AdS-CFT correspondence and the information paradox

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The information paradox in the quantum evolution of black holes is studied within the framework of the anti–de Sitter–conformal field theory (AdS-CFT) correspondence. The unitarity of the CFT strongly suggests that all information about an initial state that forms a black hole is returned in the Hawking radiation. The CFT dynamics implies an information retention time of the order of the black-hole lifetime. This fact determines many qualitative properties of the nonlocal effects that must show up in a semiclassical effective theory in the bulk. We argue that no violations of causality are apparent to local observers, but the semiclassical theory in the bulk duplicates degrees of freedom inside and outside the event horizon. Nonlocal quantum effects are required to eliminate this redundancy. This leads to a breakdown of the usual classical-quantum correspondence principle in Lorentzian black-hole spacetimes. [S0556-2821(99)02618-1]

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I. INTRODUCTION

Hawking's information paradox [1] is an important theoretical problem, which must be resolved by any theory that claims to provide a fundamental description of quantum gravity. The usual argument for information loss in black-hole evolution is made in the context of a low-energy effective theory, defined on a set of smooth spacelike hypersurfaces in a geometry describing the formation and subsequent evaporation of a large mass black hole. The key assumption in the argument is that the effective theory is a conventional local quantum field theory. If, on the other hand, we assume that black-hole evolution is a unitary process we are led to the conclusion that spacetime physics is nonlocal on macroscopic length scales. The nature of this nonlocality must be subtle, for it is certainly not apparent in our everyday low-energy activities. It should only become manifest under extreme kinematic circumstances, such as those that relate inertial and fiducial observers in a black-hole geometry.

Some evidence for the required sort of nonlocal behavior has been found in string theory [2]. The commutator of operators corresponding to an observer inside the event horizon and an observer, who measures low-energy Hawking radiation well outside the black hole, is nonvanishing in string theory, in spite of the fact that these observers are spacelike separated. These observers are related by a trans-Planckian boost, but if one instead considers a pair of spacelike separated observers with low-energy kinematics the effect is strongly suppressed and one recovers conventional local causality. These results suggest that the usual reasoning for information loss fails in the context of string theory but they have a limited range of validity, being obtained by perturbative, off-shell calculations in light-front string field theory. Similar arguments can also be made by convolving appropriate wavepackets with the *S* matrix [3]. In this case a macroscopic characteristic length scale appears in the amplitude, indicating nonlocal effects. The analyticity of the *S* matrix is, however, consistent with conventional causality.

Recent progress towards a nonperturbative formulation of string theory has provided new tools with which to explore these issues. In the present paper we re-examine the information problem from a modern point of view, using in particular the anti–de Sitter–conformal field theory (AdS-CFT) correspondence [4], which states that string theory in a certain background spacetime is equivalent to a supersymmetric gauge theory that lives on the boundary of the spacetime. The fact that the gauge theory is unitary strongly supports the view that no information is lost in the quantum-mechanical evolution of black holes, but it is less clear how the unitarity is implemented from the spacetime point of view.

In Sec. II we briefly review the AdS-CFT correspondence in Euclidean space, and discuss the definition of the Lorentzian correspondence by straightforward analytic continuation. Black-hole backgrounds, and their CFT descriptions are described in Sec. III. A simple gedanken experiment is considered in Sec. IV, which allows us to infer qualitative features of the nonlocality that must be present in the effective theory in the bulk. The dual gauge theory description implies an information retention time for the black hole, which plays a crucial role in these arguments. We conclude that local observers inside or outside the black hole see no violation of causality. We argue, however, that the semiclassical effective theory duplicates degrees of freedom inside and outside the event horizon, and that nonlocal quantum effects are required to eliminate this redundancy. Classically there are no such nonlocal effects, thus the usual classical-quantum correspondence principle breaks down in black-hole spacetimes.

