

## Conserved variable in the perturbed hydrodynamic world model

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We introduce a scalar-type perturbation variable  $\Phi$  which is conserved in the large-scale limit considering a general sign of the three-space curvature ( $K$ ), the cosmological constant ( $\Lambda$ ), and the time varying equation of state. In a pressureless medium  $\Phi$  is *exactly conserved* in all scales. [S0556-2821(99)04418-5]

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### I. INTRODUCTION

Relativistic cosmological perturbations with a hydrodynamic energy-momentum tensor were originally studied by Lifshitz in 1946 [1] based on the synchronous gauge. More convenient analyses based on better suited gauge conditions were made by Harrison in 1967 [2] and Nariai in 1969 [3]. Although the variables used by Harrison and Nariai are free of gauge degrees of freedom (thus, equivalent to corresponding gauge-invariant variables), these gauge conditions became widely known from a seminal paper by Bardeen in 1980 [4]. We believe that these are the brighter side of the history concerning gauge conditions in cosmological perturbations: Lifshitz carefully traced the remaining gauge solutions; Harrison and Nariai in fact hit the correct gauge conditions for handling aspects of hydrodynamic perturbations [see below Eq. (8)]; Bardeen showed the gauge invariance of the variables used by Harrison and Nariai and demonstrated the diversity of gauge conditions and a gauge-invariant way of handling them. On the other hand, there exist persistent algebraic errors in the literature which are often claimed to be due to the wrong gauge conditions [5]. These errors probably gave some researchers an (inappropriate) impression that the field is “plagued with gauge problems.” In any case, there exist *many* gauge conditions waiting to be employed with possible advantages in exploring certain aspects of problems. In 1988 Bardeen [8] made a practical suggestion concerning the gauge condition which allows the maximal use of the various different gauge conditions depending on problems. In [9] we elaborated on the suggestion and recently termed our approach a *gauge-ready method*; see Sec. II.

We may not need to emphasize the importance of conserved quantities in physical processes. In cosmological perturbations the conserved variable can provide an easy connection between the final results and the initial conditions. Aspects of conserved perturbation variables were discussed in [10–14]. In this paper, based on the gauge-ready method, we will derive a scalar-type perturbation variable which is conserved in the large-scale limit independently of the changing background world model with general  $K$ ,  $\Lambda$ , and the perfect fluid equation of state.

Besides the scalar-type perturbation we also have vector-type (rotation) and tensor-type (gravitational wave) perturbations which evolve independently to linear order in our simple background world model. We also have conserved quantities for these additional perturbations: ignoring aniso-

tropic stresses, the angular momentum of the rotation is generally conserved, whereas the non transient solution of the gravitational wave is conserved in the super horizon scale in the near flat case. These two conservation properties were already noticed in [1], and recently, we have elaborated on these conservation properties in some general situations [15]. In the following we concentrate on the scalar-type perturbation with the hydrodynamic energy-momentum tensor.

Although redundant, in order to make this paper self-contained, in the Appendix we present the complete set of perturbed equations based on the Einstein equations. Our new result is the conserved variable in Eq. (12) with the equation and the large-scale solution in Eqs. (16), (14).

### II. NOTATION AND STRATEGY

We consider *the most general* scalar-type perturbations in the spatially homogeneous and isotropic world model. Our notation for the metric and the energy-momentum tensor is

$$ds^2 = -a^2(1 + 2\alpha)d\eta^2 - a^2\beta_{,\alpha}d\eta dx^\alpha + a^2[g_{\alpha\beta}^{(3)}(1 + 2\varphi) + 2\nabla_\alpha\nabla_\beta\gamma]dx^\alpha dx^\beta, \quad (1)$$

$$T_0^0 = -(\bar{\mu} + \delta\mu), \quad T_\alpha^0 = -\frac{1}{k}(\mu + p)v_{,\alpha},$$

$$T_\beta^\alpha = (\bar{p} + \delta p)\delta_\beta^\alpha + \frac{1}{a^2}\left(\nabla^\alpha\nabla_\beta - \frac{1}{3}\Delta\delta_\beta^\alpha\right)\sigma, \quad (2)$$

where  $0 = \eta$ , and an overbar indicates a background order quantity and will be ignored unless necessary. Spatial indices ( $\alpha, \beta, \dots$ ) and  $\nabla_\alpha$  are based on  $g_{\alpha\beta}^{(3)}$  which is the three-space metric of the homogeneous and isotropic space.  $\beta$  and  $\gamma$  always appear in a spatially gauge-invariant combination  $\chi \equiv a(\beta + a\dot{\gamma})$ ; an overdot denotes the time derivative based on  $t$  with  $dt \equiv ad\eta$ . Using  $\chi$ , all the perturbed metric and energy-momentum tensor variables in Eqs. (1), (2) are spatially gauge invariant.

