

Cosmological implications of a relic neutrino asymmetry

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We consider some consequences of the presence of a cosmological lepton asymmetry in the form of neutrinos. A relic neutrino degeneracy enhances the contribution of massive neutrinos to the present energy density of the Universe, and modifies the power spectrum of radiation and matter. Comparing with current observations of cosmic microwave background anisotropies and large-scale structure, we derive some constraints on the neutrino degeneracy and on the spectral index in the case of a flat universe with a cosmological constant. [S0556-2821(99)03222-1]

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I. INTRODUCTION

It is generally assumed that our Universe contains an approximately equal amount of leptons and antileptons. Lepton asymmetry would be of the same order as baryon asymmetry, which is very small as required by big bang nucleosynthesis (BBN) considerations. The existence of large lepton asymmetry is restricted to be in the form of neutrinos from the requirement of universal electric neutrality, and the possibility of large neutrino asymmetry is still open. From a particle physics point of view, lepton asymmetry can be generated by an Affleck-Dine mechanism [1] without producing large baryon asymmetry (see Ref. [2] for a recent model) or even by active-sterile neutrino oscillations after the electroweak phase transition (but in this last case, it might not be of order unity) [3]. This lepton asymmetry can postpone symmetry restoration in nonsupersymmetric or supersymmetric models [4] (note that this is also true for other charges [5]).

In this paper we study some cosmological implications of relic degenerate neutrinos.¹ We do not consider any specific model for generating such an asymmetry, and just assume that it was created well before neutrinos decoupled from the rest of the plasma. An asymmetry of order 1 or larger can have crucial effects on the global evolution of the Universe. Among other effects, it changes the decoupling temperature of neutrinos, the primordial production of light elements at BBN, the time of equality between radiation and matter, or the contribution of relic neutrinos to the present energy density of the Universe. The latter changes affect the evolution of perturbations in the Universe. We focus on anisotropies of the cosmic microwave background (CMB) and on the distribution of large-scale structure (LSS). We calculate the power spectrum of both quantities, in the case of massless degenerate neutrinos, and also for neutrinos with a mass of 0.07 eV, as suggested to explain the experimental evidence of atmospheric neutrino oscillations at Super-Kamiokande [6]. The

cosmological implications of neutrinos with such a small mass are known to be very small, but we will see that this conclusion is modified if a large neutrino degeneracy exists. We also include in our analysis the possibility that the dominant contribution to the present energy density in the Universe is due to a cosmological constant $\Omega_\Lambda \sim 0.6-0.7$, keeping the Universe flat ($\Omega_0 + \Omega_\Lambda = 1$), as suggested by recent observations (see [7] and references therein).

The effect of neutrino degeneracy on the LSS power spectrum was studied in Ref. [8] as a way of improving the agreement with observations of mixed dark matter models with eV neutrinos, in the case of high values of the Hubble parameter. Also, Adams and Sarkar [9] calculated the CMB anisotropies and the matter power spectrum, and compared them with observations in the $\Omega_\Lambda = 0$ case for massless degenerate neutrinos. More recently, Kinney and Riotto [10] also calculated the CMB anisotropies for massless degenerate neutrinos in the $\Omega_\Lambda = 0.7$ case.

This paper is organized as follows. In Sec. II, we calculate the contribution of massive degenerate neutrinos to the present energy density of the Universe. In Sec. III, we explain how to calculate the power spectra, with the help of the code CMBFAST [11]. In Sec. IV, we discuss the effect of the degeneracy on CMB anisotropies and the matter power spectrum, both for massless neutrinos and $m_\nu = 0.07$ eV. Finally, in Sec. V, we derive some constraints on the neutrino degeneracy from CMB and LSS data in the particular case of a flat universe with an arbitrary cosmological constant and for standard values of other cosmological parameters.

II. ENERGY DENSITY OF MASSIVE DEGENERATE NEUTRINOS

The energy density of one species of massive degenerate neutrinos and antineutrinos, described by the distribution functions f_ν and $f_{\bar{\nu}}$, is (here and throughout the paper we use $\hbar = c = k_B = 1$ units)

$$\begin{aligned} \rho_\nu + \rho_{\bar{\nu}} &= \int \frac{d^3\vec{p}}{(2\pi)^3} E_\nu [f_\nu(p) + f_{\bar{\nu}}(p)] \\ &= \frac{1}{2\pi^2} \int_0^\infty dp p^2 \sqrt{p^2 + m_\nu^2} [f_\nu(p) + f_{\bar{\nu}}(p)], \quad (1) \end{aligned}$$

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¹Here, by degeneracy, we mean that there exists large neutrino-antineutrino asymmetry, or vice versa, and not a degeneracy in the mass sense.