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II. REVIEW OF AdS-CFT CORRESPONDENCE

As described in [5,6], the natural relation between CFT and AdS correlators is

$$\left\langle \exp \int_{S^d} \phi_0 \mathcal{O} \right\rangle \mathrm{CFT} = Z(\phi_0), \tag{2.1}$$

where $\phi_0(\Omega)$ is the boundary value of a field in the bulk, $\mathcal{O}$ is the dual operator in the CFT, and Z is the string partition function on AdS space with boundary conditions ϕ_0. This statement is made in the Euclidean formulation. It is made plausible by a remarkable theorem of Graham and Lee [7] that says that for any sufficiently smooth metric boundary values, there exists a unique smooth solution in the bulk.

In the following we will need to formulate the AdS-CFT correspondence in Lorentzian signature spacetime. Luscher and Mack [8] have shown that the Euclidean Green's functions of a quantum field theory invariant under the Euclidean conformal group can be analytically continued to the Lorentzian signature, and the resulting Hilbert space of states carries a unitary representation of the infinite-sheeted universal covering group of the Lorentzian conformal group. The natural Lorentzian spacetime is an infinite-sheeted covering of Minkowski space and thus the natural spacetime to consider in the bulk is the universal covering space of AdS.

Let us review how this analytic continuation proceeds for the case of AdS_5. Projective coordinates for anti–de Sitter space ξ^a with $a=1,\ldots,6$ satisfy

$$(\xi^6)^2-(\xi^4)^2-(\xi^k)^2=R^2, \tag{2.2}$$

where $k=1$, 2, 3, 5, and R is the radius of curvature of the AdS space. In the boundary limit we can drop the R^2 term, and parametrize the coordinates as

$$\xi^6=r\cosh\sigma,\quad \xi^4=r\sinh\sigma,\quad \xi^k=re^k, \tag{2.3}$$

where e^k is a unit four vector. The Euclidean conformal group SO(5,1) acts in an obvious way on the ξ coordinates. The set of Euclidean coordinates to be used for the boundary field theory are $(x^4,\mathbf{x})$ (with $\mathbf{x}=x^j$, $j=1$, 2,3)

$$x^4=\frac{\sinh\sigma}{\cosh\sigma+e^5},\quad \mathbf{x}=\frac{\mathbf{e}}{\cosh\sigma+e^5}. \tag{2.4}$$

The analytic continuation corresponds to taking $\sigma=i\tau$ with $-\infty<\tau<\infty$, which leads to an infinite-sheeted covering of Minkowski space $\tilde{M}$ with coordinates (τ,e^k).

A single copy of Minkowski space can be embedded in $\tilde{M}$ by taking the subspace $-\pi<\tau<\pi$ and $e^5>-\cos\tau$. The usual Minkowski coordinates are

$$x^0=\frac{\sin\tau}{\cos\tau+e^5},\quad \mathbf{x}=\frac{\mathbf{e}}{\cos\tau+e^5}. \tag{2.5}$$

This space is conformal to the boundary at infinity of the Poincare patch of anti–de Sitter space, which is defined by the coordinates (x^μ,z)

$$ds^2=\frac{R^2}{z^2}(dx^2-dz^2). \tag{2.6}$$

The boundary at infinity corresponds to $z=0$. Scale-radius duality is manifest in this set of coordinates as illustrated by the D-instanton–Yang-Mills instanton duality. The scale factor of the Yang-Mills instanton translates into the position along the radial z coordinate of the D instanton in the bulk [9].

To generate translations with respect to the global time τ, one acts with the conformal Hamiltonian of the field theory $H=1/2(P^0+K^0)$. Luscher and Mack show H is positive and self-adjoint [8], and that there is a unique vacuum state annihilated by H, invariant under conformal transformations.

In the following, we will take the point of view that the Lorentzian theory in the bulk is defined by this analytic continuation of the Euclidean correlation functions. As we will see later, this is a rather subtle point. Alternative proposals for the Lorentzian version of the AdS-CFT correspondence have appeared in the literature. An example is the eternal black-hole solution considered in [10], where the dynamics is described instead by two disconnected boundary field theories.

