# Leptogenesis in theories with large extra dimensions

Apostolos Pilaftsis
*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
*and Department of Theoretical Physics, University of Thessaloniki, GR 54006 Thessaloniki, Greece*
(Received 7 June 1999; published 25 October 1999)

We study the scenario of baryogenesis through leptogenesis in higher-dimensional theories, in which the scale of quantum gravity is many orders of magnitude smaller than the usual Planck mass. The minimal realization of these theories includes an isosinglet neutrino which feels the presence of large compact dimensions, whereas all the SM particles are localized on a $(1+3)$-dimensional subspace. In the formulation of minimal leptogenesis models, we pay particular attention to the existence of Majorana spinors in higher dimensions. After compactification of the extra dimensions, we obtain a tower of Majorana Kaluza-Klein excitations which act as an infinite series of $CP$-violating resonators and derive the necessary conditions for their constructive interference. Based on this $CP$-violating mechanism, we find that the decays of the heavy Majorana excitations can produce a leptonic asymmetry which is reprocessed into the observed baryonic asymmetry of the universe by means of out-of-equilibrium sphaleron interactions, provided the reheat temperature is above 5 GeV. [S0556-2821(99)04020-5]

PACS number(s): 11.30.Fs, 11.10.Kk

## I. INTRODUCTION

Superstring theories have been advocated to provide a consistent theoretical framework that could lead to the quantization of gravity, including its possible unification with all other fundamental forces in nature. The quantum nature of gravity is expected to play a central role at energy scales close to the Planck mass, $M_P = 1.2 \times 10^{19}$ GeV. The formulation of superstring theories requires the embedding of our well-established $(1+3)$-dimensional world into a higher-dimensional space, in which the new spatial dimensions must be highly curved for both phenomenological and theoretical reasons. In typical string theories, the fundamental string scale is generically of order $M_P$. However, Witten [1] and Hořava and Witten [2] presented an interesting alternative, in which the string scale may be considerably lowered to $\sim 10^{16}$ GeV, thereby enabling the unification of all interactions within the minimal supersymmetric model. An analogous scenario was subsequently discussed by Lykken [3], in which the string scale was further lowered to the TeV range,[1] but the fundamental Planck scale was kept intact to $M_P$.

Recently, Arkani-Hamed, Dimopoulos, and Dvali [5] have considered a more radical scenario, in which the fundamental scale of quantum gravity, $M_F$, may be as low as few TeV, thereby proposing an appealing solution to the known gauge hierarchy problem [6]. The observed weakness of gravity may then be attributed to the presence of a number $\delta$ of large extra spatial dimensions, within which only gravity can propagate and, most probably, fields that are singlets under the standard model (SM) gauge group, such as isosinglet neutrinos [7,8]. This higher $[1 + (3 + \delta)]$-dimensional space is usually termed the bulk. On the other hand, all the ordinary SM particles reside in the conventional

$(1+3)$-dimensional Minkowski subspace, which is called the wall. In such a theoretical framework, the ordinary Planck mass $M_P$ must be viewed as an effective parameter, which is related to the genuinely fundamental scale $M_F$ via a kind of generalized Gauss's law:

$$M_P \approx M_F (R M_F)^{\delta/2}, \qquad (1.1)$$

where we have assumed, for simplicity, that the additional $\delta$-dimensional volume has the configuration of a torus, with all of its radii being equal. Many astrophysical [9–16] and phenomenological [17] analyses have already appeared in the recent literature for such low-string-scale theories.

As has been mentioned already, it is conceivable to assume that isosinglet neutrinos exist in addition to gravitons and that also feel the presence of large extra space dimensions. In particular, we wish to study novel scenarios, in which the existence of higher-dimensional singlet fields may account for the observed baryonic asymmetry of the universe (BAU) by means of the Fukugita-Yanagida mechanism of leptogenesis [18]. According to this mechanism, an excess of the lepton number ($L$) is first generated by out-of-equilibrium $L$-violating decays of heavy Majorana neutrinos, which is then converted into an asymmetry of the baryon number ($B$) through ($B+L$)-violating sphaleron interactions [19]. Such an $L$-to-$B$ conversion of asymmetries stays unsuppressed, as long as the heavy Majorana-neutrino masses lie above the critical temperature $T_c$ of the electroweak phase transition where sphalerons are supposed to be in thermal equilibrium. Such a scenario of explaining the BAU is often called baryogenesis through leptogenesis.

The presence of large extra dimensions introduces a number of alternatives for leptogenesis which may even have no analogue in the conventional four-dimensional theories. We shall focus our attention on minimal realizations of higher-dimensional leptogenesis models which, after compactification of the extra dimensions, lead to scenarios that admit renormalization assuming a finite number of Kaluza-Klein

---

[1]In a different context, Antoniadis [4] had made an earlier suggestion of a low compactification scale of order TeV in string theories.

(KK) excitations. Such models of leptogenesis are therefore endowed with enhanced predictive power. For definiteness, we will consider minimal four-dimensional extensions of the SM, augmented by one singlet Dirac neutrino, which propagates in the bulk. Parenthetically, we should notice that massive Majorana neutrinos are not defined for spaces with any space-time dimensions, but only for those with 2, 3, and 4 mod 8 dimensions [20,21]. For instance, unlike in four dimensions, true Majorana spinors cannot be defined in five dimensions. This topic will be discussed in detail in Sec. II.

After compactification of the extra dimensions, the kinetic term of the bulk neutrino gives rise to an infinite series of massive KK excitations, with equally spaced Dirac masses; i.e., the mass difference between two neighboring KK states is of order $1/R$. In order to make the mechanism of leptogenesis work, it is necessary that the model under consideration violate both the lepton number $L$ and the product of symmetries of charge conjugation ($C$) and (parity) space reflection ($P$), also known as $CP$ symmetry. The violation of $L$ can be introduced into the theory by simultaneously coupling the different spinorial states of the higher-dimensional Dirac neutrino to the lepton doublets of the SM and to their $C$-conjugate counterparts. As we will see in Sec. III, however, this is not sufficient for the theory to be $CP$ violating. $CP$ nonconservation can be minimally realized in two different ways: one has to either (i) include additional higher-dimensional fermionic bilinears or (ii) extend the Higgs sector of the SM. Obviously, one may also consider more involved models based on combinations of these two minimal scenarios. The first scenario may be regarded as a higher-dimensional extension of the ordinary leptogenesis model [18]. Of most interest is, however, the second alternative, which has no analogue in four dimensions, as it does not require the inclusion of any explicit heavy Majorana or isosinglet mass scale in the Lagrangian. The characteristic feature of these extensions is that each of the Dirac KK neutrino states splits into two nearly degenerate Majorana neutrinos either at the tree level in the first scenario or at one loop in the second one.

There are generically two distinct mechanisms that give rise to $CP$ nonconservation in the decays of heavy Majorana KK states. In the first mechanism, $CP$ violation is induced by the interference of the tree-level decay graph with the absorptive part of a one-loop vertex diagram [18,22]; we call the latter $\varepsilon'$-type $CP$ violation in connection with the established terminology of the kaon system. In the second mechanism, which we call it $\varepsilon$-type $CP$ violation, the tree-level diagram interferes with the absorptive part of the one-loop self-energy transition between two heavy Majorana neutrinos [23–25], i.e., between heavy Majorana KK states. If the mass difference of two heavy Majorana states is of the order of their respective widths, the description of $\varepsilon$-type $CP$ violation becomes more subtle field theoretically [24]. In this case, finite-order perturbation theory no longer applies, and one is therefore compelled to resort to a resummation approach which consistently takes the instability of the mixed heavy Majorana states into account. This issue has extensively been discussed in [24].

Furthermore, it was shown [24] that $\varepsilon$-type $CP$ violation induced by the mixing of two nearly degenerate heavy Majorana states can be resonantly enhanced up to order of unity. As we shall discuss in more detail in Sec. IV, an analogous dynamics exhibits the system of the Majorana KK excitations. In fact, each KK pair of the two nearly degenerate Majorana states behaves as an individual $CP$-violating resonator. In this way, we shall characterize a two-level system that satisfies the resonant conditions of order-unity $CP$ violation. We find that the spacing in mass for two adjacent KK pairs of Majorana states governs the dynamics for constructive or destructive interference of the complete tower of the $CP$-violating resonators. Owing to cancellations among the different $CP$-violating vertex contributions, we can explicitly demonstrate that $\varepsilon'$-type $CP$ violation is vanishingly small.

A crucial requirement for successful baryogenesis through leptogenesis is that the temperature of reheating $T_r$ [26] due to the late decays of gravitons into photons be not much smaller than the critical temperature $T_c$, namely, the temperature above which the $(B+L)$-violating sphalerons are in thermal equilibrium. If $T_r < T_c$, sphalerons are out of equilibrium, and the conversion of the generated leptonic asymmetry into the baryon asymmetry becomes exponentially suppressed. In particular, it has been argued [12] that it may be difficult to obtain a reheat temperature above $T_c$ in theories with a low scale of quantum gravity $M_F$, such that sphalerons can effectively reprocess an excess in $L$ into $B$. For two extra large dimensions, the authors [9,12] derive the mass limit $M_F \gtrsim 100$ TeV, assuming that the reheat temperature is larger than a few MeV, so as to ensure that primordial nucleosynthesis proceeds as usual. This bound is also in qualitative agreement with recent constraints derived from considerations of rapid supernovas cooling due to graviton emission [13] and of the cosmic diffuse gamma radiation [14].

Nevertheless, several possibilities have already been reported in the literature that one might think of to avoid possible difficulties associated with a low $T_r$. For example, one could imagine that the bulk singlet neutrino only resides in a subspace of a multidimensional space spanned by a number $\delta = 6$ of extra dimensions and higher, in which gravity propagates [7]. This could lead to rather suppressed production rates of gravitons, thus allowing much larger reheat temperatures [9,12]. Another way of resolving the problem of a low $T_r$ is to assume that the compactification radii [15,16] of gravity are not all equal, but possess a large hierarchy. Such a possibility would completely change the usual cosmological picture of the previous analyses. In this context, it has been further advocated that gravitons might decay faster on a hidden wall than the observable wall we live on [9] or even a novel type of rapid asymmetric inflation could take place [11]. Because of the variety of the solutions suggested in the literature, in our analysis we shall not put forward a specific mechanism of increasing the reheat temperature close to $T_c$. Instead, we will simply assume that $T_r$ is a free parameter and place a lower limit on it, based on the requirement that the observed amount of $B$ asymmetry be produced. Especially, we shall see that the resonantly enhanced $CP$ asym-

metries in the decays of the KK states are very important to overcome part of the low-$T_r$ problem.

The paper is organized as follows: Section II reviews the topic related to the ability of defining true Majorana spinors in higher-dimensional theories. In Sec. III, we formulate minimal renormalizable higher-dimensional models that can lead to successful scenarios of leptogenesis. In Sec. IV, we derive the necessary conditions for order-unity $CP$ asymmetries due to the constructive interference of the tower of the KK $CP$-violating resonators and show that $\varepsilon'$-type $CP$-violating contributions are negligible. In Sec. V, we give an estimate of the baryonic asymmetry that arises from a sphaleron-converted leptonic asymmetry and derive a lower bound on $T_r$ and $M_F$ for successful baryogenesis. Finally, Sec. VI presents our conclusions.

## II. MAJORANA SPINORS IN HIGHER DIMENSIONS

The violation of the lepton number in leptogenesis models or supersymmetric theories is naturally mediated by Majorana fields, e.g., heavy Majorana neutrinos, neutralinos, etc. The KK formulation of these theories necessitates an analogous extension of the notion of the Majorana spinor to higher dimensions [20,21]. The ability of defining true Majorana neutrinos in any dimensions plays a key role in the construction of higher-dimensional leptogenesis models. Here we shall review this topic from a more practical, for our purposes, point of view.