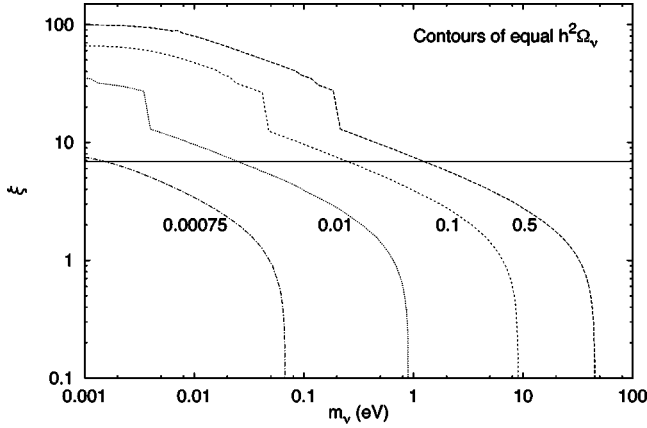


FIG. 1. Present energy density of massive degenerate neutrinos as a function of the degeneracy ξ . The curves correspond to different values of $h^2\Omega_\nu$, and the horizontal line is the upper bound from Eq. (6).

valid at any moment. Here p is the magnitude of the three-momentum and m_ν is the neutrino mass.

When the early Universe was hot enough, neutrinos were in equilibrium with the rest of the plasma via weak interactions. In that case the distribution functions f_ν and $f_{\bar{\nu}}$ changed with the Universe expansion, keeping the form of a Fermi-Dirac distribution,

$$f_\nu(p) = \frac{1}{\exp\left(\frac{p}{T_\nu} - \frac{\mu}{T_\nu}\right) + 1}, \quad f_{\bar{\nu}}(p) = \frac{1}{\exp\left(\frac{p}{T_\nu} + \frac{\mu}{T_\nu}\right) + 1}. \quad (2)$$

Here μ is the neutrino chemical potential, which is nonzero if a neutrino-antineutrino asymmetry has been previously produced. Later the neutrinos decoupled when they were still relativistic,² and from that moment the neutrino momenta just changed according to the cosmological redshift. If a is the expansion factor of the Universe, the neutrino momentum decreases, keeping ap constant. At the same time the neutrino degeneracy parameter $\xi \equiv \mu/T_\nu$ is conserved, with a value equal to that at the moment of decoupling. Therefore one can still calculate the energy density of neutrinos now from Eqs. (1) and (2), replacing μ/T_ν by ξ and p/T_ν by $p/(y_\nu T_0)$, where $T_0 \approx 2.726$ K and y_ν is the present ratio of neutrino and photon temperatures, which is not unity because once decoupled the neutrinos did not share the entropy transfer to photons from the successive particle annihilations that occurred in the early Universe. In the standard case, the massless nondegenerate neutrinos decoupled just before the electron-positron pairs annihilated to photons, from which one gets the standard factor $y_\nu = (4/11)^{1/3}$.

In the presence of a significant neutrino degeneracy ξ the decoupling temperature $T(\xi)$ is higher than in the standard case [12,13]. The reaction rate Γ of the weak processes,

²Unless the neutrino mass is comparable to the decoupling temperature, $\mathcal{O}(m_\nu) \sim 1$ MeV.

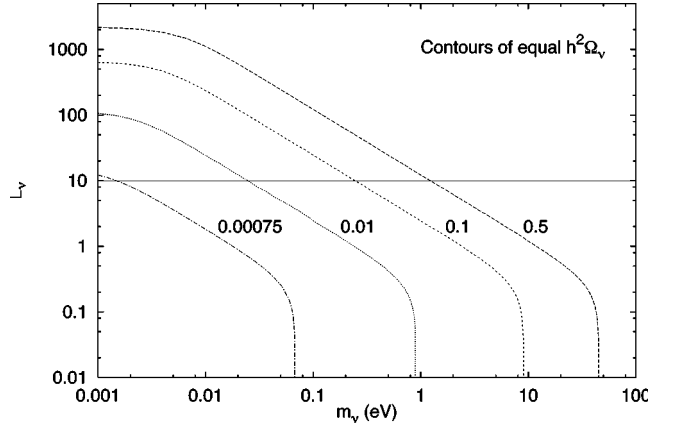


FIG. 2. Same as Fig. 1 for the neutrino asymmetry L_ν .

which keep the neutrinos in equilibrium with the other species, is reduced because some of the initial or final neutrino states will be occupied. The authors of Ref. [13] used the Boltzmann equation to calculate Γ for the process $\nu_d + \bar{\nu}_d \leftrightarrow e^+ + e^-$ (here ν_d denotes degenerate neutrinos), including the corresponding Fermi blocking factors. It was found that the neutrino decoupling temperature is $T_{dec}(\xi) \approx 0.2\xi^{2/3} \exp(\xi/3)$ MeV (for ν_μ or ν_τ). Therefore, if ξ is large enough, the degenerate neutrinos decouple before the temperature of the Universe drops below the different mass thresholds, and are not heated by the particle-antiparticle annihilations. The ratio of neutrino and photon temperatures is thus reduced accordingly.

