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Constructing hybrid baryons with flux tubes

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Hybrid baryon states are described in quark potential models as having explicit excitation of the gluon degrees of freedom. Such states are described in a model motivated by the strong coupling limit of Hamiltonian lattice gauge theory, where three flux tubes meeting at a junction play the role of the glue. The adiabatic approximation for the quark motion is used, and the flux tubes and junction are modeled by beads which are attracted to each other and the quarks by a linear potential, and vibrate in various string modes. Quantum numbers and estimates of the energies of the lightest hybrid baryons are provided. [S0556-2821(99)50221-X]

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Experiments at new electron-scattering facilities, such as those in Hall B at Jefferson Lab, are expected to produce many new baryon states, including those which are described in quark potential models as having explicit excitation of the gluon degrees of freedom. Low-lying baryon states present in analyses of πN elastic and inelastic scattering, such as the P_{11} Roper resonance $N(1440)$ which has the same quantum numbers as the nucleon, have been proposed [1,2] as hybrid candidates. This is based on extensions of the MIT bag model [3] to states where a constituent gluon in the lowest energy transverse electric mode combines with three quarks in a color octet state to form a colorless state, and on a calculation using QCD sum rules [2]. Hybrid baryons have also been constructed recently in the large-N_c limit of QCD [4].

With the assumption that the quarks are in an S-wave spatial ground state, and considering the mixed exchange symmetry of octet color wave functions of the quarks, bag-model constructions show that adding a $J^P=1^+$ gluon to three light quarks with total quark-spin 1/2 yields both $N(I=\frac{1}{2})$ and $\Delta(I=\frac{3}{2})$ hybrids with $J^P=\frac{1}{2}^+,\frac{3}{2}^+$. Quark-spin 3/2 hybrids are N states with $J^P=\frac{1}{2}^+,\frac{3}{2}^+$, and $\frac{5}{2}^+$. Energies are estimated using the usual bag Hamiltonian plus gluon kinetic energy, additional color-Coulomb energy, and one-gluon exchange plus gluon-Compton $O(\alpha_s)$ corrections. Mixings between q^3 and q^3g states from gluon radiation are evaluated. If the gluon self-energy is included, the lightest N hybrid state has $J^P=\frac{1}{2}^+$ and a mass between that of the Roper resonance and the next observed $J^P=\frac{1}{2}^+$ state, $N(1710)$. A second $J^P=\frac{1}{2}^+N$ hybrid and a $J^P=\frac{3}{2}^+N$ hybrid are expected to be 250 MeV heavier, with the Δ hybrid states heavier still. A similar mass estimate of about 1500 MeV for the lightest hybrid is attained in the QCD sum rules calculation of Ref. [2].

For this reason there has been considerable interest in the presence or absence of light hybrid states in the P_{11} and other positive-parity partial waves in πN scattering. Interestingly, quark potential models which assume a q^3 structure for the Roper resonance [5] predict an energy which is roughly 100 MeV too high, and the same is true of the $\Delta(1600)$, the lightest radial recurrence of the ground state $J^P=\frac{3}{2}^+$ $\Delta(1232)$. Furthermore, models of the electromagnetic couplings of baryons have difficulty accommodating the substantial Roper resonance photocoupling extracted from pion photoproduction data [6]. Evidence for two resonances near 1440 MeV in the P_{11} partial wave in πN scattering was cited [7], which would indicate the presence of more states in this energy region than required by the q^3 model, but this has been interpreted as due to complications in the structure of the P_{11} partial wave in this region, and not an additional physical state [8].

This article will show that when the glue in a baryon is given the flux-tube structure expected [9] from an expansion around the strong-coupling limit of the Hamiltonian formulation of lattice QCD (HLGT), that the quantum numbers of the lightest hybrid baryons largely coincide, but the lightest hybrid baryons have substantially higher energies and different internal structure than predicted using bag models. This structure of the glue, where the gluon degrees of freedom collectively condense into flux-tubes, is very different from the constituent-gluon picture of the bag model and large-N_c constructions. The basis of this model is the assumption that the dynamics relevant to the structure of hybrids is that of confinement. At large interquark separations, there is evidence that the flux-tube model hybrid potential is consistent with that evaluated from lattice QCD [9–11], whereas the adiabatic bag model does not reproduce the lattice results there [12], in contrast to its success at small separations. It has also been shown [13] that a constituent-gluon model is not able to reproduce these lattice results [11].