III. THE INFORMATION PUZZLE AND AdS BLACK HOLES

Hawking's information paradox arises when one considers the quantum-mechanical evolution of black holes [1,11]. The issues are most sharply defined for a black hole that is formed by gravitational collapse from nonsingular initial data and subsequently evaporates by emitting apparently thermal Hawking radiation. If the initial configuration is described by a pure quantum state and the Hawking radiation is truly thermal then this process involves evolution from a pure state to a mixed one, which violates quantum-mechanical unitarity. A related problem involves perturbations on a background extremal black hole. The resulting nonextremal black hole will emit Hawking radiation until it approaches extremality once more and the question of unitarity arises in this context. In both of the above settings one can also consider an equilibrium configuration where energy is fed into a black hole at the same rate that it evaporates. In this case the paradox arises from the fact that an arbitrary amount of information can be encoded into the infalling matter over time and most of this information will be absent from the outgoing Hawking radiation if it is truly thermal.

In order to take advantage of some of the recent developments in fundamental gravitational theory one would like to formulate analogous questions in the context of black-hole evolution in anti–de Sitter spacetime. This presents us with some immediate problems. For one thing AdS gravity does not have a well posed initial value problem due to the global causal structure of the AdS spacetime. While spacelike slices of AdS spacetime have infinite volume, a timelike observer can send a null signal to infinity and receive a return signal within a finite proper time. As a result it is problematic to define unitary quantum-mechanical evolution in AdS space even in the absence of black holes.

This problem can be circumvented in a number of ways. We can, for example, impose boundary conditions at infinity, which in the absence of black-hole formation lead to unitary evolution. The important point is that within some such framework we can study the formation and evaporation of a black hole whose lifetime is short compared to the light-crossing time of the AdS geometry. We can choose parameters in such a way that this black hole is nevertheless large compared to the Planck scale and thus carries a significant amount of information. The question of possible information loss associated with the evolution of the black hole is then effectively decoupled from the unitarity problem of the underlying AdS geometry.

There also exist black-hole solutions in AdS space where the Schwarzschild radius is large compared to the characteristic AdS length scale. For such black holes there is no separation of scales and thus difficult to disentangle the two unitarity problems. On the other hand, as we shall see below, such black holes are extremely unstable and not so useful for studying the information problem in the first place.

Another way to proceed would be to consider the asymptotically flat geometry of an extremal D brane, whose near-horizon limit is locally isomorphic to AdS spacetime. The asymptotically flat region then regulates the infrared pathology of the AdS geometry and one can consider evolution from initial data (subject to physical boundary conditions at the D-brane horizon). One would again want to study black holes which are small compared to the characteristic AdS length scale which in this case is the size of the extremal D-brane throat. There, of course, also exist solutions describing black holes that are larger than the throat scale. These are just the nonextremal p branes of the higher-dimensional supergravity theory. They have nonvanishing Hawking temperature and their evolution leads to an information problem of the usual type. On the other hand, once we are far from extremality the gauge theory correspondence, which is the main new tool at our disposal, is no longer useful. There is a rather subtle problem with regulating the infrared problems in this manner. The thermodynamic behavior is sensitive to the asymptotic boundary conditions, i.e., whether the conformal boundary in Euclidean signature is taken to be $S^n\times S^1$ or $\mathbf{R}^n\times S^1$ [12]. In the following, we make the choice $S^n\times S^1$, which is appropriate for the black holes we want to study, whereas the near-horizon limit of a D brane gives rise to $\mathbf{R}^n\times S^1$.