The present contribution of these degenerate neutrinos to the energy density of the Universe can be parametrized as $\rho_\nu = 10^4 h^2 \Omega_\nu$ eV cm⁻³, where Ω_ν is the neutrino energy density in units of the critical density $\rho_c = 3H^2 M_p^2 / 8\pi$, $M_p = 1.22 \times 10^{19}$ GeV is the Planck mass, and $H = 100h$ Km s⁻¹ Mpc⁻¹ is the Hubble parameter. The value of ρ_ν can be calculated as a function of the neutrino mass and the neutrino degeneracy ξ or, equivalently, the present neutrino asymmetry L_ν defined as the following ratio of number densities:

$$L_\nu \equiv \frac{n_\nu - n_{\bar{\nu}}}{n_\gamma} = \frac{1}{12\xi(3)} y_\nu^3 [\xi^3 + \pi^2 \xi]. \quad (3)$$

We show³ in Figs. 1 and 2 the contours in the (m_ν, L_ν) and (m_ν, ξ) planes that correspond to some particular values of $h^2\Omega_\nu$. One can see from the figures that there are two limits: massive nondegenerate neutrinos and massless degenerate neutrinos. The first case corresponds to the vertical lines, when one recovers the well-known bound on the neutrino mass $m_\nu \leq 46$ eV for $h^2\Omega_\nu = 0.5$. On the other hand, for very light neutrinos, the horizontal lines set a maximum value on the neutrino degeneracy that would correspond to a present neutrino chemical potential $\mu_0 \leq 7.4 \times 10^{-3}$ eV, also for

³Here we assume $\xi > 0$, but the results are also valid for $\xi < 0$ provided that ξ and L_ν are understood as moduli.

$h^2\Omega_\nu=0.5$. In the intermediate region of the figures the neutrino energy density is $\rho_\nu \simeq m_\nu n_\nu(\xi)$ and the contours follow roughly the relations

$$L_\nu \left(\frac{m_\nu}{\text{eV}} \right) \simeq 24.2 h^2 \Omega_\nu, \quad (\pi^2 \xi + \xi^3) \left(\frac{m_\nu}{\text{eV}} \right) \simeq \frac{350}{y_\nu^3} h^2 \Omega_\nu. \quad (4)$$

A similar calculation has been recently performed in Ref. [14]. However, the difference between neutrino and photon temperatures was not properly taken into account for large ξ . It was argued that, since the number density of highly degenerate neutrinos is larger than in the nondegenerate case, the neutrinos would have been longer in thermal contact with e^+e^- , therefore sharing with photons the entropy release. However, this is not the case [13] as we discussed before.

The presence of a neutrino degeneracy can modify the outcome of big bang nucleosynthesis (for a review see [15]). First a larger neutrino energy density increases the expansion rate of the Universe, thus enhancing the primordial abundance of ${}^4\text{He}$. This is valid for a nonzero ξ of any neutrino flavor. In addition, if the degenerate neutrinos are of electron type, they have a direct influence over the weak processes that interconvert neutrons and protons. This last effect depends on the sign of ξ_{ν_e} . Both effects may be simultaneously important and it could be possible in principle to explain the observed primordial abundances with a large baryon density, $\Omega_B h^2 \approx 1$ [12,13]. However, this possibility is ruled out by the fact that in that case our Universe would have been radiation dominated during a longer period and the observed large-scale structure would be difficult to explain. From BBN one gets the constraint [13]

$$-0.06 \lesssim \xi_{\nu_e} \lesssim 1.1, \quad (5)$$

while a sufficiently long matter-dominated epoch requires

$$|\xi_{\nu_\mu, \nu_\tau}| \lesssim 6.9. \quad (6)$$

This estimate from [13] agrees with our analysis in Sec. V. Assuming that the degenerate neutrinos are ν_μ or ν_τ , this places a limit on the degeneracy as shown by the horizontal line in Figs. 1 and 2.

III. POWER SPECTRA CALCULATION

We compute the power spectra of CMB anisotropies and large-scale structure using the Boltzmann code CMBFAST by Seljak and Zaldarriaga [11], adapted to the case of one family of degenerate neutrinos ($\nu, \bar{\nu}$), with mass m_ν and degeneracy parameter ξ . Let us first review the required modifications. We use the notation of Ma and Bertschinger [16], and for all issues not specific to our case, we refer the reader to this review.

Background quantities can be rewritten in terms of two dimensionless parameters (M, Q)

$$M = \frac{m_\nu}{T_{\nu 0}} = \frac{m_\nu(\text{eV})}{8.6170 \times 10^{-5} \times (4/11)^{1/3} T_0(\text{K})}, \quad Q = \frac{ap}{T_{\nu 0}} \quad (7)$$