We shall consider $d$-dimensional theories with one time component and $d-1$ spatial ones. We assume that the Lagrangian describing these theories is invariant under the generalized Lorentz transformations of the $SO(1,d-1)$ group. In such an extended $d$-dimensional Minkowski space, the corresponding Clifford algebra reads

$$\{\gamma_\mu^{(d)},\gamma_\nu^{(d)}\}=2g_{\mu\nu}^{(d)}\mathbf{1}, \tag{2.1}$$

where $g_{\mu\nu}^{(d)}=\mathrm{diag}(+1,-1,\ldots,-1)$, for $\mu,\nu=0,1,\ldots,d-1$, and $\gamma_\mu^{(d)}$ are the generalized Dirac's gamma matrices. The construction of these matrices to any number of dimensions may be found recursively. Our starting point is the representation of gamma matrices for $d=2$ and $d=3$, i.e.,

$$\gamma_0^{(2,3)}=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix},\quad \gamma_1^{(2,3)}=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix},\quad \gamma_2^{(3)}=\begin{bmatrix}i & 0\\0 & -i\end{bmatrix}. \tag{2.2}$$

The procedure for constructing gamma matrices to higher dimensions is then as follows. If $d=2m$ ($m=1,2,\ldots$), we may then define

$$\gamma_0^{(d)}=\begin{bmatrix}0 & \mathbf{1}_m\\\mathbf{1}_m & 0\end{bmatrix},$$

$$\gamma_k^{(d)}=\begin{bmatrix}0 & \gamma_0^{(d-1)}\gamma_k^{(d-1)}\\-\gamma_0^{(d-1)}\gamma_k^{(d-1)} & 0\end{bmatrix},\quad k=1,\ldots,d-2, \tag{2.3}$$

$$\gamma_{d-1}^{(d)}=\begin{bmatrix}0 & \gamma_0^{(d-1)}\\-\gamma_0^{(d-1)} & 0\end{bmatrix},\quad \gamma_P^{(d)}=\begin{bmatrix}\mathbf{1}_m & 0\\0 & -\mathbf{1}_m\end{bmatrix}, \tag{2.4}$$

where $\mathbf{1}_m$ is the unity matrix in $m$ dimensions. Note that the dimensionality of the representation of the gamma matrices for $d=2m+1$ coincides with that of $d=2m$. The matrix $\gamma_P^{(d)}$ is the generalization of the usual $\gamma_5$ matrix in four dimensions, i.e., $\gamma_P^{(d)}=c\Pi_{\mu=0}^{d-1}\gamma_\mu^{(d)}$, where the constant $c$ is defined such that $\gamma_P^{(d)2}=\mathbf{1}$. The matrix $\gamma_P^{(d)}$ anticommutes with all $\gamma_\mu^{(d)}$ for $d=2m$, whereas it commutes with all $\gamma_\mu^{(d)}$ for $d=2m+1$; i.e., it is proportional to the unity matrix. If we know the representation of gamma matrices for $d=2m$, we can easily construct the respective one for $d=2m+1$, just by including

$$\gamma_d^{(d+1)}=i\gamma_P^{(d)}. \tag{2.5}$$

In fact, Eqs. (2.3)–(2.5) are sufficient to construct all $\gamma_\mu^{(d)}$ in any number $d$ of dimensions, starting from the known expressions (2.2) for $d=2,3$. In addition, we should notice that the adopted representations of $\gamma_\mu^{(d)}$ are of the Weyl type, having the properties

$$\gamma_0^{(d)}=\gamma_0^{(d)\dagger},\quad \gamma_k^{(d)}=-\gamma_k^{(d)\dagger},\quad k=1,\ldots,d-1. \tag{2.6}$$

Finally, a useful property of the above construction is the fact that $\gamma_\mu^{(d)}$ are self-adjoint under the known bar operation, i.e.,

$$\bar{\gamma}_\mu^{(d)}\equiv\gamma_0^{(d)}\gamma_\mu^{(d)\dagger}\gamma_0^{(d)}=\gamma_\mu^{(d)}, \tag{2.7}$$

and $\overline{i\gamma_P^{(d)}}=i\gamma_P^{(d)}$.

Let us now define by $\psi(x)$ a massive fermionic free field in a multidimensional Minkowski space, which satisfies the free Dirac equation of motion, i.e., $(i\gamma^\mu\partial_\mu-m)\psi=0$. Here and henceforth, we shall drop the superscript $(d)$ on the gamma matrices to simplify notation. The Lorentz adjoint of $\psi$ is then given by $\bar{\psi}=\psi^\dagger\gamma_0$, while the invariance of the Dirac equation under generalized Lorentz transformations requires

$$\bar{S}=\gamma_0 S^\dagger\gamma_0=S^{-1}, \tag{2.8}$$

where

$$S=\exp\left(-\frac{i}{4}\sum_n\omega_n\sigma_{\mu\nu}I_n^{\mu\nu}\right), \tag{2.9}$$

with $\sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]$, is the $d$-dimensional spinorial representation of an arbitrary Lorentz rotation with angles $\omega_n$, and $I_n^{\mu\nu}$ are the generators of $SO(1,d-1)$. It is easy to see that Eq. (2.8) is equivalent to $\gamma_0\sigma_{\mu\nu}^\dagger\gamma_0=\sigma_{\mu\nu}$. The last equality is true by virtue of Eq. (2.7).

To define charge-conjugate fermionic fields in theories with many dimensions, we proceed as follows. We start with the classical Dirac equation by including a background electromagnetic field $A_\mu$ coupled to $\psi$, i.e., $[i\gamma^\mu(\partial_\mu+eA_\mu)-m]\psi=0$, and then seek for a solution of the respective

Dirac equation for the antiparticle field, denoted as $\psi^C$, which is of the form $[i\gamma^\mu(\partial_\mu - eA_\mu) - m]\psi^C = 0$. In case $\psi$ is a neutral field, e.g., a neutrino, one should initially assume that $e \neq 0$ and then take the limit $e \to 0$ at the very end of the consideration. In this way, we find that $\psi^C$ may be determined in terms of $\psi$ as follows:

$$\psi^C = C\bar{\psi}^T = C\gamma_0\psi^*, \qquad (2.10)$$

where $C$ is the charge-conjugation operator that satisfies the property

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T \qquad (2.11)$$

for massive fermionic fields. For massless fermions, we may also allow the equality

$$C^{-1}\gamma_\mu C = \gamma_\mu^T. \qquad (2.12)$$

Furthermore, consistency of charge conjugation with Lorentz invariance implies that

$$C^{-1}SC = (S^{-1})^T \qquad (2.13)$$

or, equivalently, that $C^{-1}\sigma_{\mu\nu}C = -\sigma_{\mu\nu}^T$, which holds true because of Eq. (2.11) or (2.12). At this point, we should remark that the transformations

$$\gamma_\mu' = U\gamma_\mu U^{-1}, \quad C' = UCU^T \qquad (2.14)$$

preserve all the relations of the gamma matrices given above, including Eqs. (2.8) and (2.13).

The necessary and sufficient condition for the existence of a Majorana spinor in any number of dimensions reads

$$\psi = \psi^C, \qquad (2.15)$$

which amounts to

$$C\gamma_0^T(C\gamma_0^T)^* = 1. \qquad (2.16)$$

This last equality may be rewritten as

$$C^{-1}\gamma_0 C = (C^*C)\gamma_0^T. \qquad (2.17)$$

Consequently, massive [massless] Majorana spinors in $d$ dimensions are admitted, if both the construction of a $C$ matrix satisfying Eq. (2.11) [Eq. (2.12)] and $C^*C = -1$ [$C^*C = +1$] is possible. As we will see below, this is not always the case.

For this purpose, it is important to be able to construct a matrix that obeys the identity (2.11) or (2.12). There are only two candidates that could be of interest:

$$C_A = \prod_i^p \gamma_i \quad \text{with} \quad \gamma_i = -\gamma_i^T = -\gamma_i^\dagger, \qquad (2.18)$$

$$C_S = \prod_r^s \gamma_r \quad \text{with} \quad \gamma_r = \gamma_r^T, \quad \gamma_0 = \gamma_0^\dagger,$$

$$\gamma_r = -\gamma_r^\dagger \quad (r \neq 0). \qquad (2.19)$$

TABLE I. Existence of massive Majorana spinors in $d$ dimensions.

| $d$ | $s$ even | $\varepsilon(s) = -1$ | $p$ odd | $\varepsilon(p) = 1$ | Existence of massive Majorana spinor |
|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 1 | Yes |
| 3 | 2 | $-1$ | 1 | 1 | Yes |
| 4 | 2 | $-1$ | 2 | $-1$ | Yes |
| 5 | 3 | $-1$ | 2 | $-1$ | No |
| 6 | 3 | $-1$ | 3 | $-1$ | No |
| 7 | 4 | 1 | 3 | $-1$ | No |
| 8 | 4 | 1 | 4 | 1 | No |
| 9 | 5 | 1 | 4 | 1 | No |
| 10 | 5 | 1 | 5 | 1 | Yes |
| 11 | 6 | $-1$ | 5 | 1 | Yes |
| 12 | 6 | $-1$ | 6 | $-1$ | Yes |
| 13 | 7 | $-1$ | 6 | $-1$ | No |

Specifically, $C_A$ ($C_S$) is formed by the product of all $p$ ($s$) in number gamma matrices that are pure antisymmetric (symmetric). Employing the identity $\gamma_\mu\gamma_\mu^\dagger = 1$, we can easily find the following relations for the two $C$-conjugation matrices:

$$C_A^{-1} = C_A^\dagger = (-1)^P\varepsilon(p)C_A,$$

$$C_A^T = (C_A^\dagger)^* = (-1)^p\varepsilon(p)C_A^*, \quad C_A = C_A^*,$$

$$(2.20)$$

$$C_S^{-1} = C_S^\dagger = (-1)^{s-1}\varepsilon(s)C_S,$$

$$C_S^T = (C_S^\dagger)^* = (-1)^{s-1}\varepsilon(s)C_S^*,$$

$$C_S^* = (-1)^{s-1}C_S, \qquad (2.21)$$

with $\varepsilon(z) = (-1)^{z(z-1)/2}$. As advertised, it can be shown that the two $C$-conjugation matrices satisfy the relations

$$C_A^{-1}\gamma_\mu C_A = (-1)^p\gamma_\mu^T, \quad C_S^{-1}\gamma_\mu C_S = (-1)^{s+1}\gamma_\mu^T. \qquad (2.22)$$

On the other hand, the Majorana condition given by Eq. (2.17) may now be translated into

$$C_A^{-1}\gamma_0 C_A = (-1)^p\varepsilon(p)\gamma_0^T, \quad C_S^{-1}\gamma_0 C_S = \varepsilon(s)\gamma_0^T. \qquad (2.23)$$

As a consequence, the existence of a massive Majorana spinor in any number of dimensions is ensured if

$$\varepsilon(p) = 1 \quad \text{and} \quad p \text{ is odd} \qquad (2.24)$$

or if

$$\varepsilon(s) = -1 \quad \text{and} \quad s \text{ is even.} \qquad (2.25)$$

Based on Eqs. (2.24) and (2.25), we can generate Table I. As can be seen from this table, true *massive* Majorana neutrinos exist only in 2, 3, and 4 mod 8 dimensions [20].

If we also allow the possibility of massless Majorana-Weyl spinors, we only need to impose the restriction

TABLE II. Existence of massless Majorana-Weyl spinors in $d$ dimensions.

| $d$ | $\varepsilon(p)=1$ | $\varepsilon(s) = (-1)^{s+1}$ | | Existence of massless Majorana spinor |
|---|---|---|---|---|
| 5 | $-1$ | $-1$ | 1 | No |
| 6 | $-1$ | $-1$ | 1 | No |
| 7 | $-1$ | 1 | $-1$ | No |
| 8 | 1 | 1 | $-1$ | Yes |
| 9 | 1 | 1 | 1 | Yes |

$$\varepsilon(p)=+1 \quad \text{or} \quad \varepsilon(s)=(-1)^{s+1}. \tag{2.26}$$

The so-generated Table II shows that in addition to the result found in the massive case, the definition of massless Majorana fields can be extended to 8 and 9 mod 8 dimensions [21]. For example, our analysis explicitly demonstrates that, as opposed to four dimensions, true Majorana neutrinos cannot be defined in theories with five, six, and seven dimensions. In fact, in the latter theories, $C$ loses its very same meaning of being a genuine charge-conjugation matrix. We shall pay special attention to this issue in the next section, while formulating different minimal models of leptogenesis.

## III. HIGHER-DIMENSIONAL MODELS OF LEPTOGENESIS

If the SM contains a singlet neutrino that feels large extra dimensions, this additional volume factor of the new spatial dimensions introduces a new possibility to naturally suppress the Higgs Yukawa coupling to neutrinos [7,8]. After spontaneous symmetry breaking (SSB) of the SM Higgs potential, the resulting neutrino masses may naturally be of the order of $10^{-2}$ eV, which turns out to be in the right ballpark for explaining the solar and atmospheric neutrino data [16]. Here we shall formulate minimal models of leptogenesis which, after compactification of the extra dimensions, give rise to theories containing four-dimensional operators only and are therefore renormalizable for finitely many KK states. Even though the number of KK excitations is formally infinite, on theoretical grounds, however, one expects the presence of an ultraviolet (UV) cutoff close to the string scale where gravity is supposed to set in. The issue of renormalization will become more clear while describing the leptogenesis models.

