Cosmology versus holography

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The most radical version of the holographic principle asserts that all information about physical processes in the world can be stored on its surface. This formulation is at odds with inflationary cosmology, which implies that physical processes in our part of the universe do not depend on the boundary conditions. Also, there are some indications that the radical version of the holographic theory in the context of cosmology may have problems with unitarity and causality. Another formulation of the holographic principle, due to Fischler and Susskind, implies that the entropy of matter inside the post-inflationary particle horizon must be smaller than the area of the horizon. Their conjecture was very successful for a wide class of open and flat universes, but it did not apply to closed universes. Bak and Rey proposed a different holographic bound on entropy which was valid for closed universes of a certain type. However, as we will show, neither proposal applies to open, flat, and closed universes with matter and a small negative cosmological constant. We will argue, in agreement with Easther, Lowe, and Veneziano, that whenever the holographic constraint on the entropy inside the horizon is valid, it follows from the Bekenstein-Hawking bound on the black hole entropy. These constraints do not allow one to rule out closed universes and other universes which may experience gravitational collapse, and do not impose any constraints on inflationary cosmology. [S0556-2821(99)03818-7]

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I. INTRODUCTION

Recently a new set of ideas was put forward, which was called ''the holographic principle'' [1,2]. According to this set of ideas, under certain conditions all the information about a physical system is coded on its boundary, implying that the entropy of a system cannot exceed its boundary area in Planck units.

This principle was motivated by a well-known result in black hole theory: the total entropy S_m of matter inside a black hole cannot be greater than the Bekenstein-Hawking entropy, which is equal to a quarter of the area of the event horizon in Planck units, $S_m \leqslant S_{BH} = A/4$ [3]. One can interpret this result as a statement that all information about the interior of a black hole is stored on its horizon.

The main aim of the holographic principle is to extend this statement to a broader class of situations. This principle, in its most radical form, would imply that our world is two-dimensional in a certain sense, because all the information about physical processes in the world is stored at its surface. This conjecture is very interesting, and physical implications of its most radical version could be quite significant. There has been a lot of activity related to the use of the holographic principle in quantum gravity, string theory and M theory. For example, there is a conjecture that the knowledge of a super-symmetric Yang-Mills theory at the boundary of an Anti-de Sitter space may be sufficient to restore the information about supergravity/string theory in the bulk [4].

However, if one tries to apply the holographic principle to cosmology, one immediately recognizes several problems. For example, a closed universe has finite size, but it does not have any boundary. What is the meaning of the holographic principle in such a case? If the universe is infinite (open or flat), then it does not have boundaries either. In these cases, one may try to compare the entropy inside of a box of size R with its area, and then take the limit as $R \to \infty$. But in this limit the entropy is always larger than the area [5].

Another possibility is to compare the area of a domain of the size of the particle horizon (the causally connected part of the universe) with the entropy of matter inside this domain. But this is also problematic. The entropy produced during reheating after inflation is proportional to the total volume of inflationary universe. During inflation, the volume inside the particle horizon grows as e^{3Ht}, whereas the area of the horizon grows as e^{2Ht}. Clearly, the entropy becomes much greater than the area of the horizon if the duration of inflation is sufficiently large. This means that an inflationary universe is not two dimensional; information stored at its ''surface'' is not rich enough to describe physical processes in its interior. In fact, one of the main advantages of inflation is the possibility to study each domain of size H^{-1} as an independent part of the universe, due to the no-hair theorem for de Sitter space. This makes the events at the boundaries of an inflationary universe irrelevant for the description of local physics [6]. Thus, the most radical version of the holographic principle seems to be at odds with inflationary cosmology.

One may try to formulate a weaker form of this principle, which may still be quite useful. For example, Fischler and Susskind proposed to put constraints only on the part of the entropy which passed through the backward light cone [5]. This formulation does not confront inflationary cosmology because it eliminates from the consideration most of the entropy produced inside the light cone during the post-inflationary reheating of the universe. They further concentrated on investigation of those situations where cosmological evolution is adiabatic. From the point of view of inflationary cosmology, this means that they considered the evolution of the universe after reheating. The largest domain in which all of the entropy crossed the boundary when the evolution is adiabatic is bounded by the light cone emitted *after* inflation and reheating. In what follows we will loosely call this light cone of size $O(H^{-1})$ ''particle horizon,'' even though the true particle horizon, describing the

light cone emitted at the beginning of inflation, is exponentially large.

