

Inflation and preheating in nonoscillatory models

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We study inflationary models in which the effective potential of the inflaton field does not have a minimum, but rather gradually decreases at large ϕ . In such models the inflaton field does not oscillate after inflation, and its effective mass becomes vanishingly small, so the standard theory of reheating based on the decay of the oscillating inflaton field does not apply. For a long time the only mechanism of reheating in such non-oscillatory (NO) models was based on gravitational particle production in an expanding universe. This mechanism is very inefficient. We will show that it may lead to cosmological problems associated with large isocurvature fluctuations and overproduction of dangerous relics such as gravitinos and moduli fields. We also note that the setting of initial conditions for the stage of reheating in these models should be reconsidered. All of these problems can be resolved in the context of the recently proposed scenario of instant preheating if there exists an interaction $g^2\phi^2\chi^2$ of the inflaton field ϕ with another scalar field χ . We show that the mechanism of instant preheating in NO models is much more efficient than the usual mechanism of gravitational particle production even if the coupling constant g^2 is extremely small, $10^{-14} \ll g^2 \ll 1$. [S0556-2821(99)08318-6]

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I. INTRODUCTION

Usually it is assumed that the inflaton field ϕ after inflation rolls down to the minimum of its effective potential $V(\phi)$, oscillates, and eventually decays. The stage of oscillations of the inflaton field is a necessary part of the standard mechanism of reheating of the universe [1,2].

However, there exist some models where the inflaton potential $V(\phi)$ gradually decreases at large ϕ and does not have a minimum. In such theories the inflaton field ϕ does not oscillate after inflation, so the standard mechanism of reheating does not work there.

Investigation of inflationary models of this type has been rather sporadic [3–6], and each new author has given them a new name, such as deflation [4], kination [5], and quintessential inflation [6]. However, the universe does not deflate in these models, and in general they are not related to the theory of quintessence. From our perspective, the main distinguishing feature of inflationary models of this type is the non-oscillatory behavior of the inflaton field, which makes the standard mechanism of reheating inoperative. Therefore we will call such models non-oscillatory (NO) models. In addition to describing the most essential feature of this class of theories which makes reheating problematic, this name reflects the rather negligent attitude towards these models which existed until now.

One of the reasons why NO models have not attracted much attention was the absence of an efficient mechanism of reheating. For a long time it was believed that the only mechanism of reheating possible in NO models was the gravitational particle production [3–6] which occurs because of the changing metric in the early universe [7]. This mechanism

is very inefficient, which may lead to certain cosmological problems.

However, recently the situation changed. The mechanism of instant preheating which was found in [8] is very efficient, and it works in NO models even better than in the models where $V(\phi)$ has a minimum.

In this paper we will describe various features of NO models. First of all, we will discuss the problem of initial conditions in these models, which in our opinion has not been properly addressed before. The standard assumption made in [3–6] is that at the end of inflation in NO models one has a large and heavy inflaton field ϕ which rapidly changes and creates light particles χ minimally coupled to gravity from a state where the classical value of the field χ vanishes. We will show that this setting of the problem should be reconsidered. If the fields ϕ and χ do not interact (which was the standard assumption of Refs. [3–6]), then at the end of inflation the field χ typically does not vanish. Usually the last stages of inflation are driven by the light field χ rather than by the heavy field ϕ . But in this case reheating occurs due to oscillations of the field χ , as in the usual models of inflation.

In addition to reexamining the problem of initial conditions, we will point out potential difficulties associated with isocurvature perturbations and gravitational production of gravitinos and moduli fields in NO models.

In order to provide a consistent setting for the NO models one needs to introduce an interaction between the fields ϕ and χ . This resolves the problem of initial conditions in these models and makes it possible to have a non-oscillatory behavior of the inflaton field after inflation. We show that all of these problems can be resolved in the context of the recently

proposed scenario of instant preheating [8] if there is an interaction $(g^2/2)\phi^2\chi^2$ of the inflaton field ϕ with another scalar field χ , with $g^2 \gg 10^{-14}$. In this case the mechanism of instant preheating in NO models is much more efficient than the usual mechanism of gravitational particle production studied in [3–6].

II. ON THE INITIAL CONDITIONS IN NO MODELS WITHOUT INTERACTIONS

NO models considered in [3–6] described an inflaton field which does not interact with other fields except gravitationally. As an example, we will consider here the simplest theory of the inflaton field ϕ with an effective potential $V(\phi)$ which behaves as $(\lambda/4)\phi^4$ at $\phi < 0$, and (gradually) vanishes when ϕ becomes positive. In addition, in accordance with [6], we will consider a light scalar field χ which is not coupled to the inflaton field ϕ , and which is minimally coupled to gravity. Reheating in this model occurs because of the gravitational production of χ particles. Application of the general theory of gravitational particle creation [7] to the last stages of inflation and the immediate stages after inflation was considered in many papers; see in particular [3,9]. However, this theory (and the interpretation of its results) may change dramatically if one investigates initial conditions for inflation and studies quantum fluctuations produced during inflation.

In particular, in all previous works on NO models it was assumed that at the beginning of inflation $|\phi|$ is very large and $\chi=0$. Let us show that in this case at the end of the stage of inflation driven by the field ϕ the long-wavelength fluctuations of the field χ typically become so large that it leads to a new stage of inflation which is driven not by the field ϕ , but by the field χ . This conclusion is rather general and may be extended to other models of $V(\phi)$. The explanation goes back to the work of Ref. [10], where it was found that in the presence of several scalar fields the last stage of multiple inflation is typically driven by the lightest scalar field.

