

Brane baldness versus superselection sectors

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The search for intersecting brane solutions in supergravity is a large and profitable industry. Recently, attention has focused on finding localized forms of known “delocalized” solutions. However, in some cases, a localized version of the delocalized solution simply does not exist. Instead, localized separated branes necessarily delocalize as the separation is removed. This phenomenon is related to black hole no-hair theorems, i.e., “baldness.” We continue the discussion of this effect and describe how it can be understood, in the case of Dirichlet branes, in terms of the corresponding intersection field theory. When it occurs, it is associated with the quantum mixing of phases and lack of superselection sectors in low dimensional field theories. We find surprisingly wide agreement between the field theory and supergravity both with respect to which examples delocalize and with respect to the rate at which this occurs. [S0556-2821(99)07418-4]

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I. INTRODUCTION

It is a well-known fact that, while string perturbation theory in intersecting D-brane backgrounds is discussed in terms of localized branes [1], classical supergravity solutions representing such localized branes are typically difficult to construct. One often works with supergravity solutions where the branes are less well localized than one might like, and in fact have extra translational symmetries [2–22]; see [23] for a thorough review. One says that the branes have been “smeared” over some of the directions that were, *a priori*, transverse to their world volume. In some cases, with a bit more work, one can construct explicit fully localized solutions, at least in the “near-core” limit [24,25].

However, there are some situations where fully localized supergravity solutions of the desired type simply do not exist. This happens when the world volume of one type of brane (B) is contained inside that of another (A), or inside the volume through which brane A has been smeared, and when the branes intersect on a manifold of sufficiently low dimension; the details will be explained below. If the branes are separated in a direction transverse to both types (A and B), then localized solutions exist. Nonetheless, as the transverse separation between the branes is removed, the type B brane delocalizes. This phenomenon does not occur when the dimension of the intersection manifold is sufficiently high.

This delocalization was studied in [26] for one-branes parallel to five-branes. As discussed there, this effect is related to a black hole no-hair theorem. Black hole no-hair results tell us that, in certain cases, black hole horizons must be uniform. A pertinent example occurs in Einstein-Maxwell theory: when an electric charge q is brought near a black hole, the charge q appears to be delocalized over the black hole horizon [27]. The situation discussed below is similar as, when delocalization occurs, the charge of the type B brane appears to be delocalized over the entire horizon of the type A brane. For such cases, the type A branes are “bald” and unable to support such hair.

Now, recall that there are certain dualities between supergravity and brane gauge theories, as in the AdS conformal field theory (CFT) correspondence [28] and its p -brane generalizations [29]; see also e.g., [30]. The general picture of a duality is that there is a single quantum theory which has several distinct classical limits. In our case, one of these limits would give classical supergravity while the other would give classical brane gauge theory. This means that aspects of the dynamics which are classical from the point of view of one theory correspond to strongly quantum mechanical effects in terms of the other. We will further generalize the AdS-CFT limit to describe interacting A and B branes, and we will see that the delocalization of classical supergravity corresponds to large fluctuations of a modulus field on the gauge theory side. In particular, its occurrence is related to the quantum mixing between phases and the lack of superselection sectors associated with the asymptotic values of massless fields in low dimensional quantum field theories; i.e., on the gauge theory side, delocalization is controlled by the Coleman-Mermin-Wagner theorem [31,32]. The relevant quantum mixing is between the gauge theory phase which describes type A branes separated from type B branes in an overall transverse direction, which we will refer to generally as the “separated” branch, and the phase that describes such branes with no transverse separation, which we call the “coincident” branch.¹ That some correspondence of this sort should occur was suggested in [35]. The consistency of this picture will be explained in Sec. III. We will see that, not only will this interpretation successfully predict the cases for which the type B branes delocalize, it will also give the correct rate at which the delocalization occurs as the transverse separation is removed.

Before progressing to the main topic, it is perhaps worth-

¹This result is not, in fact, in conflict with [33,34], as we will discuss.

while to display an explicit and tractable example of the supergravity delocalization phenomenon. It turns out that this is much easier to see for a D-instanton located near a D3-brane than for the systems studied in [26]. The point is that the classical supergravity solution for a localized Bogomol'nyi-Prasad-Sommerfield (BPS) D-instanton in $\text{AdS}_5 \times S^5$ is known in terms of elementary functions [36–38] while, for the other examples, the supergravity solution is known only as a (convergent) infinite series.

The near D3-brane geometry is just $\text{AdS}_5 \times S^5$. For simplicity, we take the case [36] where the D-instanton is smeared over the S^5 , but the solution localized on the S^5 is also known explicitly [37]. By writing the $\text{AdS}_5 \times S^5$ metric in Poincaré coordinates,

$$ds^2 = R^2(U^{-2}dU^2 + U^2\delta_{ij}dy^i dy^j + d\Omega_5^2), \quad (1.1)$$

we may interpret the solution of [36] as the near-core part of an asymptotically flat (Euclidean) spacetime with a spherical (S^5) shell of D-instantons located near the three-branes. In Eq. (1.1), R is the radius of the AdS_5 and of the S^5 . In the presence of the D-instantons, the (Einstein) metric is unchanged, and the dilaton is given by

$$e^\phi = c_0 + c_1 \frac{(d^2 + 2)(d^4 + 4d^2 - 2)}{d^3(d^2 + 4)^{3/2}}, \quad (1.2)$$

where

$$d^2 = U_0 U \left[(U^{-1} - U_0^{-1})^2 + \sum_{i=0}^3 (y^i - y_0^i)^2 \right], \quad (1.3)$$

with (U_0, y_0^i) the location of the D-instantons. This setup allows a separation between the D-instantons and the origin of AdS_5 . Note that far from the D-instantons e^ϕ tends to a finite and nonzero constant, $c_0 + c_1$. The constant c_0 is arbitrary while the constant c_1 is proportional to the instanton charge. Consider now the limit in which the instantons are moved onto the three-brane; that is, the limit in which $U_0 \rightarrow 0$ with fixed c_0, c_1, y_0^i . Note that U, y^i should also remain fixed as we wish to examine the solution at a given location relative to the three-brane. In this limit, Eq. (1.3) diverges so that we have

$$(e^\phi)_{\text{lim}} = c_0 + c_1. \quad (1.4)$$

Therefore the dilaton is now constant (and it is in fact the same constant as the asymptotic value of e^ϕ at generic D-instanton position). In particular, the solution no longer has any dependence on the coordinates y^i along the three-brane. We see that the D-instantons have delocalized as they were moved toward the three-brane. Note that this does not happen suddenly at $U_0 = 0$, but rather gradually. As viewed from a fixed point relative to the three-brane, the field created by the instantons smoothly blurs out as we decrease U_0 to zero. In this context, we see that our delocalization is related to the well-known scale-radius duality of this system [36–38]. The behavior of the D-instanton solution [37] that is localized on the S^5 as well as in the AdS_5 is similar. Much

the same behavior was seen in [26] for a localized one-brane as it is moved onto a localized five-brane and, as we will see below, it occurs in many other cases as well.

The outline of this paper is as follows. In Sec. II, we extend the analysis of [26] to consider the full range of two charge intersecting brane solutions whose mathematical form is similar to that of the localized D1-branes parallel to D5-branes (D1||D5). For example, the analysis includes the localized D0||D4 system, and the D2||D6 system studied in [24]. We will see that the D0||D4 system delocalizes as does the D1||D5 system, but that the same methods find the D2||D6 system to remain localized. This is a useful check of the method and resolves the apparent conflict² between the results of [26] for the D1||D5 system and the explicit construction of the localized D2||D6 solutions in [24]. The D2||D6 system is simply different from the D1||D5 system, in a way that will be discussed below. Other solutions considered below are similar to a D1-brane orthogonally intersecting a D3-brane, where (say) the D1-brane is smeared along the D3-brane world volume. In this case, we will see that when the transverse separation between a (localized) D3 brane and the smeared D1 branes is removed, the D3 brane delocalizes along the D1-brane world volume. To be quite general, we will allow arbitrary transverse separation between the two types of branes, although we will still refer to these solutions as “intersecting” brane spacetimes. For many of these systems, certain “near-core” solutions were constructed in [39]; we argue in Appendix A that, while these are certainly valid solutions to the supergravity equations, due to the subtleties of boundary conditions they are not the appropriate ones to consider in our context.

After studying the supergravity solutions, we turn in Sec. III to a discussion of the corresponding intersection field theories and the appropriate AdS-CFT limit [29]. We will see that our delocalization phenomenon (when it occurs) corresponds to a quantum mixing between phases of the Yang-Mills theory and to the fact that 0+1 and 1+1 field theories are not superselected by the asymptotic values of the massless fields. Finally, we discuss a few remaining issues in Sec. IV.

II. DELOCALIZATION IN SUPERGRAVITY

Here, we consider BPS solutions with two types of branes, A and B, each localized at fixed, but different, values of isotropic coordinates x_\perp which label the space transverse to both branes.³ That is, we consider BPS solutions with the branes separated in the transverse direction. The branes may

²In [26], it was stated that there was a normalization problem with the solutions of [24]. This is not, in fact, correct. For completeness, the normalizations are discussed in the Appendix.