[we are assuming $y_\nu = (4/11)^{1/3}$, and therefore $\xi \leq 12$ [13]; the scale factor is defined so that $a=1$ today]. For Super-Kamiokande neutrinos with $m_\nu = 0.07$ eV, $M \approx 417$. We then get, for the mean density, pressure, and phase-space distributions,

$$\bar{\rho}_\nu + \bar{\rho}_{\bar{\nu}} = \frac{T_\nu^4}{2\pi^2} \int Q^2 dQ \sqrt{Q^2 + a^2 M^2} [f_\nu(Q) + f_{\bar{\nu}}(Q)],$$

$$\bar{P}_\nu + \bar{P}_{\bar{\nu}} = \frac{T_\nu^4}{6\pi^2} \int Q^2 dQ \frac{Q^2}{\sqrt{Q^2 + a^2 M^2}} [f_\nu(Q) + f_{\bar{\nu}}(Q)], \quad (8)$$

$$f_\nu(Q) = \frac{1}{e^{Q-\xi} + 1}, \quad f_{\bar{\nu}}(Q) = \frac{1}{e^{Q+\xi} + 1}.$$

In the case of massive degenerate neutrinos, these integrals must be calculated for each value of the scale factor, and also at the beginning of the code in order to find Ω_ν today. On the other hand, for massless neutrinos, there is an exact analytic solution

$$\bar{\rho}_\nu + \bar{\rho}_{\bar{\nu}} = 3(\bar{P}_\nu + \bar{P}_{\bar{\nu}}) = \frac{7}{8} \frac{\pi^2}{15} T_\nu^4 \left[1 + \frac{30}{7} \left(\frac{\xi}{\pi} \right)^2 + \frac{15}{7} \left(\frac{\xi}{\pi} \right)^4 \right]. \quad (9)$$

So, if we define an effective number of massless neutrino families, $N_{eff} \equiv 3 + 30/7(\xi/\pi)^2 + 15/7(\xi/\pi)^4$, the mean density and pressure for all neutrinos will be given by these ones for one massless nondegenerate family, multiplied by N_{eff} .

Let us now consider perturbed quantities. We define Ψ_ν and $\Psi_{\bar{\nu}}$, the perturbations of the phase-space distribution for ν and $\bar{\nu}$, through

$$\delta f_\nu(\vec{x}, Q, \hat{n}, \tau) = f_\nu(Q) \Psi_\nu(\vec{x}, Q, \hat{n}, \tau),$$

$$\delta f_{\bar{\nu}}(\vec{x}, Q, \hat{n}, \tau) = f_{\bar{\nu}}(Q) \Psi_{\bar{\nu}}(\vec{x}, Q, \hat{n}, \tau) \quad (10)$$

(\hat{n} is the momentum direction: $\vec{p} \equiv p\hat{n}$). For our purpose, which is to integrate the linearized Einstein equations, it can be shown that only the following linear combination is relevant:

$$\Psi \equiv \frac{f_\nu \Psi_\nu + f_{\bar{\nu}} \Psi_{\bar{\nu}}}{f_\nu + f_{\bar{\nu}}}. \quad (11)$$

Using the Boltzmann equations for Ψ_ν and $\Psi_{\bar{\nu}}$, it is straightforward to show that the evolution of Ψ [in Fourier space and in the synchronous gauge; see [16], Eq. (40)] obeys

$$\frac{\partial \Psi}{\partial \tau} + i \frac{Q}{\sqrt{Q^2 + a^2 M^2}} (\vec{k} \cdot \hat{n}) \Psi + \frac{d \ln(f_\nu + f_{\bar{\nu}})}{d \ln Q} \times \left[\dot{\eta} - \frac{\dot{h} + 6 \dot{\eta}}{2} (\hat{k} \cdot \hat{n})^2 \right] = 0. \quad (12)$$

This equation depends on ξ only through the last term, which is the gravitational source term.

In the case $\xi=0$, the quantity $[d \ln(f_\nu)/d \ln Q]$ has a simple interpretation: it is the Q dependence of a Planckian perturbation of the phase-space distribution. In other words, a shift of the blackbody temperature $\Delta T/T(\vec{x}, \hat{n}, \tau)$ corresponds to a perturbation

$$\Psi(\vec{x}, Q, \hat{n}, \tau) = - \frac{\Delta T}{T}(\vec{x}, \hat{n}, \tau) \frac{d \ln(f_\nu)}{d \ln Q}. \quad (13)$$

Since the gravitational source term in the Boltzmann equation is proportional to this quantity, the Planckian shape is unaltered for massless neutrinos and also for massive neutrinos when they are still relativistic [indeed, when $Q^2 \gg a^2 M^2$, the Q dependence of the Boltzmann equation (12) vanishes in the second term, and remains only in the third term]. When $\xi \neq 0$, the source term in Eq. (12) is proportional to

$$\frac{d \ln(f_\nu + f_{\bar{\nu}})}{d \ln Q} = - \frac{Q(1 + \text{ch } \xi \text{ ch } Q)}{(\text{ch } \xi + e^{-Q})(\text{ch } \xi + \text{ch } Q)}. \quad (14)$$

When neutrinos are still relativistic, Ψ is proportional to this quantity, even if it cannot be simply interpreted in terms of blackbody temperature perturbations.