Indeed, the field ϕ during inflation obeys the following equation:

$$3H\dot{\phi} = -\lambda_\phi\phi^3. \quad (1)$$

Here

$$H = \sqrt{\frac{2\pi\lambda_\phi}{3}} \frac{\phi^2}{M_p}. \quad (2)$$

These two equations yield the solution [11]

$$\phi = \phi_0 \exp\left(-\sqrt{\frac{\lambda_\phi}{6\pi}} M_p t\right). \quad (3)$$

If the field χ is very light, then in each time interval H^{-1} during inflation fluctuations $\delta\chi = H/2\pi$ will be produced. The equation describing the growth of fluctuations of the field χ can be written as follows:

$$\frac{d\langle\chi^2\rangle}{dt} = \frac{H^3}{4\pi^2}. \quad (4)$$

In de Sitter space with $H = \text{const}$ this equation would give [12]

$$\langle\chi^2\rangle = \frac{H^3 t}{4\pi^2}. \quad (5)$$

For the theory under consideration H depends on time, and Eq. (4) reads

$$\frac{d\langle\chi^2\rangle}{dt} = \frac{\lambda_\phi\sqrt{\lambda_\phi}}{3\sqrt{6\pi}} \frac{\phi_0^6}{M_p^3} \exp\left(-\sqrt{\frac{6\lambda_\phi}{\pi}} M_p t\right). \quad (6)$$

The result of integration at large t converges to

$$\langle\chi^2\rangle = \frac{\lambda_\phi\phi_0^6}{18M_p^4}. \quad (7)$$

These fluctuations from the point of view of a local observer look like a classical scalar field χ which is homogeneous on the scale H^{-1} and which has a typical amplitude

$$\bar{\chi} = \sqrt{\langle\chi^2\rangle} = \sqrt{\frac{\lambda_\phi}{18}} \frac{\phi_0^3}{M_p^2}. \quad (8)$$

This quantity is greater than M_p for

$$\phi_0 \gtrsim \lambda_\phi^{-1/6} M_p. \quad (9)$$

This condition is quite natural. For example, if, in accordance with [11], inflation begins at $V \sim M_p^4$, one has $\phi_0 \sim \lambda_\phi^{-1/4} M_p$, which is much greater than $\lambda_\phi^{-1/6} M_p$.

If the field χ has a shallow polynomial potential such as $m_\chi^2\chi^2/2$ (with a small mass m_χ), or $\lambda_\chi\chi^4/4$ (with small λ_χ), then the existence of a homogeneous field $\bar{\chi} \gtrsim M_p$ leads to a new stage of inflation. This stage will be driven by the light field χ , and it will begin after the end of the stage of inflation driven by the field ϕ .

Condition (9) coincides with the condition that chaotic inflation with respect to the field ϕ enters the stage of self-reproduction [13]. In this regime the field ϕ may stay large even much longer than is suggested by our classical equations which do not take self-reproduction into account. As a result, fluctuations of the field χ will be even greater, and the last stage of inflation will be driven not by the field ϕ but by the lighter field χ .

In such a case, the results of gravitational particle production obtained in all previous papers on NO models do not apply. Instead of particles χ produced at the end of inflation driven by the field ϕ (or in addition to these particles), we have long-wavelength fluctuations of the field χ which initiate a new stage of inflation driven by the field χ .

A similar result can be obtained in the model $V(\phi) = (m^2/2)\phi^2$. In this case after inflation driven by the field ϕ one has $\bar{\chi} = \sqrt{\langle\chi^2\rangle} = m\phi_0^2/(\sqrt{3}M_p)$. This leads to inflation with respect to the field χ (i.e., one has $\bar{\chi} > M_p$) for ϕ_0

$\geq M_p \sqrt{M_p/m}$. Again, according to [11], the most natural initial value of ϕ is $\phi_0 \sim M_p^2/m \gg M_p \sqrt{M_p/m}$.

This suggests that if the field χ is very light (which was assumed in [4–6]), then it is this field rather than the field ϕ that is responsible for the end of inflation. Therefore instead of studying gravitational production of the field χ due to the non-oscillatory motion of the field ϕ during preheating in this NO model, one should study the mechanism of production of particles of the field ϕ by the oscillating field χ .

Of course, one can avoid some of these problems by choosing a specific set of scalar fields and a specific version of the theory which does not allow any of these scalar fields except the field ϕ to drive inflation. For example, if the field χ is an axion field (and no other light scalar fields are present), then it simply cannot be large enough to be responsible for inflation. One can also assume that the field χ is nonminimally coupled to gravity and has a large effective mass $O(H)$ during inflation (see Sec. IV). Then the long-wavelength fluctuations of this field will not be produced. Thus there are some ways to overcome the problem mentioned above. But in general this problem is very serious, and one should be aware of its existence.

III. ISOCURVATURE PERTURBATIONS IN THE NO MODELS

In the previous section we showed that even if one assumes that $\chi=0$ at the beginning of inflation, the assumption that one still has $\chi=0$ at the beginning of reheating in general is incorrect. The long-wavelength perturbations of the field χ generated during inflation typically are very large, and they look like a large homogeneous classical field χ .

Now we will consider a more general question: If the two fields ϕ and χ do not interact, then why should we assume that one of them should vanish in the beginning of inflation? And if it does not vanish, then how does it change the whole picture?

Suppose for example that the field χ is a Higgs field [6] with a relatively small mass and with a large coupling constant $\lambda_\chi \gg \lambda_\phi$. The total effective potential in this theory (for $\phi < 0$) is given by

$$V(\phi, \chi) = \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_\chi}{4} (\chi^2 - v^2)^2. \quad (10)$$

Here v is the amplitude of spontaneous symmetry breaking, $v \ll M_p$. During inflation and at the first stages of reheating this term can be neglected, so we will study the simplified model

$$V(\phi, \chi) = \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_\chi}{4} \chi^4. \quad (11)$$

This model was analyzed in [10]. It is directly related to the Peebles-Vilenkin model [6] if the field χ is the Higgs boson field with a small mass m . This model exhibits the following unusual feature.

In general, at the beginning of inflation one has both $\phi \neq 0$ and $\chi \neq 0$. Thus, unlike in the previous section, we will

not assume that $\chi=0$, and instead of studying quantum fluctuations of this field which can make it large, we will assume that it could be large from the very beginning.