The metric of a static Schwarzschild black hole in $n+1$-dimensional asymptotically AdS spacetime can be written

$$ds^2=-\left(\frac{r^2}{R^2}+1-\frac{\mu}{r^{n-2}}\right)dt^2+\left(\frac{r^2}{R^2}+1-\frac{\mu}{r^{n-2}}\right)^{-1}dr^2 +r^2 d\Omega_{n-1}^2, \tag{3.1}$$

where R is the AdS radius of curvature and μ is proportional to Newton's constant in $n+1$ spacetime dimensions times the black-hole mass,

$$\mu=\frac{8\Gamma(n/2)G_N M}{(n-1)\pi^{(n-2)/2}}. \tag{3.2}$$

As we approach the black hole from large r the metric has a coordinate singularity at the AdS-Schwarzschild radius $r=r_s$, where

$$\frac{r_s^2}{R^2}+1-\frac{\mu}{r_s^{n-2}}=0. \tag{3.3}$$

In the limit of small black-hole mass, $\mu\ll R^{n-2}$, the black-hole parameters approach those of a black hole of equal mass in asymptotically flat spacetime,

$$r_s\approx\mu^{1/(n-2)}, \tag{3.4}$$

while in the large mass limit, $\mu\gg R^{n-2}$, we instead have

$$r_s\approx(\mu R^2)^{1/n}. \tag{3.5}$$

One obtains the Hawking temperature in the standard way by continuing to the Euclidean section and requiring the horizon to be smooth,

$$T_h=\frac{n r_s^2+(n-2)R^2}{4\pi R^2 r_s}. \tag{3.6}$$

In the small mass limit this reduces to $T_h\approx(n-2)/4\pi r_s$, which is the usual Hawking temperature of a Schwarzschild black hole in asymptotically flat spacetime, but in the large mass limit we find that the AdS black hole has positive specific heat, $T_h\approx n r_s/4\pi R^2$.

We can now use the Stefan-Boltzmann law to estimate the lifetime of an AdS black hole. For small black holes we find

$$\frac{d\mu}{dt}\sim\mu^{-2/(n-2)}, \tag{3.7}$$

leading to a lifetime which grows as a power of the black-hole mass,

$$\tau\sim\mu^{n/(n-2)}. \tag{3.8}$$

If we choose parameters so that r_s is a macroscopic length (in an AdS background with even larger radius of curvature) then the black hole will slowly evaporate, at a rate reliably predicted by semiclassical considerations.

In the large-mass limit the behavior is very different. In this case the evaporation rate grows with mass,

$$\frac{d\mu}{dt}\sim\mu^2, \tag{3.9}$$

leading to a lifetime that is bounded from above[1]

$$\tau_0-\tau\sim 1/\mu. \tag{3.10}$$

[1]Here we assume the boundary conditions at infinity correspond to zero incoming flux. For reflecting boundary conditions instead, the black hole will rapidly come into equilibrium with the thermal radiation. We thank G. Horowitz for discussions on this point.

The approximation of large mass breaks down as $r_s \to R$, so τ should be interpreted as the time that elapses before the black hole has evaporated to a size of order the AdS length scale. The subsequent evaporation rate will be independent of the original black hole mass so the total lifetime is obtained by adding some constant to τ. The parameter τ_0 in Eq. (3.10) is the value of τ in the limit where the original black-hole mass becomes infinite. A black hole of arbitrarily large initial mass will reduce to a size of order the AdS scale within this time, which means that such objects are violently unstable. They do not provide us with the slowly evolving background geometries that are required for setting up the information puzzle. In fact the instability will most likely prevent them from forming in gravitational collapse in the first place. This does not preclude their existence in thermal equilibrium with a high-temperature thermal bath but since the heat bath is already in a mixed quantum state that is not the ideal configuration for studying the information problem.

The upshot of all this is that, for the purpose of studying information issues in black-hole evolution, we want to consider black holes that are macroscopic, i.e., large compared to the string scale, but at the same time small compared to the AdS scale. This means that their Schwarzschild radius is also small compared to the radius of the transverse compact space that accompanies the AdS geometry. The favored configuration in this case is, in fact, not the AdS-Schwarzschild black hole that we have been discussing but rather a higher-dimensional black hole that is localized somewhere on the transverse compact space [13]. This is not really a problem for our discussion. The semiclassical expressions (3.4) for the Schwarzschild radius, Eq. (3.7) for the evaporation rate, and Eq. (3.8) for the black-hole lifetime still remain valid if we remember to replace the n of AdS_n by the number of space dimensions of the higher-dimensional geometry.