Fischler and Susskind argued that in the case of adiabatic evolution the total entropy of matter within the particle horizon must be smaller than the area of the horizon, $S \lesssim A$ [5]. This conjecture is rather nontrivial. Indeed, the origin of the Bekenstein-Hawking constraint on the entropy of a black hole is the existence of the *event* horizon, which serves as a natural boundary for all processes inside a black hole. But there is no event horizon in a noninflationary universe, and the idea to replace it by the *particle* horizon requires some justification. Also, the Bekenstein-Hawking constraint on the entropy is valid even if the processes inside a black hole are nonadiabatic. Thus it would be desirable to investigate this proposal and find a way to apply it to the situations when the processes can be nonadiabatic.

Remarkably, Fischler and Susskind have shown that their conjecture is valid for a flat universe with all possible equations of state satisfying the condition $0 \leq p \leq \rho$. This result suggests that there may be some deep reasons for the validity of holography. However, they also noticed that their version of the holographic principle is violated in a closed universe. One may consider this observation either as an indication that closed universes are impossible or as a warning, showing that the holographic principle may require additional justification and/or reformulation. Indeed, this principle is not a rigid scheme but a theory in the making. It may be quite successful in many respects, but one should not be surprised to see some parts of its formulation change. For example, Bak and Rey suggested to replace the particle horizon by an apparent horizon in the formulation of the holographic principle, claiming that their proposal does not suffer from any problems in the closed universe case [7].

There were many attempts to apply various formulations of the holographic principle to various cosmological models, but the existing literature on cosmic holography is somewhat controversial. The entropy of the observed component of matter (such as photons) is well below 10^{90} [6]. Meanwhile the constraint $S \lesssim A$ applied to our part of the universe implies that $S < 10^{120}$ [5], which does not look particularly restrictive. Holography could be quite important if it were able to rule out some types of cosmological models, but this possibility depends on the formulation and the range of validity of the holographic principle. One may try to use holography to solve the cosmological constant problem [8,9], but the progress in this direction was very limited. Recently it was claimed that holography puts strong constraints on inflationary theory [10], but the authors of Ref. [11] argued that this is not the case. Holographic considerations were used in investigation of the pre-big bang theory [12–14], and on the basis of this investigation it was claimed that this theory solves the entropy problem in the pre-big bang theory [14], which is at odds with the results of [15].

The main goal of this paper is to examine the basic assumptions of cosmic holography and check which of them may require modifications. We will try to find out whether holography indeed puts constraints on various cosmological models. We will show, in particular, that the original formulation of the holographic principle should be reconsidered

more generally, and not only when applied to closed universes. The holographic entropy bound proposed in [5], as well as the formulation proposed in [7], is violated at late stages of evolution of open, flat, and closed universes containing usual matter and a small amount of negative vacuum energy density. At the beginning of their evolution, such universes cannot be distinguished from the universe with a positive or vanishing vacuum energy density. Thus there is no obvious reason to consider such universes unphysical and rule them out. However, when the density of matter becomes diluted by expansion, a universe with a negative vacuum energy collapses, and the condition $S \lesssim A$ becomes violated long before the universe reaches the Planck density.

The investigation of universes with a negative cosmological constant gives an additional reason to look for a reformulation of the cosmological holographic principle. Our approach will be most closely related to the approach outlined by Easther and Lowe [11], and by Veneziano [14]. They argued that the entropy of the interior of a domain of size H^{-1} cannot be greater than the entropy of a black hole of a similar radius. We will extend their discussion and propose a justification for the entropy bound obtained in Ref. [5] for the case of an expanding noninflationary (or post-inflationary) universe. We will argue, in agreement with [11,14], that in those cases where the holographic bound of Ref. [5] is valid, it is equivalent to the Bekenstein-Hawking bound, which does not require any assumptions about adiabatic evolution. This bound alone cannot resolve the entropy problem for the pre-big bang cosmology and does not lead to any constraints on inflation.