The model used here to describe the glue is the nonrelativistic flux-tube model of Refs. [9,14], coupled with the adiabatic approximation, where the quarks do not move in response to the motion of the glue (apart from moving as a rigid body in order to maintain the center-of-mass position). Although exact only in the heavy-quark limit, this approximation has been shown to be good for light-quark mesons in the flux-tube model [15]. In the strong coupling limit of HLGT, flux lines (strings) with energy proportional to their length play the role of the glue, which are modeled here by equal mass beads with a linear potential between nearest neighbors [9]. The total mass of all of the beads is given by the energy in the flux lines, which is fixed by the string tension.

Perturbations to the strong-coupling limit are provided by the plaquette operator, which moves the flux lines, and so the beads, perpendicular to their rest positions. Global color gauge invariance requires that the three flux lines which emanate from the quarks must meet at a junction, which is also modeled by a bead. Lattice QCD studies [16] support confinement from a Y-shaped linear string rather than pairwise linear strings. It can be shown in HLGT that a single plaquette operator cannot move the junction and leave the Y-string in its ground state, and so in general the junction bead can have a mass which is different to that of those on the strings.

The ground state energy of this configuration of beads representing the Y-string for definite quark positions $\mathbf{r}_i$ defines an adiabatic potential $V_B(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ for the quarks, which consists of the string energy $b\Sigma_i l_i$, where b is the string tension and l_i is the magnitude of the vector $\mathbf{l}_i$ from the equilibrium junction position to the position of quark i, plus the zero-point energy of the beads. The energy of the first excited state defines a new adiabatic potential $V_H(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$. The simplest model incorporating the essential degrees of freedom has one bead per string, plus a junction bead. The mass of a bead on string i is taken to be proportional to bl_i, and the junction bead is given a higher mass. This model has nine degrees of freedom: three string bead transverse positions (ξ_i) within, and three (z_i) out of the plane of the quarks, and three junction positions, in (x,y) and out (z) of this plane.

A nonrelativistic Hamiltonian is constructed in the small oscillations approximation, and has the form

$$H_{\text{string}}=T_{\text{j}}(\dot{\mathbf{r}})+T_{\text{b}}(\dot{\xi}_i,\dot{z}_i)+T_{\text{bb}}+T_{\text{bj}}+V_{\text{j}}(\mathbf{r})+V_{\text{b}}(\xi_i,z_i), \tag{1}$$

where in the potential the string-bead (b) and junction-bead (j) coordinates decouple, but there are terms T_{bb} and T_{bj} in the kinetic energy which couple these motions. This Hamiltonian is corrected for the center-of-mass (c.m.) motion due to the string motion, by allowing the quarks to move rigidly to maintain the c.m. position. This gives effective masses similar to reduced masses to the string and junction beads, which depend on the quark masses.

Diagonalization of the resulting coupled-oscillator Hamiltonian for a wide variety of quark configurations shows that neglect of the coupling terms T_{bb} and T_{bj} does not significantly affect the lowest string energies. Examination of the eigenfunction for the lowest energy mode shows that it is always predominantly in-plane junction motion. This is illustrated for three quark positions (specified by the lengths l_i of the three strings) in Table I.

Accordingly, the string ground state and first excited state adiabatic surfaces may be found by allowing the junction to move, while the strings connecting the junction to the quarks follow without excitation. This gives the junction an effective mass which in the limit of a large number of beads becomes

TABLE I. Lowest three non-interacting (NI) and exact frequencies in GeV for three quark configurations, for $m_{\text{junction}}=m_{\text{quark}}=0.33$ GeV, and $b=0.18$ GeV^2.