³We describe locations in the spacetime in terms of isotropic coordinates. In all cases below, this may be translated into a coordinate invariant statement by referring, for example, to the surface in spacetime on which the various gauge field strengths take a given value. However, it is simpler to discuss the solutions directly in terms of isotropic coordinates.

carry Ramond-Ramond (RR) or Neveu-Schwarz–Neveu-Schwarz (NS)-(NS) charge in type IIA or IIB string theory or be M-branes, and the A- and B-branes need not be of the same sort. For example, the type A brane may be R-R while the type B branes are NS-NS. Thus, we include the case of a fundamental string intersecting (but not ending on) a D-brane. The equations of motion for all of these cases have a similar structure as they are related by T- and S- duality. A similar analysis also applies to Kaluza-Klein monopoles and various sorts of waves. In this section, we refer to all such cases as “branes.”

The world-volumes of the type A and B branes will in general have certain directions in common. We label the directions common to both by coordinates t, z_I . Here, the I is a label denoting the “intersection” directions. Similarly, we denote by z_a the world volume directions of brane A not in I , and we denote by z_b the worldvolume directions of brane B not in I . We take brane A to be smeared in the z_b directions, as well as in any directions in which brane B has been smeared. We will have no need to refer explicitly to directions in which both branes have been smeared, as these can be removed by a T-duality symmetry transformation, but we label directions in which brane A (but not B)⁴ has been smeared by w . Finally, x_\perp labels directions orthogonal to both branes along which neither brane has been smeared. Our conventions are conveniently summarized by the following table, where (●) denotes a direction along a brane, (≡) denotes a direction in which a brane has been smeared, and an empty space denotes a direction orthogonal to the brane in which it has not been smeared.

	t	z_I	z_a	z_b	w	x_\perp
A	●	●	●	≡	≡	
B	●	●		●		

We assume that the z_a , z_b , and w directions are compactified to form tori of volumes V_a , V_b , and V_w respectively (the noncompact case just corresponds to the $V \rightarrow \infty$ limit). The z_I directions are taken to be uncompactified, or compactified on a manifold of very large volume. We will also use the symbols a, b, w to denote the number of z_a, z_b, w coordinates. Our discussion also applies to BPS branes at angles, though we will not consider this case explicitly.

Familiar examples of this class are $D(p-4)$ -branes (B) oriented parallel to Dp -branes (A) as well as various intersecting brane solutions [9,10,20,23] with one brane (A) smeared along the world-volume directions of the other brane (B). Additional cases are generated by further smearing. Thus, we may refer to brane A as “the bigger brane,” in the sense that the *smeared* type A branes fill out a higher dimensional volume.

For BPS configurations we make the ansatz

⁴We could alternatively smear brane B, but this will not affect the delocalization behavior.

$$ds^2 = \frac{1}{\sqrt{H_A H_B}} (-dt^2 + dz_I^2) + \sqrt{\frac{H_A}{H_B}} (dz_b)^2 + \sqrt{\frac{H_B}{H_A}} (dz_a)^2 + \sqrt{H_A H_B} (dx_\perp^2), \quad (2.1)$$

and

$$e^\Phi = g_s H_A^{(3-A)/4} H_B^{(3-B)/4}. \quad (2.2)$$

We have not written the gauge fields explicitly, but they have the standard form. Constructing the supergravity solutions then reduces (see, e.g., [10,13,17,40]) to solving the following equations of motion:

$$\partial_{x_\perp}^2 H_A(x_\perp) = \frac{q_A}{V_b V_w} \delta(x_\perp), \quad (2.3)$$

$$(\partial_{x_\perp}^2 + \partial_w^2) H_B(x_\perp, w, z_a) + H_A(x_\perp) \partial_{z_a}^2 H_B(x_\perp, w, z_a) = q_B \delta(x_\perp, x_{\perp 0}) \delta(w, w_0) \delta(z_a, z_{a0}), \quad (2.4)$$

where $\partial_{x_\perp}^2$ represents the flat space Laplacian in the x_\perp directions, and similarly for ∂_w^2 and $\partial_{z_a}^2$. Note that, by construction, the solution has no dependence on t , z_I or z_b . The delta-function sources, each of the appropriate dimension, ensure that the solution carries gauge field fluxes corresponding to charges q_A, q_B at the locations specified. Note that we have taken brane A to lie at the origin of the transverse coordinates x_\perp , while we have placed brane B at the location $(x_{\perp 0}, w_0, z_{a0})$. We have chosen to let q_A, q_B denote the total charge of the type A and B branes, although only the charge density of the type A branes appears in the above equations of motion (2.3).

The first equation of motion (2.3) is just Laplace’s equation. Solutions appropriate to asymptotically flat boundary conditions may be found if the x_\perp coordinates label a d dimensional space with $d \geq 3$, and we confine ourselves to this case. The type A branes are associated with a “charge radius” r_A proportional to $q_A^{1/(d+b+w-2)}$, but the behavior of the supergravity solution is controlled by the length scale

$$\hat{r}_A = \left(\frac{r_A^{d+b+w-2}}{V_b V_w} \right)^{1/(d-2)}. \quad (2.5)$$

The appropriate solution may thus be written

$$H_A(x_\perp) = 1 + \frac{\hat{r}_A^{d-2}}{r_\perp^{d-2}}, \quad (2.6)$$

where $r_\perp = |x_\perp|$.

The method of [26] then uses the symmetries and the linearity of equation (2.4) to solve for H_B as an infinite sum (in the case where the w, z_a directions are compact) over modes. This series turns out to be absolutely convergent, and so is a useful representation of the full solution.

We proceed here in the same way. As stated above, we suppose that the coordinates z_a, w label compact tori of volume V_a, V_w . If these directions are not in fact compact, then a similar argument follows simply by replacing the

mode sums with integrals. We Fourier transform over the z_a , w directions and decompose the solution into harmonics on the $d-1$ sphere associated with the d transverse coordinates x_\perp .

In fact, it is only necessary to consider modes which are constant over the S^{d-1} sphere. The higher spherical harmonics would tell us about localization of the type B branes in the angular directions, but we already expect that, when we move the type B branes to $r_\perp=0$, the limiting solution becomes symmetric over the S^{d-1} sphere. Including a discussion of the higher spherical harmonics, as was done in [26], verifies this conclusion, but does not impact the question of localization in the z_a and w directions. Thus, we consider here only modes which are uniform over the S^{d-1} sphere. This is equivalent to replacing the point source in B with an S^{d-1} shell of source: $q_B r_\perp^{-(d-1)} \delta(r_\perp, r_{\perp 0}) \delta(w, w_0) \delta(z_a, z_{a0})$, where $r_{\perp 0} = |x_{\perp 0}|$.

We express the function $H_B(r_\perp, w, z_a)$ as a sum over Fourier modes in w and z_a multiplied by radial modes $H_{B, p_w, p_a}(r_\perp)$, where p_w and p_a denote the relevant (discrete) momenta in the w , z_a directions. These radial modes satisfy the following second order ordinary differential equation (ODE):

$$\begin{aligned} r_\perp^{-(d-1)} \partial_{r_\perp} (r_\perp^{(d-1)} \partial_{r_\perp} H_{B, p_w, p_a}(r_\perp)) - p_w^2 H_{B, p_w, p_a}(r_\perp) \\ - H_A(r_\perp) p_a^2 H_{B, p_w, p_a}(r_\perp) = \frac{q_B}{V_a V_w} r_\perp^{-(d-1)} \delta(r_\perp, r_{\perp 0}). \end{aligned} \quad (2.7)$$

Let us, for the moment, fix our attention on one choice of p_w, p_a , so that we need not indicate these labels explicitly. The $p_w=0, p_a=0$ mode has special boundary conditions which we will discuss later. For the other modes, imposing boundedness at large r_\perp will determine the solution at $r_\perp > r_{\perp 0}$ to be a constant α_+ times some particular solution ϕ_+ , while continuity at the origin will determine the solution for $r_\perp < r_{\perp 0}$ to be a constant α_- times some particular solution ϕ_- . The constants α_\pm are then determined by the matching conditions dictated by the delta-function in Eq. (2.7). Namely, as the source in Eq. (2.7) has no derivatives of delta functions, we must have $\alpha_+ \phi_+(r_{\perp 0}) = \alpha_- \phi_-(r_{\perp 0})$, while the discontinuity in the first derivatives must reproduce the delta function source. Using ϕ_\pm and ϕ'_\pm to denote the values of the solutions and their r_\perp derivatives evaluated at $r_\perp = r_{\perp 0}$, we have as usual

$$\begin{aligned} \alpha_+ &= \frac{q_B r_{\perp 0}^{-(d-1)} \phi_-}{V_a V_w (\phi_- \phi'_+ - \phi_+ \phi'_-)}, \\ \alpha_- &= \frac{q_B r_{\perp 0}^{-(d-1)} \phi_+}{V_a V_w (\phi_- \phi'_+ - \phi_+ \phi'_-)}. \end{aligned} \quad (2.8)$$

Our main task is to study the behavior of α_+ as $r_{\perp 0} \rightarrow 0$. If the coefficient α_+ vanishes in this limit, then the corresponding mode will not appear in the limiting solution for H_B .

