -\epsilon' V_{16} & 3\epsilon V_{16} & 0 \\ 0 & r\,\epsilon\,(1-\sigma)\,T_{\bar{\nu}} V_{16} & V'_{16} \end{pmatrix}, \qquad (13)$$

$$M_N=\begin{pmatrix} 0 & 0 & 0 \\ 0 & S & \phi \\ 0 & \phi & 0 \end{pmatrix}.$$

V_{16}, V'_{16} are proportional to the VEV of $\overline{16}$ (with different implicit Yukawa couplings) and S, ϕ are up to couplings the VEV's of S^{22}, ϕ^2, respectively.

Since both V and M_N are of order the GUT scale, the states $\bar{\nu}$, N may be integrated out of the effective low-energy theory. In this case, the effective neutrino mass matrix is given (at M_G) by[10] (the matrix is written in the $(\nu,\bar{N})$ *flavor* basis where charged lepton masses are diagonal)

$$m_\nu=\tilde{U}_e^T\begin{pmatrix} m\,(V^T)^{-1}\,M_N\,V^{-1}\,m^T & -m\,(V^T)^{-1}\,\tilde{\mu} \\ -\tilde{\mu}^T\,V^{-1}\,m^T & 0 \end{pmatrix}\tilde{U}_e \qquad (14)$$

with

$$\tilde{U}_e=\begin{pmatrix} U_e & 0 \\ 0 & 1 \end{pmatrix}, \qquad (15)$$

$$e_0=U_e e;\ \nu_0=U_e\,\nu.$$

U_e is the 3×3 unitary matrix for left-handed leptons needed to diagonalize Y_e [Eq. (2)] and $e_0,\ \nu_0\,(e,\ \nu)$ represent the three families of left-handed leptons in the charged-weak (-mass) eigenstate basis.

The neutrino mass matrix is diagonalized by a unitary matrix $U=U_{\alpha\,i}$;

$$m_\nu^{\rm diag}=U^T\,m_\nu\,U, \qquad (16)$$

where $\alpha=\{\nu_e,\nu_\mu,\nu_\tau,\nu_{s_1},\nu_{s_2},\nu_{s_3}\}$ is the flavor index and $i=\{1,\ldots,6\}$ is the neutrino mass eigenstate index. $U_{\alpha\,i}$ is observable in neutrino oscillation experiments. In particular, the probability for the flavor state ν_α with energy E to oscillate into ν_β after traveling a distance L is given by

$$P(\nu_\alpha\to\nu_\beta)=\delta_{\alpha\beta}-4\sum_{k<j}U_{\alpha\,k}U^*_{\beta\,k}U^*_{\alpha\,j}U_{\beta\,j}\sin^2\Delta_{j\,k}, \qquad (17)$$

where $\Delta_{j\,k}=\delta m^2_{jk}\,L/4E$ and $\delta m^2_{jk}=m_j^2-m_k^2$.

[10]In fact, at the GUT scale M_G we define an effective dimension-five supersymmetric neutrino mass operator where the Higgs VEV is replaced by the Higgs doublet H_u coupled to the entire lepton doublet. This effective operator is then renormalized using one-loop renormalization-group equations to M_Z. It is only then that H_u is replaced by its VEV.

In general, neutrino masses and mixing angles have many new parameters so that one might expect to have little predictability. However, as we shall now see, the U(2)×U(1) family symmetry of the theory provides a powerful constraint on the form of the neutrino mass matrix. In particular, the matrix has many zeros and few arbitrary parameters. Before discussing the four neutrino case, we show why three neutrinos cannot work without changing the model.

A. Three neutrinos

Consider first m_ν for three active neutrinos. We find (at M_G) in the (ν_e,ν_μ,ν_τ) basis

$$m_\nu=m'\,U_e^T\begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 1 \\ 0 & 1 & 0 \end{pmatrix}U_e \qquad (18)$$

with

$$m'=\frac{\lambda^2 v^2 \sin^2\beta\,\omega\,\phi}{2V_{16}V'_{16}}\approx\frac{m_t^2\,\omega\,\phi}{V_{16}V'_{16}}, \qquad (19)$$

$$b=\omega\frac{SV'_{16}}{\phi V_{16}}+2\sigma r\epsilon,$$

where in the approximation for m' we use

$$m_t(\equiv m_{\rm top})\approx\lambda\frac{v}{\sqrt{2}}\sin\beta, \qquad (20)$$

valid at the weak scale.

m_ν is given in terms of two independent parameters $\{m',b\}$. Note, this theory in principle solves two problems associated with neutrino masses. It naturally has small mixing between $\nu_e-\nu_\mu$ since the mixing angle comes purely from diagonalizing the charged lepton mass matrix which, like quarks, has small mixing angles. While, for $b\leqslant1$, $\nu_\mu-\nu_\tau$ mixing is large without fine tuning. Also note, in this theory one neutrino (predominantly ν_e) is massless.