l_i(fm)	E_1(NI)	E_1	E_2(NI)	E_2	E_3(NI)	E_3
0.5, 0.5, 0.5	0.614	0.607	0.614	0.607	0.869	0.828
0.5, 0.5, 0.1	0.623	0.616	1.069	0.985	1.069	1.005
0.5, 1.0, 0.1	0.520	0.483	0.544	0.534	0.544	0.590

$$M_{\text{eff}}=b\sum_i l_i\left[\frac{1}{3}-\frac{b\sum_i l_i}{4\sum_i (bl_i+M_i)}\right], \tag{2}$$

where the second term is the center of mass correction with quark masses M_i. Here M_{eff} is independent of the junction-bead to string-bead mass ratio. The potential is the string tension times the length of the lines connecting the displaced junction to the quarks, so that the junction Hamiltonian is

$$H_{\text{flux}}=\frac{1}{2}M_{\text{eff}}\dot{\mathbf{r}}^2+b\sum_{i=1}^{3}|\mathbf{l}_i-\mathbf{r}|. \tag{3}$$

The adiabatic surfaces are found numerically *via* a variational calculation. This is made necessary by a singularity in the small oscillations expansion for quark configurations where the triangle made by joining them contains an angle θ_i of at least 120°, so that the equilibrium position of the junction is such that the corresponding distance l_i between the junction and the quark is zero. This variational calculation agrees with the analytic small oscillations results when the l_i are all large, but shows the small oscillations approximation to be poor for some other configurations. It was shown in Ref. [14] that when the small oscillations approximation is removed, as has been done here, hybrid meson masses go down, and when the adiabatic approximation is removed, as has been done partially here, hybrid meson masses go up. This same behavior is seen here.

As $T_{\text{j}}(\dot{\mathbf{r}})+V_{\text{j}}(\mathbf{r})$ is even under $z\to -z$, one of the three first excited modes of the junction always involves motion along $\hat{\boldsymbol{\eta}}_z=\hat{z}$. Analytic results in the small oscillations approximation show that the frequency for motion along $\hat{\boldsymbol{\eta}}_z$ is always higher than those in the plane of the quarks. Trial wave functions for the ground and first excited states are taken to be the anisotropic harmonic oscillator wave functions

$$\Psi_B(\mathbf{r})=\frac{(\alpha_+\alpha_-\alpha_z)^{1/2}}{\pi^{3/4}}\exp\{-[(\alpha_+\hat{\boldsymbol{\eta}}_+\cdot\mathbf{r})^2+(\alpha_-\hat{\boldsymbol{\eta}}_-\cdot\mathbf{r})^2+(\alpha_z z)^2]/2\},$$

$$\Psi_H(\mathbf{r})=\sqrt{2}\,\alpha_-\hat{\boldsymbol{\eta}}_-\cdot\mathbf{r}\Psi_B(\mathbf{r}), \tag{4}$$

with variational parameters α_-, α_+, α_z, and θ, the latter giving the direction $\hat{\boldsymbol{\eta}}_-$ in the plane of the quarks of the lowest-energy oscillation relative to a body-fixed axis in that plane. For every configuration of the quarks, specified by the magnitudes of the Jacobi coordinates ρ,λ, and the cosine of the angle $\theta_{\rho\lambda}$ between them, the ground (V_B) and first excited state (V_H) string energies are independently minimized. The variational wave functions in Eq. (4) are exact in the long-string limit, and given their four parameters are expected to yield energies within a percent or so of the true energies away from this limit.

In order to compare to the relativized model calculation of baryon masses of Ref. [17], hybrid baryon masses are found by allowing the quarks to move in a confining potential given by the linear potential $b\Sigma_i l_i$ of Ref. [17], plus $V_H - V_B$. Since a typical string configuration when two quarks are near to each other involves a string extending from the distant quark to the junction nearby the other two quarks, nearby quarks are in a color $\overline{\mathbf{3}}$ configuration in both conventional and hybrid baryons. For this reason, and since our flux-tube model makes no predictions for the short-distance interquark potential, the Coulomb potential from one-gluon exchange is taken to be the same in conventional and hybrid baryons. Spin-dependent terms are neglected. Quark wave functions are expanded in a large oscillator basis, and energies for baryons (linear confinement) and hybrids (linear plus $V_H - V_B$ confinement) composed of light quarks are evaluated. When added to the spin-averaged mass of the N and Δ which is 1085 MeV, hybrids with quark orbital angular momenta $L_q=0,1,2$ have masses 1980, 2340 and 2620 MeV respectively. Hyperfine (contact plus tensor) interactions, discussed below, split the N hybrids down and the Δ hybrids up by similar amounts, so that the N hybrid mass becomes 1870 MeV. The error in this mass due to uncertainties in the parameters $(\delta b/b = \pm 10\%, \delta M_i/M_i = +50\%)$ is estimated to be less than ± 100 MeV. This lightest $(L_q=0)$ hybrid level is substantially higher than the roughly 1.5 GeV estimated from bag model and QCD sum rules calculations. This calculation also shows that the size of the quark core of hybrid baryons is roughly 20% larger for hybrid baryons than for conventional baryons.