The denominator of Eq. (2.8) is just the Wronskian (W) of Eq. (2.7) evaluated at $r_\perp = r_{\perp 0}$ and may be computed by

standard techniques. Up to a constant (which does not depend on $r_{\perp 0}$), it is given by $W(r_\perp) = r_\perp^{-(d-1)}$. Thus, up to an irrelevant constant we have

$$\alpha_+ = \frac{q_B \phi_-(r_{\perp 0})}{V_a V_w}, \quad \alpha_- = \frac{q_B \phi_+(r_{\perp 0})}{V_a V_w}. \quad (2.9)$$

All that remains is to determine the behavior of ϕ_- for small $r_{\perp 0}$.

Let us suppose that p_a is nonzero, so that the behavior near $r_\perp=0$ is controlled by the term proportional to $H_A(r_\perp) p_a^2$ in the radial equation (2.7). Recall that H_A describes some power law potential in r_\perp that diverges at $r_\perp=0$. For the r_\perp^{-2} potential ($d=4$), the radial equation may be solved exactly in terms of Bessel functions as was done in [26]. Given the explicit solution in [26], one can see that the sum over p_a converges absolutely at any $r_\perp \neq r_{\perp 0}$ so that the series gives an accurate description of the physics. Solutions to the source-free equation behave like $r_\perp^{-1 \pm \sqrt{1+p_a^2 \hat{r}_A^2}}$ near $r_\perp=0$, so that continuity at the origin forces ϕ_- to vanish there. Thus, only modes with $p_a=0$ contribute in the $r_{\perp 0} \rightarrow 0$ limit and the type B branes delocalize in the z_a directions. This case corresponds to, for example, D1 branes parallel to D5 branes.

Still considering the case $d=4$ (e.g., D1-branes and D5-branes), it is interesting to ask about the rate at which the type B branes delocalize. Since we are interested in the limiting behavior as $r \rightarrow 0$, we consider the delocalization as it reaches the largest length scales (small p_a). Note that, as measured by the asymptotic fields, the solution has delocalized on a length scale δx when the coefficient α_+ corresponding to the momentum $p_a \sim 1/\delta x$ becomes small relative to its value at large $r_{\perp 0}$. For convenience, we will measure α_- relative to its value ($q_B \phi_-(\hat{r}_A)/V_a V_w$) at $r_{\perp 0} = \hat{r}_A$. A comparison with [26] or an approximate solution for small r_\perp shows that α_+ vanishes like

$$\frac{V_a V_w \alpha_+}{q_B \phi_-(\hat{r}_A)} \sim \left(\frac{r_{\perp 0}}{\hat{r}_A} \right)^{-1 + \sqrt{1+p_a^2 \hat{r}_A^2}} \quad (2.10)$$

for $r_\perp \ll \hat{r}_A$. Specifically, for small p_a , α_+ vanishes like $(r_{\perp 0}/\hat{r}_A)^{(1/2)p_a^2 \hat{r}_A^2}$. Thus, for $d=4$, the type B brane appears to be delocalized on a length scale $\delta x \sim \hat{r}_A \sqrt{\ln \hat{r}_A / r_{\perp 0}}$ as viewed from infinity. Note that, for large \hat{r}_A , the B-branes are quite well delocalized before they reach any strong curvature region.

Let us now consider the case $d > 4$, e.g., D0-branes approaching D4-branes. Since ϕ_- vanishes at the origin for the r^{-2} potential, one may expect the same behavior for the stronger potentials $r^{-(d-2)}$. This may be verified by looking for a solution of the form $\phi = r_\perp^{-(d-1)/2} e^{\Psi(r_\perp)}$, using the WKB approximation, and again imposing continuity at $r_\perp=0$. Note that the WKB approximation is self-consistent for such strong potentials. Thus, the supergravity solutions delocalize in the z_a directions for these cases as well. The delocalization is even faster than for $d=4$ as α_+ now vanishes like

$$\alpha_+ \sim \exp \left[-p_a \left(\frac{d-2}{2} - 1 \right)^{-1} \hat{r}_A \left(\frac{\hat{r}_A}{r_{\perp 0}} \right)^{d-2/2-1} \right], \quad (2.11)$$

as opposed to the power law behavior for the r_{\perp}^{-2} potential. When the type B branes are at $r_{\perp 0}$, the coefficients α_+ are small for $\hat{r}_A p_a > (r_{\perp 0} / \hat{r}_A)^{d-2/2-1}$ and the type B brane has delocalized to a size scale $\delta x \sim \hat{r}_A (\hat{r}_A / r_{\perp 0})^{d-2/2-1}$. Again, at least within the domain of the WKB approximation, the sum over momenta converges absolutely. The case $d > 4$ includes D0 branes approaching D4 branes, as well as intersecting brane solutions (either R-R, NS-NS, or M) with 5 or more directions transverse to both branes. For example, it addresses the case of a D3-brane (B) and an orthogonal D1-brane (A) smeared in the three z_b directions along the D3-brane. As the transverse separation is removed, the D3 brane delocalizes along the D1-brane. The case $d > 4$ also includes the case of D-instantons in a D3 brane, which was explicitly seen to delocalize in Sec. I. As before, for large \hat{r}_A , the type B branes are well delocalized before they reach any strong curvature region.

The remaining case is when the bigger brane has only three transverse directions ($d=3$), which is exactly the situation that arises in the solutions of [24]. In this case, the WKB approximation is not self-consistent, but we may study the radial equation by a related technique. Writing $\phi(r_{\perp}) = r^{-1} e^{\Psi(r_{\perp})}$, the sourceless radial equation (2.7) becomes

$$\partial_{r_{\perp}}^2 \Psi + (\partial_{r_{\perp}} \Psi)^2 - p_w^2 - p_a^2 \frac{\hat{r}_A}{r_{\perp}} = 0. \quad (2.12)$$

This may be analyzed by assuming that $\partial_{r_{\perp}}^2 \Psi$ is much larger than $(\partial_{r_{\perp}} \Psi)^2$, which turns out to be self-consistent for $r_{\perp} \ll \hat{r}_A$. Within this approximation, the general solution behaves near the origin like

$$\phi = C_1 \left(\frac{r_{\perp}}{\hat{r}_A} \right)^{[p_a^2 \hat{r}_A r_{\perp}^{-1}]} + C_2 \left(\frac{r_{\perp}}{\hat{r}_A} \right)^{[p_a^2 \hat{r}_A r_{\perp}]} . \quad (2.13)$$

Thus, if ϕ is to be continuous at the origin we must have $C_1 = 0$. As a result, ϕ_{-} behaves like $(r_{\perp} / \hat{r}_A)^{[p_a^2 \hat{r}_A r_{\perp}]}$, which is finite and nonzero at $r_{\perp} = 0$. This time we find that α_+ does not vanish as $r_{\perp 0} \rightarrow 0$. The result is that the type B brane remains localized in the z_a directions in agreement with [24]. Again, within the domain of validity of this approximation ($\hat{r}_A \gg r_{\perp 0}, r_{\perp}$), one finds that the sum over modes converges absolutely. The infinite series discussed here should sum to the solution of [24] in the near-core region.

We have not yet addressed localization in the w directions for any value of d . Let us therefore consider a mode with $p_a = 0, p_w \neq 0$. The radial equations in this case are just those for the Coulomb potential of a massive field and are easily studied. One finds that ϕ_{-} does not vanish at $r_{\perp} = 0$, and the

sum over modes once again converges absolutely.⁵ Thus, localization is always possible in any directions transverse to brane B along which brane A has been smeared.

The last mode to consider is the case $p_a = 0, p_w = 0$; i.e., the spatially homogeneous mode. This mode does not tell us about delocalization of the branes; instead, it is the entire field remaining once complete delocalization has occurred. This mode is special as its boundary condition at infinity differs from that of any other mode. The point is that, for $q_B = 0$, the appropriate solution is $H_B = 1$ and not $H_B = 0$. Thus, the correct solution for the homogeneous mode is a constant plus a function that vanishes for large r_{\perp} . We now consider the case where the type B brane delocalizes completely and make several observations. Recall that complete delocalization occurs when $d \geq 4$ and there are no w directions. First, note that the complete solution for $r_{\perp 0} = 0$ is of the form $H_B = 1 + (r_B^{d-2+a} / V_a r_{\perp}^{d-2})$, since only the spatially homogeneous mode survives. Second, since all modes with $p_a \neq 0$ vanish at $r_{\perp} = 0$ for any location $r_{\perp 0}$ of the type B branes, we may evaluate H_B exactly at the origin:

$$H_B(r_{\perp} = 0) = 1 + \left(\frac{r_B^{d-2+a}}{V_a r_{\perp 0}^{d-2}} \right). \quad (2.14)$$

Note that this result also holds in the infinite volume limit ($V_a \rightarrow \infty$) in which $r_B^{d-2+a} / V_a = 0$. Thus, if the z_a directions are not compactified, we have simply $H_B = 1$ at $r_{\perp} = 0$, independent of $r_{\perp 0}$. These observations will prove useful in the following sections.