We can now specify all the changes required in CMBFAST, first in the case of massive degenerate neutrinos. As usual, Ψ can be expanded in a Legendre series: $\Psi = \sum_{l=0}^{\infty} (-i)^l (2l+1) \Psi_l P_l$. It is easy to show that for each multipole Ψ_l , the evolution equation and the initial condition are both identical to those of the nondegenerate case, provided that we replace $[d \ln(f_\nu)/d \ln Q]$ by Eq. (14). So, in summary, one only needs to modify the homogeneous phase-space distribution, its logarithmic derivative with respect to Q , and the initial calculation of Ω_ν . Also, in order to obtain a good precision in the CMB anisotropy spectra, one must set $l=5$ for the number of multipoles Ψ_l to be time integrated. For transfer functions, the value $l=25$ proposed by the code is sufficient.

In the case of massless degenerate neutrinos, the situation is even simpler. The Q dependence of the Boltzmann equation can be integrated away, just like in the nondegenerate case. For this purpose, we must introduce the Q -independent variable F_ν ,

$$F_\nu(\vec{k}, \hat{n}, \tau) \equiv \frac{\int Q^3 dQ (f_\nu + f_{\bar{\nu}}) \Psi}{\int Q^3 dQ (f_\nu + f_{\bar{\nu}})} \equiv \sum_{l=0}^{\infty} (-i)^l (2l+1) F_{\nu l} P_l, \quad (15)$$

and integrate by part the last term in Eq. (12). The multipoles $F_{\nu l}$ are exactly identical for degenerate and nondegenerate

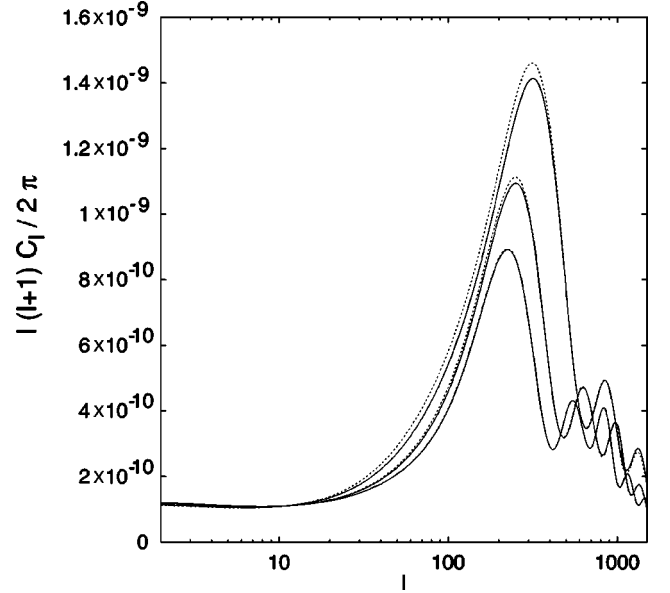


FIG. 3. CMB anisotropy spectrum for different models with one family of degenerate neutrinos. Solid lines account for the case of massless degenerate neutrinos and correspond, from bottom to top, to $\xi=0,3,5$. Dashed lines correspond to degenerate neutrinos with mass $m_\nu=0.07$ eV. Other parameters are fixed to $h=0.65$, $\Omega_b=0.05$, $\Omega_\Lambda=0.70$, $\Omega_{CDM}=1-\Omega_b-\Omega_\nu-\Omega_\Lambda$, $Q_{rms-ps}=18 \mu\text{K}$, $n=1$. We neglect re-ionization and tensor contribution.

massless neutrinos, because they share the same evolution equations and initial conditions. So the effect of ξ arises only through the background quantities in Eq. (8) and is completely described by introducing an effective number of massless neutrinos.

IV. RESULTS

First, as a consistency check, we compute CMB anisotropies and transfer functions for different values of ξ , choosing a very small mass $m_\nu \leq 0.001$ eV. We check that the results match exactly those obtained with the unmodified version of CMBFAST, when the appropriate effective neutrino number N_{eff} is specified.

The effect of ξ and m_ν on the CMB anisotropy spectrum can be seen in Fig. 3. We choose a set of cosmological parameters ($h=0.65$, $\Omega_b=0.05$, $\Omega_\Lambda=0.70$, $\Omega_{CDM}=1-\Omega_b-\Omega_\nu-\Omega_\Lambda$, $Q_{rms-ps}=18 \mu\text{K}$, flat primordial spectrum, no re-ionization, no tensor contribution), and we vary ξ from 0 to 5, both in the case of massless degenerate neutrinos (solid lines) and degenerate neutrinos with $m_\nu=0.07$ eV (dashed lines). Let us first comment on the massless case. The main effect of ξ is to boost the amplitude of the first peak.⁴ Indeed,

⁴In fact, this is not true for very large values of ξ . In such cases, recombination can take place still at the end of radiation domination, and anisotropies are suppressed. For our choice of cosmological parameters, this happens for $\xi \geq 7$, but in such a case the location of the first peak is $l \geq 450$, and the matter power spectrum is strongly suppressed.