Even though the fields ϕ and χ do not interact with each other directly, they move towards the state $\phi=0$ and $\chi=0$ in a coherent way. The reason is that the motion of these fields is determined by the same value of the Hubble constant H .

The equations of motion for both fields during inflation appear as follows:

$$3H\dot{\phi} = -\lambda_\phi \phi^3, \quad (12)$$

$$3H\dot{\chi} = -\lambda_\chi \chi^3. \quad (13)$$

These equations imply that

$$\frac{d\phi}{\lambda_\phi \phi^3} = \frac{d\chi}{\lambda_\chi \chi^3}, \quad (14)$$

which yields the general solution

$$\frac{1}{\lambda_\phi \phi^2} = \frac{1}{\lambda_\chi \chi^2} + \frac{1}{\lambda_\phi \phi_0^2} - \frac{1}{\lambda_\chi \chi_0^2}. \quad (15)$$

Since the initial values of these fields are much greater than the final values, at the last stages of inflation one has [10]

$$\frac{\phi}{\chi} = \sqrt{\frac{\lambda_\chi}{\lambda_\phi}}. \quad (16)$$

Suppose $\lambda_\phi \ll \lambda_\chi$. In this case the ‘‘heavy’’ field χ rapidly rolls down, and then from the last equation it follows, rather paradoxically, that the Hubble constant at the end of inflation is dominated by the ‘‘light’’ field ϕ . Thus we can consistently consider the creation of fluctuations of the field χ (χ particles) at the end of and after the last inflationary stage driven by the ϕ field. But now these fluctuations occur on top of a nonvanishing classical field χ .

To study the behavior of the classical fields ϕ and χ and their fluctuations analytically, one should remember that during the inflationary stage driven by ϕ one has $H = \sqrt{2\lambda_\phi \pi/3} (\phi^2/M_p)$, as in the previous section. In this case, as before, the solution for the equation of motion of the field ϕ is given by [11]

$$\phi = \phi_0 \exp\left(-\sqrt{\frac{\lambda_\phi}{6\pi}} M_p t\right). \quad (17)$$

Meanwhile, according to Eq. (16),

$$\chi = \phi_0 \sqrt{\frac{\lambda_\phi}{\lambda_\chi}} \exp\left(-\sqrt{\frac{\lambda_\phi}{6\pi}} M_p t\right), \quad (18)$$

whereas for perturbations of the field χ one has

$$\delta\chi = \delta\chi_0 \exp\left(-3\sqrt{\frac{\lambda_\phi}{6\pi}} M_p t\right). \quad (19)$$

Let us consider, for example, the behavior of the fields and their fluctuations at the end of inflation, starting from the moment $\phi = \phi_i$. One may take, for example, $\phi_i \sim 4M_p$, which corresponds to a point approximately 60 e-folds before the end of inflation. The fluctuations $\delta\chi_i \sim H(\phi_i)/2\pi$ decrease according to Eq. (19), and at the end of inflation one gets

$$\frac{\delta\chi}{\chi} = \frac{H(\phi_i)}{2\pi\chi_i} \exp\left(-2\sqrt{\frac{\lambda_\phi}{6\pi}} M_p t\right) = \frac{\sqrt{\lambda_\chi} \phi_i}{\sqrt{6\pi} M_p} \frac{\phi_e^2}{\phi_i^2}. \quad (20)$$

Here $\phi_e \sim 0.3M_p$ corresponds to the end of inflation.¹ After that moment the fields ϕ and χ begin oscillating, and the ratio of $\delta\chi$ to the amplitude of oscillations of the field χ remains approximately constant. This gives the following estimate for the amplitude of isocurvature perturbations in this model:

$$\frac{\delta V(\chi)}{V(\chi)} \sim 4 \frac{\delta\chi}{\chi} = 2 \times 10^{-2} \sqrt{\lambda_\chi}. \quad (21)$$

Initially perturbations of $V(\chi)$ give a negligibly small contribution to perturbations of the metric because $V(\chi) \ll V(\phi)$; that is why they are called isocurvature perturbations. However, the main idea of preheating in NO models is that eventually χ fields or the products of their decay will give the dominant contribution to the energy-momentum tensor because the energy density of the field ϕ rapidly vanishes due to the expansion of the universe ($\rho_\phi \sim a^{-6}$). However, because of the inhomogeneity of the distribution of the field χ (which will be imprinted in the density distribution of the products of its decay on scales greater than H^{-1}), the period of the dominance of matter over the scalar field ϕ will happen at different times in different parts of the universe. In other words, the epoch when the universe begins expanding as $a \sim \sqrt{t}$ or $a \sim t^{2/3}$ instead of $a \sim t^{1/3}$ will begin at different moments t (at different total densities) in different parts of the universe. Starting from this time the isocurvature fluctuations (21) will produce metric perturbations, and, as a result, perturbations of cosmic microwave background (CMB) radiation.

Note that if the equation of state of the field χ or of the products of its decay coincided with the equation of state of the scalar field ϕ after inflation, fluctuations of the field χ would not induce any anisotropy of CMB radiation. For example, these fluctuations would be harmless if the field χ decayed into ultrarelativistic particles with the equation of state $p = \rho/3$ and if the equation of state of the field ϕ at that time were also given by $p = \rho/3$. However, in our case the field ϕ has equation of state $p = \rho$, which is quite different from the equation of state of the field χ or of its decay products.

Isocurvature fluctuations lead to approximately six times greater large scale anisotropy of the cosmic microwave ra-

diation as compared with adiabatic perturbations [14]. To avoid cosmological problems, one would need to have $\delta V(\chi)/V(\chi) \leq 5 \times 10^{-6}$. If χ is the Higgs field with $\lambda_\chi \geq 10^{-7}$, then the perturbations discussed above will be unacceptably large. This may be a rather serious problem. Indeed, one may expect to have many scalar fields in realistic theories of elementary particles. To avoid large isocurvature fluctuations each of these fields must be extremely weakly coupled, with $\lambda_\chi \leq 10^{-7}$.