II. COSMOLOGY AND HOLOGRAPHY

A. Flat universe with $p = \gamma \rho$

Let us begin with a brief review of [5]. We will restrict our attention to the case when gravitational dynamics is given by the Einstein's equations, and the evolution is adiabatic. First we will consider flat homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universes, whose metric is

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega). \tag{1}$$

We will use the units $8\pi G_N = 1$. For simplicity we will consider matter with the energy-momentum tensor $T_{\mu\nu} = \text{diag}(\rho, p, p, p)$. The independent equations of motion are

$$H^2 = \rho/3, \quad \dot{\rho} + 3H(\rho + p) = 0, \tag{2}$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ and p are the energy density and pressure, and the overdot denotes the time derivative. We will assume that $\rho > 0$, $p = \gamma \rho$, and that the energy-momentum tensor satisfies the dominant energy condition $|\gamma| \leq 1$. This will generalize the results of [5] obtained for $0 \leq \gamma \leq 1$, and is in fact the correct sufficient condition for the validity of the holographic bounds in flat and open FRW universes.

The solutions of (2) for $\gamma > -1$ can be written as

$$a(t) = t^{2/[3(\gamma+1)]}. \tag{3}$$

Here we took by definition $a=1$ at the Planck time $t=1$. Density decreases as $\rho = \rho_0/[a^{3(\gamma+1)}]$, where $\rho_0 = 4/[3(\gamma+1)^2]$ is the density at $t=1$. (For $\gamma=-1$ one has the usual de Sitter solution.) The particle horizon is defined by the distance covered by the light cone emitted at the singularity $t=0$:

$$L_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = a(t)r_H(t), \tag{4}$$

where r_H is the comoving size of the horizon defined by the condition $dt/a = dr_H$. Suppose first that $\gamma > -1/3$. Then the comoving horizon is

$$r_H = L_H/a = \frac{3(\gamma+1)}{3\gamma+1} t^{(3\gamma+1)/[3(\gamma+1)]}, \tag{5}$$

and

$$L_H = \frac{3(\gamma+1)}{3\gamma+1} t = \frac{2}{3\gamma+1} H^{-1}. \tag{6}$$

At the Planck time $t=1$ one has $L_H = [3(\gamma+1)]/(3\gamma+1)$ which generically is $O(1)$. The volume of space within the distance L_H from any point was also $O(1)$. The entropy density at that time could not be greater than $O(1)$, so one may say that initially $(S/A)_0 = \sigma \lesssim 1$. Later the total entropy inside the horizon grows as $\sigma L_H^3/a^3$, whereas the total area A of the particle horizon grows as L_H^2. Therefore

$$\frac{S}{A} \sim \sigma \frac{L_H}{a^3} = \sigma \frac{r_H}{a^2}. \tag{7}$$

This yields

$$\frac{S}{A} \sim \sigma t^{(\gamma-1)/(\gamma+1)}. \tag{8}$$

Thus the ratio S/A does not increase in time for $1 \geq \gamma > -1/3$, so if the holographic constraint $S/A \lesssim 1$ was satisfied at the Planck time, later on it will be satisfied even better [5].

A similar result can be obtained for $-1 \leq \gamma \leq -1/3$. However, investigation of this case involves several subtle points. First of all, in this case the integral in Eq. (4) diverges at small t. This is not a real problem though. It is resolved if one defines the particle horizon as an integral not from $t=0$, but from the Planck time $t=1$.

A more serious issue is the assumption of adiabatic expansion of the universe. If one makes this assumption, then one can show that the holographic bound is satisfied for all γ in the interval $-1 \leq \gamma \leq 1$, which generalizes the result obtained in [5]. However, the universe with $1+\gamma \lesssim 2/3$ (i.e., with $\gamma \approx -1$) is inflationary. The density of matter after inflation becomes negligibly small, so it must be created again in the process of reheating of the universe. This process is strongly nonadiabatic.

As we already mentioned in the Introduction, in inflationary cosmology the bounds of Ref. [5] refer to the post-inflationary particle horizon, which means that the integration in Eq. (4) should begin not at $t=0$ or at $t=1$ but after reheating of the universe. One can easily verify that the bounds obtained in [5] are valid in this case as well.