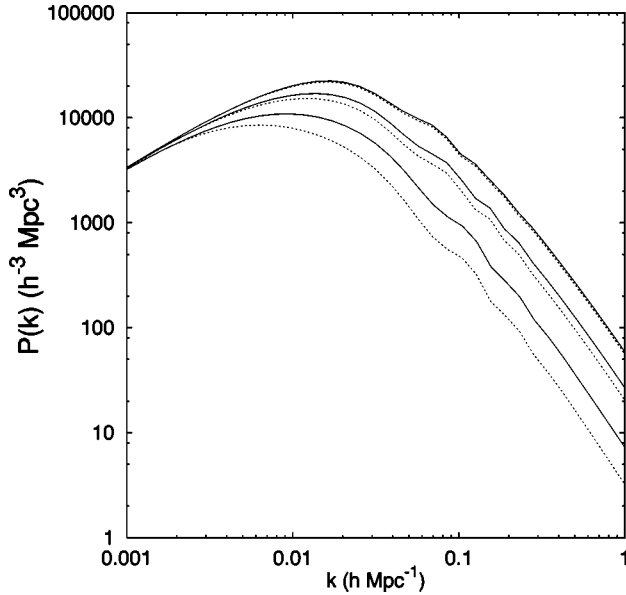


FIG. 4. Present power spectrum of matter density, for the same parameters as in the previous figure. From top to bottom, to $\xi = 0, 3, 5$.

increasing the energy density of radiation delays matter-radiation equality, which is known to boost the acoustic peaks [17] (the same explanation holds for the effect of Ω_Λ in flat models). For the same reason, all peaks are shifted to higher multipoles, by a factor $[(1 + a_{eq}/a_*)^{1/2} - (a_{eq}/a_*)^{1/2}]^{-1}$ [17] (a_{eq} is the scale factor at equality, and increases with ξ , while the recombination scale factor a_* is almost independent of the radiation energy density). Secondary peaks are then more affected by diffusion damping at large l , and their amplitude can decrease with ξ .

In the case of degenerate neutrinos with $m_\nu = 0.07$ eV, the results are quite similar in a first approximation. Indeed, the effects described previously depend on the energy density of neutrinos at equality. At that time, they are still relativistic and identical to massless neutrinos with equal degeneracy parameter. However, with a large degeneracy, Ω_ν today becomes significant: for $\xi = 5$, one has $\Omega_\nu = 0.028$, i.e., the same order of magnitude as Ω_b . Since we are studying flat models, $\Omega_\nu = 0.028$ must be compensated by fewer baryons, cold dark matter (CDM), or Ω_Λ . In our example, Ω_b and Ω_Λ are fixed, while Ω_{CDM} slightly decreases. This explains the small enhancement of the first peak compared to the massless case. Even if this effect is indirect, it is nevertheless detectable in principle (even if one does not impose the flatness condition, the effect of Ω_ν will be visible through a modification of the curvature). In Fig. 3, for $\xi = 0$, the first peak maximum is enhanced by only 0.37%, while for $\xi = 5$, there is an increase of 3.4%, detectable by the future satellite missions the Microwave Anisotropy Probe (MAP) and Planck, unless there are large parameter degeneracies. It is well known that such degeneracies are generally removed when CMB and LSS data are combined for parameter extraction [18].

We plot in Fig. 4 the power spectrum $P(k)$ for the same models as in Fig. 3, normalized on large scales to the Cosmic

Background Explorer (COBE). The effect of both parameters ξ and m_ν is now to suppress the power on small scales. Indeed, increasing ξ postpones matter-radiation equality, allowing less growth for fluctuations crossing the Hubble radius during radiation domination. Adding a small mass affects the recent evolution of fluctuations, and has now a direct effect: when the degenerate neutrinos become nonrelativistic, their free streaming suppresses the growth of fluctuations for scales within the Hubble radius. For nondegenerate neutrinos, this effect is known to reduce the power on those scales by a relative amount $\Delta P/P \sim 8\Omega_\nu/\Omega_0$ [19] (we introduced $\Omega_0 = 1 - \Omega_\Lambda$). So, even with $m_\nu = 0.07$ eV and $\xi = 0$, it is significant, especially at low Ω_0 . In the models of Fig. 4, $P(k)$ decreases by $\sim 5\%$, in agreement with the theoretical prediction ($\Omega_\nu = 1.8 \times 10^{-3}$, $\Omega_0 = 0.3$). However, at $\xi = 5$ (i.e., $\Omega_\nu = 0.028$), this effect is even larger: $P(k)$ decreases by a factor of 2.2, instead of an expected 1.7. This effect is likely related to the phase-space distribution of neutrinos with a chemical potential: their average momentum is shifted to larger values, making the free-streaming suppression mechanism even more efficient.

Let us compare our results with those of previous works. The effect of ξ on the CMB for massless neutrinos and $\Omega_\Lambda = 0$ is the same as that one found in [9]. We also agree with the revised results in [10].