Unfortunately this theory cannot simultaneously fit both solar and atmospheric neutrino data. This problem can be solved at the expense of adding a new family symmetry breaking VEV[11]

$$\langle\phi^1\rangle=\kappa\langle\phi^2\rangle. \qquad (21)$$

We discuss this three neutrino solution in a future paper [19]. With $\kappa\neq0$ the massless eigenvalue in the neutrino mass matrix is now lifted. This allows us to obtain a small mass difference between the first and second mass eigenvalues which was unattainable before in the large mixing limit for $\nu_\mu-\nu_\tau$. Hence, a good fit to both solar and atmospheric neutrino data can now be found for $\kappa\leqslant0.2$. In addition, note

[11]This additional VEV was necessary in the analysis of Carone and Hall [14].

TABLE II. Fit to atmospheric and solar neutrino oscillations. Initial parameters: (four neutrinos w/large $\tan\beta$), $m' = 7.11\times10^{-2}$ eV, $b=-0.521$, $c=0.278$, $\Phi_b=3.40$ rad.

Observable	Computed value
δm^2_{atm}	3.2×10^{-3} eV^2
$\sin^2 2\theta_{atm}$	1.08
δm^2_{sol}	4.2×10^{-6} eV^2
$\sin^2 2\theta_{sol}$	3.0×10^{-3}

that this small value of κ moderately improves the global fits to charged fermion masses and mixing angles [19].

In the next section we discuss a four neutrino solution to both solar and atmospheric neutrino oscillations in the theory with $\kappa=0$.

B. Neutrino oscillations (3 active+1 sterile)

In the four neutrino case the mass matrix (at M_G) is given by[12]

$$m'\begin{bmatrix} U_e^T\begin{pmatrix}0&0&0\\0&b&1\\0&1&0\end{pmatrix}U_e & -U_e^T\begin{pmatrix}0\\u\,c\\c\end{pmatrix}\\ -(0 \quad u\,c \quad c)\,U_e & 0\end{bmatrix}, \tag{22}$$

where m' and b are given in Eq. (19) and

$$u=\sigma\, r\, \epsilon, \tag{23}$$

$$c=\frac{\sqrt{2}\mu_3\, V_{16}}{\omega\,\lambda\, v\sin\beta\phi}\approx\frac{\mu_3\, V_{16}}{\omega\, m_t\phi}.$$

In the analysis of neutrino masses and mixing angles we use the fits for charged fermion masses as input. Thus the parameter u is fixed. We then look for the best fit to solar and atmospheric neutrino oscillations. For this we use the latest Super-Kamiokande data for atmospheric neutrino oscillations [2] and the best fits to solar neutrino data including the possibility of ''just so'' vacuum oscillations or both large and small angle MSW oscillations [1]. Our best fit is found in Tables II and III. It is obtained in the following way.

For atmospheric neutrino oscillations we have evaluated the probabilities $[P(\nu_\mu\to\nu_\mu),\ P(\nu_\mu\to\nu_x)$ with $x=\{e,\tau,s\}]$ as a function of $\mathrm{x}\equiv\ln[(L/\mathrm{km})/(E/\mathrm{GeV})]$. In order to smooth out the oscillations we have averaged the result over a bin size, $\Delta\mathrm{x}=0.5$. In Fig. 2(a) we have compared the results of our model with a two neutrino oscillation model. We see that our result is in good agreement with the values of δm^2_{atm} and $\sin^2 2\theta_{atm}$ as given.

An approximate formula for the effective atmospheric mixing angle is defined by

TABLE III. Neutrino masses and mixings. Mass eigenvalues (eV): 0.0, 0.002, 0.04, 0.07. Magnitude of neutrino mixing matrix $U_{\alpha i}$, $i=1,\ldots,4$ labels mass eigenstates, $\alpha=\{e,\mu,\tau,s\}$ labels flavor eigenstates.