We now make a few final remarks about variations on the above theme. Consider, for example, solutions representing not branes in asymptotically flat space, but branes in the near horizon geometry associated with the type A branes; i.e., the solutions obtained by taking a limit $\hat{r}_A \gg r_{\perp}, r_{\perp 0}$. This is really the only part of the asymptotically flat geometry of which we have made significant use, so the discussion is not changed. Note that taking this limit is equivalent to setting $H_A = (\hat{r}_A / r_{\perp})^{d-2}$ and fixing the boundary conditions by imposing boundedness at infinity and specifying the value of H_B at $r_{\perp} = 0$ to be given by Eq. (2.14) above. The point we wish to emphasize is that the $p_a = 0, p_w = 0$ mode still approaches a nonzero constant far from the type B branes and we have $H_B \rightarrow 1$ as $r \rightarrow \infty$. One can see this explicitly in the D-instanton example from the Introduction, which already resides in the near-horizon geometry of the three-brane.

In addition, one might ask how the analysis would change if a given set of (type IIA) branes were lifted to eleven dimensional supergravity. Such a lift has a translational symmetry in x_{11} , so that x_{11} does not become a transverse direction. As the behavior of the classical solution is determined by the number of dimensions transverse to the type A brane, it is not affected by this process. One might also ask about related solutions where the M-brane is localized in x_{11} . An

⁵Of course, the full sum over modes includes a sum over p_w even for $p_a \neq 0$. For $p_w^2 \gg p_a^2 (\hat{r}_A / r_{\perp})^{d-2}$, the analysis is identical to the $p_a = 0$ case and shows convergence of the sum over p_w .

example where this is possible is lifting a solution with D2-branes (with one brane smeared over the other) intersecting in points to a solution with M2-branes intersecting in points (with one brane still smeared over the other but otherwise fully localized). In such cases, the M-theoretic solution will generally delocalize faster than the type IIA supergravity solution due to the increased number of transverse dimensions. Note that a qualitative change from delocalization to localization could only happen if the number of transverse directions crosses the threshold at $d=4$, i.e., the lifting of a D6 brane to M-theory. But this lift is a Kaluza-Klein monopole and, while there are only 6 directions along the corresponding M-theoretic ‘‘brane,’’ the Kaluza-Klein monopole has a nontrivial structure in the remaining 4 spatial directions. Due to the structure of these dimensions, the corresponding potential H_A behaves only as r^{-1} even for the M-theoretic solution, and therefore has the same properties with regard to delocalization as the IIA D6-brane. One can, of course, consider a Kaluza-Klein monopole in any dimension and, as it always has an r^{-1} potential, the type B branes always remain localized. This is of course to be expected from the method of [24,25], which realizes the ‘‘near-core’’ Kaluza-Klein monopole as an orbifold of flat space and inserts the type B branes before taking the orbifold quotient.

Finally, within supergravity, it is clear that the above analysis can be extended to branes that are not asymptotically flat. They correspond to potentials that are softer than r^{-1} and therefore allow localization of the directions parallel to the A brane. However, this case is not our primary concern and we will not discuss it in detail.

III. THE SUPERGRAVITY-FIELD THEORY CORRESPONDENCE

In the last section we studied the delocalization behavior of a class of asymptotically flat supergravity brane solutions of type IIA/B string theory. We would now like to understand this behavior from a field theory perspective by using a generalization of the p -brane AdS-CFT correspondence, which is obtained by taking a certain low-energy limit of a system of N_p R-R charged p -branes. We first study the supergravity solutions and define the AdS-CFT limit, and then move on to a field theory explanation of the supergravity (de)localization phenomena.

Let us first orient ourselves with a lightning review of the salient features of the AdS-CFT correspondence of [29] for DA-branes. Here, A denotes the number of spatial dimensions of what in Sec. II was a type A (Dirichlet) brane. We will ignore numerical factors in this entire section; the relevant precise normalizations may be found in [29] or in the previous section. The AdS-CFT correspondence for N_A DA-branes is obtained by starting in string theory with the coupled bulk-brane dynamics, and taking the low-energy limit

$$(El_s) \rightarrow 0. \quad (3.1)$$

This energy E is measured in the gauge theory. The gauge theory coordinates are the isotropic supergravity coordinates

in directions parallel to the brane. To see this, one can compute the associated moduli space metric for motion of a probe DA-brane in the background of the others; it is trivial. The next step is to keep the dimensionless expansion parameter in the $U(N_A)$ gauge theory fixed; since we are interested in large numbers of branes for comparison to the supergravity, this expansion parameter is

$$\lambda_A^2(E) = g_{YM,A}^2 N_A E^{A-3} = g_s N_A (l_s E)^{A-3}. \quad (3.2)$$

The field theory is perturbative when this parameter is small. One also keeps fixed the mass of strings stretched perpendicular to the branes, $E = |x_\perp|/l_s^2 \equiv U$, where x_\perp is the displacement between the branes.

In the AdS-CFT low-energy limit the physics on the brane decouples from the physics in the bulk, provided that $A < 6$. This decoupling happens essentially because supergravity is a weak interaction at low energy compared to the gauge interaction.

The supergravity metric is

$$ds^2 = \frac{1}{\sqrt{H_A}} (-dt^2 + dz_A^2) + \sqrt{H_A} dx_\perp^2, \quad (3.3)$$

where $H_A = 1 + \left(\frac{r_A}{|x_\perp|} \right)^{7-A}$.

Here, we have taken all of the DA-branes to be ‘‘clumped’’ together at the origin $r=0$ and have not allowed any smearing of the branes. The symbol r_A denotes the charge radius of the brane, i.e. the radius where the 1 in the harmonic function is comparable to the other term:

$$r_A = l_s (g_s N_A)^{1/(7-A)}. \quad (3.4)$$

Converting x_\perp to U and the other factors using Eq. (3.2) the harmonic function may be written $H_A = 1 + \lambda_A^2(U)/(l_s U)^4$. Assuming that the supergravity U , which is an energy, scales the same as the gauge theory energy E , we see that the 1 in H_A is lost in the low-energy limit. This means that we have lost the asymptotically flat part of the supergravity geometry. In considering the physical validity of this near-horizon supergravity solution, there are two types of corrections to worry about, α' and g_s . A measure of the first type of correction is the Ricci scalar measured in string units, which is found to be $l_s^2 \mathcal{R} = 1/\lambda_A(U)$ and so the supergravity is weakly coupled for $\lambda_A(U) \gg 1$. In comparing this to the gauge theory regime $\lambda_A(E) \ll 1$, as pointed out in [41], we must be careful in specifying the type of probe we are using, which in turn gives a relationship between the gauge theory energy E and the supergravity radius U . The simplest type of probe to consider is the one originally studied in [29], the stretched fundamental string, which is a BPS state in the gauge theory and has $E=U$ as above. One then sees that the supergravity and gauge theory breakdowns happen in a consistent fashion. Here it was important that this result for the stretched string mass is not corrected gravitationally. The gravitational ‘‘warpage’’ for a fundamental string is computed from the DA-brane metric in the string frame (3.3), and since the

(BPS) stretched string points in a direction perpendicular (\perp) to the branes the warpage is unity:

$$\perp \text{ warpage} = \int \sqrt{-g_{\tau\tau}g_{\sigma\sigma}} = \int \sqrt{H_A^{-1/2}H_A^{+1/2}} = 1. \quad (3.5)$$

At still higher values of λ , other descriptions of the physics are appropriate [29], such as S-dual supergravity geometries. Overall one finds different descriptions of the physics to be valid at different λ^2 's, and they are dual to one another. We will be concentrating on the super Yang-Mills and ten-dimensional supergravity phases.

Lastly, let us make a brief remark contrasting the near-horizon spacetime with the asymptotically flat solution. If we compute the curvature in string units of the asymptotically flat $A < 3$ solutions, we find that it starts from zero at $x_\perp = 0$, peaks at around the charge radius at a value of order $(g_s N_A)^{-2/(7-p)}$ which is small for large $g_s N_A$, and falls off to zero again at $r \rightarrow \infty$. For $A > 3$, a similar behavior occurs for the dilaton. This is to be contrasted with the near-horizon spacetime for which the curvature and dilaton are monotonic [29]. For $A < 3$, the curvature diverges near the branes while for $A > 3$ the dilaton diverges there.