V. COMPARISON WITH OBSERVATIONS

Since the degeneracy increases dramatically the amplitude of the first CMB peak, we expect large ξ values to be disfavored in the case of cosmological models known to predict a fairly high peak. On the other hand, a high ξ is likely to be allowed (or even favored) for models that predict systematically a low peak, unless a large scalar spectral index $n \geq 1.2$ (*blue tilt*) is invoked. For instance, the degeneracy is likely to be favored by (i) a large contribution of tensor perturbations, (ii) a significant effect from re-ionization, (iii) a low baryon density, (iv) a large h ($h \geq 0.7$), (v) flat models with $\Omega_\Lambda \leq 0.6$, etc. For such models, the peak amplitude can be boosted by ξ , keeping n close to 1, which is more natural from the point of view of inflation. However, a careful case-by-case analysis is required, since the effects of ξ and n on CMB and LSS spectra are far from being equivalent. Our goal here is not to explore systematically all possibilities, but to briefly illustrate how ξ can be constrained by current observations for flat models with different values of Ω_Λ . Recent results from supernovas [20], combined with CMB constraints, favor flat models with $\Omega_\Lambda \sim 0.6 - 0.7$ [7].

We choose a flat model with $h = 0.65$, $\Omega_b = 0.05$, $Q_{rms-ps} = 18 \mu\text{K}$, no re-ionization, and no tensor contribution, and look for the allowed region in the space of free parameters (Ω_Λ, ξ, n) . The allowed region will not be defined using a maximum likelihood analysis, but with the more conservative technique called ‘‘concordance’’ by Wang *et al.* [21], which consists in taking the intersection of regions allowed by each experiment.

For simplicity, we take into account only a few constraints on the matter power spectrum, known to be representative of the large amount of available data: the value of σ_8

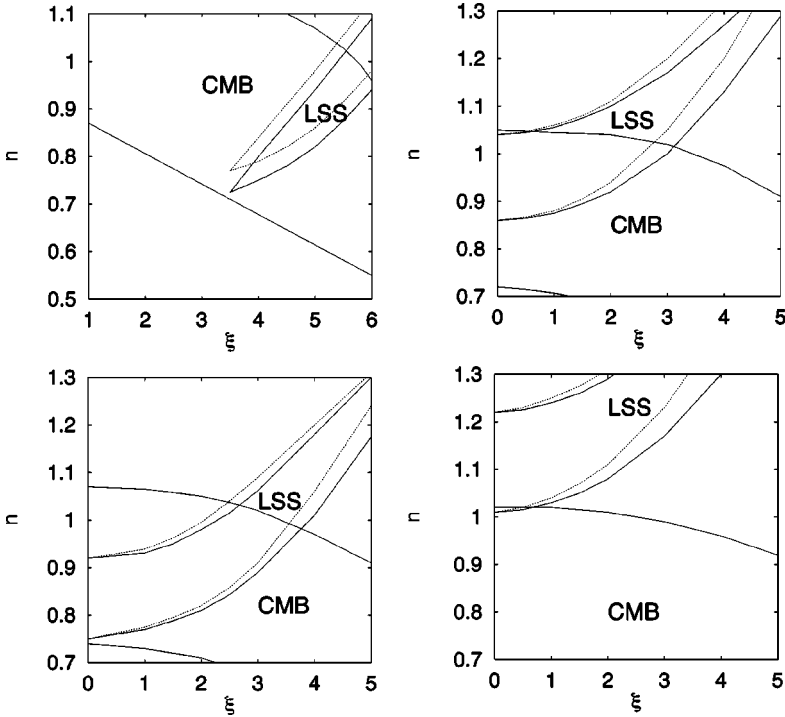


FIG. 5. LSS and CMB constraints in (ξ, n) space, for different choices of Ω_Λ : from top left to bottom right, $\Omega_\Lambda = 0, 0.5, 0.6, 0.7$. The underlying cosmological model is flat, with $h = 0.65$, $\Omega_b = 0.05$, $Q_{rms-ps} = 18 \mu\text{K}$, no re-ionization, no tensor contribution. The allowed regions are those where the labels are. For LSS constraints, we can distinguish between degenerate neutrinos with $m_\nu = 0$ (solid lines) and $m_\nu = 0.07 \text{ eV}$ (dotted lines).

(the variance of mass fluctuations in a sphere of radius $R = 8h^{-1} \text{ Mpc}$) given for flat models in [22], at 95% confidence level (C.L.);⁵ a χ^2 comparison with the STROMLO-APM redshift survey [24], at scales well within the linear regime, also with 95% CL;⁶ and finally, the constraint on bulk velocity at $R = 50h^{-1} \text{ Mpc}$ [25], taking into account the cosmic variance. Except for the updated σ_8 constraint, we use exactly the same experimental tests as in [26], and refer the reader to this paper for details. For CMB data, we perform a χ^2 analysis based on 19 experimental points and window functions, taking into account the Saskatoon calibration uncertainty, in the way suggested by [27]. The list of data that we use is given in [26], and again allowed regions correspond to 95% C.L.⁷ We do not take into account the most recent experiments, for which window functions are still unpublished; they are anyway in good agreement with the data considered here.