For simplicity, we shall consider a five-dimensional model. The generalization of the results to higher dimensions is then straightforward. Following [7,8], we assume that all particles with nonzero SM charges reside in a subspace of $(1+3)$ dimensions. Also, we introduce one Dirac isosinglet neutrino $N(x_\mu,y)$ that propagates in the bulk of all five dimensions. We denote by $x_\mu=(x_0,x_1,x_2,x_3)$ the one time and the three spatial coordinates of our observable world and by $y \equiv x_4$ the new spatial dimension. The $y$ coordinate is to be compactified on a circle of radius $R$ by applying the periodic identification $y \equiv y+2\pi R$. Specifically, the minimal field content of a one-generation model of leptogenesis is

$$L(x)=\begin{pmatrix} \nu_L(x) \\ l_L(x) \end{pmatrix}, \quad l_R(x), \quad N(x,y)=\begin{pmatrix} \xi(x,y) \\ \bar{\eta}(x,y) \end{pmatrix}, \tag{3.1}$$

where $\nu_L$, $l_L$, and $l_R$ are four-dimensional Weyl spinors, and $\xi$ and $\eta$ are two-component spinors in five dimensions. Depending on the model, we shall also assume that $\xi$ ($\eta$) is symmetric (antisymmetric) under a $y$ reflection:[2] $\xi(x,y)=\xi(x,-y)$ and $\eta(x,y)=-\eta(x,-y)$. Following the procedure outlined in Sec. II, the gamma matrices in five dimensions may be represented by

$$\gamma_\mu=\begin{pmatrix} 0 & \bar{\sigma}_\mu \\ \sigma_\mu & 0 \end{pmatrix}, \quad \gamma_4=\begin{pmatrix} i\mathbf{1}_2 & 0 \\ 0 & -i\mathbf{1}_2 \end{pmatrix}, \tag{3.2}$$

where $\sigma^\mu=(\mathbf{1}_2,\vec{\sigma})$ and $\bar{\sigma}^\mu=(\mathbf{1}_2,-\vec{\sigma})$, with $\vec{\sigma}_{1,2,3}$ being the usual Pauli matrices.

As we have mentioned in the Introduction, there are two representative minimal scenarios of leptogenesis.

(i) The first scenario may be viewed as a higher-dimensional generalization of the usual leptogenesis model of Ref. [18], in which the Lorentz- and gauge-invariant fermionic bilinears $\bar{N}N$ and $N^T C^{(5)-1}N$ are included. As we will see, however, if a $Z_2$ discrete symmetry is imposed on $N(x,y)$, the former bilinear mass term $\bar{N}N$ does not contribute to the effective action. According to Eqs. (2.18) and (2.19), the matrix $C^{(5)}$ satisfies Eqs. (2.12) and (2.13), but not Eq. (2.11), which defines the true $C$-conjugation matrix for a massive Dirac field. Despite its close analogy to four dimensions, the operator $N^T C^{(5)-1}N$ does not represent a genuine bare Majorana mass in five dimensions. Nevertheless, after KK compactification, the effective Lagrangian of this scenario displays a dynamics rather analogous to the known scenario of leptogenesis due to Fukugita and Yanagida [18].

(ii) The second scenario of leptogenesis requires, in addition to the bulk Dirac singlet field $N(x,y)$, that the Higgs sector of the SM be extended by two more Higgs doublets. The first Higgs doublet, denoted as $\Phi_1$, couples to the lepton isodoublet $L$, and the second Higgs doublet $\Phi_2$ couples to its charge-conjugate counterpart $C\bar{L}^T$, while the last one $\Phi_3$ has no coupling to matter. The so-extended Higgs potential admits $CP$ nonconservation which originates from the bilinear mixing of the three Higgs doublets. In fact, in this model, both the Majorana masses of the KK excitations and $CP$ violation are generated via loop effects. Most interestingly, as we will detail below, this scenario of leptogenesis has no analogue in four dimensions.

Of course, one may consider more involved models of leptogenesis that are based on combinations of the basic scenarios (i) and (ii), including their possible supersymmetric extensions. Therefore, it is very instructive to analyze in more detail these two representative models of leptogenesis,

---

[2]With the imposition of such a symmetry which might be justified within the context of a $Z_2$ orbifold compactification [8], one may avoid a twofold mass degeneracy in the spectrum of the KK states for the leptogenesis models under study.

as well as a hybrid scenario that includes both the extensions mentioned above, i.e., fermionic bilinears and two additional Higgs doublets.

### A. Leptogenesis model with fermionic bilinears

In this scenario, the SM is augmented by a higher-dimensional Dirac singlet neutrino $N(x,y)$, while the SM particles are considered to be confined to a four-dimensional hypersurface which describes our world and is often termed as a 3-brane.[3] In this picture, the bulk Dirac neutrino field $N(x,y)$ intersects the 3-brane at a position $y=a$, which naturally gives rise to small Yukawa couplings suppressed by the volume of the extra dimensions. This suppression mechanism is very analogous to the one that gravity owes its weakness at long distances in theories with a low scale of quantum gravity [5]. The most general effective Lagrangian of the scenario under discussion is given by

$$
\mathcal{L}_{\text{eff}} = \int_0^{2\pi R} dy \left\{ \bar{N}(i\gamma^\mu \partial_\mu + i\gamma_4 \partial_y)N - m\bar{N}N \right.
$$
$$
- \frac{1}{2}(MN^T C^{(5)-1}N + \text{H.c.})
$$
$$
+ \delta(y-a)[\bar{h}_1 \tilde{L}\tilde{\Phi}\xi + \bar{h}_2 \bar{L}\tilde{\Phi}\eta + \text{H.c.}]
$$
$$
\left. + \delta(y-a)\mathcal{L}_{\text{SM}} \right\}, \tag{3.3}
$$

where $\mathcal{L}_{\text{SM}}$ denotes the SM Lagrangian and

$$
C^{(5)} = -\gamma_1\gamma_3 = \gamma_0\gamma_2\gamma_4 = \begin{bmatrix} -i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{bmatrix}. \tag{3.4}
$$

In Eq. (3.3), $\tilde{\Phi} = i\sigma_2\Phi^*$ is the hypercharge conjugate of the SM Higgs doublet, and $\xi$ and $\eta$ are higher-dimensional two-component spinors defined in Eq. (3.1). Note that $\bar{h}_1$ and $\bar{h}_2$ are dimensionful kinematic parameters, which may be related to the dimensional-less Yukawa couplings $h_1$ and $h_2$ through

$$
\bar{h}_{1,2} = \frac{h_{1,2}}{(M_F)^{\delta/2}}, \tag{3.5}
$$

with $\delta=1$. Here one must remark that the fundamental scale of quantum gravity, $M_F$, occurs naturally in Eq. (3.5), as it is the only available energy scale of the effective Lagrangian to normalize these higher-dimensional Yukawa couplings.

Given that $N(x,y)$ is a periodic function of $y$, with a period $2\pi R$ and with its two-component spinorial modes being

---

[3]In a field-theoretic context, Rubakov and Shaposhnikov [27] presented the possibility of dynamically localizing four-dimensional fermions on a solitonic brane embedded in a higher-dimensional space by employing the index theorem in a solitonic background [28]

constrained by the aforementioned $Z_2$ discrete symmetry [8], we may expand the two-component spinors $\xi$ and $\eta$ in a Fourier series as follows:

$$
\xi(x,y) = \frac{1}{\sqrt{2\pi R}}\xi_0(x) + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty} \xi_n(x)\cos\left(\frac{ny}{R}\right), \tag{3.6}
$$

$$
\eta(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty} \eta_n(x)\sin\left(\frac{ny}{R}\right), \tag{3.7}
$$

where the chiral spinors $\xi_n(x)$ and $\eta_n(x)$ form an infinite tower of KK modes. After integrating out the $y$ coordinate, the effective Lagrangian takes on the form

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{\xi}_0(i\bar{\sigma}^\mu\partial_\mu)\xi_0 + \left(\bar{h}_1^{(0)}\bar{L}\tilde{\Phi}\xi_0 - \frac{1}{2}M\xi_0\xi_0 + \text{H.c.}\right)
$$
$$
+ \sum_{n=1}^{\infty}\left[\bar{\xi}_n(i\bar{\sigma}^\mu\delta_\mu)\xi_n + \bar{\eta}_n(i\bar{\sigma}^\mu\partial_\mu)\eta_n\right.
$$
$$
+ \frac{n}{R}(\xi_n\eta_n + \bar{\xi}_n\bar{\eta}_n) - \frac{1}{2}M(\xi_n\xi_n + \bar{\eta}_n\bar{\eta}_n + \text{H.c.})
$$
$$
\left. + \sqrt{2}(\bar{h}_1^{(n)}\bar{L}\tilde{\Phi}\xi_n + \bar{h}_2^{(n)}\bar{L}\tilde{\Phi}\eta_n + \text{H.c.})\right], \tag{3.8}
$$

where we have chosen the weak basis in which $M$ is positive, and

$$
\bar{h}_1^{(n)} = \frac{h_1}{(2\pi M_F R)^{\delta/2}}\cos\left(\frac{na}{R}\right) = \frac{M_F}{M_P}h_1\cos\left(\frac{na}{R}\right) \quad (n\geq 0), \tag{3.9}
$$

$$
\bar{h}_2^{(n)} = \frac{h_2}{(2\pi M_F R)^{\delta/2}}\sin\left(\frac{na}{R}\right) = \frac{M_F}{M_P}h_2\sin\left(\frac{na}{R}\right) \quad (n\geq 1). \tag{3.10}
$$

In deriving the last equalities on the right-hand sides (RHS's) of Eqs. (3.9) and (3.10), we have employed the basic relation given in Eq. (1.1). In agreement with [7,8], we find that independently of the number $\delta$ of the extra dimensions, the four-dimensional Yukawa couplings $\bar{h}_1^{(n)}$ and $\bar{h}_2^{(n)}$ are naturally suppressed by an extra volume factor $M_F/M_P \lesssim 10^{-10}$. We also observe that the mass term $m\bar{N}N$ drops out from the effective Lagrangian, as a result of the $Z_2$ discrete symmetry.

In the symmetric (unbroken) phase of the theory, the part of the Lagrangian describing the KK masses is given by

$$-\mathcal{L}_{\mathrm{mass}}^{\mathrm{KK}} = \frac{1}{2} M \xi_0 \xi_0 + \frac{1}{2} \sum_{n=1}^{\infty} (\xi_n, \eta_n)$$

$$\times \begin{pmatrix} M & -n/R \\ -n/R & M \end{pmatrix} \begin{pmatrix} \xi_n \\ \eta_n \end{pmatrix} + \mathrm{H.c.}$$

$$= \frac{1}{2} \mu \chi_1^{(0)} \chi_1^{(0)} + \frac{1}{2} \sum_{n=1}^{\infty} (\chi_1^{(n)}, \chi_2^{(n)})$$

$$\times \begin{pmatrix} n/R-\mu & 0 \\ 0 & n/R+\mu \end{pmatrix} \begin{pmatrix} \chi_1^{(n)} \\ \chi_2^{(n)} \end{pmatrix} + \mathrm{H.c.}, \qquad (3.11)$$

where $\chi_{1(2)}^{(n)} = (1/\sqrt{2}) \exp(i\phi_{1(2)}^n) [\xi_n + (-)\eta_n]$. As in Ref. [8], we have defined that $\mu = \min(|M-k/R|)$ be the smallest mass eigenvalue for some given value of $k$ and have relabelled the remaining KK mass eigenstates $\chi_1^{(n)}$ and $\chi_2^{(n)}$ with respect to $k$. Thus, after compactification, we see how the heavy isosinglet mass $M$ gets replaced by the small Majorana mass $\mu$, with $\mu \lesssim 1/R$. Further technical details and discussion may be found in [8]. After expressing the effective Lagrangian in Eq. (3.8) in the newly introduced Majorana-mass basis, we obtain

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{mass}}^{\mathrm{KK}} + \bar{\chi}_1^{(0)} (i\bar{\sigma}^\mu \partial_\mu) \chi_1^{(0)} + (h_1^{(0)} \bar{L} \tilde{\Phi} \chi_1^{(0)} + \mathrm{H.c.})$$

$$+ \sum_{n=1}^{\infty} [\bar{\chi}_1^{(n)} (i\bar{\sigma}^\mu \partial_\mu) \chi_1^{(n)} + \bar{\chi}_2^{(n)} (i\bar{\sigma}^\mu \partial_\mu) \chi_2^{(n)}$$

$$+ (h_1^{(n)} \bar{L} \tilde{\Phi} \chi_1^{(n)} + h_2^{(n)} \bar{L} \tilde{\Phi} \chi_2^{(n)} + \mathrm{H.c.})], \qquad (3.12)$$

where

$$h_{1(2)}^{(n)} = e^{i\phi_1^n} \bar{h}_1^{(n)} + (-1) e^{i\phi_2^n} \bar{h}_2^{(n)}, \qquad (3.13)$$

and the Yukawa couplings $\bar{h}_{1,2}^{(n)}$ are given in Eqs. (3.9) and (3.10).