The hyperfine interaction derived from the Coulomb interaction between the quarks has the same sign in conventional and hybrid baryons, so that the hybrids with quark-spin-$\frac{3}{2}$ will be heavier than those with quark-spin-$\frac{1}{2}$. Given the increased size of the hybrid states, the quark-spin-$\frac{1}{2}$ hybrids are not expected to split as far from the spin averaged level as in the usual baryons.

For every set of quark positions $\mathbf{r}_i$ the potential in which the junction moves is anisotropic, which means that the solutions of the Schrödinger equation for the junction motion do not have definite angular momentum. However, in the absence of the adiabatic approximation the combined wave function of the quark and junction motions must be a state of good angular momentum. Although it is possible to project states of good angular momentum out of the combined junction and quark motion states, this is technically difficult and will not be reported on here. Instead, a simplified argument is given to justify that the total orbital angular momentum of the lowest-lying hybrid baryons (H) is unity.

The H hybrid wave function in Eq. (4) is proportional to $\hat{\boldsymbol{\eta}}_- \cdot \mathbf{r}$, and since $\hat{\boldsymbol{\eta}}_-$ lies in the plane of the quarks, $\Psi_H(\mathbf{r})$ is proportional to a linear combination of $Y_{l1}(\hat{\mathbf{r}})$ and $Y_{l-1}(\hat{\mathbf{r}})$, with the junction position $\mathbf{r}$ defined relative to a (body) z-axis perpendicular to the quark plane. However, $\Psi_H(\mathbf{r})$ is better than 99.75% in a linear combination of $Y_{11}(\hat{\mathbf{r}})$ and $Y_{1-1}(\hat{\mathbf{r}})$. When the quarks are in their lowest energy $L_q = 0$ state in the hybrid confining potential, the total orbital angular momentum is $L=1$.

The junction Hamiltonian in Eq. (3) is invariant under the inversion of coordinates, $\mathbf{r}_i \to -\mathbf{r}_i$ which implies $\mathbf{l}_i \to -\mathbf{l}_i$, and $\mathbf{r} \to -\mathbf{r}$, so that the flux wave function must be a state of good parity. Since $\hat{\boldsymbol{\eta}}_-$ is a vector in the plane of the quarks it can be written as a linear combination of the $\mathbf{l}_i$, with coefficients which are functions of the parity-invariant lengths l_i. It follows that $\hat{\boldsymbol{\eta}}_-$ is parity-odd, and that the hybrid-baryon wave function from Eq. (4) has even parity.

Permutations P_{ij} of the quark labels exchange the quark positions $\mathbf{r}_i \leftrightarrow \mathbf{r}_j$, and so $\mathbf{l}_i \leftrightarrow \mathbf{l}_\mathbf{j}$. Since the junction Hamiltonian in Eq. (3) is symmetric under such a relabeling, then H_{flux} and P_{ij} must commute. This implies that Ψ and $P_{ij}\Psi$, where Ψ is an eigenfunction of H_{flux} with energy $V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$, must be degenerate. Since the baryon B and hybrid H have different energies, $P_{ij}\Psi$ must be a multiple of Ψ. As the only one-dimensional representations of the permutation group are either totally symmetric (S) or totally antisymmetric (A), it follows that for the hybrid Ψ is either S (hybrids H^S) or A (hybrids H^A) under quark label exchange. With the above flux Hamiltonian, the logical choice is an exchange-symmetric baryon trial wave function Ψ_B.