B. Closed universe

The metric of a closed FRW universe is

$$ds^2 = -dt^2 + a^2(t)(d\chi^2 + \sin^2\chi d\Omega), \tag{9}$$

where the spatial part represents a three-sphere, with χ being the azimuthal angle and $d\Omega$ the line element on the polar two-spheres. The lightcones are still bounded by the particle horizon. However, due to the curvature of the three-sphere, the light rays must now travel along the azimuthal direction in order to maximize the sphere of causal contact. The comoving horizon is the extent of the azimuthal angle traveled by light between times 0 and t:

$$\chi_H = \frac{L_H}{a} = \int_0^t \frac{dt'}{a(t')}. \tag{10}$$

The boundary area of the causal sphere is then given by

$$A \sim 4\pi a^2(t)\sin^2\chi_H. \tag{11}$$

The volume inside of this sphere is

$$V = \int_0^{\chi_H} d\chi \, \sin^2\chi \, d\Omega = \pi(2\chi_H - \sin 2\chi_H). \tag{12}$$

Assuming a constant comoving entropy density σ, we find

$$\frac{S}{A} = \sigma \frac{2\chi_H - \sin 2\chi_H}{4a^2(t)\sin^2\chi_H}. \tag{13}$$

Here we have explicitly retained the contribution from the comoving entropy density σ, which was ignored in [5].

Consider for simplicity a cold dark matter dominated universe, with $p \ll \rho$. In this case $a = a_{max}\sin^2(\chi_H/2)$. The moment $\chi_H = \pi$ corresponds to the maximal expansion, $a = a_{max}$. But at that time the light cone emitted from the ''North pole'' of the universe converges at the ''South pole,'' the area of the horizon (10) vanishes, and the holographic bound on the ratio S/A becomes violated [5]. Note that in all other respects the point $\chi_H = \pi$ is regular, so one cannot argue, for example, that the violation of the holographic bound is a result of violent quantum fluctuations of the light cone.

C. Open, closed, and flat universes with $\Lambda < 0$

Let us return to the discussion of the flat universe case and look at Eq. (7) again. The size of the comoving horizon r_H can only grow. Despite this growth, the holographic bound is satisfied for $\rho > 0$, $p > -\rho$, because the value of a^2 grows faster than r_H in this regime. But this bound can be

violated if a^2 grows more slowly than r_H, and it will definitely be violated in all cases where a flat space can collapse.

Usually, cosmologists believe that closed universes collapse, whereas open or flat universes expand forever. But the situation is not quite so simple. If there is a sufficiently large positive cosmological constant, then even a closed universe will never collapse. On the other hand, if the cosmological constant is negative, then, even if it is extremely small, eventually it becomes dominant, and the universe collapses, independently of whether it is closed, open or flat. In all of these cases the holographic principle, as formulated in [5], will be violated.

For simplicity, we will consider a flat universe ($k=0$) with a negative vacuum energy density $-\lambda<0$, so that $\rho = p/\gamma - \lambda$. We will assume that $\lambda \ll 1$ in Planckian units. For example, in our universe λ cannot be greater than 10^{-122}. In an expanding universe $\rho = \rho_0/a^{3(\gamma+1)} - \lambda$, and the Friedmann equation

$$3H^2 = \frac{\rho_0}{a^{3(\gamma+1)}} - \lambda \qquad (14)$$

can be rewritten as

$$\dot{a} = \pm \frac{1}{\sqrt{3}} \sqrt{\frac{\rho_0}{a^{3\gamma+1}} - \lambda a^2}. \qquad (15)$$

Because of the presence of the cosmological term, in general we cannot write the integrals in a simple form. However, the exact form of the solutions is not necessary for our purpose here.

First of all, we see that $\dot{a}$ vanishes at $\lambda a^{3(\gamma+1)} = \rho_0$, after which $\dot{a}$ becomes negative and the universe collapses. This happens within a finite time after the beginning of the expansion. From the definition of the particle horizon and (15), one can find the value of L_H at the turning point:

$$L_H(\text{turning}) = \frac{B\left(\dfrac{\gamma}{2(\gamma+1)},\dfrac{1}{2}\right)}{3(\gamma+1)\sqrt{\lambda}}, \qquad (16)$$

where $B(p,q)$ is the Euler beta function. Putting these formulas together, we see that at the turning point

