Comment on ''Observational constraints on power-law cosmologies''

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''Power-law cosmologies'' are defined by their growth of the cosmological scale factor as t^α regardless of the matter content or cosmological epoch. Constraints from the current age of the universe and from the high redshift supernovae data require ''large'' α (≈ 1). We reinforce this with the latest available observations. Such a large α is also consistent with the right amount of helium and the lowest observed metallicity in the universe for a model with the baryon entropy ratio $\approx 8.1\times 10^{-9}$. [S0556-2821(99)08620-8]

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A power-law growth $a(t)=t^\alpha$ for the cosmological scale factor $a(t)$ is a generic feature of a class of models that attempt to dynamically solve the cosmological constant problem [1]. Another example of a power-law cosmology is the linear scaling produced by Allen [6] in a model determined by an SU(2) cosmological instanton dominated universe. As pointed out by Kaplinghat *et al.* [2] (hereinafter referred to as paper I), constraints on all such ''power-law cosmologies'' from the present age of the universe and from the high redshift data are consistent with large $\alpha\approx 1$. However, paper I considered the primordial light element abundances from early universe nucleosynthesis and concluded that α is forced to lie in a very narrow range with an upper limit ≈ 0.55. It was thus concluded in paper I that power-law cosmologies are not viable. In this Comment, while we reinforce the constraints for a large $\alpha\approx 1$ from the more recent data for type Ia supernovae (SNIa) reported by the supernovae cosmology project [3], we demonstrate that the nucleosynthesis constraints on α arrived at in paper I are seriously in error. A large value $\alpha\approx 1$ is consistent with the right amount of helium observed in the universe in a model with the baryon to entropy ratio $\approx 8.1\times 10^{-9}$.

In general, for a power-law cosmology, the present hubble parameter H_0 is related to the present epoch t_0 by $H_0 t_0=\alpha$. In what follows we shall restrict ourselves to the case where the scale factor evolves linearly with time; i.e., $\alpha=1$. This would include a Milne cosmology for which $a(t)=t$, as well as a general coasting cosmology for which $a(t)=kt$ [5]. The Hubble parameter is precisely the inverse of the age $t_0=(H_0)^{-1}$. In the standard big bang (SBB) model, $t_0\approx 2/3H_0$. Thus the age of the universe inferred from a measurement of the Hubble parameter is 1.5 times the age inferred by the same measurement in standard matter-dominated model. With the best reported value for H_0 standing at $(H_0)=100h\ \mathrm{km(s)^{-1}(Mpc)^{-1}}$, with $h=0.65$ [7,8], the age of the universe turns out to be ≈ 15 Gyr. Such an age is comfortably consistent with age estimates for old clusters.

Paper I put constraints on the value of α using the data of Perlmutter *et al.* [4,9] on SNIa at $z=0.83$. The quoted value of the figure of merit for SNIa favors a large $\alpha\geqslant 1$. For a more recent data set [3], we find the best fit for $\alpha=1.02\pm 0.05$. It was also noted in [3] that the curve for $(\Omega_\Lambda=\Omega_M=0)$ is ''practically identical to a *best fit* plot for an unconstrained cosmology.'' This is nothing but the Milne model $\alpha=1$ and further reinforces I as far as the concordance of an $\alpha=1$ power-law cosmology with age and the $m-z$ relations are concerned.

As regards nucleosynthesis, with the expansion scale factor evolving linearly with time, the temperature scales as $aT=tT=\text{const}$ as long as we are in an era where the photon entropy is not changing much. (The small entropy change at the time of e^+, e^- annihilation does not alter the following argument as well as the results substantially). The hubble expansion rate at a given temperature is much smaller than its corresponding value at the same temperature in standard cosmology. Taking the present age as the inverse of the hubble parameter and the present effective cosmic microwave background ''temperature'' as 2.7 K, it is easily seen that the universe would be some 50 years old at temperatures $\approx 10^9$ K. Such a universe would take some 5000 years to cool to 10^7 K. With the neutron decay rate around 888 s at low enough temperatures, it would seem that all neutrons would have decayed by the time nucleosynthesis may be expected to commence at around 10^9 K. This is precisely the argument paper I used to label nucleosynthesis as spelling ''*disaster*'' for such cosmologies—and thus ruling them out. However, if we consider weak interaction rates of neutrons and protons, it is easily seen that the inverse (proton's) β decay remains effective and is not frozen until temperatures of even slightly less than 10^9 K. The weak interactions of the leptons also remain in equilibrium until temperatures even lower: 10^8 K [10]. This has interesting consequences. First, the equality of photon and neutrino temperature $(T_\nu=T)$ is ensured even after the electron-positron annihilation. With temperature measured in units of $T_9=10^9$ K, this leads to an exact expression for the p going to n rate as $\approx\exp[-15/T_9]$ times the n going to p rate. Figure 1 exhibits the $p\rightarrow n$ rate in comparison to the hubble parameter near $T_9\approx 1$. It is clear that by inverse β decay a proton's conversion into a neutron is not decoupled at temperatures as low as 10^9 K. The n/p ratio is expected to follow its equilibrium value irrespective of the neutron decay rate as long as both n going to p, and p going to n rates are large in comparison to the expansion rate of the universe and the rate of nucleon