The color structure of baryons and hybrid baryons is motivated by the strong-coupling limit of lattice QCD, where the quarks are triplet sources of color joined by strings connected to a totally antisymmetric junction, so that the color wave functions of both kinds of state are totally antisymmetric under quark-label exchange. The Pauli principle therefore requires that the ground state symmetric hybrids H^S have combined flavor-spin wave functions for the quarks which are totally symmetric, since these will have symmetric quark orbital wave functions. This implies that flavor-symmetric Δ states must have $S=\frac{3}{2}$, and that flavor-mixed-symmetry N states have $S=\frac{1}{2}$. The H^A hybrids require totally antisymmetric quark flavor-spin wave functions, which are impossible for Δ states, and possible only for N states with $S=\frac{1}{2}$. When these flavor-spin configurations are put together with the $L^P = 1^+ \otimes 0^+$ combined flux-quark orbital wave function for the H hybrids, the result is the set of configurations shown in Table II. Although both the present model and the bag model [3] predict the presence of seven low-lying hybrid baryons, only the states $N^2\frac{1}{2}^+$ and $N^2\frac{3}{2}^+$ have the same flavor, quark spin S, total angular momentum, and parity as low-lying hybrids in the bag model [3]. Flavor assignments for the H^S hybrids are reversed in the bag model, due to the different (mixed) exchange symmetry of the quark color wave functions required by the addition of a single colored gluon. These differences emphasize the collective nature of the gluonic excitation in the flux-tube model. With

TABLE II. Quantum numbers of low-lying hybrid baryons for the adiabatic surface H, degenerate in the absence of spin-dependent forces.

Hybrid Baryon	L	S	$(N,\Delta)^{2S+1}J^P$
H^S	1	$\frac{1}{2},\frac{3}{2}$	$N^2\frac{1}{2}^+, N^2\frac{3}{2}^+, \Delta^4\frac{1}{2}^+, \Delta^4\frac{3}{2}^+, \Delta^4\frac{5}{2}^+$
H^A	1	$\frac{1}{2}$	$N^2\frac{1}{2}^+, N^2\frac{3}{2}^+$
Bag model	1	$\frac{1}{2},\frac{3}{2}$	$\Delta^2\frac{1}{2}^+, \Delta^2\frac{3}{2}^+, N^4\frac{1}{2}^+, N^4\frac{3}{2}^+, N^4\frac{5}{2}^+$
	1	$\frac{1}{2}$	$N^2\frac{1}{2}^+, N^2\frac{3}{2}^+$

the addition of spin-dependent forces, the lowest-lying hybrid states are predicted to be two pairs of quark-spin-$\frac{1}{2}$ nucleon states in the P_{11} and P_{13} partial waves, with more massive quark-spin-$\frac{3}{2}\Delta$ states. The central conclusion of this article is that the lightest of these states are at 1870 ± 100 MeV, considerably higher than previous mass estimates from the bag model [3] and QCD sum rules [2] of about 1500 MeV.

If hybrid baryons obey suppression of certain decay modes similar to the decay selection rules for hybrid mesons [18], they may be distinguishable from conventional baryons in the same mass range on the basis of their strong decays; in addition, their electromagnetic couplings are expected to be distinctive [1]. There are q^3 baryon states predicted by quark potential models to be present in these partial waves at these energies which are missing from the analyses. A signal for the presence of hybrid baryons would be the discovery, in analyses of data expected from the new experiments, of more states than predicted by models which constrain the glue to be in its ground state. Furthermore, in order to understand experiments which are designed to find these missing q^3 baryons, states with excited glue must be considered. An obvious place to look for signals for hybrid baryons would be in electro- and photoproduction of ρ and ω in Hall B at TJNAF; there are also planned πN scattering experiments [19] by the Crystal Ball Collaboration at the new D-line at Brookhaven National Laboratory which will examine the final states $N\eta$, $N\rho$ and $N\omega$ in the 2 GeV mass region.

The decays $\psi\to p\bar{p}\omega$ and $\psi\to p\bar{p}\eta'$ have been observed [20] with branching ratios of roughly 10^{-3}. Since gluonic hadron production is expected to be enhanced above conventional hadron production in the glue-rich decay of the ψ, it is possible that a partial wave analysis of the $p\omega$ or $p\eta'$ invariant masses would yield evidence for hybrid baryons. Future work in this area at the Beijing Electron Positron Collider and an upgraded τ-charm factory could be of critical importance.

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