Perturbed order variables can be expanded in eigenfunctions of the Laplace-Beltrami operator ( $\Delta$  based on  $g_{\alpha\beta}^{(3)}$ ) with eigenvalues  $-k^2$  where  $k$  is a comoving wave number [4]; to linear order each eigenmode decouples, and without any confusion we can assume variables in either configuration space or phase space. For the flat ( $K=0$ ) and the hyperbolic ( $K$

$= -1$ ) backgrounds  $k^2$  takes a continuous value with  $k^2 \geq 0$ , whereas in the spherical ( $K = +1$ ) background we have  $k^2 = n^2 - K$  ( $n = 1, 2, 3, \dots$ ) [1,2,16,17]. The situation in the hyperbolic background may deserve special attention. Probably because any square integrable function can be expanded using harmonics with  $k^2 \geq 1$  (subcurvature) modes only, most of the cosmology literature ignored  $0 \leq k^2 < 1$  (supercurvature) modes, and gave the wrong impression that supercurvature modes do not exist [1,2]: the state of affairs was well summarized in [17]. In the spherical background, the lack of physically relevant perturbations for the lowest two harmonics  $n = 1$  ( $k^2 = 0$ ) and  $n = 2$  ( $k^2 - 3K = 0$ ) was pointed out in [1]. Later, we will use a vanishing  $c_s^2 k^2$  term as the large-scale limit. Thus the results in such a limit will be relevant in all scales for general  $K$  in the pressureless medium ( $c_s^2 = 0$ ), and in the large-scale limit ( $k^2 \rightarrow 0$ ) for flat and hyperbolic situations in a medium with  $c_s^2 \sim 1$  ( $\equiv c^2$ ).

Complete sets of equations for the background and the perturbed orders are presented in the Appendix. Equations (A2)–(A8) are written in a gauge-ready form. In this form we still have a *right* to choose a temporal gauge condition [9]. Equations are designed so that imposing the gauge condition is as simple as the following: the synchronous gauge chooses  $\alpha \equiv 0$ , the comoving gauge chooses  $v/k \equiv 0$ , the zero-shear gauge chooses  $\chi \equiv 0$ , the uniform-curvature gauge chooses  $\varphi \equiv 0$ , the uniform-expansion gauge chooses  $\kappa \equiv 0$ , the uniform-density gauge chooses  $\delta\mu \equiv 0$ , etc. Except for the synchronous gauge, since any of the other gauge conditions completely fixes the temporal gauge degree of freedom [see Eq. (A10)], any variable in such a gauge condition is equivalent to a unique gauge-invariant combination of the variable concerned and the variable used in the gauge condition. Examples of some useful gauge-invariant combinations can be constructed using Eq. (A10):

$$\begin{aligned}
 \varphi_v \equiv \varphi - \frac{aH}{k} v, \quad \varphi_\chi \equiv \varphi - H\chi, \\
 \delta_v \equiv \delta + 3(1+w) \frac{aH}{k} v, \quad v_\chi \equiv v - \frac{k}{a} \chi.
 \end{aligned} \quad (3)$$

In this way, we can flexibly use the gauge degree of freedom as an *advantage* in handling problems [8,9].

### III. CONSERVED VARIABLE

From Eqs. (A2), (A3), (A5), (A6) we can derive [18]

$$\frac{\mu+p}{H} \left[ \frac{H^2}{(\mu+p)a} \left( \frac{a}{H} \varphi_\chi \right) \right] + c_s^2 \frac{k^2}{a^2} \varphi_\chi = \text{stresses}. \quad (4)$$