The general conclusion is that the theory of reheating in NO models, as well as their consequences for the creation of the large-scale structure of the universe, may be quite different from what was anticipated in the first papers on this subject. In the simplest versions of such models inflation typically does not end in the state $\chi = 0$, and large isocurvature fluctuations are produced.

IV. COSMOLOGICAL PRODUCTION OF GRAVITINOS AND MODULI FIELDS

If the inflaton field ϕ is sterile, not interacting with any other fields, the elementary particles constituting the universe should be produced gravitationally due to the variation of the scale factor $a(t)$ with time. This was one of the basic assumptions of all papers on NO models [3–6]. Not all species can be produced this way, but only those which are not conformally invariant. Indeed, the metric of the Friedmann universe is conformally flat. If one considers, for example, massless scalar particles χ with an additional term $-1/12\chi^2 R$ in the Lagrangian (conformal coupling), one can make conformal transformations of χ simultaneously with transformations of the metric and find that the theory of χ particles in the Friedmann universe is equivalent to their theory in flat space. That is why such particles would not be created in an expanding universe.

Since conformal coupling is a rather special requirement, one expects a number of different species to be produced. An apparent advantage of gravitational particle production is its universality [9]. There is a kind of ‘‘democracy’’ rule for all particles non-conformally coupled to gravity: the density of such particles produced at the end of inflation is $\rho_\chi \sim \alpha_\chi H^4$, where $\alpha_\chi \sim 10^{-2}$ is a numerical factor specific for different species and H is the Hubble parameter at the end of inflation.

Unfortunately, democracy does not always work; there may be too many dangerous relics produced by this universal mechanism. One of the potential problems is related to the overproduction of gravitons mentioned in [6]. In order to solve it one needs to have models with a very large number of types of light particles. This is difficult but not impossible [6]. However, even more difficult problems will arise if NO models are implemented in supersymmetric theories of elementary particles.

For example, in supersymmetric theories one may encounter many flat directions of the effective potential associated with moduli fields. These fields usually are very stable. Moduli particles decay very late, so in order to avoid cosmological problems the energy density of the moduli fields must be many orders of magnitude smaller than the energy density of other particles [15,16].

¹We are grateful to Peebles and Vilenkin for pointing out that the factor ϕ_e^2/ϕ_i^2 should be present in this equation.

Moduli fields typically are not conformally invariant (though some exceptions are possible). There are several different effects which add up to give them masses CH during expansion of the universe, with $C=O(1)$ (in general, C is not a constant) [15]. This is very similar to what happens if, for example, one adds a term $-(\xi/2)R\phi^2$ to the Lagrangian of a scalar field. Indeed, during inflation $R=12H^2$, so this term leads to the appearance of a contribution to the mass of the scalar field $\Delta m^2=12\xi H^2$. Conformal coupling would correspond to $m^2=2H^2$.

According to [3], the energy density of scalar particles produced gravitationally at the end of inflation is given by $10^{-2}H^4(1-6\xi)^2$. Thus, unless the constant C is fine-tuned to mimic conformal coupling, we expect that in addition to the energy of classical oscillating moduli fields, at the end of inflation one has gravitational production of moduli particles with energy density $\sim 10^{-2}H^4$, just as for all other conformally noninvariant particles.

In usual inflationary models one also encounters the moduli problem if the energy of classical oscillating moduli fields is too large [15]. Here we are discussing an independent problem which appears even if there are no classical oscillating moduli. Indeed, in NO models all particles created by gravitational effects at the end of inflation will have similar energy density $\sim 10^{-2}H^4$. But if the number density of moduli particles is not suppressed as compared to the number density of other particles by about 12–14 orders of magnitude, such models are ruled out [15,16]. We do not see any reason for such a strong suppression of production of moduli fields in the models of Refs. [3–6].

A similar problem appears if one considers the possibility of gravitational (nonthermal) production of gravitinos. Usually it is assumed that gravitinos have mass $m_{3/2}=10^2-10^3$ GeV, which is much smaller than the typical value of the Hubble constant at the end of inflation. Therefore naively one could expect that gravitinos, just like massless fermions of spin 1/2, are (almost exactly) conformally invariant and should not be produced due to expansion of the Friedmann universe.

However, in the framework of supergravity, the background metric is generated by inflaton field(s) ϕ_j with an effective potential constructed from the superpotential $W(\phi_j)$. The gravitino mass in the early universe acquires a contribution proportional to $W(\phi_j)$. Depending on the model, the gravitino mass soon after the end of inflation may be of the same order as H or somewhat smaller, but typically it is much greater than its present value $m_{3/2}$.

A general investigation of the behavior of gravitinos in the Friedmann universe shows that the gravitino field in a self-consistent Friedmann background supported by scalar fields is not conformally invariant [17]. For example, the effective potential $\lambda\phi^4$ can be obtained from the superpotential $\sqrt{\lambda}\phi^3$ in the global supersymmetry limit. This leads to a gravitino mass $\sim\sqrt{\lambda}\phi^3/M_p^2$. At the end of inflation $\phi\sim M_p$, and therefore the gravitino mass is comparable to the Hubble constant $H\sim\sqrt{\lambda}\phi^2/M_p$. This implies strong breaking of conformal invariance.

The theory of gravitational production of gravitinos is

strongly model-dependent, and in some models it might be possible to achieve a certain suppression of their production as compared to the production of other particles. The problem is that, just like in the situation with the moduli fields, this suppression must be extraordinary strong. Indeed, to avoid cosmological problems one should suppress the number of gravitinos as compared to the number of other particles by a factor of about 10^{-14} [20]. Investigation of gravitino production in the models proposed in Refs. [3–6] has shown no such suppression [17]. This implies that the number of gravitinos produced in such models typically violates the cosmological constraints by about 14 orders of magnitude.