0.998	0.0204	0.0392	0.0529
0.0689	0.291	0.567	0.767
0.317×10^{-3}	0.145	0.771	0.620
0.284×10^{-3}	0.946	0.287	0.154

$$P(\nu_\mu\to\nu_\mu)\equiv 1-\text{`}\sin^2 2\theta_{atm}\text{`}\sin^2\left(\frac{\text{`}\delta m^2_{atm}\text{`}L}{4E}\right) \tag{24}$$

with

$$\text{`}\sin^2 2\theta_{atm}\text{`}\approx 4[\|U_{\mu4}\|^2(1-\|U_{\mu4}\|^2) + \|U_{\mu3}\|^2(1-\|U_{\mu3}\|^2-\|U_{\mu4}\|^2)] \tag{25}$$

using the approximate relation

$$\text{`}\delta m^2_{atm}\text{`}=\delta m^2_{43}\approx\delta m^2_{42}\approx\delta m^2_{41}\approx\delta m^2_{32}\approx\delta m^2_{31}. \tag{26}$$

Note, '$\sin^2 2\theta_{atm}$' may be greater than one. This is consistent with the definition above and also with Super-Kamiokande data where the best fit occurs for $\sin^2 2\theta_{atm}=1.05$. We obtain a good fit to the data.

In Fig. 2(b) we see, however, that although the atmospheric neutrino deficit is predominantly due to the maximal mixing between $\nu_\mu-\nu_\tau$, there is nevertheless a significant (~10% effect) oscillation of $\nu_\mu-\nu_s$. This effect may be observable at Super-Kamiokande. It would appear as a deficit of neutrinos in the ratio of experimental to theoretical muon (single ring events) plus tau (multiring events) as a function of L/E.

The oscillations $\nu_\mu\to\nu_\tau$ or ν_s may also be visible at long baseline neutrino experiments. For example at K2K [21], the mean neutrino energy $E=1.4$ GeV and distance $L=250$ km corresponds to a value of x=2.3 in Fig. 2(b) and hence $P(\nu_\mu\to\nu_\tau)\sim0.4$ and $P(\nu_\mu\to\nu_s)\sim0.1$. At Minos [22] low-energy beams with hybrid emulsion detectors are also being considered. These experiments can first test the hypothesis of muon neutrino oscillations by looking for muon neutrino disappearance [for x=2.3 we have $P(\nu_\mu\to\nu_\mu)\sim0.5$]. Verifying oscillations into sterile neutrinos is, however, much more difficult. For example at K2K, if only quasielastic muon neutrino interactions (single ring events at SuperK) are used, then this cannot be tested. Minos, on the other hand, may be able to verify the oscillations into sterile neutrinos by using the ratio of neutral current to charged current measurements [22] (the so-called T test).

For solar neutrinos we plot, in Figs. 3(a) and 3(b), the probabilities $[P(\nu_e\to\nu_e),P(\nu_e\to\nu_x)$ with $x=\{\mu,\tau,s\}]$ for neutrinos produced at the center of the sun to propagate to the surface (and then without change to earth), as a function

[12] This expression defines the effective dimension-five neutrino mass operator at M_G which is then renormalized to M_Z in order to make contact with data.