In the large-scale limit, super-sound-horizon scale where we

ignore the  $c_s^2 k^2/a^2$  term,<sup>1</sup> and ignoring stresses ( $e$  and  $\sigma$ ), Eq. (4) has an exact solution valid for general  $K$ ,  $\Lambda$ , and time-varying equation of state  $p(\mu)$  [19,7]:

$$\varphi_\chi(\mathbf{x}, t) = C(\mathbf{x}) 4\pi G \frac{H}{a} \int_0^t \frac{(\mu+p)a}{H^2} dt + d(\mathbf{x}) \frac{H}{a}, \quad (8)$$

where  $C(\mathbf{x})$  and  $d(\mathbf{x})$  are coefficients of the growing and decaying solutions, respectively.  $\varphi_\chi$  most closely resembles the behavior of perturbed Newtonian potential [7]. The variables most closely resembling the Newtonian behaviors of the density and velocity perturbations are  $\delta_v$  and  $v_\chi$ , respectively [2–4,7]. From Eqs. (A3), (A4), and Eqs. (A2), (A4), (A5), we can derive, respectively,

$$\frac{k^2 - 3K}{a^2} \varphi_\chi = 4\pi G \mu \delta_v, \quad (9)$$

$$\dot{\varphi}_\chi + H \varphi_\chi = -4\pi G(\mu+p) \frac{a}{k} v_\chi - 8\pi G H \sigma. \quad (10)$$

Solutions for  $\delta_v$  and  $v_\chi$  follow from these equations. We can similarly derive a solution for  $\varphi_v$  using

$$\varphi_v \equiv \varphi - \frac{aH}{k} v = \varphi_\chi - \frac{aH}{k} v_\chi. \quad (11)$$

Introduce

$$\Phi \equiv \varphi_v - \frac{K/a^2}{4\pi G(\mu+p)} \varphi_\chi = \varphi_v - \frac{K}{k^2 - 3K} \frac{\delta_v}{1+w}, \quad (12)$$

where we used Eq. (9) [20]. Using Eqs. (11), (10), ignoring  $\sigma$ , we can show that

$$\dot{\Phi} = \frac{H^2}{4\pi G(\mu+p)a} \left( \frac{a}{H} \varphi_\chi \right). \quad (13)$$

Thus, using the large-scale solution in Eq. (8) we have

$$\Phi(\mathbf{x}, t) = C(\mathbf{x}), \quad (14)$$

where the decaying solution has vanished. Therefore,  $\Phi$  is *generally conserved in the super-sound-horizon scale consid-*

<sup>1</sup>We would like to comment on our large-scale limit used in Eqs. (4), (16). Consider an equation of a form

$$\frac{1}{Z} (Z\dot{\phi}) + c_s^2 \frac{k^2}{a^2} \phi = 0. \quad (5)$$

In terms of  $\psi \equiv \sqrt{Z/a} \phi$  we have ( $' \equiv \partial/\partial\eta$ )

$$\psi'' + [c_s^2 k^2 - (\sqrt{Z/a})''/\sqrt{Z/a}] \psi = 0. \quad (6)$$

By ignoring the  $c_s^2 k^2$  term in either equation we have

$$\phi \propto \text{const}, \quad \int_0^t dt/Z. \quad (7)$$

Thus, when we ignore the  $c_s^2 k^2/a^2$  term in Eq. (5) as the large-scale limit, we assume that the second term in Eq. (6) is negligible compared with the third term.

ering general  $K$ ,  $\Lambda$ , and time-varying  $p(\mu)$ . On the other hand, from Eqs. (4), (13), thus ignoring stresses, we have

$$\dot{\Phi} = -\frac{Hc_s^2}{4\pi G(\mu+p)} \frac{k^2}{a^2} \varphi_\chi. \quad (15)$$

Thus, in the pressureless case  $\Phi(\mathbf{x},t) = C(\mathbf{x})$  is an exact solution valid in general scale; this was noticed in [13]. Combining Eqs. (13), (15) we have a closed form equation for  $\Phi$ :

$$\frac{H^2 c_s^2}{(\mu+p)a^3} \left[ \frac{(\mu+p)a^3}{H^2 c_s^2} \dot{\Phi} \right] + c_s^2 \frac{k^2}{a^2} \Phi = 0, \quad (16)$$

which is valid for  $c_s^2 \neq 0$ . Thus, for the vanishing  $c_s^2 k^2/a^2$  term<sup>1</sup> we have a general solution

$$\Phi(\mathbf{x},t) = C(\mathbf{x}) + \tilde{d}(\mathbf{x}) \int_0^t \frac{H^2 c_s^2}{4\pi G(\mu+p)a^3} dt, \quad (17)$$

which includes the  $c_s^2 = 0$  limit. We can show easily that the  $\tilde{d}$  term in Eq. (17) is higher order in the large-scale expansion compared with the  $d$  term in Eq. (8): by comparing the two solutions of Eqs. (8),(17) in Eq. (15) we can show  $\tilde{d} = -k^2 d = \Delta d$ . Therefore, the solution in Eq. (14) is valid in the large scale (super-sound-horizon scale) with vanishing dominating decaying solution.