$$\frac{S}{A} \sim \sigma \lambda^{(1-\gamma)/[2(1+\gamma)]} \qquad (17)$$

up to factors of order unity. For $1 \geq \gamma > -1$, the power of λ is positive and so the ratio S/A is very small at the turning point. Now, we can consider what happens near the final stages of collapse, where the energy density reaches the Planckian scales. By symmetry, $L_H \sim 2[a_0/a(\text{turning})]L_H(\text{turning}) \sim \lambda^{-(3\gamma+1)/[6(\gamma+1)]}$ at this time, whereas $\sigma/a^3 \sim 1$. Hence, Eq. (8) yields $S/A \sim \lambda^{-(3\gamma+1)/[6(\gamma+1)]} \gg 1$ when $\gamma > -1/3$. Therefore, we see that the ratio S/A reaches unity at some time after the turning point, and that the holographic bound becomes violated thereafter, but still well in the classical phase, when the uni-

verse is still very large. Indeed, we can estimate the density of matter at that time to be $\rho \sim \lambda^{(\gamma+1)/2} \ll 1$.

A universe where the only energy density is in the form of a negative cosmological constant is called the anti-de Sitter space (AdS). In string theory, AdS spaces typically emerge after compactifying string or M theory on an internal, compact, Einstein space of positive constant curvature. Many interesting applications of the holographic principle have been elaborated for the pure AdS space. It is therefore quite interesting that in the cosmological context an AdS background containing matter describes a collapsing Friedmann universe with a negative vacuum energy, in which the cosmological holographic principle is violated.

D. AdS spaces with matter and an alternative formulation of cosmic holography

In order to cure the problems of the original formulation of the cosmological holographic principle, Bak and Rey proposed a different formulation [7]. They suggested to consider the so-called apparent horizon instead of the particle horizon and claimed that in this case the holographic bound holds even in a closed universe. We will not present here a detailed discussion of their proposal. Instead we will consider here their holographic bound in the three-dimensional spatially flat universe (d = 3), see Eq. (16) of [7]:

$$\frac{4\sigma}{3a^2(t)\dot{a}(t)} \leq 1. \qquad (18)$$

This condition is violated when the universe approaches the turning point at $\lambda a^{3(\gamma+1)} = \rho_0$, when one has $\dot{a}=0$. This violation occurs even much earlier than in the original formulation of the cosmological holographic principle of Ref. [5].

One can propose two possible interpretations of these results. First of all, one may argue that closed universes are impossible, and that the universes with a negative cosmological constant are also impossible. We do not see how one could justify such a statement. After all, the main reason why the holographic constraint was violated in both cases studied above was related to the possibility of gravitational collapse. It would be very odd to expect that the holographic principle which was motivated by the study of black holes should imply that gravitational collapse cannot occur.

Another possibility is that the formulations of the cosmic holography proposed in [5,7] should be somewhat modified in the cases when the universe may experience collapse. It would also be interesting to understand the reasons why the holographic inequalities were correct in the flat universe case. We will discuss this issue in the next section.

III. BLACK HOLES AS BIG AS A UNIVERSE

The simplest way to understand the holographic bound on the entropy of the observable part of the universe is related to the theory of black holes. In what follows we will develop further an argument given by Easther and Lowe [11], and by Veneziano [14].

The simplest cosmological models are based on the as-

sumption that our universe is homogeneous. But how do we know that it is indeed homogeneous if the only part of the universe that we can see[1] has size H^{-1}? We cannot exclude the possibility that if we wait for another 10 billion years, we will see that we live near the center of an expanding but isolated gravitational system of size $O(H^{-1})$ in an asymptotically flat space. Then we can apply the Bekenstein bound to the entropy of this system, $S \lesssim ER$, where $E \sim \rho R^3$ is the total energy and $R \sim H^{-1}$ is the size of this system, with $H^2 \sim \rho$, in Planck units. This gives $S \lesssim H^{-2}$, which coincides with the holographic bound.

Of course, the idea that our part of the universe is a small isolated island of size H^{-1} is weird, but we do not really advocate this view here. Rather, we simply say that since we cannot tell whether the universe is homogeneous, or it is an island of a size somewhat greater than H^{-1}, the bound $S \lesssim H^{-2}$ must hold for a usual homogeneous universe as well.