The $2+1$-dimensional black hole is a rather special case. The metric for the nonrotating Bañados-Teitelboim-Zanelli (BTZ) black hole [14] takes the form

$$ds^2 = \left(\frac{r^2}{R^2} - m\right) dt^2 - \left(\frac{r^2}{R^2} - m\right)^{-1} dr^2 - r^2 d\varphi^2, \tag{3.11}$$

where φ has period 2π. The relationship between m and the black-hole mass depends on which geometry one uses as a zero-mass reference. Two choices offer themselves. One is to define AdS_3 to have zero mass, in which case we have

$$m = -1 + 8G_N M_{\mathrm{adS}}. \tag{3.12}$$

Since m is required to be positive we see that $2+1$-dimensional black holes have a nonvanishing minimum mass with this definition. The other definition, which is perhaps more natural from the point of view of black-hole physics, is to define the $m=0$ geometry in Eq. (3.11) to have vanishing mass, so that

$$m = 8G_N M_{\mathrm{BTZ}}. \tag{3.13}$$

In this case the Schwarzschild radius, $r_s = \sqrt{m}R$, goes to zero when the mass is taken to zero, and AdS_3 appears as an isolated smooth geometry in a family of solutions with naked singularities which formally have negative mass.

The Hawking temperature of the black hole (3.11) is $T_h = \sqrt{m}/2\pi R$ and the entropy is $S = \pi\sqrt{m}R/2G_N$. The lifetime of such $(2+1)$-dimensional black holes is formally infinite. This is because the rate of Hawking radiation slows down as m approaches zero. This is not a problem because such small $2+1$-dimensional black holes are not relevant to the physics. Here the AdS-CFT correspondence involves string theory on a background of the form $\mathrm{AdS}_3 \times S^3 \times M$ and black holes with a Schwarzschild radius small compared to the size of the S^3 are unstable to form $5+1$-dimensional black holes that are localized on the S^3. Those black holes have a finite lifetime, given by Eq. (3.8) with $n=5$.

Let us now consider the description of macroscopic black holes, that are nevertheless small compared to the scale of the transverse geometry, from the gauge theory point of view. We begin with the AdS_5 case. In the canonical ensemble, Witten [15] has shown there is a phase transition from a large mass AdS-Schwarzschild solution to an AdS space with certain discrete identifications, generalizing the work of Hawking and Page for $3+1$ dimensions [16]. In the gauge theory, this is reflected as a deconfinement transition in the gauge theory as the temperature is lowered. For the AdS_3 case, there is no analog of the Hawking-Page phase transition, and instead there is a smooth crossover as the temperature decreases.

To obtain a stable phase containing the intermediate mass black holes that we will be interested in, it is convenient to consider the microcanonical ensemble instead. In general it will be necessary to impose additional constraints on the ensemble, by requiring that the energy density be sufficiently well-localized, to ensure that only single black-hole states dominate the ensemble. The gauge theory version of this ensemble will likewise be a microcanonical ensemble with additional constraints. To determine the form of these constraints one must follow through the mapping of the energy-momentum tensor of the gravity theory into operators in the gauge theory. This is of course a difficult task once one wishes to go beyond the linearized approximation, but is nevertheless a well-defined procedure. Analogous constraints appear in the discussion of black-hole entropy in matrix theory given in [17].

IV. UNITARITY VS LOCALITY

The static Schwarzschild solution (3.1) describes a black hole in equilibrium with a thermal gas in an AdS background. In order to study the information problem we want instead to consider the evolving geometry of a black hole which forms by gravitational collapse in AdS space and then evaporates as it emits Hawking radiation. We will not attempt to write down such a solution but rather choose parameters in such a way that the black hole evaporates slowly compared to all microscopic timescales and has a long lifetime. The geometry is then described to a good approximation by Eq. (3.1) with μ varying slowly with time. In order to separate the issue of black-hole information loss from the usual unitarity problem in AdS space we will also assume that this macroscopic black hole is formed in an AdS background with a very large radius of curvature so that the lifetime Eq. (3.8) is small compared to the light-crossing time.