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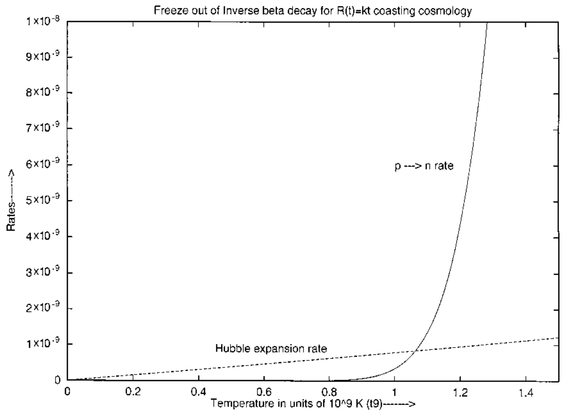

FIG. 1. The inverse β decay rate $p \rightarrow n$ and Hubble expansion rate as a function of temperature in units of 10^9 K. Inverse β decay decouples only at $T_9 \approx 1.08$ K.

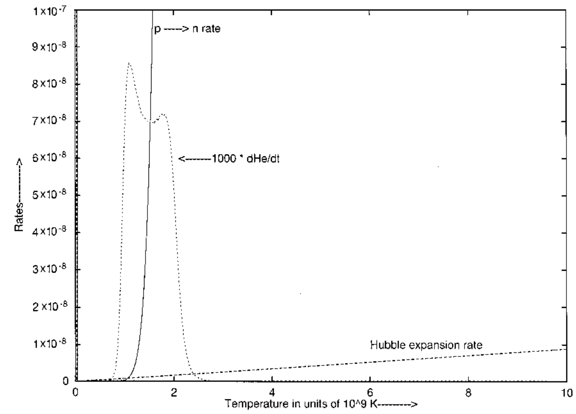

FIG. 2. Comparison of helium precipitation and neutron production rates as a function of temperature. The helium production rate, which is identically equal to the nucleon precipitation rate out of the $n-p$ equilibrium buffer, at these temperatures is some 1000 times smaller than $n \leftrightarrow p$ conversion rates by β decay.

leak into the nucleosynthesis channel. Although the n/p ratio is small at temperatures $T_9 \approx 1$, every time any neutron branches off into the nucleosynthesis channel, the n/p ratio will be replenished by the inverse β decay of the proton. Simple chemical kinetics shows that if we remove one of the reactants or the products of a reaction in equilibrium at a rate slower than the relaxation period of the equilibrium buffer, reactions proceed in an equilibrium restoring direction. As long as we keep precipitating a product at a small enough rate, reversible reactions that maintain a solution in equilibrium would restore the buffer to an equilibrium configuration. This is just what is referred to as ''the law of mass action'' in chemistry.

What actually happens is that, depending on the baryon entropy ratio, helium starts precipitating out at temperatures around 7×10^9 K. The rate of precipitation of helium is exhibited in Fig. 2, where it is clear that the amount of nucleon precipitation into the helium synthesis channel is negligible in comparison to the neutron formation and destruction due to inverse and forward β decay, respectively. This is sufficient to maintain n/p to its equilibrium value. Even in the SBB model, at such temperatures much higher than the so-called deuterium ''bottleneck'' temperature, there is a tiny amount of helium always forming. However, the universe keeps to such temperatures in the SBB model for less than 100's of seconds only and so the amount formed before the ''bottleneck'' temperature is negligible. In the case at hand, the universe is at such temperatures for some 100 years and the tiny amounts of helium steadily builds up. This is conclusively demonstrated by resorting to a numerical integration of Boltzmann equations incorporating the entire network of reactions. It is easy to implement the required modifications in Kawano's standard nucleosynthesis code to suit the linear expansion of the scale factor. Stability of the code requires it to be compiled in a higher precision than required in the SBB model. Runs for different values of the baryon-to-entropy ratio (η), and with the currently favored value of 65 km/s/Mpc for the hubble parameter, yield the result that an $\eta \approx 8.1\times10^{-9}$ gives just the right amount of helium (23.8%) as observed in the universe [10,11]. As also pointed out in [10], nucleosynthesis in a power-law cosmology yields a metallicity quite close to the lowest observed metallicities.

We conclude that helium-4 synthesis does not rule out a power-law cosmology as claimed in paper I. The only problem one has to contend with is the significantly low yields of other light elements in such a cosmology. The low yields for deuterium and helium-3, for example, turn out to be of the order of 10^{-18} and 10^{-14}, respectively, and are clearly unacceptable. If one can contend with the other light element production by alternate mechanisms (such as spallation, shock waves, inhomogeneities, etc. that are under investigation by the authors for characteristic properties that a power-law cosmology allows), the higher primordial metallicity may well be a bonus for such a model.

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