#### IV. OTHER CONSERVATION VARIABLES

$\Phi$  differs from the well-known conserved variable  $\zeta$  in [10,8]. In [10]  $\zeta$  was introduced in the flat background and in that background it is the same as

$$\zeta = \varphi + \frac{\delta}{3(1+w)} \equiv \varphi_\delta, \quad (18)$$

whereas in the flat background we have  $\Phi = \varphi_v$  [21]. It is interesting to note that  $\varphi_\delta$  is also conserved in the large-scale limit considering general  $K$  [7]: from Eqs. (9), (12), (18), we can derive

$$\varphi_\delta = \Phi + \frac{1}{12\pi G(\mu+p)} \frac{k^2}{a^2} \varphi_\chi. \quad (19)$$

According to the solutions in Eqs. (8), (14), or Eq. (17), we can see that for vanishing  $k^2$  order the term  $\varphi_\delta$  is conserved considering general  $K$ . However, we also see that due to the second term the conservation property of  $\varphi_\delta$  breaks down near and inside the horizon ( $k/[aH] \geq 1$ ) even for  $K=0$ , whereas  $\Phi$  is conserved independently of the horizon crossing in the matter dominated era. We can also show the conservation property of  $\varphi_\kappa$ : from Eqs. (A3), (A10) we can show that

$$\varphi_\kappa \equiv \varphi + \frac{H\kappa}{3\dot{H} - k^2/a^2} = \frac{\varphi_\delta}{1 + \frac{k^2 - 3K}{12\pi G(\mu+p)a^2}}. \quad (20)$$

Thus, the conservation property of  $\varphi_\kappa$  breaks down for  $K \neq 0$  or near and inside the horizon. In the  $K=0$  situation,  $\varphi$  in many different gauge conditions shows conserved behavior in the large-scale limit: for an ideal fluid see Eqs. (41),(73) in [22] and Eqs. (34), (35) in [23], for the scalar field see Eqs. (92) in [24], and for the generalized gravity see Sec. VI in [25]. Conservation properties of  $\varphi$  in various gauge conditions were also discussed in [13,14] where the arguments were based on first order equations of the type in Eq. (15).

#### V. DISCUSSIONS

We would like to emphasize again that the conservation property of the  $\Phi$  is valid in the limit of the vanishing  $c_s^2 k^2/a^2$  term. Thus, in the pressureless medium ( $c_s^2 = 0$ ) it applies in all scales for general  $K$ , and in the medium with dominant pressure ( $c_s^2 \sim 1$ ) it applies in the large-scale limit ( $k^2 \rightarrow 0$ ) for the flat and the hyperbolic situation. As long as these conditions are met,  $\Phi$  is conserved independently of the time-varying equation of states  $p(\mu)$ . Since we anticipate a time-varying equation of states during an equal time from the radiation-dominated era ( $p = (1/3)\mu$ ) to the matter-dominated era ( $p = 0$ ), and during the (pre)heating period from the acceleration era ( $p < -(1/3)\mu$ ) to the radiation era, and since the observationally relevant scales stayed in the large-scale limit during the transitions,  $\Phi$  is a practically important quantity in tracing the evolution of scalar-type perturbation from the early universe until the recent era before nonlinear evolution takes over.

We would like to conclude with remarks on the history of the variables in Eq. (12):  $\varphi_\chi$  and  $\delta_v$  in their gauge-invariant forms became widely known from Bardeen's work in 1980 (these are  $\Phi_H$  and  $\epsilon_m$  in [4]). However,  $\varphi_\chi$  is the same as  $\varphi$  in the zero-shear gauge [which fixes  $\chi = 0$  as the temporal gauge condition; see Eq. (3)] which was first used by Harrison in 1967 [2], and  $\delta_v$  is the same as  $\delta$  in the comoving gauge (which fixes  $v/k = 0$ ) which was first used by Nariai in 1969 [3]. For  $K=0$ , the variable  $\varphi_v$  is widely recognized as a conserved variable in the literature. To our knowledge it was first introduced as  $\varphi$  in the comoving gauge by Lyth in 1985 [11], and later was used in the context of the scalar field and generalized gravity as the large-scale conserved variable [26].