A. The supergravity side

We now wish to define a generalized AdS-CFT correspondence for intersecting D-branes appropriate to the R-R supergravity solutions that we constructed in the last section. Our conventions differ from the last section only in that, for the moment, we take there to be no w directions. One special case in which we will need to reintroduce w directions will be discussed near the end of subsection B. So, our setup is

	t	z_I	z_a	z_b	x_\perp
DA	●	●	●	≡	
DB	●	●		●	

There are $d = (9 + I - A - B)$ of the x_\perp , $a = (A - I)$ of the z_a , and $b = (B - I)$ of the z_b . Without loss of generality, we have chosen the DA-branes to be smeared over the z_b . If there are no z_b , then the intersection is parallel, otherwise it is orthogonal. In addition, we take any smeared configuration to be irreducibly smeared in the sense that it cannot be reduced via a T-duality symmetry transformation to an un-smeared configuration. We work in the large-volume limit, $V_{a,b} \rightarrow \infty$, but we keep a finite charge density of the A-branes, $N_A / (V_b / l_s^b)$. In contrast, the charge density $N_B / (V_a / l_s^a) \rightarrow 0$. Our analysis therefore differs from the special case considered in [42] in which there was a volume infinity of D-instantons on D3-branes.⁶ It also differs from previous analyses such as [30] where AdS-CFT for smeared intersecting branes was studied.

⁶For this case, in contrast to what we study later, the theory on the D-instantons is not dynamical.

Let us now study our intersecting brane supergravity solutions from Sec. II in a generalization of the low-energy limit of [29]. Our concern is to find the relevant region of the supergravity geometry. If the DA-branes and DB-branes are very far from each other, then each has its own near-horizon region as if the other collection of branes did not exist. The more interesting, and new, situation arises when the branes influence each other strongly, i.e. when the separation $x_{\perp 0}$ is near-horizon in a sense which we now explain. In this section, we keep the localization in the $(d-1)$ -sphere explicit, and do not use spherical shells.

An important subtlety in determining the low-energy limit for intersecting D-branes concerns the kind of probe we will use to connect a gauge theory energy E with a supergravity radius U . As before, the most straightforward probes to consider are the stretched strings. In the intersecting brane geometry, with metric (2.1), and ‘‘harmonic’’ functions $H_A(x_\perp), H_B(x_\perp, z_a)$, the only BPS stretched strings are the ones running in the transverse (x_\perp) directions. Strings stretched in directions other than x_\perp , for example tethering two clumps of B branes separated in a z_a (\parallel) direction, are not BPS; a symptom of this is that they experience large warpage

$$\parallel \text{ warpage} = \int \sqrt{-g_{\tau\tau}g_{\sigma\sigma}} = \int d\tau d\sigma \sqrt{l/H_A}. \quad (3.6)$$

Because of this warpage, the non-transverse strings are attracted to the A branes. So we will concentrate only on the transverse BPS stretched strings in the following; the dynamics for other types of probes is much more complicated and we will not discuss it. As a consequence, when we take the low-energy limit, we perform the scaling of coordinates only in the transverse directions: $U l_s = x_\perp / l_s \rightarrow 0$.

If $A \geq B$ we hold the A-brane coupling $\lambda_A^2(U)$ fixed; if $A < B$ we hold $\lambda_B^2(U)$ fixed. Effectively, this means we are taking the AdS-CFT limit just as for the larger brane. In a sense, including the physics of the smaller brane is like a ‘‘perturbation’’ on this larger-brane AdS-CFT correspondence, although we are not in any way treating the smaller brane physics perturbatively. A simple example of this is the case of the D-instanton in the D3-brane near-horizon geometry which we discussed in the Introduction; however, for the examples with larger branes such as D1||D5, the theory on the smaller branes is in fact dynamical. In addition, the intersecting brane gauge theories will decouple from the bulk as long as the branes are ‘‘small enough;’’ here this means that there are more than three directions transverse to both branes. For the cases where there are only three, even though the metric does not have the same form as for a D6-brane, we can compute the curvature and Hawking temperature and they turn out to behave as for six-branes. Therefore we suspect that holography is breaking down, i.e. that making predictions about the classical supergravity from the quantum brane field theory may be problematic. We will remark on these cases explicitly when we encounter them.

Now, doing the AdS-CFT scaling in x_\perp and holding λ^2 of the larger brane fixed gives the the near-horizon geometry of

the DA-branes. That this is true no matter whether $A \geq B$ or $A < B$ can be seen as follows. From

$$H_A = 1 + \frac{r_A^{7-A}}{V_b} \frac{1}{|x_\perp|^{7-A-b}} \quad (3.7)$$

we find

$$H_A \rightarrow \frac{[g_s N_A (l_s U)^{A-3}]}{(V_b/l_s^b)(l_s U)^{4-b}} = \frac{N_A}{(V_b/l_s^b)N_B} \frac{[g_s N_B (l_s U)^{B-3}]}{(l_s U)^{4-a}}. \quad (3.8)$$

Recall that there are several constraints of our setup, such as asymptotic flatness of the intersecting brane solution, the smearing of the DA-branes over z_b , and the fact that both intersecting D-branes live in either type IIA or IIB string theory. As a result, it is always true that $b < 4$ if $A \geq B$ where we hold $\lambda_A^2(U)$ fixed, or $a < 4$ if $A < B$ where we hold $\lambda_B^2(U)$ fixed. Thus we see that, as previously advertised, in either case only the near-horizon part of H_A remains. That this happens even for $A < B$ is due to the smearing of the A-branes over z_b .

We now need to check whether the supergravity curvature and dilaton develop any additional strong coupling regions due to the presence of the B branes. First of all, we should not be tempted to just throw away the 1 in H_B by analogy with what happened to H_A . Recall that in the infinite volume limit, we have $H_B = 1$ at the location of the DA-branes for any value of $x_{\perp 0}$, and that this provides a boundary condition which forces $H_B \rightarrow 1$ far from the DB-branes as well. (We also discussed this phenomenon explicitly for the $A = 3, B = -1$ example in the Introduction.) We conclude that, far from the DB-branes, the geometry and dilaton are just as they were for the near-horizon DA-branes in isolation. Thus, one expects that new strong coupling regions could arise only near the DB-branes. We examine this possibility now, though we need to treat the $x_{\perp 0} = 0$ and $|x_{\perp 0}| > 0$ cases separately. Let us study $x_{\perp 0} = 0$ first.

For $x_{\perp 0} = 0$ we go directly to the exact solution of the previous section. For all cases with potentials stronger than $1/r$, where we expect holography to hold, delocalization occurs, and the solution depends only on the d overall transverse coordinates. For $x_{\perp 0} = 0$ the solution is

$$H_B = 1 + \frac{r_B^{7-B}}{V_a} \frac{1}{r^{7-B-a}} = 1 + \frac{g_s N_B}{V_a/l_s^a} \left(\frac{l_s}{|x_\perp|} \right)^{7-A-b}, \quad (3.9)$$

where V_a is the volume of the z_a directions, and we have used the identity $a + B = A + b$. Since we do not have a (V_a) volume infinity of DB-branes, we have just $H_B = 1$, everywhere. As a result, the curvature and dilaton behave exactly as they would for the isolated DA-branes. We can therefore simply use our DA-brane intuition to tell us where the $x_{\perp 0} = 0$ intersecting brane supergravity solution is valid.

At generic separations $x_{\perp 0}$, the supergravity solution is complicated, and is not known in terms of elementary functions. We will instead use an approximation scheme to study the curvature. As usual, we consider DA-branes with world

volume $\{t, z_I, z_a\}$ and DB-branes with $\{t, z_I, z_B\}$. The supergravity equation of motion for H_A is

$$[\partial_{x_\perp}^2] H_A = \frac{q_A}{V_b} \delta(x_\perp) \quad (3.10)$$

i.e. the A-branes are all at $x_\perp = 0$. The near-horizon solution is

$$H_A = \frac{r_A^{7-A}}{V_b |x_\perp|^{7-A-b}}, \quad (3.11)$$

where the b appears because of the smearing of the DA-branes along the DB-branes. (Recall that, in our low-energy limit, the 1 in H_A has dropped out.)

We chose the origin of coordinates such that the DB-branes are located at $z_a = 0$ and at $x_\perp = x_{\perp 0}$. The equation satisfied by H_B is

$$[\partial_{x_\perp}^2 + H_A \partial_{z_a}^2] H_B = q_B \delta(x_\perp, x_{\perp 0}) \delta(z_a). \quad (3.12)$$

We will solve this equation in the region near the DB-branes, in particular for x_\perp satisfying $|x_\perp - x_{\perp 0}| \ll |x_{\perp 0}|$. Over such a region, the function H_A does not vary much, and we can approximate it by a constant in the equation for H_B . After a change of coordinates,

$$y_\perp = x_\perp - x_{\perp 0}, \quad y_a = \frac{z_a}{\sqrt{H_A}}, \quad (3.13)$$

we see that an approximate solution to the H_B equation of motion is given by

$$H_B = f_0 + \frac{r_B^{7-B}}{[(y_a)^2 + (y_\perp)^2]^{(7-B)/2}}, \quad (3.14)$$

where f_0 is a solution of the $q_B = 0$ version of Eq. (3.12). In particular, f_0 is smooth at the location of the DB-branes, so that the singular term will dominate in this region. For convenience, we pretend that $f_0 = 1$ in order to borrow results from the familiar DB-brane metric, but our conclusions will not depend on this choice.