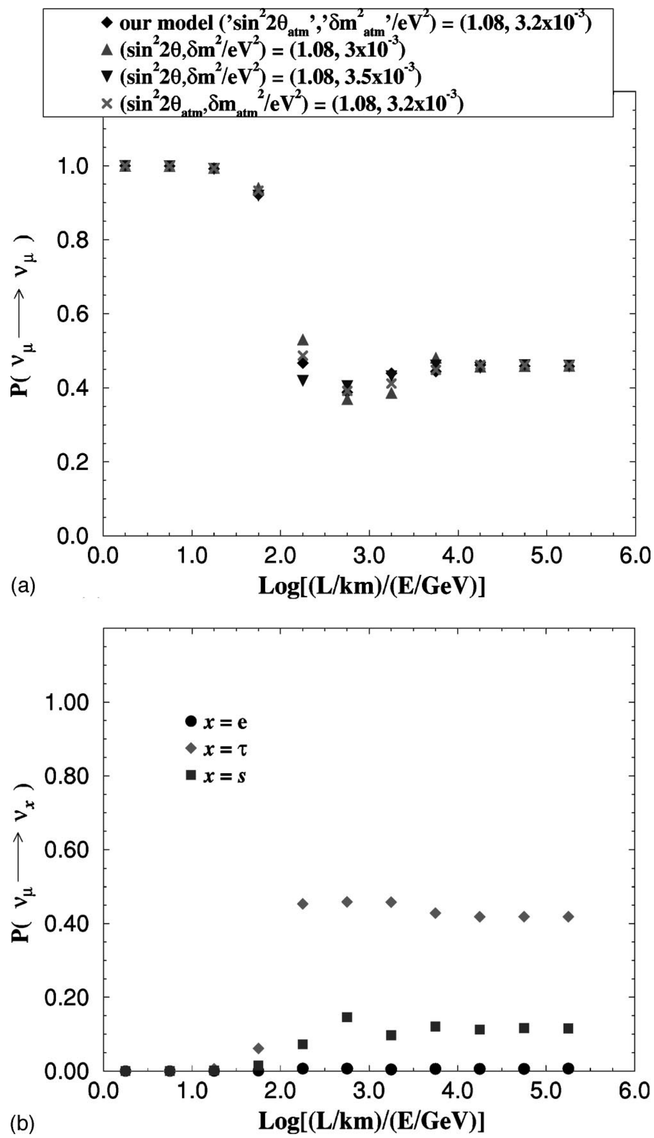

FIG. 2. (a) Probability $P(\nu_\mu \rightarrow \nu_\mu)$ for atmospheric neutrinos. For this analysis, we neglect the matter effects. (b) Probabilities $P(\nu_\mu \rightarrow \nu_x)$ $(x = e,\ \tau,$ and $s)$ for atmospheric neutrinos.

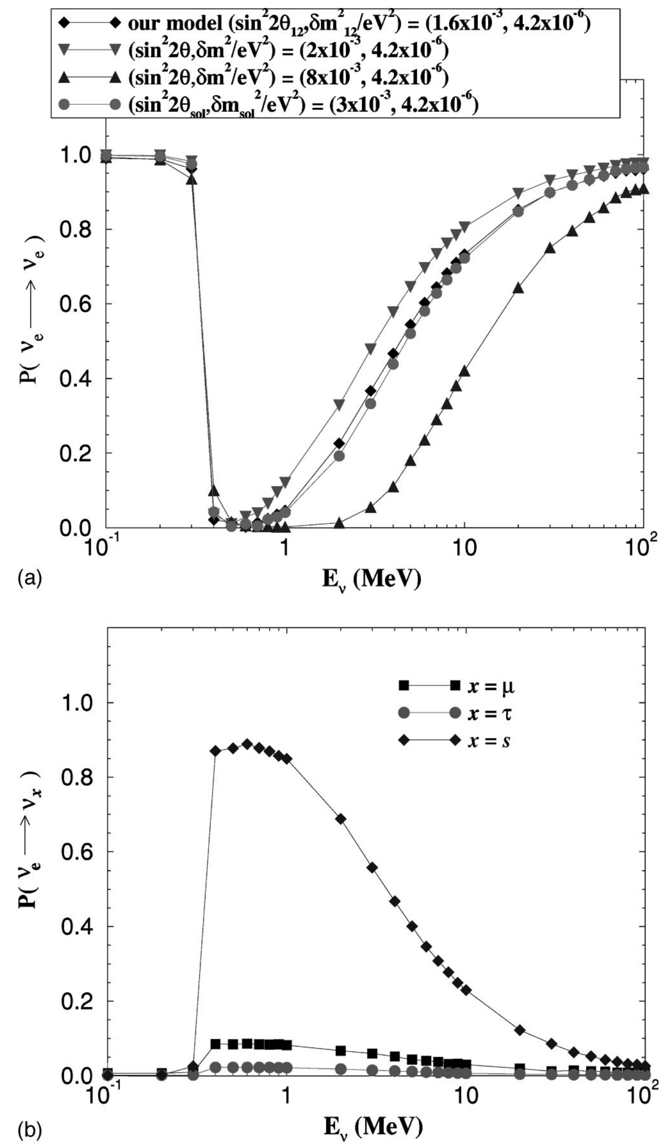

FIG. 3. (a) Probability $P(\nu_e \rightarrow \nu_e)$ for solar neutrinos. (b) Probabilities $P(\nu_e \rightarrow \nu_x)$ $(x = \mu,\ \tau,$ and $s)$ for solar neutrinos.

of the neutrino energy E_ν (MeV).[13] We compare our model to a two neutrino oscillation model with the given parameters. We see that the solar neutrino deficit is predominantly due to the small mixing angle MSW solution for $\nu_e - \nu_s$ oscillations. The results are summarized in Tables II and III.