The gravitino/moduli problem and the problem of isocurvature perturbations are interrelated in a rather nontrivial way. Indeed, the gravitino and moduli problems are especially severe if the density of gravitinos and/or moduli particles produced during reheating is of the same order of magnitude as the energy density of scalar fields χ . We assumed, according to [3,6], that the energy density of the fields χ after inflation is $O(10^{-2}H^4)$. But this statement is not always correct. It was derived in [3] under an assumption that particle production occurs during a short time interval when the equation of state changes. Meanwhile in inflationary cosmology the long-wavelength fluctuations of the field χ minimally coupled to gravity are produced during inflation all the time when the Hubble constant $H(t)$ is smaller than the mass of the χ particles m_χ . The energy density of χ particles produced during inflation will contain a contribution $\rho_0=(m_\chi^2/2)\langle\chi^2\rangle$, which may be many orders of magnitude greater than $10^{-2}H^4$.

For the sake of argument, one may consider inflation in the theory $\lambda/4\phi^4$ and take m_χ equal to the value of H at the end of inflation, $m_\chi\sim\sqrt{\lambda}\phi M_p$. Then, according to Eq. (7), after inflation one has $\rho_0\sim\lambda\phi m_\chi^2\phi_0^6/36M_p^4\sim 10^{-2}H^4(\phi_0/M_p)^6\gg 10^{-2}H^4$, because $\phi_0\gg M_p$.

This is the same effect which we discussed in Sec. II: If ϕ_0 is large enough, we may even have a second stage of inflation driven by the large energy density of the fluctuations of the field χ . But even if ϕ_0 is not large enough to initiate the second stage of inflation, it still must be much greater than M_p to drive the first stage of inflation, which makes the standard estimate $\rho\sim 10^{-2}H^4$ incorrect [18].

There is one more effect which should be considered, in addition to gravitational particle production. The effective mass of the particles ϕ at $\phi<0$ is given by $\sqrt{3}\lambda\phi$. At the end of inflation, at $\phi\sim M_p$, this mass is of the same order as the Hubble constant $\sim\sqrt{\lambda}\phi^2/M_p$. Then, within the Hubble time H^{-1} the field ϕ rolls to the valley at $\phi>0$ and its mass vanishes. This is a non-adiabatic process; the mass of the scalar field changes by $O(H)$ during the time $O(H^{-1})$. As a result, in addition to gravitational particle production there is an equally strong production of particles ϕ due to the nonadiabatic change of their mass [18,19].

This may imply that the fraction of energy in gravitinos will be somewhat smaller than previously expected, simply because the fraction of energy in the fluctuations of the field χ will be larger. However, this is a relatively small effect,

which cannot account for the 14 orders of magnitude disagreement between the cosmological constraints on the gravitino abundance and the theory of gravitino production in NO models [17].

Also, there is no free lunch. For example, the production of large number of nearly massless particles ϕ may lead to problems with nucleosynthesis. Large inflationary fluctuations of the field χ can create large isocurvature fluctuations. In the end of Sec. II we mentioned that one can avoid this problem if one assumes, for example, that the fields χ acquire effective mass $O(H)$ in an expanding universe. Then their fluctuations will not be produced during inflation. But in such a case their density after inflation will be given by $10^{-2}H^4$, and therefore we do not have any relaxation of the gravitino and the moduli problems.

V. SAVING NO MODELS: INSTANT PREHEATING

As we will see, the problems discussed above will not appear in theories of a more general class, where the fields ϕ and χ can interact with each other. We will consider a model with the interaction $(g^2/2)\phi^2\chi^2$. First we will show that in this case it really makes sense to study preheating assuming that $\chi=0$. Then we will describe the scenario of instant preheating, which allows a very efficient energy transfer from the inflaton field to particles χ .

A. Initial conditions for inflation and reheating in the model with interaction $(g^2/2)\phi^2\chi^2$

Consider a theory with an effective potential dominated by the term $V(\phi, \chi) = (g^2/2)\phi^2\chi^2$. This means that we will assume that the constant g is large enough for us to temporarily neglect the terms $(\lambda_\phi/4)\phi^4 + (\lambda_\chi/4)\chi^4$ in the discussion of initial conditions.

In this case the Planck boundary is given by the condition

$$\frac{g^2}{2}\phi^2\chi^2 \sim M_p^4, \quad (22)$$

which defines a set of four hyperbolas

$$g|\phi||\chi| \sim M_p^2. \quad (23)$$

At larger values of ϕ and χ the density is greater than the Planck density, so the standard classical description of space-time is impossible there. On the other hand, the effective masses of the fields should be smaller than M_p , and consequently the curvature of the effective potential cannot be greater than M_p^2 . This leads to two additional conditions:

$$|\phi| \leq g^{-1}M_p, \quad |\chi| \leq g^{-1}M_p. \quad (24)$$

We assume that $g \ll 1$. Suppose for definiteness that initially the fields ϕ and χ belong to the Planck boundary (23) and that $|\phi|$ is half-way towards its upper bound Eq. (24): $|\phi| \sim g^{-1}M_p/2$. The choice of the coefficient 1/2 here is not essential; we only want to make sure that the field χ initially is of order M_p , but it can be slightly greater than M_p . This

allows for an extremely short stage of inflation when the field χ rolls down towards $\chi=0$.