One can look at this constraint from a different perspective. It is well known that if our universe is locally overdense on a scale of horizon with $\delta\rho/\rho = O(1)$, the overdense part will collapse and form a black hole of a size H^{-1} [16]. Then the entropy of this part of the universe will satisfy the black hole bound $S \lesssim H^{-2}$. Again, there is no indication that $\delta\rho/\rho = O(1)$ on the horizon scale, but since we cannot exclude this possibility on a scale somewhat greater than the present value of H^{-1}, the bound should apply to the homogeneous universe as well.

Instead of debating the homogeneity of our universe, one can imagine adding a sufficient amount of cold dark matter to a part of our universe of size R. This would not change its entropy, but it would lead to black hole formation. Then one can find an upper bound on the entropy of a black hole of size R: $S \lesssim R^2$. If one takes $R \sim H^{-1}$, one again finds that $S \lesssim H^{-2}$.

The bound $S \lesssim R^2$ implies that the density of entropy satisfies the constraint $s = S/R^3 < 1/R$. Thus one could expect that it is possible to get a more stringent constraint on the density of entropy by considering black holes of size greater than H^{-1}. However, according to Carr and Hawking [16], black holes formed in a flat universe cannot have size greater than $O(H^{-1})$. This constraint has a dynamical origin, and is not related to the size of the particle horizon. Usually the difference between H^{-1} and the particle horizon is not very large, but during inflation this difference is very significant: H^{-1} remains nearly constant, whereas the particle horizon grows exponentially.

If an inflationary domain is homogeneous on a scale $O(H^{-1})$, then it is going to expand exponentially, independently of any inhomogeneities on a larger scale. Such a domain is not going to collapse and form a black hole until

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[1]If one takes into account inflation, then particle horizon is exponentially large. Still we can see (by means of electromagnetic radiation) only a small part of the universe of size $\sim H^{-1} \sim t$. It is important that this scale, rather than the particle horizon, determines the largest size of a black hole which can be formed in an expanding universe.

inflation ends and we wait long enough to see the boundaries of the domain. But this will not happen for an exponentially long time. Nevertheless the holographic constraints on the entropy can be derived for the processes after inflation, just as in the case considered above. These constraints will be related to the size of the largest black hole which can be formed during the expansion of the post-inflationary universe, $R \sim H^{-1}$, rather than to the exponentially large size of the particle horizon in an inflationary universe. As a result, the holographic bounds do not lead to the constraints on the duration of inflation, inflationary density perturbations, and other parameters of inflationary theory discussed in [10].

If the universe is non-inflationary and closed, or if it has a negative cosmological constant, then, prior to the point of maximal expansion, the holographic constraints on the entropy within the regions of size $H^{-1} \sim t$ coincide with the constraints for the flat universe case. Once the universe begins to collapse, the constraints cannot be further improved because the typical time of formation of a black hole of size $O(t)$ at that stage will be of the same order of magnitude as the lifetime of the universe. But this fact does not imply the impossibility of collapsing universes.

Note that in our consideration we did not make any assumptions about the adiabatic evolution of the universe. Thus, the cosmological holographic constraints on entropy are as general as their black hole counterparts. In fact, we believe that these two constraints have the same origin.

IV. HOLOGRAPHY VERSUS INFLATION

As we already mentioned, all holographic constraints discussed in this paper apply only to the post-inflationary universe. Inflationary cosmology in its spirit is somewhat opposite to holography. The possibility of solving the horizon, homogeneity, isotropy, and flatness problems is related to the superluminal stretching of the universe, which erases all memory about the boundary conditions. The speed of rolling of the inflaton scalar field approaches an asymptotic value which does not depend on its initial speed. The gradients of the fields and the density of particles which existed prior to inflation (if there were any) become exponentially small. All particles (and all entropy) which exist now in the universe have been created after inflation in the process of reheating. This process occurs locally, so the properties of particles as well as their entropy do not depend on the initial conditions in the universe.

In order to investigate this issue in a more detailed way, let us consider the simplest version of inflationary cosmology where the universe during inflation expands only 10^{30} times (the minimal amount which is necessary for inflation to work). We will also assume for simplicity that inflation occurs at the grand unified theory (GUT) scale, so that $H \sim 10^{-6}$ and the temperature after reheating is $T \sim 10^{-3}$ in Planck units.