We can now imagine describing the bulk of the evolution of the black hole in terms of a low-energy effective theory defined on a set of ''nice'' slices [18,19,2] that foliate the slowly evolving spacetime and approach the local free-fall frame of infalling matter at (and inside) the event horizon but also approach the frame of fiducial observers far away from the black hole. If the low-energy effective theory on the nice slices is a local quantum field theory then it follows from standard arguments [11] that the quantum state of the Hawking radiation will be correlated to that of the infalling matter. If we further assume that the black hole completely evaporates, leaving only outgoing Hawking radiation behind, then the final state cannot be a pure quantum state. On the other hand, the evolution of states in a local quantum theory is unitary and therefore the final state should be a pure state if the initial configuration before the black hole forms is described by a pure state.

This apparent contradiction must somehow be resolved in a fundamental theory and a number of scenarios have been put forward [20]. The conjectured AdS-CFT correspondence supports the view that black-hole evolution is unitary since the gauge theory is manifestly unitary. This in turn means that the low-energy effective theory cannot be a local quantum field theory.[2] This is not a problem in the AdS-CFT context because the duality map that relates the gauge theory and spacetime physics is quite nonlocal, as has been emphasized in recent work [21–23].

In the following we will assume the validity of the AdS-CFT conjecture and ask what its implications are for the propagation of information in black-hole spacetimes. For this purpose it is useful to consider gedanken experiments which highlight the conflict between unitarity and locality [24]. Let us in particular examine a simple experiment which involves correlated degrees of freedom inside and outside the event horizon. Imagine a pair of spins prepared in a singlet state well outside the black hole. Here ''spin'' should be understood as some internal label because conventional spin can, in principle, be detected by its long-range gravitational field and is, therefore, not suitable for this experiment. One of the spins is then carried inside the black hole, where a measurement of the spin is made, at some point $\mathcal{A}$. Meanwhile, an observer $\mathcal{O}$ outside the black hole makes measurements on the Hawking radiation. If all the information about the quantum state inside the black hole is encoded in the Hawking radiation, this observer can effectively measure a component of the spin that went inside the black hole. The observer then passes inside the event horizon, where he can receive a signal from point $\mathcal{A}$, which potentially contradicts his previous measurement, in violation of the laws of quantum mechanics. There is no real paradox here from the spacetime point of view, for if $\mathcal{O}$ is to learn of the contradiction before hitting the singularity then the signal sent by $\mathcal{A}$ has to involve frequencies beyond the Planck scale [24]. If, on the other hand, the signal from $\mathcal{A}$ is generated using only low-energy physics then $\mathcal{O}$ will have entered a region of strong curvature before receiving it. Either way the analysis of the gedanken experiment requires knowledge of physics beyond the Planck scale and the apparent contradiction only arises if we make the (unwarranted) assumption that this physics is described by local quantum field theory.

Let us reconsider this experiment using the boundary gauge theory. There is only a single quantum state on any given time slice from the boundary point of view, so no contradiction can arise between the spin measurements. During the evaporation phase, the Hamiltonian of the boundary theory, to a good approximation, generates evolution in the asymptotic time t in the AdS–Schwarzschild spacetime (3.1). This time variable belongs to a coordinate system which only covers a region of spacetime exterior to the black hole and is therefore awkward for describing the fate of observers that enter the black hole. The history of such an observer is more economically described if we instead evolve our quantum state using a different timelike generator, one which is associated with the free-fall frame at the horizon and connects the interior and exterior regions of the black hole [25]. It is, however, an important matter of principle that the evolution in asymptotic time must contain all information about the infalling matter, even inside the black-hole region. This is guaranteed by the unitarity of the boundary gauge theory.