The equations for the two fields are

$$\ddot{\phi} + 3H\dot{\phi} = -g^2\phi\chi^2. \quad (25)$$

and

$$\ddot{\chi} + 3H\dot{\chi} = -g^2\phi^2\chi. \quad (26)$$

The curvature of the effective potential in the ϕ direction initially is $\sim g^2\chi^2 \sim g^2M_p^2$, which is very small compared to the initial value of $H^2 \sim M_p^2$. Thus the field ϕ will move very slowly, so one can neglect the term $\ddot{\phi}$ in Eq. (25).

$$3H\dot{\phi} = -g^2\phi\chi^2. \quad (27)$$

If the field ϕ changes slowly, then the field χ behaves as in the theory $(m_\chi^2/2)\chi^2$ with $m_\chi \sim g|\phi|$ being slightly smaller than M_p and with the initial value of χ being slightly greater than M_p . This leads to a very short stage of inflation which ends within a few Planck times. After this short stage the field χ rapidly oscillates. During this stage the energy density of the oscillating field drops down as a^{-3} , the universe expands as $a \sim t^{2/3}$, and $H = (2/3t)$. Thus the square of the amplitude of the oscillations of the field χ decreases as follows: $\chi^2 \sim \chi_0^2 a^{-3} \sim t^{-2}$. This leads to the following equation for the field ϕ :

$$\frac{\dot{\phi}}{\phi} \sim -\frac{g^2}{t}. \quad (28)$$

The solution of this equation is $\phi = \phi_0(t/t_0)^{-g^2}$ with $t_0 \sim M_p^{-1}$, and $\phi_0 \sim -M_p/g$. (The condition $t_0 \sim M_p^{-1}$ follows from the fact that the initial value of $H = 2/3t$ is not much below M_p .) This gives

$$\phi \sim -\frac{M_p}{g} (M_p t)^{-g^2}. \quad (29)$$

The inflaton field ϕ becomes equal to $-M_p$ after the exponentially large time

$$t \sim M_p^{-1} \left(\frac{1}{g}\right)^{g^{-2}}. \quad (30)$$

During this time the energy of oscillations of the field χ becomes exponentially small, and the small term $(\lambda_\phi/4)\phi^4$ which we neglected until now becomes the leading term driving the scalar field ϕ . At this stage we will have the usual chaotic inflation scenario with $|\phi| > M_p$ and with the fields evolving along the direction $\chi=0$.

Thus in the presence of the interaction term $(g^2/2)\phi^2\chi^2$ one can indeed consider inflation and reheating with $\chi=0$. As we have seen, this possibility was rather problematic in the models where ϕ and χ interacted only gravitationally.

The effective mass of the field χ during inflation is $g|\phi|$, which is much greater than the Hubble constant $\sim \sqrt{\lambda}\phi^2/M_p$ for $g^2/\lambda \gg \phi^2/M_p^2$. In realistic versions of this model one has

$\lambda \sim 10^{-13}$ [12], and $g^2 \gg \lambda$. Therefore long-wavelength fluctuations of the field χ are not produced during the last stages of inflation, when $\phi \sim M_p$.

A similar conclusion is valid if at the last stages of inflation the effective potential of the field ϕ is quadratic, $V(\phi) = m^2/2\phi^2$. In this case $H \sim m\phi/M_p$, and inflationary fluctuations of the field χ are not produced for $g \gg m/M_p$. In realistic versions of this model one has $m \sim 10^{-6}M_p$ [12], and fluctuations $\delta\chi$ are not produced if $g^2 \gg 10^{-12}$. This means that the problem of isocurvature fluctuations does not appear.

B. Instant preheating in NO models

To explain the main idea of the instant preheating scenario in NO models, we will assume for simplicity that $V(\phi) = (m^2/2)\phi^2$ for $\phi < 0$, and that $V(\phi)$ vanishes for $\phi > 0$. We will discuss a more general situation later. We will assume that the effective potential contains the interaction term $(g^2/2)\phi^2\chi^2$, and that χ particles have the usual Yukawa interaction $h\bar{\psi}\psi\chi$ with fermions ψ . For simplicity, we will assume here that χ particles do not have any bare mass, so that their effective mass is equal to $g|\phi|$.

In this model inflation ends when the field ϕ rolls from large negative values down to $\phi \sim -0.3M_p$ [12]. Production of particles χ begins when the effective mass of the field χ starts to change nonadiabatically, $|\dot{m}_\chi| \gtrsim m_\chi^2$, i.e., when $g|\dot{\phi}|$ becomes greater than $g^2\phi^2$.

This happens only when the field ϕ rolls close to $\phi = 0$, and the velocity of the field is $|\dot{\phi}_0| \approx mM_p/10 \approx 10^{-7}M_p$ [8]. [In the theory $(\lambda/4)\phi^4$ with $\lambda = 10^{-13}$ one has a somewhat smaller value $|\dot{\phi}_0| \approx 6 \times 10^{-9}M_p^2$.] The process becomes nonadiabatic for $g^2\phi^2 \lesssim g|\dot{\phi}_0|$, i.e., for $-\phi_* \lesssim \phi \lesssim \phi_*$, where $\phi_* \sim \sqrt{(|\dot{\phi}_0|/g)}$ [2]. Note that for $g \gg 10^{-5}$ the interval $-\phi_* \lesssim \phi \lesssim \phi_*$ is very narrow: $\phi_* \ll M_p/10$. As a result, the process of particle production occurs nearly instantaneously, within the time

$$\Delta t_* \sim \frac{\phi_*}{|\dot{\phi}_0|} \sim (g|\dot{\phi}_0|)^{-1/2}. \quad (31)$$

This time interval is much smaller than the age of the universe, so all effects related to the expansion of the universe can be neglected during the process of particle production. The uncertainty principle implies in this case that the created particles will have typical momenta $k \sim (\Delta t_*)^{-1} \sim (g|\dot{\phi}_0|)^{1/2}$. The occupation number n_k of χ particles with momentum k is equal to zero all the time when it moves toward $\phi = 0$. When it reaches $\phi = 0$ (or, more exactly, after it moves through the small region $-\phi_* \lesssim \phi \lesssim \phi_*$) the occupation number suddenly (within the time Δt_*) acquires the value [2]

$$n_k = \exp\left(-\frac{\pi k^2}{g|\dot{\phi}_0|}\right), \quad (32)$$

and this value does not change until the field ϕ rolls to the point $\phi = 0$ again.