In such a case the size of the particle horizon after inflation will be $L_H \sim H^{-1} \times 10^{30} \sim 10^{36}$, the area $A \sim L_H^2 \sim 10^{72}$, and the entropy $S \sim T^3 L_H^3 \sim 10^{99}$, which clearly violates the bound $S < A$. This means that the information stored at the

surface of an inflationary domain cannot describe dynamics in its interior.

In practice, it is extremely difficult to invent inflationary theories where the universe grows only by a factor of 10^{30} because typically in such models $\delta\rho/\rho = O(1)$ at the scale of the horizon. In the simplest versions of chaotic inflation the universe grows more than $10^{1\,000\,000}$ times during inflation. The situation becomes especially dramatic in those versions of inflationary cosmology which lead to the process of eternal self-reproduction of inflationary domains. In such models the universe is not an expanding ball of a huge size, but a growing fractal consisting of many exponentially large balls. In the process of eternal self-reproduction of the universe all memory about the boundary conditions and initial conditions becomes completely erased [6].

Of course, one can use the version of the holographic principle describing the post-inflationary evolution of the universe, as discussed in the previous sections. However, in realistic inflationary models the energy density at the end of inflation falls more than 15 orders of magnitude below the Planck density, and the most interesting part of dynamics of the universe where quantum gravity could play a significant role is already over.

There is another interesting aspect of relations between inflation and holography. The holographic bound on the present entropy of the universe is $S \lesssim H^{-2}$. One has $H^{-1} \sim 10^{60}$ in Planck units. This gives the constraint

$$S \lesssim H^{-2} \sim 10^{120}. \tag{19}$$

Meanwhile, the entropy of matter in the observable part of the universe is smaller than 10^{90}. If one thinks about cosmology in terms of the information which can be stored on the horizon (or, to be more accurate, on a surface of a sphere of size H^{-1}), one can be encouraged by the fact that the holographic bound is satisfied with a wide safety margin, $S/A \lesssim 10^{-30}$. On the other hand, if, as we have argued, the information stored on the sphere of size H^{-1} is not related to the initial conditions at the beginning of inflation, then its importance is somewhat limited. In such a case the only information about the universe that we gained is the bound $S \lesssim 10^{120}$, which is 30 orders of magnitude less precise than the observational constraint on the entropy. But what is the origin of these 30 orders of magnitude?

Let us look back in time and assume that there was no inflation and the evolution of the universe was adiabatic. Our part of the universe today has size $\sim 10^{28}$ cm. At the Planck time its size l would be 10^{28} cm multiplied by T_p/T_0, where T_0 is the present value of the temperature of the universe, and $T_p \sim 1$ is the Planck temperature. (Note that the scale of the universe is inversely proportional to T during adiabatic expansion.) One therefore finds $l \sim 10^{-3}$ cm, which is 10^{30} times greater than the Planck length. That is exactly the reason why we need the universe to inflate by the factor of 10^{30}. (The true number depends on the value of reheating temperature after inflation.)

If the universe did not inflate at all, it would be very holographic. A typical homogeneous part of the universe soon after the big bang would have Planck size, it would contain just one or two particles, and the constraint $S < A$ would be saturated. But we would be unable to live there.

Let us assume, for the sake of the argument, that inflation starts and ends at the Planck density, and it has Planckian temperature after reheating. If the universe during this period inflated by more than 10^{30} times, then our part of the universe after inflation would have the size 10^{-3} cm, i.e., 10^{30} in Planck units, just as we estimated above. Its entropy would be 10^{90}. Then the universe expands by $T_p/T_0 \sim 10^{30}$ times, and the area of our domain becomes 10^{120}. This makes it clear that the factor of 10^{30} which characterizes the discrepancy between the holographically natural value of entropy 10^{120} and the observed value 10^{90} is the same factor which appears in the formulations of the entropy problem and flatness problem [14,17].

Thus, in the final analysis, the reason why one has $S \lesssim 10^{-30}A$ in our universe is related to inflation. Without inflation one would have $S \sim A$, and a typical locally homogeneous patch of the universe would collapse within the Planck time. The safety margin of 30 orders of magnitude created by inflation makes the universe very large and long living, but simultaneously prevents the holographic constraint on entropy from being very informative.