The scale-radius duality [4] tells us that an object far outside the black hole is described by a localized configuration in the gauge theory, but as the black hole is approached the same object will be represented by an excitation of much larger transverse size in the gauge theory if the system evolves in asymptotic time. This should be a correct approximate statement in asymptotically AdS backgrounds, but note that there we are going beyond the application of scale-radius duality in the unperturbed AdS background. The black hole is represented by a system of particles in the gauge theory with fixed total energy. As an object falls into the black hole the gauge theory configuration that describes it spreads in transverse size and at the same time gets entangled with the particles that make up the black hole. While the exact quantum state of the system of the object plus black hole contains the information that the object continues its plunge towards the singularity, this information is not readily available to outside observers. In fact the only way to access it is through careful observations of correlations in the entire train of outgoing Hawking radiation, as we discuss further below.

Let us return to our gedanken experiment. In order for an outside observer to conduct a measurement on the spin that entered the black hole, he or she measures correlations between Hawking particles emerging from the black hole at different times. One could imagine a more active type of measurement where the outside observer attempts to probe the black hole in various ways. This would only serve to excite the black hole and be counterproductive since the state

[2]A possible loophole to this argument would be that information is stored in Planck-mass remnant states. The existence of such remnants would imply an enormous peak in the density of states around the Planck scale. This, however, is in conflict with the gauge theory calculations of black-hole entropy. Furthermore, the uniqueness of the conformally invariant ground state of the CFT rules out zero energy remnants.

of the spin would now be entangled with that of the probes in addition to the original black hole.

The gauge theory configuration that describes the black hole containing the spin at $\mathcal{A}$ behaves as a conventional thermodynamic system. Ideally we would like to calculate the entropy and lifetime using the CFT description, but such a calculation requires a better understanding of the strongly coupled CFT in the large-N limit. But we stress these are nevertheless completely well-defined computations in the CFT. The best we can do at present is to assume the validity of the AdS-CFT correspondence and infer that the gauge theory answers coincide with the semiclassical gravity results.

With an understanding that the entropy and the rate of Hawking radiation can be obtained from the CFT point of view, we can then invoke a result of Page [26,27]. This states that no useful information is emitted from a thermodynamic system that is radiating, until its coarse-grained entropy has been reduced by a factor of 2. The time this takes for an evaporating black hole is of the order of the black-hole lifetime. In other words, there is an information retention time in the gauge theory description of the black hole. The existence of an information retention time was postulated in Refs. [24] and [28] but the AdS-CFT correspondence now provides a concrete realization of that idea.

The information transfer between inside the event horizon and outside the event horizon thus effectively takes place only when a time of the order of the lifetime of the black hole has elapsed, from the point of view of a distant outside observer. It is also important to determine at what point the outside measurements begin to have significant influence on infalling observers inside the black hole. Let us think of this, for the moment, in the context of a low-energy effective theory, defined on a set of ''nice'' slices. If we assume that the effective theory is a local field theory that has been evolved forward from a nonsingular initial configuration described by some pure quantum state, and we further stipulate that all information about the initial state is to be found in the outgoing Hawking radiation, then the degrees of freedom on that part of the nice slice that is inside the black hole can carry no information about the initial state [29]. In other words, all information about the initial state must be ''bleached'' out of the infalling matter immediately upon crossing the event horizon, in blatant violation of the equivalence principle. For a large mass black hole the horizon sits in a region of weak curvature where tidal effects are small and an object in free fall should pass through more or less unaffected. This is, of course, just a statement of the information paradox in the context of a local effective field theory of gravity.

In the boundary theory this appears in the form of a somewhat different puzzle. There the infalling object is described by a spreading field configuration which is getting entangled with the ambient fields describing the black hole. This entanglement gets very complicated as the object is ''thermalized'' into the black-hole configuration, yet it is very delicate. The slightest change in the combined configuration could drastically change the results of subsequent correlation measurements on the Hawking radiation, leading the outside observers to conclude that the infalling object did indeed get bleached as it passed through the horizon.