It is now easy to recognize that the Lagrangian in Eq. (3.12) is the known four-dimensional model of leptogenesis with an infinite number of pairs of Majorana neutrinos $\chi_1^{(n)}$ and $\chi_2^{(n)}$ [18]. With the help of a method based on generalized $CP$ transformations [29], we can derive the sufficient and necessary condition for the theory to be $CP$ invariant. Adapting the result found in [24] to the model under discussion, we find the condition

$$\mathrm{Im}\, \mathrm{Tr}(h^\dagger h \hat{M}_\chi^\dagger \hat{M}_\chi \hat{M}_\chi^\dagger h^T h^* \hat{M}_\chi) = 0, \qquad (3.14)$$

where $h = (h_1^{(0)}, h_1^{(1)}, h_2^{(1)}, \ldots, h_1^{(n)}, h_2^{(n)}, \ldots)$ and $\hat{M}_\chi = \mathrm{diag}(\mu, 1/R-\mu, 1/R+\mu, \ldots, n/R-\mu, n/R+\mu, \ldots)$ are formally infinite-dimensional matrices that contain the Higgs Yukawa couplings and the KK mass eigenvalues, respectively. It is a formidable task to analytically calculate the LHS of Eq. (3.14). Instead, we notice that if one of the following equalities holds true,

$$\mu = 0, \quad a = (0 \text{ or } \pi R), \quad \mathrm{Im}(h_1 h_2^*)^2 = 0, \quad (3.15)$$

the theory is then invariant under $CP$ transformations. Consequently, $CP$ violation requires that $\mu \neq 0$ and a nonzero shifting of the brane, $a \neq 0$, apart from a relative $CP$-violating phase between the original Yukawa couplings $h_1$ and $h_2$. Finally, we should remark that if the $Z_2$ discrete symmetry were not imposed on $N(x,y)$, the resulting Lagrangian would predict a dangerous twofold mass degeneracy in the spectrum of the would-be Majorana KK states that would effectively correspond to the $\mu = 0$ case and, hence, would lead to the absence of $CP$ violation as well.

### B. Leptogenesis model with extended Higgs sector

The second scenario that we will be discussing does not involve the inclusion of any heavy isosinglet mass scale. Instead, in addition to the higher-dimensional Dirac field $N(x,y)$ supplemented by the $Z_2$ discrete symmetry, we shall extend the Higgs sector by two more Higgs doublets that carry the same hypercharge as the SM Higgs doublet. As we will demonstrate below, such an extension of the Higgs potential by three Higgs doublets, denoted hereafter as $\Phi_1$, $\Phi_2$, and $\Phi_3$, is dictated by the necessity of introducing sufficient $L$ and $CP$ violation into the theory. Specifically, this scenario is governed by the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = \int_0^{2\pi R} dy \{ \bar{N}(i\gamma^\mu \partial_\mu + i\gamma_4 \partial_y) N$$

$$+ \delta(y-a) [\bar{h}_1 \bar{L} \tilde{\Phi}_1 \xi + \bar{h}_2 \bar{L} \tilde{\Phi}_2 \eta + \mathrm{H.c.}]$$

$$+ \delta(y-a) [\mathcal{L}_{\mathrm{SM}}'(\Phi_1) + \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3)] \}, \qquad (3.16)$$

where $\tilde{\Phi}_i = i\sigma_2 \Phi_i^*$ $(i=1,2,3)$, and $\mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3)$ and $\mathcal{L}_{\mathrm{SM}}'(\Phi_1)$ describe the Higgs potential and the residual standard part of the model, respectively. Furthermore, the model is invariant under the transformations

$$N \to iN, \quad \Phi_1 \to -i\Phi_1, \quad \Phi_2 \to i\Phi_2, \quad \Phi_3 \to \Phi_3,$$

$$l_R \to -il_R, \quad u_R \to iu_R, \quad d_R \to -id_R, \qquad (3.17)$$

where $l_R$, $u_R$, and $d_R$ denote the right-handed charged leptons, and the up- and down-type quarks, respectively. Obviously, only $\Phi_1$ couples to the observed SM particles, whereas $\Phi_3$ does not couple to matter at all. The discrete symmetry in Eq. (3.17) is very crucial, as it ensures the renormalizability of the model; the discrete symmetry is only broken softly by operators of dimension 2:

$$\mathcal{L}_V^{\mathrm{soft}} = \sum_{i<j=1}^{3} m_{ij}^2 \Phi_i^\dagger \Phi_j + \mathrm{H.c.} \subset \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3). \qquad (3.18)$$

Notice that the Higgs potential of this scenario is very similar to that of Weinberg's three-Higgs-doublet model [30].

One might now naively argue that the third Higgs doublet $\Phi_3$ is not compelling for introducing $CP$ violation into the theory, e.g., $\mathrm{Im}(\bar{h}_1 \bar{h}_2^* m_{12}^2) \neq 0$. However, this is not true. Not-

FIG. 1. Feynman graphs giving rise to UV-finite KK kinetic terms.

withstanding that $\bar{h}_1$ and $\bar{h}_2$ might initially be complex in the basis in which $m_{12}^2$ is real, one can always rephase $L \to e^{i\phi_l}L$ and $N \to e^{i\phi}N$ to make both real. If $\phi_{h_1}$ and $\phi_{h_2}$ denote the phases of the two Higgs Yukawa couplings, these phases can be eliminated by choosing $\phi_l = (\phi_{h_1} + \phi_{h_2})/2$ and $\phi = (\phi_{h_2} - \phi_{h_1})/2$. In this scenario, CP violation gets communicated radiatively to the neutrino sector via bilinear Higgs-mixing effects. To be precise, CP nonconservation in the symmetric phase of the Higgs potential $\mathcal{L}_V$ is manifested by the nonvanishing of the following rephasing-invariant quantity [31]:

$$\text{Im}(m_{12}^2 m_{23}^2 m_{13}^{*2}) \neq 0. \qquad (3.19)$$

In addition, CP violation can only occur on a shifted brane, i.e., $a \neq 0$. The latter amounts to nonzero values for both compactified Higgs Yukawa couplings $\bar{h}_1^{(n)}$ and $\bar{h}_2^{(n)}$.

Proceeding as in Sec. III A, we integrate out the compact coordinate $y$ in Eq. (3.16) to eventually arrive at

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}'(\Phi_1) + \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3) + \bar{\xi}_0(i\bar{\sigma}^\mu \partial_\mu)\xi_0$$

$$+ (\bar{h}_1^{(0)}\tilde{L}\tilde{\Phi}_1\xi_0 + \text{H.c.}) + \sum_{n=1}^{\infty}\left[ \bar{\xi}_n(i\bar{\sigma}^\mu\partial_\mu)\xi_n \right.$$

$$+ \tilde{\eta}_n(i\bar{\sigma}^\mu\partial_\mu)\eta_n - \frac{n}{R}(\xi_n\eta_n + \bar{\xi}_n\bar{\eta}_n) + \sqrt{2}(i\bar{h}_1^{(n)}\tilde{L}\tilde{\Phi}_1\xi_n$$

$$\left. + i\bar{h}_2^{(n)}\tilde{L}\tilde{\Phi}_2\eta_n + \text{H.c.}) \right], \qquad (3.20)$$

where $\bar{h}_1^{(n)}$ and $\bar{h}_2^{(n)}$ are given by Eqs. (3.9) and (3.10), respectively. Observe that the effective Lagrangian in Eq. (3.20) still preserves the original discrete symmetry in Eq. (3.17), where the KK components of $N(x,y)$ transform as $\xi_n \to i\xi_n$ and $\eta_n \to -i\eta_n$. At the tree level, the model predicts an infinite number of KK Dirac states that have masses which are equally spaced by an interval $1/R$. Once radiative corrections are included, however, as shown in Fig. 1, each KK Dirac state splits into a pair of nearly degenerate Majorana neutrinos. In fact, radiative effects induce new UV-finite kinetic terms involving the KK states. The new KK kinetic terms are given by

$$\mathcal{L}_{\text{rad}} = \sum_{n,m=1}^{\infty} \kappa_{nm}\bar{\eta}_n(i\bar{\sigma}^\mu\partial_\mu)\xi_m + \text{H.c.}, \qquad (3.21)$$

where simple dimensional analysis of the Feynman graphs displayed in Fig. 1 suggests[4]

$$\kappa_{nm} \sim \frac{\bar{h}_2^{(n)*}\bar{h}_1^{(m)}}{8\pi^2}\left[\frac{m_{12}^2}{m_{11}^2 + m_{22}^2} + \frac{m_{13}^2 m_{23}^{*2}}{m_{33}^2(m_{11}^2 + m_{22}^2)}\right]. \qquad (3.22)$$

Since $\kappa_{nm} \ll 1/(RM_F)$, we find that to a good approximation, only the diagonal kinetic transitions $\xi_n \to \eta_n$ contribute predominantly to the splitting of a KK Dirac state into a pair of Majorana states. After canonically normalizing the KK kinetic terms, the KK mass spectrum is determined by

$$-\mathcal{L}_{\text{mass}}^{\text{KK}} = \sum_{n=1}^{\infty}\frac{n}{2R}(\chi_1^{(n)}, \chi_2^{(n)})\begin{pmatrix} \dfrac{1}{1+|\epsilon_n|} & 0 \\ 0 & \dfrac{1}{1-|\epsilon_n|} \end{pmatrix}\begin{pmatrix} \chi_1^{(n)} \\ \chi_2^{(n)} \end{pmatrix}$$

$$+ \text{H.c.}, \qquad (3.23)$$

where $\epsilon_n \approx \kappa_{nn}$ and

$$\begin{pmatrix} \xi_n \\ \eta_n \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\phi_\epsilon^n/2} & -e^{-i\phi_\epsilon^n/2} \\ e^{i\phi_\epsilon^n/2} & e^{i\phi_\epsilon^n/2} \end{pmatrix}\begin{pmatrix} \dfrac{1}{\sqrt{1+|\epsilon_n|}} & 0 \\ 0 & \dfrac{1}{\sqrt{1-|\epsilon_n|}} \end{pmatrix}$$

$$\times \begin{pmatrix} \chi_1^{(n)} \\ \chi_2^{(n)} \end{pmatrix}, \qquad (3.24)$$

with $\phi_\epsilon^n = \arg(\epsilon_n)$. From Eq. (3.24), we readily see that the radiatively induced KK Majorana states $\chi_1^{(n)}$ and $\chi_2^{(n)}$ mix strongly with one another, and so form a two-level CP-violating system, namely, a CP-violating resonator. The striking feature of the present scenario is that both the lifting of the dangerous mass degeneracy of the KK Majorana states and CP violation occurs through loop effects. This model of leptogenesis has no analogue in four dimensions, since the inclusion of an explicit heavy Majorana mass is theoretically not necessary.

### C. Hybrid leptogenesis model

We shall now consider a model based on the two scenarios discussed in Secs. III A and III B, in which we include the fermionic bilinears $m\bar{N}N$ and $MN^TC^{(5)-1}N$, as well as the three Higgs doublets $\Phi_1$, $\Phi_2$, and $\Phi_3$. As opposed to the previous two cases, we shall not impose the $Z_2$ discrete

___

[4]We should remark that our renormalization procedure consists of two steps. In the first step, all UV infinities are absorbed by off-diagonal wave function and mixing renormalizations of the KK states in the on-shell scheme [32]. To leading order, such a rescaling does not generally affect the original form of the tree-level effective action. The second step, which is of our interest here, consists of a finite renormalization of the kinetic terms.

symmetry on the bulk Dirac neutrino $N(x,y)$. As we will see, the absence of the $Z_2$ symmetry yields a distinct prediction for the mass spectrum of the KK states. In particular, we find that the heavy mass scales $m$ and $M$ neither decouple completely from the KK mass spectrum nor get replaced by other small quantities of order $1/R$.

The effective Lagrangian of the hybrid model reads

$$
\mathcal{L}_{\text{eff}} = \int_0^{2\pi R} dy \left\{ \bar{N}(i\gamma^\mu \partial_\mu + i\gamma_4 \partial_y)N - m\bar{N}N \right.
$$

$$
- \frac{1}{2}(MN^T C^{(5)-1} N + \text{H.c.})
$$

$$
+ \delta(y-a)[\bar{h}_1 \bar{L} \tilde{\Phi}_1 \xi + \bar{h}_2 \bar{L} \tilde{\Phi}_2 \eta + \text{H.c.}]
$$

$$
\left. + \delta(y-a)[\mathcal{L}'_{\text{SM}}(\Phi_1) + \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3)] \right\}.
$$
(3.25)

The above Lagrangian possesses a global symmetry given by Eq. (3.17) which is only broken softly by the Higgs mass terms in Eq. (3.18) and by $MN^T C^{(5)-1}N$. This is a crucial fact that ensures the renormalizability of the model.