Now notice that the fields of the intersecting brane system do not depend on any of $\{t, z_I, z_b\}$. We may therefore rescale these coordinates without affecting the supergravity physics. Defining

$$(T, y_I, y_b) = \frac{(t, z_I, z_b)}{\sqrt{H_A}}, \quad (3.15)$$

the metric becomes

$$ds^2 = \sqrt{H_A} \left[\frac{1}{\sqrt{H_B(y_a, y_\perp)}} (-dT^2 + dy_I^2 + dy_b^2) + \sqrt{H_B(y_a, y_\perp)} (dy_a^2 + dy_\perp^2) \right]. \quad (3.16)$$

With the metric near the DB -branes in the above form, the scalar curvature \mathcal{R} of the intersecting brane solution is easy to compute. We have

$$l_s^2 \mathcal{R} \sim \frac{1}{\sqrt{H_A}} l_s^2 \mathcal{R}_B \quad (3.17)$$

where \mathcal{R}_B is the curvature of the DB -brane metric by itself. At large $g_s N_B$, this curvature is small everywhere, as we saw at the beginning of this section. The factor of $1/\sqrt{H_A}$ only makes this conclusion stronger, by a factor $\sqrt{(g_s N_A)/(x_{\perp 0}/l_s)^{7-A-b}} \rightarrow \infty$.

We also consider the dilaton. For the intersecting solution it is

$$e^\Phi = g_s H_A^{(3-A)/4} H_B^{(3-B)/4}. \quad (3.18)$$

For small B , this will become large near the DB -branes. For $A > 3$, it will be damped by a power of H_A . However, due to the change of coordinates above from z_a to y_a , H_B is large over a range of z_a that is larger than for the B -branes in isolation. For small B , the supergravity solution also breaks down near the DB -branes, over a range of z_a that is significantly stretched relative to what occurs for the B -branes by themselves. If we were interested in the region where the dilaton were too large, we would switch to an S-dual description.

However, the most important point for us follows from the observation of the previous section that we have $H_B \rightarrow 1$ far from the DB -branes. In fact, when the separation $|x_{\perp 0}|$ is small and if we do not have a volume (V_a) infinity of DB -branes, H_B approaches 1 quite rapidly. Now let us pick a point significantly further out than the DB -branes but close enough that it would lie inside the region of validity of the near-horizon supergravity solution for the DA -branes alone. Then in the combined system, H_B is close to 1 at this point, and the DB -branes do not affect the validity of the supergravity solution. The previous section then tells us that the delocalization is visible in this region, and also that, by adjusting parameters, we can make the delocalization arbitrarily large without placing the DB -branes in the strong-curvature region of the near- A -brane geometry. We conclude that delocalization is a reliable prediction in this intersecting brane spacetime, and so it should have a dual description in terms of the gauge field theory on the branes.

B. The field theory side

On the field theory side of our generalized correspondence we have the coupled field theory of the DA - and DB -branes. There are three sectors of open strings, the $A-A$ strings, the $B-B$ strings, and the $A-B$ strings. The action for this system in the low-energy limit at weak gauge couplings is well known; it is T-dual to that for the $D0||D4$ system and has eight real supercharges.

Since the dimensions of the low-energy field theories for the $A-A$, $B-B$, and $A-B$ strings are all different, we need to know how the couplings scale relatively in the low-energy limit ($El_s \rightarrow 0$). The gauge couplings on the A - and

B -clumps are built out of the same string theory parameters. Let us study the expansion parameters λ^2 closely. For the A -clump we have

$$\lambda_A^2(E) = g_s N_A (l_s E)^{A-3}, \quad (3.19)$$

and for the B -clump

$$\lambda_B^2(E) = g_s N_B (l_s E)^{B-3}. \quad (3.20)$$

Now let us take the low-energy limit of a system of two clumps of branes, the DA -branes and the DB -branes, as we did in the previous subsection. In general the expansion parameters will develop a hierarchy for energies E of similar order:

$$\frac{\lambda_A^2(E)}{\lambda_B^2(E)} \sim \frac{N_A}{N_B} (l_s E)^{A-B}. \quad (3.21)$$

We see that a large hierarchy arises because we are taking the low-energy limit ($l_s E \rightarrow 0$). Now recall the condition that we keep $\lambda^2(E)$ fixed for the bigger brane; this would be $\lambda_A^2(U)$ for $A \geq B$ or $\lambda_B^2(U)$ for $A < B$. Therefore we see that the coupling of the smaller brane becomes much stronger than that of the larger one,

$$\lambda_{\text{small}}^2(E) \gg \lambda_{\text{big}}^2(E). \quad (3.22)$$

This means that the physics on the bigger brane does not significantly influence the physics on the smaller brane. This is similar in spirit to the AdS-CFT decoupling of the bulk theory from the brane theory. So whenever we consider two intersecting branes, orthogonal or parallel, we need only study the gauge theory on the smaller brane and on the intersection, and we can ignore the physics on the bigger brane as it is essentially frozen out. The one exception occurs, of course, when $A=B$, in which case the dynamics on both clumps of branes is equally relevant.

Now, for Dp -branes with $p < 3$, we have from the formula for $\lambda_p^2(E)$ that the perturbative Super Yang-Mills (SYM) regime is the high-energy or ultraviolet regime [29]. So in a Wilsonian sense the SYM description is the fundamental one. Now, note that the dimension of the theory on the intersection for our intersecting D-brane configurations has an upper bound of $d=2+1$. This happens because there are not enough dimensions of spacetime to have intersecting branes (parallel or orthogonal) which are asymptotically flat and which have an intersection theory with $d > 2+1$. Therefore, even at strong coupling, we may rely on conclusions that follow from general properties of the field theory on the smaller branes (with $p \leq 3$) and on the intersection, such as locality and dimensionality. On the other hand, the SYM physics on the $d=p+1$ world volume of $p > 3$ branes is at best a low-energy effective field theory, and gets replaced in the UV by a more complicated theory which may not even be a local quantum field theory [29]. Taking into account our previous finding that the gauge physics is relevant only on the intersection or the smaller brane, or at worst on both clumps for the $A=B$ case, the only case where the fact that SYM_{p+1} is not the UV theory might bother us is $D4 \perp D4(2)$.

In fact, since in this case we do not know the fully localized supergravity solution but only an irreducibly smeared one, we are in effect dealing with $1/r$ potentials. Therefore, we suspect that holography may be breaking down for this particular case.

We see that the strongly quantum mechanical coupled DA, DB field theory describes the supergravity solution in the near-A-horizon regime. This concludes our discussion of the regime of validity of the gauge theory and its relation to the supergravity regime of validity. Now we turn to the gauge theory considerations, with which we want to explain our previous supergravity results on (de)localization.

In the supergravity section we saw (de)localization occur as we brought the B-branes in to the A-branes from finite transverse separation to zero separation. We now want to see how this happens from the field theory perspective. Since the type A and B branes are initially separated, we are *a priori* on the “separated” branch of moduli space. For the D5||D1 (and T-dual D4||D0) case this is usually called the Coulomb branch, and the “coincident” branch the Higgs branch. Now, if we were to integrate out the 1–5 strings to study the Coulomb branch, we would find [33,34] that the Coulomb and Higgs branches of moduli space are separated by an infinite distance and decouple. However, these 1–5 strings become light in the limit in which the separation between the A and B branes is removed, which is our situation of interest. In particular, in our setup we have kept the mass of such strings fixed relative to our gauge theory energy scale. Thus, the moduli space is simply not sufficient to describe physics in the region of interest. An analogy to our Ramond-Ramond case is the S-dual situation of fundamental strings approaching N-S five-branes. There, the infinite distance in moduli space corresponds to the infinite throat of the five-brane. However, we know that this is no obstacle to a fundamental string reaching and crossing the five-brane horizon. Again, what one finds is that the moduli space approximation is simply not sufficient to describe this part of the dynamics. In the same way, there is no conflict between our picture and the results of [33,34]; a mixing between the Coulomb and Higgs branches is allowed in our setup of the AdS-CFT limit for the intersecting branes when the separation is small.⁷