For a resolution of this puzzle we again have to appeal to the AdS-CFT correspondence. Since we know that an infalling object encounters no obstacle at the horizon of a large black hole in the supergravity description, the gauge theory dynamics must somehow miraculously preserve the integrity of infalling matter even if it appears, to a casual outside observer, to be thermalized as it interacts with the black-hole configuration. This is very reminiscent of the discussion in Ref. [21] of low-energy, large impact parameter gravitational scattering. On the supergravity side the particles hardly interact at all and move past each other with only a small deflection, but on the gauge theory side the objects completely merge during the collision and interact strongly, but then somehow disentangle themselves and go their separate ways.

Some recent work [23,30–32] has shown explicitly in certain examples how the gauge theory dynamics, at large N, leads to a local and causal description of semiclassical low-energy supergravity. Building on this, we can argue that the information transfer to the outgoing Hawking radiation must take place near the black-hole singularity from the point of view of infalling observers. Consider once again the gedanken experiment involving spins. The observer inside the black hole cannot be influenced by the outside measurements until their result has been communicated inside, but the outside measurements cannot be completed until at least of the order of the black-hole information retention time has passed according to asymptotic AdS–Schwarzschild clocks. In order to receive a signal carrying outside results before hitting the singularity, the observer inside would have to undergo a proper acceleration of order $\exp(M^{2/(n-2)})$ (in $n+1$ dimensions). Such an acceleration would require trans-Planckian energies, which are simply not available to a low-energy observer. We, therefore, conclude that the observation of Hawking radiation does in fact bleach out a low-energy observer inside the horizon, but the bleaching only takes place at the singularity, where life is less than good anyway.

Since the singularity is by definition not in the causal past of any of the outgoing Hawking radiation it is clear that this information transfer is nonlocal on a macroscopic scale. Causality in the boundary gauge theory does not guarantee causality on the spacetime side. It does lead to an approximate causality in the bulk physics in flat or near-flat spacetime, but our black-hole example illustrates that even this macroscopic causality must break down near spacetime singularities. We note that this breakdown occurs already at the level of a perturbative $1/N$ expansion in the boundary gauge theory. The time evolution operator must be unitary, order by order in a $1/N$ expansion.

Further insight into the nature of this nonlocality in the nice slice theory can be gained by considering physics in the Euclidean continuation. The analytic continuation from Euclidean space completely determines arbitrary correlation functions in the Lorentzian CFT, and in particular correlators corresponding to the above gedanken experiment. We learn from the theorem of Graham and Lee [7] that in the Euclidean signature, the classical bulk geometry is smooth, and there is a one-to-one mapping between the field configurations in the bulk, and those on the boundary. In the Euclidean signature, we see no sign of any nonlocality in the bulk theory. Only when we analytically continue to Lorentz signature does the singularity arise, hidden behind the event horizon. Classically, this leads to too many degrees of freedom in the bulk. The degrees of freedom inside and outside the event horizon are independent. Quantum mechanically, the picture in the Lorentz signature is radically different. The CFT tells us the degrees of freedom inside are to be identified with degrees of freedom outside the event horizon. This implies the usual classical-quantum correspondence principle breaks down for black-hole spacetimes. The degrees of freedom in the correct quantum description in the bulk do not smoothly go over to the classical degrees of freedom of the supergravity theory. The information paradox arises when we ask questions involving these degrees of freedom that are duplicated in the classical theory. Nonlocal effects are required in a semiclassical description on a set of nice slices, to see that these degrees of freedom are in fact redundant.

This set of arguments also implies the CFT formulation resolves the singularity of the black hole [33,25] and allows us to propagate states smoothly past the point where the black hole has evaporated. Classically, this point looks like a timelike (or null) naked singularity in the spacetime. Thus from the Lorentz point of view, the analytic continuation from the Euclidean signature leads to definite boundary conditions on this naked singularity.

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