A nontrivial relation between the holographic constraint and inflation does not mean that one can identify the entropy problem (existence of a huge entropy $S \sim 10^{90}$ in our part of the universe) and the holography problem (discrepancy between the holography bound 10^{120} and the true value of entropy 10^{90}). For example, in one of the recent versions of the pre-big bang scenario the stage of the pre-big bang inflation begins from a state which can be identified with a black hole with a large area of the black hole horizon [18]. In this case, the initial entropy of the gravitational configuration by definition satisfies the Bekenstein-Hawking bound, which coincides with the holographic bound. If one assumes that the entropy of matter inside the black hole *saturates* the Bekenstein-Hawking bound (this is just an assumption which does not follow from the black hole theory), then the holography problem will be resolved [14]. However, one should still determine the origin of the enormously large black hole entropy in this scenario, which constitutes the entropy problem [15].

V. CONCLUSIONS

The idea that all information about physical processes in the world can be stored on its surface is very powerful. It has many interesting implications in investigation of the nonperturbative properties of M theory. However, it is rather difficult to merge this idea with cosmology. The universe may not have any boundary at all, or it may expand so fast that boundary effects become irrelevant for the description of the local dynamics. In this paper we have shown that some of the formulations of the holographic principle should be modified not only in application to a closed universe, but also for open, closed, and flat universes with a negative cosmological constant. We believe that the cosmological holographic constraints on entropy, in those cases where they are valid, can be understood using the Bekenstein-Hawking

bound on the entropy of black holes. These constraints are rather nontrivial, but if applied to our part of the universe they are much weaker than the observational constraints, as well as the constraints which follow from the theory of creation of matter after inflation. We believe that these constraints do not permit one to rule out the universes which may experience gravitational collapse, and they do not impose any additional constraints on inflationary cosmology.

The constraints on entropy represent only one aspect of the holographic principle. A stronger form which has been advocated requires the existence of a theory living on the boundary surface which would describe physical processes in the enclosed volume. Validity of this conjecture in the cosmological context has not been demonstrated, and in fact one may argue that there exists a general obstacle on the way towards the realization of this idea. In the theory of black holes, the role of the holographic surface is played by the black hole horizon. Its area, and correspondingly the number of degrees of freedom living on the horizon, remains constant if one neglects quantum gravity effects. Thus it is not unreasonable to assume that there exists a unitary quantum theory associated with the black hole horizon. However, in an expanding universe the number of degrees of freedom associated with the cosmological horizon, or with an apparent horizon, or with a horizon of a would-be black hole which provides holographic constraints on entropy, rapidly changes in time. For example, in a closed universe the initial area of the horizon is vanishingly small, then it grows until it reaches the maximum, and subsequently it disappears. Thus the number of degrees of freedom associated with such a surface strongly depends on time even when the evolution of the universe is adiabatic and the total number of degrees of freedom in the bulk is conserved [19].

Therefore one may wonder whether the holographic theory existing on such a surface will violate unitarity. In addition, the disappearance of degrees of freedom after the moment of the maximal expansion implies that the entropy measured at the holographic surface will increase during the universe expansion, but then it will decrease during its contraction, and eventually it will vanish. This means that the second law of thermodynamics may be violated in the holographic theory.

The situation with causality in such a theory is not clear as well. Indeed, information about the new degrees of freedom which are going to appear or disappear on the holographic surface is stored not on this surface but in the bulk. This information does not propagate along the surface, rather it crosses the surface when new particles enter the apparent horizon. But this suggests that the creation of the new degrees of freedom in the holographic theory will not look like an effect caused by the earlier existing conditions at the surface.

It remains to be seen whether one can overcome all of these problems and make the holographic principle a useful part of the modern cosmological theory including inflationary theory. We should note, however, the quantum cosmology is extremely complicated and counterintuitive in many respects. It is still a challenging task to unify M-theory and inflationary cosmology. Any progress in this direction would be very important. One may expect that the ideas borne out by the investigation of quantum dynamics of black holes and enriched by the study of supergravity and string theory will play the key role in the development of a nonperturbative approach to quantum cosmology.

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