Let us consider in more detail the case of D1-branes (B) and D5-branes (A), where on the Higgs branch the relevant moduli are the scale sizes and orientations of the gauge instantons which represent the D1-branes in the D5-brane gauge theory. (There are also position moduli but they will not be important in the following.) As we argued above, we expect a mixing between the Coulomb and Higgs branches as the separation goes to zero. We will use physics of the Higgs branch to study delocalization, in an approximate sense, keeping in mind that the small mass of the 1–5 strings

modifies the dynamics of the system at large length scales and thus provides an infrared (IR) cutoff. Now let us extract a length scale for instanton size fluctuations. The scale size and orientation for a single instanton form a 1 + 1 quantum field theory with a moduli space metric which is flat for $N_1 = 1$, and with a coefficient $1/(l_s^2 g_s)$ in front of the Lagrangian. The l_s^2 is associated with the fact that ρ has dimensions of a length. That the metric is not renormalized at strong coupling is a consequence of the high degree of supersymmetry in this system (it is hyper Kähler). For large N_5 , there are roughly N_5 possible orientations for the instanton in gauge space, so we have

$$\sqrt{\langle \rho^2 \rangle} = \sqrt{\langle \rho_1^2 + \dots + \rho_{N_5}^2 \rangle} \sim \sqrt{N_5 g_s l_s^2} \sqrt{\log(\Lambda_{UV}/\Lambda_{IR})}, \quad (3.23)$$

where $\Lambda_{IR}, \Lambda_{UV}$ are appropriate infrared and ultraviolet cutoffs. Now, since we take all energies low by comparison to the string scale as in Eq. (3.1), $\Lambda_{UV} \sim l_s^{-1}$. In addition, as above, the 1–5 strings have a mass U which provides an IR cutoff. If there are N_B separate instantons, the moduli space metric, although uncorrected, is not flat. What is important for our estimate is the normalization, which is the same as in the $N_B = 1$ case. In addition, the instantons all fluctuate independently, and so we may expect the above rough estimate to carry over. To translate our estimate into the quantities used in the classical supergravity discussion, recall that $U = r_{\perp 0}/l_s$ and, since the A-brane is an unsmeared five-brane, $r_5 = l_s \sqrt{N_5 g_s}$. Also, since we are holding $\lambda_5^2(U) = g_s N_5 (l_s U)^2$ fixed and $r_{\perp 0}$ is small, up to numbers of order one we may replace the l_s coming from Λ_{UV} inside the logarithm with r_5 . We then have

$$\sqrt{\langle \rho^2 \rangle} \sim r_5 \sqrt{\ln(r_{\perp 0}/r_5)}. \quad (3.24)$$

We see that this estimate matches the supergravity result.

The story is similar for D0 branes approaching D4 branes. In that case, we have a (0 + 1)-quantum field theory and the rms fluctuations will be proportional to $\sqrt{1/\Lambda_{IR}}$, but $r_4 = (g_s N_4)^{1/3} l_s$. We find

$$\sqrt{\langle \rho^2 \rangle} \sim \sqrt{N_4 g_s l_s} \sqrt{l_s^2/r_{\perp 0}} = (g_s N_4)^{1/3} l_s \sqrt{\frac{(g_s N_4)^{1/3} l_s}{r_{\perp 0}}}. \quad (3.25)$$

Again, this matches the classical supergravity delocalization rate. The case of D3-branes and D-instantons is a bit degenerate, but one finds agreement with the classical supergravity results, and with scale-radius duality, by taking $\sqrt{\langle \rho^2 \rangle}$ proportional to $1/\Lambda_{IR}$.

We note that, for every case of intersecting Ramond-Ramond branes that falls within our framework, we have $A + b \geq 4$ and the potential H_A diverges no faster than r^{-3} . Furthermore, an r^{-3} potential is always associated with a (0 + 1)-dimensional intersection and an r^{-2} potential is always associated with a (1 + 1)-dimensional intersection. Similarly, cases where H_A diverges only like r^{-1} or weaker correspond to (2 + 1)-dimensional or larger intersections, and both the quantum size moduli fluctuations in the brane

⁷Our analysis also differs in that our B-branes are not probes; we take into account their effect on the supergravity fields. In addition, the field theory description of our setup is neither the conformal field theory which appears in the extreme IR on the Higgs branch nor the one on the Coulomb branch (these theories have different R-symmetries) [33].

gauge theory and the classical supergravity delocalization are small. [If the z_I were compactified on a very large manifold, the above results hold in the infinite-volume limit, and so by continuity the (de)localization results are essentially unchanged at large but finite volume. We will, however, avoid finite volumes so as to finesse additional phenomena that occur when the sizes of the compactified manifolds get too small near the core in the supergravity geometry.] Therefore, we see agreement for both the parallel and orthogonal intersections.

In cases where $b > 0$ i.e. the type A branes are smeared, their field theory is still $(A+1)$ -dimensional, not $(A+b+1)$ -dimensional. It is therefore reasonable to replace N_A in the argument above for the delocalization rate with the volume density $N_A l_s^b / V_b$, as one may think of this case as having a large number, of order V_b / l_s^b , of separate intersections. Thus, an estimate of the instanton scale size fluctuations can always be made that agrees with the classical supergravity delocalization rate.

Let us lastly consider the qualitatively different kinds of solutions we get by additionally smearing the type A branes along the w direction. In the classical supergravity, we have a localized solution if only three transverse dimensions are left unsmeared. This is hard to explain from the field theory perspective, as can be seen by considering a prototypical example of the D0-clump with a D4-clump smeared along two of the five transverse dimensions. Then the smeared D4-clump gives rise to a $1/r$ potential and the supergravity solution localizes, even though the intersection is $(0+1)$ -dimensional. Now, recall that holography for the D6||D2(2) and D4⊥D4(2) systems from the quantum brane gauge theory is surprising. On the other hand, it may simply be that smearing one brane and not the other is not a straightforward operation from the field theory perspective. Note also that when the D4-branes are smeared over only one transverse direction, instead of two, the quantum theory on the branes and the classical supergravity agree that delocalization should occur, but do not agree with regard to the rate at which this happens.

C. Asymptotically flat orthogonal branes

We would like to add a few more comments on the cases involving orthogonal intersecting branes, and how to use our earlier results to say something about asymptotically flat, as opposed to near-horizon, spacetimes. Recall that the intersection field theory description is dual to the near-horizon supergravity description. As such, it does not directly say anything about the asymptotically flat solutions. However, the near-horizon and asymptotically flat supergravity solutions are controlled by the same equations of motion (2.3),(2.4). The only difference is in the boundary conditions imposed on H_A ; the boundary conditions on H_B are identical for both cases. Thus the two supergravity solutions must agree to

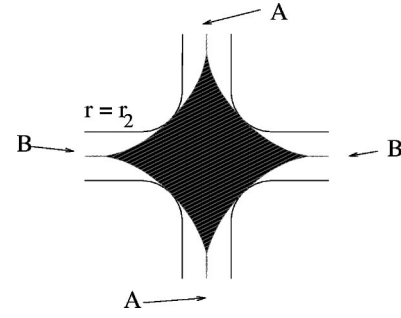


FIG. 1. Field theory delocalization arguments predict a finite-sized “neck” for the asymptotically flat supergravity solution for two orthogonal clumps (“A” and “B”) of D2-branes.

high accuracy in the region $|x_\perp| \ll r_A$. Thus, if the near-horizon geometry is delocalized, there must also be a region (perhaps, only for $|x_\perp| \ll r_A$) in which the asymptotically flat geometry is delocalized as well. We saw this explicitly for the solutions exhibited in Sec. II, in which the type A branes were initially smeared over the z_b directions. The same conclusion should hold in the case without the initial smearing, for which the supergravity solutions are not yet known. For initial progress toward constructing these solutions, see [21].

Let us now consider as a prototype of orthogonal intersections the D2⊥D2(0) system. Initially, for clarity, we refrain from smearing the A-branes over the B-brane world-volume. In the quantum gauge field theory, the instanton scale sizes become blowup modes of the orthogonally intersecting D2-branes. To see this, write each pair of spatial world volume coordinates as a complex coordinate Z , then for the combined world volumes we get the holomorphic curve $Z_A Z_B = \rho$. The smearing of ρ is infinite because the field theory on the intersection is only $(0+1)$ -dimensional. This means that the corresponding near-horizon geometry will also be smeared.

We now use the above argument about matching supergravity solutions and our knowledge of the blowup modes in the near-horizon case to draw some conclusions about delocalization in the asymptotically flat case. This solution should be delocalized, but perhaps only in some near-horizon region. A diagram giving our artistic impression of the asymptotically flat solution is included below. We can only conclude that delocalization must occur in the interior of the shaded region, which is the region inside a blowup mode that has expanded until it reaches the curve $r = r_2$. This is consistent with our expectations that, far from the intersection, the solution should reduce to the known physics of a lone D2-brane clump. Our delocalization has become a finite-sized “neck” of the supergravity solution.

In the actual D2⊥D2(0) case studied in the previous section, the D2⊥D2(0) supergravity solution has the A-branes initially smeared over z_b . The field theory on the intersection is still $(0+1)$ -dimensional, and so we conclude that the B-branes delocalize over the full near-A-horizon spacetime. In considering the implications for the asymptotically flat solutions, we recall that the near-A-horizon region has been enlarged by smearing over z_b . Figures 1 and 2 show the result for three clumps of type A branes placed close

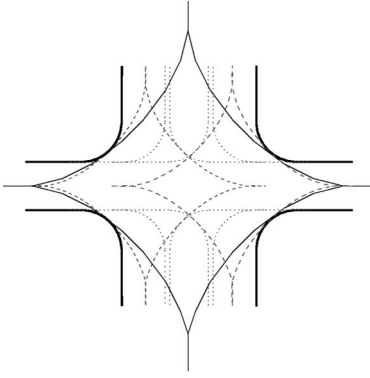


FIG. 2. Smearing of the “A” clump of D2-branes over the world volume of the “B” clump leads to delocalization in all world volume directions.

enough to each other that their charge radii (r_2 , indicated by dotted lines) overlap. The full near-horizon region is the interior of the solid heavy line. Thus, blowup modes near the center (thin solid line) are now allowed to be much larger while remaining inside the near-horizon region. When enough clumps are present to model a complete smearing of the type A branes, we may expect the field theory prediction to imply complete delocalization in the z_a directions of the type B branes in the asymptotically flat solution as well as the near-A-horizon solution.