A detailed description of this process including the derivation of Eq. (32) was given in the second paper of Ref. [2]; see in particular Eq. (55) there. This equation (32) can be written in a more general form. For example, if the particles χ have bare mass m_χ , this equation can be written as follows [8]:

$$n_k = \exp\left(-\frac{\pi(k^2 + m_\chi^2)}{g|\dot{\phi}_0|}\right). \quad (33)$$

This can be integrated to give the density of χ particles

$$n_\chi = \frac{1}{2\pi^2} \int_0^\infty dk k^2 n_k = \frac{(g\dot{\phi}_0)^{3/2}}{8\pi^3} \exp\left(-\frac{\pi m_\chi^2}{g|\dot{\phi}_0|}\right). \quad (34)$$

As we already mentioned, in the theory $(m^2/2)\phi^2$ with $m = 10^{-6}M_p$ one has $|\dot{\phi}_0| = 10^{-7}M_p^2$ [8]. This implies, in particular, that if one takes $g \sim 1$, then in the theory $(m^2/2)\phi^2$ there is no exponential suppression of production of χ particles unless their mass is greater than $m_\chi \sim 2 \times 10^{15}$ GeV. This agrees with a similar conclusion obtained in [2,21–24].

Let us now concentrate on the case $m_\chi^2 \ll g|\dot{\phi}_0|$, when the number of produced particles is not exponentially suppressed. In this case the number density of particles at the moment of their creation is given by $(g\dot{\phi}_0)^{3/2}/8\pi^3$, but then it decreases as $a^{-3}(t)$:

$$n_\chi = \frac{(g\dot{\phi}_0)^{3/2}}{8\pi^3 a^3(t)}. \quad (35)$$

Here we take $a_0 = 1$ at the moment of particle production.

Particle production occurs only in a small vicinity of $\phi = 0$. Then the field ϕ continues rolling along the flat direction of the effective potential with $\phi > 0$, and the mass of each χ particle grows as $g\phi$. Therefore the energy density of produced particles is

$$\rho_\chi = \frac{(g\dot{\phi}_0)^{3/2}}{8\pi^3} \frac{g\phi(t)}{a^3(t)}. \quad (36)$$

The energy density of the field ϕ drops down much faster, as $a^{-6}(t)$. The reason is that if one neglects backreaction of produced particles, the energy density of the field ϕ at this stage is entirely concentrated in its kinetic energy density $\frac{1}{2}\dot{\phi}^2$, which corresponds to the equation of state $p = \rho$. We will study this issue now in a more detailed way.

The equation of motion for the inflaton field after particle production looks as follows:

$$\ddot{\phi} + 3H\dot{\phi} = -g^2\phi\langle\chi^2\rangle. \quad (37)$$

We will assume for simplicity that the field χ does not have bare mass, i.e. $m_\chi = g\phi$. As soon as the field ϕ becomes greater than ϕ^* (and this happens practically instantly, when

particle production ends), the particles χ become nonrelativistic. In this case $\langle\chi^2\rangle$ can be easily related to n_χ :

$$\langle\chi^2\rangle\approx\frac{1}{2\pi^2}\int\frac{n_k k^2 dk}{\sqrt{k^2+g^2\phi^2}}\approx\frac{n_\chi}{g\phi}\approx\frac{(g\dot{\phi}_0)^{3/2}}{8\pi^3 g\phi a^3(t)}. \quad (38)$$

Therefore the equation for the field ϕ reads

$$\ddot{\phi}+3H\dot{\phi}=-gn_\chi=-g\frac{(g\dot{\phi}_0)^{3/2}}{8\pi^3 a^3(t)}. \quad (39)$$

To analyze the solutions of this equation, we will first neglect backreaction. In this case one has $a\sim t^{1/3}$, $H=1/3t$, and

$$\phi=\frac{M_p}{2\sqrt{3}\pi}\log\frac{t}{t_0}, \quad (40)$$

where $t_0=1/3H_0=M_p/2\sqrt{3}\pi\dot{\phi}_0\approx 5/\sqrt{3}\pi m$.

One can easily check that this regime remains intact and backreaction is unimportant for $t<t_1\sim 8\pi^3/\sqrt{g^5\dot{\phi}_0}$, until the field ϕ grows up to

$$\phi_1\approx\frac{5M_p}{4\sqrt{3}\pi}\log\frac{1}{g}. \quad (41)$$

This equation is valid for $g\ll 1$. For example, for $g=10^{-3}$ one has $\phi_1\sim 3M_p$. For $g=10^{-1}$ one has $\phi_1\sim M_p$. Note that the terms in the left hand side of the Eq. (39) decrease as t^{-2} when the time grows, whereas the backreaction term goes as t^{-1} . As soon as the backreaction becomes important, i.e. as soon as the field ϕ reaches ϕ_1 , it turns back, and returns to $\phi=0$. When it reaches $\phi=0$, the effective potential becomes large, so the field ϕ cannot become negative, and it bounces towards large ϕ again.

Now let us take into account interaction of the χ field with fermions. This interaction leads to decay of the χ particles with the decay rate [2]

$$\Gamma(\chi\rightarrow\psi\psi)=\frac{h^2 m_\chi}{8\pi}=\frac{h^2 g|\phi|}{8\pi}. \quad (42)$$

Note that the decay rate grows with the growth of the field $|\phi|$, so particles tend to decay at large ϕ . In our case the field ϕ spends most of the time prior to t_1 at $\phi\sim M_p$ (if it does not decay earlier, see below). The decay rate at that time is

$$\Gamma(\chi\rightarrow\psi\psi)\sim\frac{h^2 g M_p}{8\pi}. \quad (43)$$

If $\Gamma_\chi^{-1}<t_1\sim 8\pi^3/g^{5/2}\dot{\phi}_0$, then particles χ will decay to fermions ψ at $t<t_1$ and the force driving the field ϕ back to $\phi=0$ will disappear before the field ϕ turns back. In this case the field ϕ will continue to grow, and its energy density will continue decreasing anomalously fast, as a^{-6} . This happens if

$$\frac{h^2 g M_p}{8\pi}\gtrsim\frac{g^{5/2}\dot{\phi}_0^{1/2}}{8\pi^3}. \quad (44)$$

Taking into account that in our case $\dot{\phi}_0\sim mM_p/10$ and $m\sim 10^{-6}M_p$, one finds that this condition is satisfied if $h\gtrsim 5\times 10^{-3}g^{3/4}$. This is a very mild condition. For example, it is satisfied for $h>5\times 10^{-3}$ if $g=1$, and for $h>5\times 10^{-7}$ if $g=10^{-4}$.