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#### APPENDIX

In the following we derive Einstein equations based on Eqs. (1), (2). To the background order, from the,  $T_0^0$  and  $T_\alpha^\alpha - 3T_0^0$  components of the Einstein equations and  $T_{0;b}^b = 0$ , respectively, we can derive

$$H^2 = \frac{8\pi G}{3} \mu - \frac{K}{a^2} + \frac{\Lambda}{3}, \quad \dot{H} = -4\pi G(\mu + p) + \frac{K}{a^2},$$

$$\dot{\mu} = -3H(\mu + p), \quad (\text{A1})$$

where  $H \equiv \dot{a}/a$ . To the perturbed order we can derive

$$\kappa \equiv 3(-\dot{\phi} + H\alpha) + \frac{k^2}{a^2} \chi, \quad (\text{A2})$$

$$-\frac{k^2 - 3K}{a^2} \phi + H\kappa = -4\pi G \mu \delta, \quad (\text{A3})$$

$$\kappa - \frac{k^2 - 3K}{a^2} \chi = 12\pi G(\mu + p) \frac{a}{k} v, \quad (\text{A4})$$

$$\dot{\chi} + H\chi - \alpha - \phi = 8\pi G \sigma, \quad (\text{A5})$$

$$\dot{\kappa} + 2H\kappa = \left( \frac{k^2}{a^2} - 3\dot{H} \right) \alpha + 4\pi G(1 + 3c_s^2) \mu \delta + 12\pi G e, \quad (\text{A6})$$

$$\dot{\delta} + 3H(c_s^2 - w) \delta + 3H \frac{e}{\mu} = (1 + w) \left( \kappa - 3H\alpha - \frac{k}{a} v \right), \quad (\text{A7})$$

$$\dot{v} + (1 - 3c_s^2) H v = \frac{k}{a} \alpha + \frac{k}{a(1+w)}$$

$$\times \left( c_s^2 \delta + \frac{e}{\mu} - \frac{2}{3} \frac{k^2 - 3K}{a^2} \frac{\sigma}{\mu} \right), \quad (\text{A8})$$

where we have introduced

$$\delta p(\mathbf{k}, t) \equiv c_s^2(t) \delta \mu(\mathbf{k}, t) + e(\mathbf{k}, t),$$

$$\delta \equiv \frac{\delta \mu}{\mu}, \quad w(t) \equiv \frac{p}{\mu}, \quad c_s^2(t) \equiv \frac{\dot{p}}{\dot{\mu}}. \quad (\text{A9})$$

In Eq. (A2) we introduced a variable  $\kappa$  which is the perturbed part of the trace of the extrinsic curvature; similarly, for meanings of the other perturbation variables, see Sec. 2.1 of [9]. Equations (A3)–(A6) follow from the  $T_0^0$ ,  $T_\alpha^0$ ,  $T_\beta^\alpha - \frac{1}{3} \delta_\beta^\alpha T_\gamma^\gamma$ , and  $T_\alpha^\alpha - T_0^0$  components of the Einstein equations, respectively, and Eqs. (A7), (A8) follow from  $T_{0;b}^b = 0$  and  $T_{\alpha;b}^b = 0$ , respectively. This set of equations was originally derived in Eqs. (41)–(47) of [8], see also Eqs. (22)–(28) in [9].

Under the gauge transformation  $\bar{x}^a = x^a + \xi^a$ , we have (see Sec. 2.2 in [9])

$$\bar{\alpha} = \alpha - \xi^t, \quad \bar{\phi} = \phi - H\xi^t, \quad \bar{\chi} = \chi - \xi^t, \quad \bar{v} = v - \frac{k}{a} \xi^t,$$

$$\bar{\kappa} = \kappa + \left( 3\dot{H} - \frac{k^2}{a^2} \right) \xi^t, \quad \bar{\delta} = \delta + 3(1+w)H\xi^t. \quad (\text{A10})$$

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equivalent to a gauge-invariant combination with subindex  $\chi$ . Remember that  $\varphi_\chi$  is the same as  $\varphi$  in the zero-shear gauge which sets  $\chi=0$  as the gauge condition.

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