Since periodicity is the only constraint that applies to $N(x,y)$, the five-dimensional two-component spinors $\xi$ and $\eta$ may then be expressed in terms of a Fourier series expansion as follows:

$$
\xi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \xi_n(x) \exp\left(\frac{iny}{R}\right),
$$
(3.26)

$$
\eta(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \eta_n(x) \exp\left(\frac{iny}{R}\right).
$$
(3.27)

Substituting Eqs. (3.26) and (3.27) into the effective Lagrangian (3.25), we find, after $y$ integration,

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}'_{\text{SM}}(\Phi_1) + \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3) + \mathcal{L}_{\text{rad}}
$$

$$
+ \sum_{n=-\infty}^{\infty} \left\{ \bar{\xi}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_n + \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\eta_n \right.
$$

$$
- \left[ \left( m + \frac{in}{R} \right) \xi_n \eta_{-n} + \text{H.c.} \right]
$$

$$
- \frac{1}{2}M(\xi_{-n}\xi_n + \bar{\eta}_{-n}\bar{\eta}_n + \text{H.c.}) + (\bar{h}_1^{(n)} \bar{L} \tilde{\Phi}_1 \xi_n
$$

$$
\left. + \bar{h}_2^{(n)} \bar{L} \tilde{\Phi}_2 \eta_n + \text{H.c.}) \right\},
$$
(3.28)

where

$$
\bar{h}_1^{(n)} = \frac{M_F}{M_P} h_1 \exp\left(\frac{ina}{R}\right), \quad \bar{h}_2^{(n)} = \frac{M_F}{M_P} h_2 \exp\left(\frac{ina}{R}\right).
$$
(3.29)

In Eq. (3.28), $\mathcal{L}_{\text{rad}}$ indicates the UV-finite radiative contributions to the KK kinetic terms, i.e.,

$$
\mathcal{L}_{\text{rad}} = \sum_{n,m=-\infty}^{\infty} \kappa_{n,m} \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_m + \text{H.c.},
$$
(3.30)

where $\kappa_{n,m}$ is given by a formula very analogous to Eq. (3.22). To avoid excessive complication in the calculation, we consider only those radiative terms $\kappa_{n,m}$ that are expected to have a dominant effect on the KK mass spectrum. More explicitly, we have

$$
\mathcal{L}_{\text{rad}} \approx \kappa_{0,0} \bar{\eta}_0(i\bar{\sigma}^\mu \partial_\mu)\xi_0 + \sum_{n=1}^{\infty} \left[ \kappa_{n,n} \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_n \right.
$$

$$
+ \kappa_{-n,-n} \bar{\eta}_{-n}(i\bar{\sigma}^\mu \partial_\mu)\xi_{-n} + \kappa_{n,-n} \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_{-n}
$$

$$
\left. + \kappa_{-n,-n} \bar{\eta}_{-n}(i\bar{\sigma}^\mu \partial_\mu)\xi_n \right] + \text{H.c.}
$$
(3.31)

Notice that all $|\kappa_{n,m}|$ have the same absolute value and do not depend on the indices $n$ and $m$.

To evaluate the masses of the KK neutrino states, it is convenient to write the kinetic part of the KK sector as a sum of two terms:

$$
\mathcal{L}_{\text{kin}}^{\text{KK}} = \mathcal{L}_{n=0}^{\text{KK}} + \mathcal{L}_{n\geqslant 1}^{\text{KK}},
$$
(3.32)

where

$$
\mathcal{L}_{n=0}^{\text{KK}} = (\bar{\xi}_0, \bar{\eta}_0)(i\bar{\sigma}^\mu \partial_\mu)\begin{pmatrix} 1 & \kappa_{0,0} \\ \kappa_{0,0} & 1 \end{pmatrix}\begin{pmatrix} \xi_0 \\ \eta_0 \end{pmatrix} - \frac{1}{2}(\xi_0, \eta_0)
$$

$$
\times \begin{pmatrix} M & m \\ m & M \end{pmatrix}\begin{pmatrix} \xi_0 \\ \eta_0 \end{pmatrix} + \text{H.c.},
$$
(3.33)

$$
\mathcal{L}_{n\geqslant 1}^{\text{KK}} = \sum_{n=1}^{\infty} \left[ (\bar{\xi}_n, \bar{\eta}_n, \bar{\xi}_{-n}, \bar{\eta}_{-n})(i\bar{\sigma}^\mu \partial_\mu) \right.
$$

$$
\times \begin{pmatrix} 1 & \kappa_{n,n}^* & 0 & \kappa_{n,-n}^* \\ \kappa_{n,n} & 1 & \kappa_{n,-n} & 0 \\ 0 & \kappa_{n,-n}^* & 1 & \kappa_{-n,-n}^* \\ \kappa_{n,-n} & 0 & \kappa_{-n,-n} & 1 \end{pmatrix}
$$

$$
\times \begin{pmatrix} \xi_n \\ \eta_n \\ \xi_{-n} \\ \eta_{-n} \end{pmatrix} - \frac{1}{2}(\xi_n, \eta_n, \xi_{-n}, \eta_{-n})
$$

$$
\left. \times \begin{pmatrix} 0 & 0 & M & \tilde{m}_n \\ 0 & 0 & \tilde{m}_n^* & M \\ M & \tilde{m}_n^* & 0 & 0 \\ \tilde{m}_n & M & 0 & 0 \end{pmatrix}\begin{pmatrix} \xi_n \\ \eta_n \\ \xi_{-n} \\ \eta_{-n} \end{pmatrix} + \text{H.c.} \right],
$$
(3.34)

with $\tilde{m}_n = m + (in/R)$ (i.e., $\tilde{m}_0 = m$). From the Lagrangian $\mathcal{L}_{n=0}^{\text{KK}}$ in Eq. (3.33), one obtains two Majorana neutrinos $\chi_1^{(0)}$ and $\chi_2^{(0)}$ with masses

$$m_{\chi_1}^{(0)} = \frac{|M-n|}{|1-\kappa_{0,0}|} \approx |M-m|, \quad m_{\chi_2}^{(0)} = \frac{M+m}{|1+\kappa_{0,0}|} \approx M+m. \tag{3.35}$$

We now turn to the evaluation of the KK neutrino masses, for the more involved case with $n \geq 1$. To this end, we first go to a weak basis in which the mass matrix is real by rephasing the KK fields:

$$\xi_n \to e^{-i\phi_n/2}\xi_n, \quad \eta_n \to e^{i\phi_n/2}\eta_n, \quad \xi_{-n} \to e^{i\phi_n/2}\xi_{-n},$$

$$\eta_{-n} \to e^{-i\phi_n/2}\eta_{-n}, \tag{3.36}$$

with $\phi_n = \arg \tilde{m}_n$. Even though one could always work out the most general case, it is, however, very illuminating to make a further assumption that leads to much simpler analytic results. We assume that all radiative kinetic terms are predominantly real in the new weak basis in Eq. (3.36), i.e., $\mathrm{Im}\,\kappa_{n,m} \ll \mathrm{Re}\,\kappa_{n,m} \approx \epsilon$. Then, considering $m \geq M$ for definiteness, we can diagonalize the Lagrangian in Eq. (3.34) through the canonical transformation

$$\begin{pmatrix} \xi_n \\ \eta_n \\ \xi_{-n} \\ \eta_{-n} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} i & 1 & -i & 1 \\ -i & -1 & -i & 1 \\ -i & 1 & i & 1 \\ i & -1 & i & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & \dfrac{i}{\sqrt{1-2\epsilon}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \dfrac{1}{\sqrt{1+2\epsilon}} \end{pmatrix} \begin{pmatrix} \chi_1^{(n)} \\ \chi_2^{(n)} \\ \chi_1^{(-n)} \\ \chi_2^{(-n)} \end{pmatrix},$$

$$\tag{3.37}$$

which leads to the KK Majorana masses

$$m_{\chi_1}^{(n)} = \sqrt{m^2 + \frac{n^2}{R^2}} - M,$$

$$m_{\chi_2}^{(n)} = \frac{1}{1-2\epsilon}\left( \sqrt{m^2 + \frac{n^2}{R^2}} - M \right),$$

$$m_{\chi_1}^{(-n)} = M + \sqrt{m^2 + \frac{n^2}{R^2}},$$

$$m_{\chi_2}^{(-n)} = \frac{1}{1+2\epsilon}\left( M + \sqrt{m^2 + \frac{n^2}{R^2}} \right). \tag{3.38}$$

Evidently, Eq. (3.38) shows that the immediate effect of radiative corrections is to lift the dangerous twofold mass degeneracy among the KK states $\chi_1^{(\pm n)}$ and $\chi_2^{(\pm n)}$, thus rendering the theory $CP$ violating. If one considers that $m > M$, the mass of the lowest-lying KK state is $m_{\chi_1}^{(0)} \approx m-M$, which can naturally be much bigger than the compactification scale
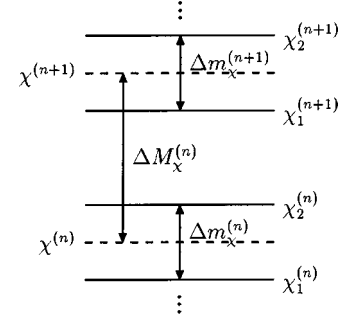


FIG. 2. Schematic representation of the mass spectrum of the KK states.

$1/R$. This is a distinctive feature of the present model, since, unlike the previous two scenarios, the heavy mass scale $m - M$ neither decouples from the complete KK mass spectrum nor gets replaced by quantities of order $1/R$.

## IV. RESONANT $CP$ VIOLATION

In addition to lepton-number violation, $CP$ nonconservation constitutes another important ingredient for leptogenesis. These two necessary conditions satisfy, by construction, the three models of leptogenesis, discussed in the previous section. However, these conditions may not be sufficient to guarantee an appreciable leptonic asymmetry that results from decays of KK Majorana states according to the standard scenario of leptogenesis [18]. In particular, in theories with a low scale of quantum gravity, we have to ensure that the total net effect of the individual $CP$-violating contributions coming from the tower of the nearly degenerate KK Majorana states does not vanish because of some kind of a Glashow, Iliopoulos, and Maiani (GIM) [33] cancellation mechanism. In fact, by making use of such a GIM-type mechanism, we can show that all the $CP$-violating vertex ($\varepsilon'$-type) terms almost cancel pairwise. On the other hand, we find that the interference of the $CP$-violating self-energy ($\varepsilon$-type) contributions is constructive or destructive, depending on the mass spacing of the KK Majorana states.

The mass spectrum of the higher-dimensional models of leptogenesis under discussion consists of an infinite series of pairs of nearly degenerate Majorana neutrinos, which we denote by $\chi_1^{(n)}$ and $\chi_2^{(n)}$. The generic pattern of the KK mass spectrum may be represented by Fig. 2. As we have discussed in Sec. III, the mass difference between $\chi_1^{(n)}$ and $\chi_2^{(n)}$ may be induced either at the tree level,

$$\Delta m_\chi^{(n)} \equiv m_{\chi_2}^{(n)} - m_{\chi_1}^{(n)} = 2\mu, \tag{4.1}$$

or through radiative kinetic terms,

$$\Delta m_\chi^{(n)} \sim \kappa_{nm} m_\chi^{(n)} \sim \frac{h_2^* h_1}{8\pi^2} \frac{M_F^2}{M_P^2} m_\chi^{(n)}, \tag{4.2}$$

with

$$m_\chi^{(n)} \equiv \frac{1}{2}(m_{\chi_1}^{(n)} + m_{\chi_2}^{(n)}). \tag{4.3}$$

Furthermore, the mass difference between two adjacent KK pairs is determined by

$$\Delta M_\chi^{(n)} \equiv m_\chi^{(n+1)} - m_\chi^{(n)} \approx \frac{1}{R} = \left(\frac{M_F}{M_P}\right)^{2/\delta} M_F. \quad (4.4)$$

In deriving the approximate equality in Eq. (4.4), we have implicitly assumed that $m \lesssim n/R$, for the hybrid scenario outlined in Sec. III C. Clearly, if the origin of a nonzero $\Delta m_\chi^{(n)}$ is due to loop effects, one naturally has $\Delta M_\chi^{(n)} \gg \Delta m_\chi^{(n)}$ for any number $\delta$ of extra dimensions. However, if $\Delta m_\chi^{(n)}$ occurs in the Born approximation, then $\Delta m_\chi^{(n)}$ and $\Delta M_\chi^{(n)}$ could be of equal order. As we will see below, the last two quantities determine the magnitude of *CP* violation that originates from the interference of the tower of the KK states.