This scenario is always 100% efficient. The initial fraction of energy transferred to matter at the moment of χ particle production is not very large, about $10^{-2}g^2$ of the energy of the inflaton field [8]. However, because of the subsequent growth of this energy due to the growth of the field ϕ , and because of the rapid decrease of kinetic energy of the inflaton field, the energy density of the χ particles and of the products of their decay soon becomes dominant. This should be contrasted with the usual situation in the theories where $V(\phi)$ has a minimum. As was emphasized in [2], efficient preheating is possible only in a subclass of such models. In many models where $V(\phi)$ has a minimum the decay of the inflaton field is incomplete, and it accumulates an unacceptably large energy density compared with the energy density of the thermalized component of matter. The possibility of having a very efficient reheating in NO models may have significant consequences for inflationary model building.

It is instructive to compare the density of particles produced by this mechanism to the density of particles created during gravitational particle production, which is given by $\rho_\chi\sim 10^{-2}H^4\sim\rho_\phi(\rho_\phi/M_p^4)$, where ρ_ϕ is the energy density of the field ϕ at the end of inflation. In the model $(\lambda_\phi/4)\phi^4$ one has $\rho_\phi\sim 10^{-16}M_p^4$, and, consequently, $\rho_\chi\sim\rho_\phi(\rho_\phi/M_p^4)\sim 10^{-16}\rho_\phi$. Meanwhile, as we just mentioned, at the first moment after particle production in our scenario the energy density of produced particles is of the order of $10^{-2}g^2\rho_\phi$ [8], and then it grows together with the field ϕ because of the growth of the mass $g\phi$ of each χ particle. Thus, for $g^2\gtrsim 10^{-14}$ the number of particles produced during instant preheating is much greater than the number of particles produced by gravitational effects. Therefore one may argue that reheating of the universe in NO models should be described using the instant preheating scenario. Typically it is much more efficient than gravitational particle production. This means, in particular, that production of normal particles will be much more efficient than the production of gravitinos and moduli.

In order to avoid the gravitino problem altogether one may consider versions of NO models where the particles produced during preheating remain nonrelativistic for a while. Then the energy density of gravitinos during this epoch decreases much faster than the energy density of usual particles. New gravitinos will not be produced if the resulting temperature of reheating is sufficiently small.

VI. OTHER VERSIONS OF NO MODELS

The mechanism of particle production described above can work in a broad class of theories. In particular, since parametric amplification of particle production is not impor-

tant in the context of the instant preheating scenario, it will work equally well if the inflaton field couples not to bosons χ but to fermions [26,27]. Indeed, the creation of fermions with mass $g|\phi|$ also occurs because of the nonadiabaticity of the change of their mass at $\phi=0$. The theory of this effect is very similar to the theory of the creation of χ particles described above; see in this respect [27].

Returning to our scenario, production of particles χ depends on the interactions between the fields ϕ and χ . For example, one can consider models with the interaction $(g^2/2)\chi^2(\phi+v)^2$. Such interaction terms appear, for example, in supersymmetric models with superpotentials of the type $W=g\chi^2(\phi+v)$ [25]. In such models the mass m_χ vanishes not at $\phi_1=0$, but at $\phi_1=-v$, where v can take any value. Correspondingly, the production of χ particles occurs not at $\phi=0$ but at $\phi=-v$. When the inflaton field reaches $\phi=0$, one has $m_\chi\sim gv$, which may be very large. If one takes $v\sim M_p$, one can get $m_\chi\sim gM_p$, which may be as great as 10^{18} GeV for $g\sim 10^{-1}$, or even 10^{19} GeV for $g\sim 1$. If one takes $v\gg M_p$, the density of χ particles produced by this mechanism will be exponentially suppressed by the subsequent stage of inflation.

In the previous section we considered the simplest model where $V(\phi)=0$ for $\phi>0$. However, in general $V(\phi)$ may become flat not at $\phi=0$, but only asymptotically, at $\phi\gtrsim M_p$. Such theories have become rather popular now in relation to the theory of quintessence; for a partial list of references see e.g. [28]. In such a case the backreaction of created particles may never turn the scalar field ϕ back to $\phi=0$. Therefore the decay of the particles χ may occur very late, and one can have very efficient preheating for any values of the coupling constants g and h .

On the other hand, if the χ particles are stable, and if the field ϕ continues rolling for a very long time, one may encounter a rather unusual regime. If the particle masses $g|\phi|$

at some moment approach M_p , the χ particles may convert to black holes and immediately evaporate.

Indeed, in conventional quantum field theory, an elementary particle of mass M has a Compton wavelength M^{-1} smaller than its Schwarzschild radius $2M/M_p^2$ if $M\gtrsim M_p$. Therefore one may expect that as soon as $m_\chi=g|\phi|$ becomes greater than M_p , each χ particle becomes a Planck-size black hole, which immediately evaporates and reheats the universe. If this regime is possible, it should be avoided. Indeed, black holes of Planck mass may produce similar amounts of all kinds of particles, including gravitinos. Therefore if reheating occurs because of black hole evaporation, then we will return to the gravitino problem again.

Thus, the best possibility is to consider those versions of the instant preheating scenario which do not lead to the creation of stable particles of Planckian mass. It may seem paradoxical that one needs to be careful about this constraint. Several years ago it would have seemed impossible to produce particles of mass greater than 5×10^{12} GeV during the decay of an inflaton field of mass $m_\phi\sim 10^{13}$ GeV. Here we consider a nonperturbative mechanism of preheating which may produce particles 5 orders of magnitude heavier than m_ϕ . It is interesting that the mechanism of instant preheating discussed in this paper works especially well in the context of NO models where all other mechanisms are rather inefficient.

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