We plot in Fig. 5 the LSS- and CMB-allowed regions in (ξ, n) parameter space, corresponding to $\Omega_\Lambda = 0, 0.5, 0.6, 0.7$. For $\Omega_\Lambda = 0.5-0.7$, the LSS window just comes out of σ_8 limits. For $\Omega_\Lambda = 0$, the lower LSS constraint is from σ_8 and the upper one from APM data. In the case of degenerate neutrinos with $m_\nu = 0.07 \text{ eV}$, the windows are

slightly shifted at large ξ , since, as we saw, the effect of ξ is enhanced (dotted lines in the figure). The CMB-allowed regions do not show this distinction, given the smallness of the effect and the imprecision of the data. One can immediately see that LSS and CMB constraints on n are shifted in the opposite direction with ξ : indeed, the effects of ξ and n both produce a higher CMB peak, while to a certain extent they compensate each other in $P(k)$. So, for $\Omega_\Lambda = 0.7$, a case in which a power spectrum normalized to both COBE and σ_8 yields a too high peak,⁸ a neutrino degeneracy can only make things worse. On the other hand, for $\Omega_\Lambda = 0.5-0.6$, a good agreement is found up to $\xi \approx 3$.

Let us finally consider the $\Omega_\Lambda = 0$ case in which, after COBE normalization of the power spectrum, there is a well-known discrepancy between the amplitude required by σ_8 and the shape probed by redshift surveys: these two constraints favor different values of n . We find that the neutrino degeneracy can solve this problem with $\xi \gtrsim 3.5$; however, the allowed window is cut at $\xi \approx 6$ by CMB data, and we are left with an interesting region in which $\Omega_0 = 1$ models are viable. This result is consistent with [9]. However, current evidence for a low Ω_0 Universe [20,28] is independent of the constraints used here, so there are not much motivation at the moment to consider this window seriously.

VI. CONCLUSIONS

We have considered some cosmological implications of a large relic neutrino degeneracy. We have shown that this

⁵The Viana-Liddle result [22] is in very good agreement with an independent derivation by Girardi *et al.* [23].

⁶This confidence level stands for the goodness of fit of the model: when χ^2 is greater than some value, the probability that we find the observed data set, assuming the model to be valid, is smaller than 5%. For the APM data, we have 9–3 degrees of freedom (DOF), and the limiting value is found in numerical tables to be $\chi^2 = 12.5$.

⁷Here we have 19–3 DOF; the 95% C.L. is given by $\chi^2 = 26$.

⁸At least, for the values of the other cosmological parameters considered here. This situation can be easily improved, for instance, with $h = 0.7$.

degeneracy enhances the contribution of massive neutrinos to the present energy density of the Universe. For instance, neutrinos with a small mass $m_\nu \sim 10^{-2}$ eV can contribute significantly to Ω_0 , provided that there is a large neutrino-antineutrino asymmetry.

Our main result is the computation of the power spectra of CMB anisotropies and matter density in presence of a neutrino degeneracy. We found, in agreement with [9], that it boosts the amplitude of the first CMB peak, shifts the peaks to larger multipoles, and suppresses small-scale matter fluctuations. These effects follow the increase of neutrino energy density, which delays matter-radiation equality.

We extended the calculation to the case of massive degenerate neutrinos, and showed the results for a mass of 0.07 eV, as suggested by the Super-Kamiokande experiment. This mass has a small effect on CMB anisotropies. Indeed, such light neutrinos are still relativistic at recombination, but in the presence of a degeneracy, they can account for a substantial part of the density today, of order $\Omega_\nu \sim 10^{-2}$. Also, we showed that small-scale matter fluctuations are much more suppressed when the degenerate neutrinos are massive, because the free streaming of non-relativistic neutrinos is more efficient when their average momentum is boosted by the chemical potential.

We compared our results with observations, in the restricted case of a flat universe with arbitrary (Ω_Λ, ξ, n) and fixed values of other cosmological parameters. We found

that for $\Omega_\Lambda \approx 0.5-0.6$, a large degeneracy is allowed, up to $\xi \approx 3$. However, this upper bound is smaller than the value $\xi \approx 4.6$ needed to explain the generation of ultrahigh-energy cosmic rays by the annihilation of high-energetic neutrinos on relic neutrinos with mass $m_\nu = 0.07$ eV [29]. We also tried smaller values of Ω_Λ , even if they are not favored by combined CMB and supernova data. It turns out that a large degeneracy can account for both CMB and LSS constraints even with $\Omega_0 = 1$, provided that $3.5 \leq \xi \leq 6$. This analysis could be extended to other cosmological models. For instance, the degeneracy is likely to be compatible with a large contribution of tensor perturbations to large-scale CMB anisotropies.

Finally, it turns out that the degeneracy parameter and the mass of degenerate neutrinos have effects within the level of detectability of future CMB observations and redshift surveys, even with $m_\nu \sim 0.07$ eV. However, a careful analysis should be performed in order to detect possible parameter degeneracy between ξ , m_ν and other cosmological parameters.

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