Let us first consider $\varepsilon'$-type *CP* violation in the decays of KK states. For our illustrations, it is sufficient to assume that the KK states decay predominantly to the SM Higgs doublet $\Phi$ or to the Higgs doublet with the smallest (thermal) mass in the model with the extended Higgs sector. Since $\Delta m_\chi^{(n)} < \Delta M_\chi^{(n)} \ll m_\chi^{(n)}$, the *CP*-violating parameter of our interest is

$$\varepsilon_\chi^{\prime(n)} = \frac{|\mathcal{T}_{\chi_1}^{(n)\varepsilon'}|^2 + |\mathcal{T}_{\chi_2}^{(n)\varepsilon'}|^2 - |\bar{\mathcal{T}}_{\chi_1}^{(n)\varepsilon'}|^2 - |\bar{\mathcal{T}}_{\chi_2}^{(n)\varepsilon'}|^2}{|\mathcal{T}_{\chi_1}^{(n)\varepsilon'}|^2 + |\mathcal{T}_{\chi_2}^{(n)\varepsilon'}|^2 + |\bar{\mathcal{T}}_{\chi_1}^{(n)\varepsilon'}|^2 + |\bar{\mathcal{T}}_{\chi_2}^{(n)\varepsilon'}|^2}, \quad (4.5)$$

where we used the shorthand notation for the transition amplitudes: $\mathcal{T}_{\chi_1}^{(n)\varepsilon'} = \mathcal{T}^{\varepsilon'}(\chi_1^{(n)} \to L\Phi^\dagger)$, $\bar{\mathcal{T}}_{\chi_1}^{(n)\varepsilon'} = \mathcal{T}^{\varepsilon'}(\chi_1^{(n)} \to L^C\Phi)$, and likewise for $\chi_2^{(n)}$. In all these amplitudes, only vertex diagrams are included. For simplicity, we assume that $h_1^{(k)}$ and $h_2^{(k)}$ are independent of $k$, although the most general case does not depend on this particular assumption. The parameter $\varepsilon_\chi^{\prime(n)}$ is then found to be

$$\varepsilon_\chi^{\prime(n)} = \frac{1}{8\pi(|h_1^{(n)}|^2 + |h_2^{(n)}|^2)} \sum_k \mathrm{Im}(h_1^{(n)*} h_2^{(k)})^2$$
$$\times \left[ f\left(\frac{m_{\chi_2}^{(k)2}}{m_{\chi_1}^{(n)2}}\right) - f\left(\frac{m_{\chi_1}^{(k)2}}{m_{\chi_2}^{(n)2}}\right) \right], \quad (4.6)$$

with

$$f(x) = \sqrt{x}\left[1 - (1+x)\ln\left(1 + \frac{1}{x}\right)\right]. \quad (4.7)$$

Note that the range of summation over the KK states explicitly depends on the model. Equation (4.6) may further be approximated as

$$\varepsilon_\chi^{\prime(n)} \approx \frac{1}{4\pi(|h_1^{(n)}|^2 + |h_2^{(n)}|^2)} \sum_k \mathrm{Im}(h_1^{(n)*} h_2^{(k)})^2$$
$$\times \left(\frac{\Delta m_\chi^{(n)}}{m_\chi^{(n)}} + \frac{\Delta m_\chi^{(k)}}{m_\chi^{(k)}}\right) \frac{m_\chi^{(k)2}}{m_\chi^{(n)2}} f'\left(\frac{m_\chi^{(k)2}}{m_\chi^{(n)2}}\right), \quad (4.8)$$

where $f'(x)$ is the derivative of the function $f(x)$, i.e.,

$$f'(x) = \frac{3}{2\sqrt{x}}\left[1 - \left(\frac{2}{3} + x\right)\ln\left(1 + \frac{1}{x}\right)\right]. \quad (4.9)$$

It is obvious that each individual KK term in Eq. (4.8) is suppressed by a factor $\Delta m_\chi^{(k)}/m_\chi^{(k)}$. This is a generic consequence of a GIM-type cancellation mechanism, as a result of interference between two pseudo-Dirac neutrinos.

To explicitly demonstrate that $\varepsilon'$-type contributions to *CP* violation are indeed small, it is instructive to offer an estimate for the sum over the KK states in Eq. (4.8), after making a few plausible assumptions. For simplicity, we consider a theory with one additional spatial dimension ($\delta = 1$), and further assume that the mass differences $\Delta m_\chi^{(k)}$ (or $\Delta m_\chi^{(k)}/m_\chi^{(k)}$) and $\Delta M_\chi^{(k)}$ are independent of $k$. In addition, we convert the sum over the KK states $k$ to an energy integral which has an UV cutoff at the fundamental scale of quantum gravity $M_F$. With these considerations, we obtain

$$\varepsilon_\chi^{\prime(n)} \approx -\frac{\mathrm{Im}(h_1^{(n)*} h_2^{(n)})^2}{16\pi(|h_1^{(n)}|^2 + |h_2^{(n)}|^2)} \frac{\Delta m_\chi^{(n)}}{\Delta M_\chi^{(n)}}$$
$$\approx -\frac{\mathrm{Im}(h_1^* h_2)^2}{16\pi(|h_1|^2 + |h_2|^2)} \frac{M_F^2}{M_P^2} \frac{\Delta m_\chi^{(n)}}{\Delta M_\chi^{(n)}}, \quad (4.10)$$

where we have used that $f'(x) \approx -1/(2x^{3/2})$, for $x \gg 1$. The above exercise shows that $\varepsilon_\chi^{\prime(n)}$ is extremely suppressed by the extra-dimensional volume factor $M_F^2/M_P^2$ and, in certain scenarios, by the ratio $\Delta m_\chi^{(n)}/\Delta M_\chi^{(n)} \ll 1$. Consequently, we can safely neglect $\varepsilon'$-type *CP* violation in the decays of the KK Majorana states.

In the following, we shall focus our attention on the self-energy ($\varepsilon$-type) contribution to *CP* violation. In analogy to Eq. (4.5), the relevant measure of $\varepsilon$-type *CP* violation may be defined by

$$\varepsilon_\chi^{(n)} = \frac{|\mathcal{T}_{\chi_1}^{(n)\varepsilon}|^2 + |\mathcal{T}_{\chi_2}^{(n)\varepsilon}|^2 - |\bar{\mathcal{T}}_{\chi_1}^{(n)\varepsilon}|^2 - |\bar{\mathcal{T}}_{\chi_2}^{(n)\varepsilon}|^2}{|\mathcal{T}_{\chi_1}^{(n)\varepsilon}|^2 + |\mathcal{T}_{\chi_2}^{(n)\varepsilon}|^2 + |\bar{\mathcal{T}}_{\chi_1}^{(n)\varepsilon}|^2 + |\bar{\mathcal{T}}_{\chi_2}^{(n)\varepsilon}|^2}. \quad (4.11)$$

Correspondingly, $\mathcal{T}_{\chi_{1,2}}^{(n)\varepsilon}$ ($\bar{\mathcal{T}}_{\chi_{1,2}}^{(n)\varepsilon}$) indicate the decays $\chi_{1,2}^{(n)} \to L\Phi^\dagger$ ($\chi_{1,2}^{(n)} \to L^C\Phi$), where only the self-energy graph has been taken into account. Since each $\varepsilon$-type term is proportional to the mass difference of the KK states involved, one has to avoid the self-energy transitions $\chi_1^{(n+1)} \to \chi_2^{(n+1)}$ cancelling against the transitions $\chi_1^{(n+1)} \to \chi_2^{(n)}$. From Fig. 2, we may schematically deduce the condition for destructive interference among adjacent KK states, which translates into the relation

$$\Delta m_\chi^{(n)} \sim \frac{1}{2}\Delta M_\chi^{(n)}. \quad (4.12)$$

Instead, if

$$\Delta M_\chi^{(n)} \gg \Delta m_\chi^{(n)}, \quad (4.13)$$

the interference of two neighboring KK states is constructive. Employing the resummation approach to the mixing of unstable particles which was developed in [24,34], we find that

$$
\varepsilon_\chi^{(n)} = \frac{2\,\mathrm{Im}(h_1^{(n)}h_2^{(n)*})^2}{(|h_1^{(n)}|^2+|h_2^{(n)}|^2)^2}\,\frac{\Delta m_\chi^{(n)}(\Gamma_{\chi_1}^{(n)}+\Gamma_{\chi_2}^{(n)})}{(\Delta m_\chi^{(n)})^2+\frac{1}{4}\Gamma_{\chi_2}^{(n)2}}
$$

$$
\times\left[1+\frac{(\Delta m_\chi^{(n)})^2+\frac{1}{4}\Gamma_{\chi_2}^{(n)2}}{(\Delta m_\chi^{(n)})^2+\frac{1}{4}\Gamma_{\chi_1}^{(n)2}}\right],
\tag{4.14}
$$

where

$$
\Gamma_{\chi_1}^{(n)}=\frac{1}{8\pi}|h_1^{(n)}|^2 m_{\chi_1}^{(n)},\quad \Gamma_{\chi_2}^{(n)}=\frac{1}{8\pi}|h_2^{(n)}|^2 m_{\chi_2}^{(n)}
\tag{4.15}
$$

are the decay widths of $\chi_1^{(n)}$ and $\chi_2^{(n)}$, respectively. In Eq. (4.14), we have neglected contributions to $\varepsilon_\chi^{(n)}$ of order $\Delta m_\chi^{(n)}/\Delta M_\chi^{(n)}$ [cf. Eq. (4.13)] and $\Delta m_\chi^{(n)}/m_\chi^{(n)}$.

In agreement with Ref. [24], we observe that the $CP$-violating parameter $\varepsilon_\chi^{(n)}$ given by Eq. (4.14) can be of order 1, if the two conditions

$$
\text{(i)}\quad \delta_{CP}^{(n)}\equiv\frac{|\mathrm{Im}(h_1^{(n)}h_2^{(n)*})^2|}{|h_1^{(n)}|^2|h_2^{(n)}|^2}\sim 1,
$$

$$
\text{(ii)}\quad \Delta m_\chi^{(n)}\sim\frac{\Gamma_{\chi_1}^{(n)}}{2}\ \text{ or }\ \frac{\Gamma_{\chi_2}^{(n)}}{2}
\tag{4.16}
$$

are satisfied. The first condition is rather model dependent. *A priori* there is no reason to believe that the phases of the original Yukawa couplings $h_1$ and $h_2$ should somehow be aligned and, as a result of this, the parameter $\delta_{CP}^{(n)}$ must be suppressed. Thus, we consider $\delta_{CP}^{(n)}\sim 1$.

In a general model, it is more difficult, however, to theoretically justify the second condition, as the mass splitting of the mixed particles involved must be of the order of their widths. For the scenarios discussed in Secs. III B and III C, the mass splitting $\Delta m_\chi^{(n)}$ is radiatively induced by integrating out the Higgs interactions and then canonically normalizing the resulting kinetic terms. Thus the width of the KK Majorana states and their respective mass difference formally occur at the same electroweak loop order. Therefore, the second condition is naturally implemented for these two models. For the leptogenesis model described in Sec. III A, one has to assume that $\mu\sim\Gamma_{\chi_1}^{(n)}$ or $\Gamma_{\chi_2}^{(n)}$. As a consequence of compactification of the extra large dimensions, the mass parameter $\mu$ always comes out to be smaller than $\Delta M_\chi^{(n)}$, so some degree of tuning $\mu$ to even smaller values is required in this case.

The models of leptogenesis we have been studying share the generic feature that each KK Dirac state decomposes into a pair of nearly degenerate Majorana neutrinos. Such a pair of KK Majorana neutrinos forms a strongly mixed two-level system that exhibits $CP$ violation of order unity; such a system was called a $CP$-violating resonator. In fact, it was shown in [24] that the resonant enhancement of $CP$ violation

is driven by the nondiagonalizable (Jordan-like) form of the effective Hamiltonian (or equivalently the resummed propagator) of the two-level system, which satisfies conditions very analogous to those of Eq. (4.16). Finally, we should stress that the constructive interference of all the individual KK $CP$-violating resonators is assured on the basis of the requirement given by Eq. (4.13). This last requirement is more naturally implemented in the models of leptogenesis with extended Higgs sector (see also discussion in Secs. III B and III C).