We draw entirely analogous conclusions for the other orthogonally intersecting cases which have $(d=0+1)$ - or $(d=1+1)$ -dimensional intersections, such as the $D1\perp D3(0)$. The only case which can localize from the field theory perspective has a $(d=2+1)$ -intersection, and this is $D4\perp D4(2)$. But since this has only three totally transverse coordinates it exerts a $1/r$ potential and its near-horizon supergravity solution is localized, so again we have agreement of the near-horizon supergravity and field theory. Following our above argument, there will be asymptotically flat solutions in which both branes are localized, in addition to the known solutions where the A branes are initially smeared over the z_b directions.

IV. DISCUSSION

We have seen that many supergravity solutions containing two types of branes (A and B) have the property that one of the branes (B) delocalizes when the transverse separation between the branes is removed. This happens when the world volume directions of the type B brane are contained in the world volume directions of the type A brane, or in directions in which the type A brane has been smeared, and when the dimension of the intersection manifold is sufficiently small.

In terms of the corresponding brane gauge theories, this phenomenon is associated with the lack of a sharp transition between the “separated” and “coincident” branches in the limit where the separation between the branes is very small. It is also associated with the fact that the asymptotic values of massless fields do not label superselection sectors in $0+1$ and $1+1$ dimensions; i.e., with the Coleman-Mermin-

Wagner theorem [31,32]. In particular, the supergravity delocalization is related to large fluctuations of some size moduli fields in the “coincident” branch of the super Yang-Mills theory. As mentioned in [26], the classical delocalization is closely related to the black hole no-hair “theorems,” recently reviewed in [43]. It is rather amusing to connect such a classic feature of black hole physics with quantum fluctuations in the super Yang-Mills theory. Up to subtleties discussed in Sec. III, we find agreement both with respect to which cases should delocalize and with respect to the rate at which this delocalization occurs as the transverse separation between the branes is removed. It would be very interesting to understand in detail exactly why this rate agrees so well.

We have also seen this delocalization to be in accord with expectations that there should in fact exist orthogonal intersecting brane supergravity solutions with *both* branes (A and B) localized in the directions along the other brane. In this case, our delocalization phenomenon may become a finite neck of the supergravity solution of the sort that is seen [44] in the Born-Infeld description of intersecting branes. Thus, when the intersection manifold is $(0+1)$ - or $(1+1)$ -dimensional, we expect only solutions with necks of some minimum finite size while, for higher dimensional cases we expect solutions with necks of all finite sizes (including zero).

There were, however, some cases that we were not able to analyze properly. In some of these cases, the type A brane has been smeared so that it covers a $(6+1)$ -dimensional volume. The corresponding spacetime then resembles, to a certain extent, that of a D6-brane and the brane gauge theory may not properly decouple from the bulk. We were therefore unable to rely on holography to draw conclusions about the classical supergravity from the quantum gauge theory. Nonetheless, we found agreement for the $D2\parallel D6$ and $D4\perp D4(2)$ cases, and we would like to understand why this happened.

As mentioned in [26], BPS supergravity solutions for many three- (and higher)-charge solutions can also be analyzed in this way. Typically, when two of the charges are smeared, we can discuss localization of the third just as was done for the type B branes above. Asymptotically flat situations of this type that involve only Ramond-Ramond branes include three sets of D2 branes, or three sets of D3 branes. In these cases, the branes are again smeared over a $(6+1)$ -dimensional volume and we do not expect decoupling from the bulk.

Other cases that could not be studied precisely include NS-NS objects. Consider first the case of fundamental strings intersecting R-R branes. Here, we are stymied by our lack of understanding of fundamental strings in R-R backgrounds.⁸ However, in the supergravity regime, either the curvature in string units or the dilaton becomes large near the core of the R-R branes. This suggests that the fundamental string will fluctuate significantly near the supergravity R-R branes, and that this should give rise to delocalization of the endpoint (necking) near the core of the R-R branes in analogy with

⁸For recent progress in this direction, see [45–47].

our discussion of solutions describing D-strings intersecting a D3-brane.

For the case of Dp -branes intersecting NS5-branes, where the intersection has $p-1$ space and one time dimension, we have little to say because we do not understand well enough the theory on the NS5-branes or the related theory on the intersection manifold. It may be described by some sector of the NS5 ‘‘little string theory’’ [48,49], but such a description is likely to require much more than a field theory. Note that there are only 3 directions transverse to both branes for any value of p , so that the supergravity solutions, with one brane smeared over the other, will always be localized.

There remains however, the annoying case of D4||D0 with the D4-branes smeared over one w direction. Here, our classical supergravity and quantum field theory analyses predict delocalization. But, despite the fact that the potential is r^{-2} and we expect holography to hold, the two descriptions disagree with regard to the rate at which this should happen. It appears that smearing just the D4 branes is a more subtle operation in the quantum brane gauge theory than the supergravity would have us believe.

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APPENDIX: A COMPARISON WITH NEAR-CORE SOLUTIONS

One potential confusion concerns the near-core solutions of [39]. While they are valid solutions to the supergravity equations, we now argue that, due to the subtleties of boundary conditions, they are not the appropriate ones to consider in our context.

To discuss those solutions, it is necessary to recall some elements of their construction. The differential equations to be solved are just Eqs. (2.3) and (2.4) in the regime $\hat{r}_A \gg r_\perp, r_{\perp 0}$ under the assumption that no w directions (associated with smearing of the type A branes in directions transverse to the type B branes) are present. In [39], $r_{\perp 0}$ was set to zero. However, we will find it useful to keep $r_{\perp 0}$ nonzero and then study the limit in which it vanishes. Following [39],

we take the z_a directions to be non-compact. We take the number of z_a directions to be D . We would like to have spherical symmetry in both the x_\perp and z_a directions so, as in Sec. II, we replace the fully localized type B brane with a spherical $(S^{d-1} \times S^{D-1})$ shell of source: $q_B r_\perp^{-(d-1)} r_a^{-(D-1)} \delta(r_\perp, r_{\perp 0}) \delta(r_a, r_{a0})$. Thus, we have spherical symmetry in both the z_a and x_\perp directions and the solution depends only on r_\perp and $r_a = |z_a|$.

In the region $\hat{r}_A \gg r_\perp, r_{\perp 0}$, [39] uses a trick first introduced in [24] and finds that the equation simplifies under a change of coordinates (correcting a typographic error in [39]):

$$r_\perp \rightarrow Y = \frac{2\hat{r}_A^{(d-2)/2}}{|4-d|} r_\perp^{4-d/2}. \quad (\text{A1})$$

In terms of Y , the equation to be solved (for $\hat{r}_A \gg r_\perp, r_{\perp 0}$) may be written

$$\begin{aligned} Y^{-d/(4-d)} (\partial_Y (Y^{d/(4-d)} \partial_Y H_B)) + r_a^{1-D} (\partial_{r_a} (r_a^{D-1} H_B)) \\ = Y^{-d/(4-d)} r_a^{1-D} \delta(Y - Y(r_{\perp 0})) \delta(r_a, r_{a0}). \end{aligned} \quad (\text{A2})$$

This equation may be solved by realizing that it is the analytic continuation (to non-integer dimensions) of Laplace’s equation on $\mathbb{R}^{[(d/4-d)+1] \times \mathbb{R}^D}$, in coordinates in which the $SO[(d/4-d)+1] \times SO(D)$ symmetry is manifested. The closed form solutions of [39] may be obtained by smearing out the source further over the $S^{(d/4-d)+D}$ sphere $Y^2 + r_a^2 = R_0^2$ and taking the limit as $R_0 \rightarrow 0$. What is important to note is that, for $d > 4$, the coordinate transformation (A1) means that $R_0 \rightarrow 0$ corresponds to taking $r_{\perp 0}$ to infinity. Thus, the type B and A branes are not in fact being placed at the same location in space. One may verify that, near the type A branes, the solutions of [39] for $d > 4$ do not depend on the z_a coordinates, and so in this sense are not localized solutions. The fact that, at a generic point in the spacetime, the solutions of [39] do depend on z_a is a reflection of the lack of an asymptotically flat region in the near-core spacetime: even though the type B branes have been taken to infinity, part of their field can still be seen. For the case $d = 4$, the coordinate transformation (A1) breaks down, but [39] constructs a logarithmic solution to which similar remarks apply.

We note that if one tries to use these methods to construct a localized solution (for $d > 4$) by taking $r_{a0}, r_{\perp 0} \rightarrow 0$, one takes the source to infinity in Y where it will have little impact. Thus, this seems to reproduce the conclusion of Sec. II that the type B branes delocalize. However, the analysis is complicated by the nontrivial mapping of surfaces in z_a, x_\perp space into z_a, Y space.

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