A naive definition of the effective solar mixing angle is given by

$$\sin^2 2\theta_{12} \equiv 4\|U_{e1}\|^2\|U_{e2}\|^2. \tag{27}$$

[13]For this calculation we assume that electron (n_e) and neutron (n_n) number densities at a distance r from the center of the sun are given by $(n_e, n_n) = (4.6, 2.2) \times 10^{11} \exp[-10.5(r/R)]$ eV^3 where R is a solar radius. We also use an analytic approximation necessary to account for both large and small oscillation scales. For the details, see the forthcoming paper [19].

In Fig. 3(a) we see that the naive definition of $\sin^2 2\theta_{12}$ underestimates the value of the effective two neutrino mixing angle. Thus we see that our model reproduces the neutrino results for $\delta m^2_{\rm sol} = \delta m^2_{12} = 4.2 \times 10^{-6}\,{\rm eV}^2$ but instead is equivalent to a two neutrino mixing angle $\sin^2 2\theta_{\rm sol} = 3 \times 10^{-3}$ instead of $\sin^2 2\theta_{12} = 1.6 \times 10^{-3}$. Our result is consistent with the fits of Bahcall *et al.* [1].

In addition, whereas the oscillation $\nu_e - \nu_s$ dominates we see in Fig. 3(b) that there is a significant ($\sim$8% effect) for $\nu_e - \nu_\mu$. This result may be observable at SNO [23] with threshold $E \geqslant 5$ MeV for which $P(\nu_e \rightarrow \nu_\mu) \sim 0.05$.

We note that, even though we have four neutrinos, we are **not** *able to simultaneously fit atmospheric, solar, and LSND data, i.e. it is not possible to get ''$\delta m^2_{\nu_e - \nu_\mu}$'' large enough to be consistent with LSND. We have also checked that introducing the new parameter κ [Eq. (21)] does not help.*

Finally let us discuss whether the parameters necessary for the fit make sense. We have three arbitrary parameters.

We have taken b complex, while any phases for m' and c are unobservable. A large mixing angle for $\nu_\mu - \nu_\tau$ oscillations is obtained with $|b| \sim 0.5$. This does not require any fine tuning; it is consistent with $S\,V'_{16}/\phi\,V_{16} \sim 1$ which is perfectly natural [see Eq. (19)]. The parameter c [Eq. (23)] $\approx 0.28 \approx \mu_3\,V_{16}/\omega\,m_t\,\phi$ implies $\mu_3 \approx 0.41\,(\phi/V_{16})\,m_t$. Thus in order to have a light sterile neutrino we need the parameter $\mu_3 \sim 70$ GeV for $\phi \sim V_{16}$. Considering that the standard μ parameter (see the parameter list in the captions to Table I) with value $\mu = 70$ GeV and μ_3 [Eq. (9)] may have similar origins, both generated once SUSY is spontaneously broken, we feel that it is natural to have a light sterile neutrino. Lastly consider the overall scale of symmetry breaking, i.e., the seesaw scale. We have $m' = 7 \times 10^{-2}$ eV (Table II) $\approx m_t^2\,\omega\,\phi/V_{16}\,V'_{16}$. Thus we find $V_{16}\,V'_{16}/\phi \sim m_t^2\,\omega/m' \sim 6 \times 10^{14}$ GeV. This is perfectly reasonable for $\langle \overline{16} \rangle \sim \langle \phi^2 \rangle \sim M_G$ once the implicit Yukawa couplings are taken into account.

IV. CONCLUSION

We have presented the results of a predictive SO(10)×U(2)×U(1) model of fermion masses. We fit charged fermion masses and mixing angles as well as neutrino masses and mixing. The model ''naturally'' gives small mixing angles for charged fermions and for $\nu_e \rightarrow \nu_{\rm sterile}$ oscillations (small-angle MSW solution to the solar neutrino problem) and the large mixing angle for $\nu_\mu \rightarrow \nu_\tau$ oscillations (atmospheric muon neutrino deficit). The model presented here may be one of a large class of models which fit charged fermion masses. The most important conclusion from our work is that predictive theories of charged fermion masses (including GUT and family symmetry) strongly constrain the neutrino sector of the theory. These theories can thus be predictive in the neutrino sector and neutrino data will strongly constrain any predictive theory of fermion masses.

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