## V. BARYONIC ASYMMETRY OF THE UNIVERSE

Astronomical observations give strong evidence that the present universe mainly consists of matter rather antimatter, viz., the universe possesses an excess in the $B$ number. The observed $B$ asymmetry may be quantified by the nonzero baryon-number–to–entropy ratio of densities [26],

$$
Y_{\Delta B}=\frac{n_{\Delta B}}{s}=(0.6\text{–}1)\times 10^{-10},
\tag{5.1}
$$

where $n_{\Delta B}=n_B-n_{\bar B}\approx n_B$ and $s$ is the entropy density. As we mentioned in the Introduction, an attractive solution that could account for the nonzero value of $Y_{\Delta B}$ by making use of all the necessary conditions imposed by Sakharov [35] may be given by means of the scenario of baryogenesis through leptogenesis [18]. Based on an analysis of chemical potentials [36], one may derive that

$$
Y_{\Delta B}(T>T_c)=\frac{8N_F+4N_H}{22N_F+13N_H}\,Y_{\Delta(B-L)}.
\tag{5.2}
$$

Almost independently of the numbers $N_F$ and $N_H$ of flavors and Higgs doublets, one finds that approximately one-third of the initial $B-L$ and/or $L$ asymmetry will be reprocessed into an asymmetry in $B$, provided sphalerons are in thermal equilibrium. If the reheat temperature $T_r$ is smaller than the critical temperature $T_c$, sphalerons are out of equilibrium, and the above $L$-to-$B$ conversion will be exponentially suppressed by a factor $\exp(-T_c/T_r)$ [26].

Let us first consider the constraints on the parameter space of the leptogenesis models coming from Sakharov's requirement that $L$- or $(B-L)$-violating processes, such as decays of KK Majorana modes, must be out of thermal equilibrium in an expanding universe. As was discussed by Abel and Sarkar [37], the presence of low-lying KK states drastically influences the evolution of the universe, as the number of the relativistic degrees of freedom increases with temperature $T$. To be more precise, if $m_{\chi_1}^{(0)}\equiv m_{\min}$ represents the mass of the lowest KK state in a given model of leptogenesis, the number of relativistic KK states below $T$ is then roughly given by $[(T-m_{\min})R]^\delta$, where $\delta$ is the number of large compact dimensions. Thus the number of active degrees of freedom at a given temperature $T$ is determined by

$$g(T) \approx g_* + S_\delta \theta(T - m_{\min}) [(T - m_{\min}) R]^\delta$$

$$\approx g_* + S_\delta \theta(T - m_{\min}) \left( \frac{M_P}{M_F} \right)^2 \left( \frac{T - m_{\min}}{M_F} \right)^\delta, \quad (5.3)$$

where $g_* \approx 100$ is the number of active degrees of freedom in usual four-dimensional extensions of the SM, and $S_\delta = 2\pi^{\delta/2}/\Gamma(\delta/2)$ is the surface area of a $(\delta+1)$-dimensional sphere of unit radius. From Eq. (5.3), we see that the part of $g(T)$ modified by the presence of KK states, $g_{KK}(T)$, can generally be much larger than $g_*$, unless $m_{\min} \sim T_c$, or $M_F$ and $\delta$ are sufficiently high for some specific model. For instance, in the hybrid leptogenesis model, one may have $m_{\min} \approx m - M > T_c$, and $g_{KK}(T)$ is of order $g_*$ for $T \gtrsim T_c$.

Sakharov's requirement that all $B$- and, because of possible equilibrated sphaleron interactions, $L$-violating processes should be out of thermal equilibrium translates into the approximate inequality for the total $T$-dependent decay rate of the KK states:

$$\Gamma_\chi(T) \equiv \sum_{n=\mathrm{int}(m_{\min}R)}^{\mathrm{int}(TR)} \Gamma_\chi^{(i)} \lesssim 2H(T), \quad (5.4)$$

where $n = (n_1, n_2, \ldots, n_\delta)$, $\Gamma_\chi^{(n)} = \frac{1}{2}(\Gamma_{\chi_1}^{(n)} + \Gamma_{\chi_2}^{(n)})$ is the average decay width of the $n$th $CP$-violating resonator, and

$$H(T) = 1.73 g^{1/2}(T) \frac{T^2}{M_P} \approx 1.73 S_\delta^{1/2} \left( \frac{T - m_{\min}}{M_F} \right)^{\delta/2} \frac{T^2}{M_F}. \quad (5.5)$$

The last approximate equality holds true, provided $g_{KK}(T) \gg g_*$. Converting the multidimensional sum over the KK modes in Eq. (5.4) into an integral, we find that

$$\Gamma_\chi(T) \approx \frac{|h_1|^2 + |h_2|^2}{16\pi^2} \frac{S_\delta}{\delta} \left( \frac{T - m_{\min}}{M_F} \right)^{\delta+1} M_F. \quad (5.6)$$

An immediate result of the out-of-equilibrium condition in Eq. (5.4) is the constraint

$$\frac{1}{2}(|h_1|^2 + |h_2|^2) \lesssim 32\pi^2 S_\delta^{-1/2} \frac{T^2}{M_F^2} \left( \frac{T - m_{\min}}{M_F} \right)^{-1 - \delta/2}, \quad (5.7)$$

which no longer depends on $M_P$. From Eq. (5.7), it is interesting to see that no serious arrangement of the parameters is necessary for all $\delta \geq 1$ and $m_{\min} < T < M_F$, even if the original Yukawa couplings $h_1$ and $h_2$ in Eq. (3.5) are taken to be of order 1. This should be contrasted with the extremely tight limits on the Yukawa couplings in conventional four-dimensional models [24], namely, $h_{1,2} \lesssim 10^{-6}$. These limits are obtained if one sets $m_{\min} = \delta = 0$, $T = 0.2 - 1$ TeV, and $M_F = M_P$, and replaces $S_\delta$ by $1/g_*$ in Eq. (5.7).

It is interesting to derive the time evolution of the universe as a function of its temperature in higher-dimensional theories. We assume that the Friedmann-Robertson-Walker metric governs the expansion of the universe after inflation [26], and that all active relativistic degrees of freedom are in chemical equilibrium and therefore have the same temperature. Imposing entropy conservation, i.e.,

$$sR^3 \propto g(T) T^3 R^3 = \mathrm{const}, \quad (5.8)$$

and differentiating with respect to time $t$, we find that

$$H \equiv \frac{1}{R} \frac{dR}{dt} = -\frac{\delta+3}{3} \frac{1}{T} \frac{dT}{dt}, \quad (5.9)$$

for $g_{KK}(T) \gg g_*$. If we differentiate the Hubble variable in Eq. (5.5) with respect to $t$ and employ Eq. (5.9), we arrive at the differential equation

$$\frac{dH}{dt} = -\frac{3}{2} \frac{\delta+4}{\delta+3} H^2. \quad (5.10)$$

Considering as initial condition $H(t \to 0) \to \infty$, the solution of Eq. (5.10) reads

$$t(T) = \frac{2}{3} \frac{\delta+3}{\delta+4} \frac{1}{H(T)} \approx (7.6 \times 10^{-28} \ \mathrm{sec}) \frac{1}{S_\delta^{1/2}} \frac{\delta+3}{\delta+4} \left( \frac{\mathrm{TeV}}{M_F} \right)$$

$$\times \left( \frac{T}{M_F} \right)^{-2 - \delta/2}. \quad (5.11)$$

If $g_{KK}(T) \sim g_*$, which happens for temperatures $T \lesssim m_{\min} + (M_F/M_P)^{2/\delta} g_*^{1/\delta} M_F$, the time-temperature relation (5.11) goes over into the canonical four-dimensional form

$$t(T) = \frac{1}{2H(T)} \approx (2.3 \ \mathrm{sec}) \times g_*^{-1/2} \left( \frac{\mathrm{MeV}}{T} \right)^2. \quad (5.12)$$

Comparing the $T$ dependences in Eqs. (5.11) and (5.12), one readily sees that the presence of large compact dimensions drastically changes the evolution of the universe.

We shall now attempt to give an estimate of the baryonic asymmetry that results from a sphaleron-converted leptonic asymmetry in KK-neutrino decays, with masses $m_\chi^{(n)} > T_c \sim M_\Phi$, including possible suppression factors due to a low reheat temperature $T_r$. As a starting point, we assume that $n_\chi^{(n)}(T) \approx n_\gamma(T)$, for $m_\chi^{(n)} \lesssim T \lesssim M_F$, where $n_\chi^{(n)}(T)$ is the number density of the $n$th KK pair of Majorana states and $n_\gamma(T) \approx 2.4 T^3/\pi^2$ is the respective number density for photons. The dominant contribution to the $L$ asymmetry is expected to be encoded in the $n$th $CP$-violating resonator at $T \approx m_\chi^{(n)}$ for $m_\chi^{(n)} > T_c \sim M_\Phi$, $m_{\min}$, when the equilibrium number density of the $n$th KK pair is of the order of $n_\chi^{(n)}(T)$ [26]. Thus, the $n$th $CP$-violating resonator gives rise to a leptonic excess

$$Y_{\Delta L}^{(n)} \approx \frac{\varepsilon_\chi^{(n)} n_\chi^{(n)}(m_\chi^{(n)})}{s(m_\chi^{(n)})} \approx \frac{\varepsilon_\chi^{(n)}}{g(m_\chi^{(n)})}. \quad (5.13)$$

In deriving the last step of Eq. (5.13), we have used that $s(T) \approx g(T) n_\gamma$. Since low-scale quantum gravity theories are plagued by the low reheat temperature problem, i.e., $T_r \ll T_c$, the conversion of an $L$-to-$B$ asymmetry can only proceed via sphaleron interactions, which are out of thermal equilibrium. Such an $L$-to-$B$ conversion mediated by out-of-equilibrium sphalerons may be taken into account by multiplying the RHS of Eq. (5.13) with an exponentially sup-

pressed factor $\exp(-T_c/T_r)$ [26]. In this qualitative picture, the total $B$ asymmetry may be estimated by

$$Y_{\Delta B} \approx -\frac{1}{3} e^{-T_c/T_r} \sum_{n=\text{int}(T_{\min}R)}^{\text{int}(M_F R)} Y_{\Delta L}^{(n)}. \qquad (5.14)$$

In the leptogenesis models under discussion, all the individual $CP$-violating asymmetries $\varepsilon_\chi^{(n)}$ are of the same order, i.e., $\varepsilon_\chi^{(n)} = -\varepsilon_\chi$ for all $n$, and their net effect is constructive, as long as the condition in Eq. (4.13) is satisfied. For generality, let us assume that the interference of the $CP$-violating resonators is constructive up to an energy scale $M_F' \lesssim M_F$. Approximating the sum over the KK states in Eq. (5.14) by an integral, we obtain

$$Y_{\Delta B} \approx \frac{1}{3} e^{-T_c/T_r} S_\delta \varepsilon_\chi \ln\left(\frac{M_F'}{T_{\min}}\right), \qquad (5.15)$$

with $T_{\min} = \max(T_r, m_{\min})$. From Eq. (5.15), we find that the generated BAU, $Y_{\Delta B}$, does crucially depend on $T_r$. Considering resonant conditions for $CP$ violation, i.e., $\varepsilon_\chi \approx 1$, one needs $T_c/T_r \lesssim 20$ in order to generate a baryonic asymmetry at the observed level, namely, $Y_{\Delta B} \approx 10^{-10}$. Thus, if the critical temperature is $T_c \approx 100$ GeV, then a reheat temperature as low as 5–10 GeV would be sufficient to account for the BAU, through the mechanism of baryogenesis through leptogenesis. According to estimates in [9,12], to get a reheat temperature as high as 10 GeV in a theory with $\delta = 6$ large extra dimensions, one must have at least $M_F \approx 100$ TeV, while for $\delta = 4$ and 2 extra dimensions, the scale of quantum gravity $M_F$ must be larger than $10^4$ and $10^6$ TeV, respectively.

Another difficulty of the leptogenesis models we have been discussing is that the late decays of the low KK neutrinos may distort the abundances related to the light elements $^4$He, D, $^7$Li, etc. Of course, for sufficiently large $M_F$ and/or $\delta$ values, the lowest KK state, with mass of order $1/R \sim (M_F/M_P)^{2/\delta} M_F$, will be heavy enough to decay just before nucleosynthesis. This may reintroduce a mild hierarchy problem in the parameters of the theory, in case we wish to identify $M_F$ with the scale of soft-supersymmetry breaking [5]. Therefore, among the leptogenesis models which were discussed in Sec. III, the hybrid scenario represents the most attractive solution to this problem, as the lowest KK state, with mass $m_{\min} = m - M$, can be made heavy enough in order to decay sufficiently rapidly. Whether such a scenario can be embedded to a more general supersymmetric theory is an issue which we shall not address in the present work.

## VI. CONCLUSIONS

We have studied the scenario of baryogenesis through leptogenesis in theories with large compact dimensions. The formulation of these theories requires the extension of the notion of the Majorana spinor to multidimensional Minkowski spaces. We have reviewed this topic in Sec. II. In particular, it was shown that genuine massive Majorana neutrinos exist in 2, 3, and 4 mod 8 dimensions only. This limitation is due to the lack of finding Clifford-algebra representations that satisfy the Majorana properties in any number of dimensions. In Sec. III, we have formulated minimal models of leptogenesis which are renormalizable if a finite number of KK states are considered. Such a truncation of the number of the KK states may not be very unrealistic, as the fundamental scale of quantum gravity, $M_F$, is expected to play the role of an UV cutoff.

The leptogenesis models that we have been discussing initially furnish the field content of the theory with an infinite series of KK Dirac states. Subsequently, each KK Dirac state splits into pairs of nearly degenerate Majorana neutrinos. Such a mass splitting occurs either at the tree level after compactification of the extra dimensions or, more interestingly, at the one-loop level by integrating out the Higgs interactions of an extended Higgs sector. As a consequence, each pair of the Majorana neutrinos behaves as a $CP$-violating resonator; i.e., it becomes a strongly mixed two-level system producing a leptonic $CP$ asymmetry of order unity. Depending on the mass difference between two adjacent pairs of KK Majorana neutrinos, the tower of the $CP$-violating resonators may have a constructive or destructive interference. In Sec. IV, we have found that such an interference is constructive, if the level spacing between any two nearby pairs of KK Majorana neutrinos is much bigger than the mass difference of the Majorana neutrinos within each pair [cf. Eq. (4.13)].

In Sec. V, we have seen that the KK Majorana neutrinos mostly decay out of thermal equilibrium in theories with large compact dimensions. Based on the aforementioned $CP$-violating mechanism, the resulting leptonic asymmetry is of order unity. However, in theories with a low scale of quantum gravity [9,11], gravitational interactions play an essential role, as they generically lead to low reheat temperatures, much below $T_c$. Then, the conversion of an $L$ into $B$ asymmetry is exponentially suppressed, as sphalerons are out of thermal equilibrium. In such a cosmological framework, the upper bound on the reheat temperature compatible with baryogenesis may be reduced by almost one order of magnitude relative to $T_c$. In fact, we can estimate that a reheat temperature $T_r$ as low as 5–10 GeV would be sufficient to account for the BAU through out-of-equilibrium sphaleron interactions. The later leads to lower limits: $M_F \gtrsim 10^6$, $10^4$ and 100 TeV for $\delta = 2$, 4, and 6 large extra dimensions, respectively. An important virtue of leptogenesis, when compared to the usual scenarios of baryogenesis in low-string scale theories, is that one does not need to worry about suppressing $B$-violating interactions which might lead to observable proton decays [38]. For this reason, we believe that embedding the minimal scenarios of leptogenesis that we have studied here into more realistic models of inflation constitutes an interesting issue for future investigations.

[1] E. Witten, Nucl. Phys. **B471**, 135 (1996).

[2] P. Hořava and E. Witten, Nucl. Phys. **B460**, 506 (1996); **B475**, 94 (1996).

[3] J. D. Lykken, Phys. Rev. D **54**, 3693 (1996).

[4] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); I. Antoniadis, K. Benakli, and M. Quiros, *ibid.* **331**, 313 (1994).

[5] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **436**, 257 (1998).

[6] See, also, K. R. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436**, 55 (1998); Nucl. Phys. **B537**, 47 (1999).

[7] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and J. March-Russell, hep-ph/9811448.

[8] K. R. Dienes, E. Dudas, and T. Gherghetta, hep-ph/9811428.

[9] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).

[10] N. Kaloper and A. Linde, Phys. Rev. D **59**, 101303 (1999); T. Banks, M. Dine, and A. Nelson, J. High Energy Phys. **06**, 014 (1999); C. Csaki, M. Graesser, and J. Terning, Phys. Lett. B **456**, 16 (1999); A. Riotto, hep-ph/9904485; J. M. Cline, hep-ph/9904495.

[11] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and J. March-Russell, hep-ph/9903224.

[12] K. Benakli and S. Davidson, Phys. Rev. D **60**, 025004 (1999).

[13] S. Cullen and M. Perelstein, Phys. Rev. Lett. **83**, 268 (1999).

[14] L. J. Hall and D. Smith, Phys. Rev. D **60**, 085006 (1999).

[15] R. Sundrum, Phys. Rev. D **59**, 085010 (1999); N. Arkani-Hamed, S. Dimopoulos, and J. March-Russell, *ibid.* (to be published), hep-th/9809124.

[16] G. Dvali and A. Yu. Smirnov, hep-ph/9904211.

[17] G. F. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys. **B544**, 3 (1999); T. Han, J. D. Jykken, and R.-J. Zhang, Phys. Rev. D **59**, 105006 (1999); J. L. Hewett, Phys. Rev. Lett. **82**, 4765 (1999); E. A. Mirabelli, M. Perlstein, and M. E. Peskin, *ibid.* **82**, 2236 (1999); S. Nussinov and R. Shrock, Phys. Rev. D **59**, 105002 (1999); T. G. Rizzo, *ibid.* **59**, 115010 (1999); K. Agashe and N. G. Deshpande, Phys. Lett. B **456**, 60 (1999); M. L. Graesser, hep-ph/9902310; P. Nath and M. Yamaguchi, hep-ph/9902323; Phys. Rev. D (to be published), hep-ph/9903298; M. Masip and A. Pomarol, *ibid.* **60**, 096005 (1999); K. Cheung and W.-Y. Keung, *ibid.* (to be published), hep-ph/9903294; P. Mathews, S. Raychaudhuri, and K. Sridhar, hep-ph/9904232; hep-ph/9905395.

[18] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986); for recent reviews, see A. Pilaftsis, Int. J. Mod. Phys. A **14**, 1811 (1999); A. Riotto, hep-ph/9901362.

[19] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. **155B**, 36 (1985).

[20] J. Scherk, in *Recent Developments in Gravitation*, proceedings of a summer institute held in Cargèse, France, 1978, edited by M. Lévy and S. Deser (Plenum, New York, 1979), p. 479; S. Weinberg, Phys. Lett. **143B**, 97 (1984); C. Wetterich, Nucl. Phys. **B234**, 413 (1984); K. Shimizu, Prog. Theor. Phys. **74**, 610 (1985); R. Finkelstein and M. Villasante, Phys. Rev. D **31**, 425 (1985); S. Chadha and M. Daniel, Rutherford Report No. RAL-85-040, Oxford, 1985.

[21] P. van Nieuwenhuizen, in *Supergravity 1981*, proceedings of a conference held in Trieste, Italy, 1981, edited by S. Ferrara and J. Taylor (Cambridge University Press, Cambridge, England, 1982), p. 151.

[22] M. A. Luty, Phys. Rev. D **45**, 455 (1992); C. E. Vayonakis, Phys. Lett. B **286**, 92 (1992); R. N. Mohapatra and X. Zhang, Phys. Rev. D **46**, 5331 (1992); K. Enqvist and I. Vilja, Phys. Lett. B **299**, 281 (1993); G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, *ibid.* **315**, 325 (1993); H. Murayama, H. Suzuki, T. Yanagida, and J. Yokoyama, Phys. Rev. Lett. **70**, 1912 (1993); T. Gherghetta and G. Jungman, Phys. Rev. D **48**, 1546 (1993); A. Acker, H. Kikuchi, E. Ma, and U. Sarkar, *ibid.* **48**, 5006 (1993); M. P. Worah, *ibid.* **53**, 3902 (1996); W. Buchmüller and T. Yanagida, Phys. Lett. B **445**, 399 (1999).

[23] J. Liu and G. Segrè, Phys. Rev. D **48**, 4609 (1993); M. Flanz, E. A. Paschos, and U. Sarkar, Phys. Lett. B **345**, 248 (1995); L. Covi, E. Roulet, and F. Vissani, *ibid.* **384**, 169 (1996); M. Flanz, E. A. Paschos, U. Sarkar, and J. Weiss, *ibid.* **389**, 693 (1996); L. Covi and E. Roulet, *ibid.* **319**, 113 (1997); E. Kh. Akhmedov, V. A. Rubakov, and A. Yu. Smirnov, Phys. Rev. Lett. **81**, 1359 (1998); E. Ma and U. Sarkar, *ibid.* **80**, 5716 (1998); J. Faridani, S. Lola, P. J. O'Donnell, and U. Sarkar, Eur. Phys. J. C **7**, 543 (1999); G. Lazarides, Q. Shafi, and N. Vlachos, Phys. Lett. B **427**, 53 (1998); W. Buchmüller and M. Plümacher, *ibid.* **431**, 354 (1998); J. Ellis, S. Lola, and D. V. Nanopoulos, *ibid.* **452**, 87 (1999); G. Lazarides, *ibid.* **452**, 227 (1999); J.-M. Frère, F.-S. Ling, M. H. G. Tytgat, and V. Van Elewyck, Phys. Rev. D **60**, 016005 (1999); E. Ma, M. Raidal, and U. Sarkar, hep-ph/9901406; M. S. Berger and B. Brahmachari, Phys. Rev. D **60**, 073009 (1999); G. Lazarides and N. D. Vlachos, hep-ph/9903511; G. F. Giudice, M. Peloso, A. Riotto, and I. Tkachev, hep-ph/9905242.

[24] A. Pilaftsis, Phys. Rev. D **56**, 5431 (1997); Nucl. Phys. **B504**, 61 (1997).

[25] For related phenomena of resonant *CP* violation at colliders, see A. Pilaftsis, Z. Phys. C **47**, 95 (1990); A. Pilaftsis and M. Nowakowski, Phys. Lett. B **245**, 185 (1990); Int. J. Mod. Phys. A **9**, 1097 (1994); R. Cruz, B. Grzadkowski, and J. F. Gunion, Phys. Lett. B **289**, 440 (1992); A. S. Joshipura and S. D. Rindani, Phys. Rev. D **46**, 3008 (1992); D. Atwood, G. Eilam, A. Soni, R. R. Mendel, and R. Migneron, Phys. Rev. Lett. **70**, 1364 (1993); T. Arens and L. M. Sehgal, Phys. Rev. D **51**, 3525 (1995); A. Skjold and P. Osland, Nucl. Phys. **B453**, 3 (1995); A. Pilaftsis, Phys. Rev. Lett. **77**, 4996 (1996); D. Bowser-Chao and W.-Y. Keung, Phys. Rev. D **56**, 3924 (1997); N. Arkani-Hamed, J. L. Feng, L. J. Hall, and H.-C. Cheng, Nucl. Phys. **B505**, 3 (1997); S.-Y. Choi and M. Drees, Phys. Rev. Lett. **81**, 5509 (1998); W. Bernreuther, A. Brandenburg, and M. Flesch, hep-ph/9812387.

[26] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, New York, 1990).

[27] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **125B**, 136 (1983).

[28] R. Jackiw and C. Rebbi, Phys. Rev. D **13**, 3398 (1976); E. Weinberg, *ibid.* **24**, 2669 (1981).

[29] J. Bernabéu, G. C. Branco, and M. Gronau, Phys. Lett. **169B**, 243 (1986); G. C. Branco, M. N. Rebelo, and J. W. F. Valle, Phys. Lett. B **225**, 385 (1989).

[30] S. Weinberg, Phys. Rev. D **42**, 860 (1990).

[31] L. Lavoura and J. P. Silva, Phys. Rev. D **50**, 4619 (1994).

[32] See, e.g., B. A. Kniehl and A. Pilaftsis, Nucl. Phys. **B474**, 286

(1996); C. Balzereit, T. Hansmann, T. Mannel, and B. Plümper, Eur. Phys. J. C **9**, 197 (1999); P. Gambino, P. A. Grassi, and F. Madricardo, Phys. Lett. B **454**, 98 (1999).

[33] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).

[34] For a detailed discussion of the pinch-technique resummation approach to the physics of unstable particles, see J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **75**, 3060 (1995); Phys. Rev. D **53**, 2128 (1996); **54**, 5315 (1996); K. Philippides and A. Sirlin, Nucl. Phys. **B477**, 59 (1996); M. Passera and A. Sirlin, Phys. Rev. Lett. **77**, 4146 (1996); N. J. Watson, Nucl.

Phys. **B494**, 388 (1997).

[35] A. D. Sakharov, Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32 (1967) [JETP Lett. **5**, 24 (1967)].

[36] J. A. Harvey and M. S. Turner, Phys. Rev. D **42**, 3344 (1990); B. A. Campbell, S. Davidson, J. Ellis, and K. A. Olive, Phys. Lett. B **297**, 118 (1992).

[37] S. A. Abel and S. Sarkar, Phys. Lett. B **342**, 40 (1995).

[38] For interesting alternatives, see G. Dvali and S. H. Henry Tye, Phys. Lett. B **450**, 72 (1999); G. Dvali and G. Gabadadze, hep-ph/9904221; G. Dvali, hep-ph/9905204.