Inhomogeneous cosmologies with Q matter and varying Λ

Luis P. Chimento* and Alejandro S. Jakubi[†]

Departamento de Física, Universidad de Buenos Aires, 1428 Buenos Aires, Argentina

Diego Pavón[‡]

Departament de Física, Universidad Autónoma de Barcelona, 08193 Bellaterra, Spain

Starting from the inhomogeneous shear-free Nariai metric we show, by solving the Einstein-Klein-Gordon field equations, how a self-interacting scalar field plus a material fluid, a variable cosmological term, and a heat flux can drive the universe to its currently observed state of homogeneous accelerated expansion. A quintessence scenario where power-law inflation takes place for a string-motivated potential in the late-time dominated field regime is proposed. [S0556-2821(99)09018-9]

I. INTRODUCTION

The theory of general relativity implies that the large scale geometry and evolution of the Universe is dictated by its content of matter and fields, including energy fluxes, shear stresses, particle production, and so on. Recently, there have been claims in the literature that the Universe, besides its content in normal matter and radiation, must possess a not yet identified component (usually called quintessence matter, Q matter for short) [1–4], characterized by a negative pressure, and possibly a cosmological term which may be constant or not [5,6]. These claims were prompted at the realization that the clustered matter component can account at most for one third of the critical density. Therefore, an additional ''soft'' (i.e., nonclustered) component is needed if the critical density predicted by many inflationary models is to be achieved.

Very often the geometry of the proposed models is very simple, just Friedmann-Lemaître-Robertson-Walker (FLRW), partly because of mathematical simplicity, and partly because they deal with not very early stages of cosmic evolution. However, up to now no convincing argument has been avanced to the effect that the geometry of the very early Universe (say, some time before nucleosynthesis) could not have been either anisotropic or inhomogeneous, or both. Note that it is very natural to assume the geometry at that primeval epoch more general than just FLRW. Moreover, recently it has been demonstrated that given any spherically symmetric geometry and any set of observations, evolution functions for the sources can be found that will make the model compatible in general with observation [7]. Further motivations to study the evolution of inhomogeneous cosmological models can be found in Refs. [8] and [9]. In constrast to FLRW models, inhomogeneous spaces are in general compatible with heat fluxes, and these might imply important consequences such as inflation [10] or the avoidance of the initial singularity [11]. Here we focus on an isotropic but inhomogeneous spherically symmetric universe which besides a material fluid contains a self-interacting scalar field (which can be interpreted as Q matter), and a cosmological term, Λ which, in general, may vary with time. As it turns out the homogenization of the Universe at large times depends very much on the value the effective adiabatic index γ, defined below [Eq. (5)], takes. When more generally this quantity varies with time, homogenization can also be achieved under rather ample conditions.

Obviously, density inhomogeneities triggered by gravitational instability must be present at any stage of evolution, even when the universe has reached a near homogeneous state describable in the mean by the FLRW metric, i.e., an average homogeneous universe housing growing as well as decaying density perturbation modes, the former leading eventually to the formation of the cosmic structures we observe today. About the evolution of scalar (i.e., density) inhomogeneities there is a rather large and still growing body of literature (see, e.g., Ref. [12], and references therein) but we shall not deal with them as they lie outside the main focus of this paper. We only mention that the negative pressure associated to Q matter and Λ will tend to slow down the growing modes (see, e.g., Refs. [1,12,13]), and shift the epoch of matter-radiation equality toward more recent times [14]. A detailed study of all this will be the subject of a future work.

Section II presents the basic equations. Section III solves the Einstein-Klein-Gordon (EKG) field equations assuming γ (but not Λ) a constant. Section IV studies the asymptotic evolution toward a Q-matter dominated era. Section V solves the EKG equations when γ is a function of time. Finally, Sec. VI summarizes the findings of this paper. Units have been chosen so that $c = G = 1$.

II. EINSTEIN-KLEIN-GORDON FIELD EQUATIONS

Let us consider a shear-free spherically symmetric spacetime with metric [15]

$$ds^2 = -A(t,r)^2 dt^2 + B(t,r)^2 [dr^2 + r^2 d\Omega^2],$$

where as usual $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$. For later purposes we introduce the functions $F(t,r) \equiv 1/B(t,r)$ and $v(t,r) \equiv AF$. Thus, the line element assumes the more convenient form

*Electronic address: chimento@df.uba.ar
[†]Electronic address: jakubi@df.uba.ar
[‡]Electronic address: diego@ulises.uab.es

$$ds^2 = \frac{1}{F(t,r)^2}[-v(t,r)^2 dt^2 + d\,r^2 + r^2 d\Omega^2]. \quad (1)$$

As sources of the gravitational field we take: a fluid of material energy density $\rho_f = \rho_f(r,t)$, hydrostatic pressure $p_f = p_f(r,t)$, with a radial heat flow ($q_r = q_r(r,t)$ and $q_t = q_\theta = q_\phi = 0$), plus a cosmological term, related to the energy density of vacuum by $\Lambda = 8\pi\rho_{vac}$, that depends only on time $\Lambda = \Lambda(t)$, and a self-interacting scalar field ϕ driven by the potential $V(\phi)$. Taking the scalar field to depend only on t, its energy-momentum tensor may be written in the perfect fluid form

$$T_{ik} = (p_\phi + \rho_\phi)u_i u_k + p_\phi g_{ik}, \quad (2)$$

where $u^i = \phi^{\cdot i}/\sqrt{-\phi_{,l}\phi^{,l}}$ together with

$$\rho_\phi = -\frac{1}{2}\phi_{,l}\phi^{,l} + V(\phi),$$

$$p_\phi = -\frac{1}{2}\phi_{,l}\phi^{,l} - V(\phi). \quad (3)$$

The fluid interpretation of the scalar field, while not compulsory, has proven very useful in the study of the inflationary phase and reheating of the universe—see, for instance, Refs. [16] and [17]. In particular it leads us to consider its equation of state $p_\phi = (\gamma_\phi - 1)\rho_\phi$. Hence the scalar field can be interpreted as Q-matter since, depending on $V(\phi)$, γ_ϕ may be lower than 1, see, e.g., Ref. [3]. The stress energy-tensor of the normal matter, with a heat flow, plus Q matter (scalar field) and the cosmological term is

$$T^i_k = (\rho_f + \rho_\phi + p_f + p_\phi)u^i u_k + (\Lambda - p_f - p_\phi)\delta^i_k + q^i u_k + q_k u^i, \quad (4)$$

where the comoving four-velocity is normalized so that $u^i u_i = -1$, with $u^i = \delta^i_t(g_{tt})^{-1/2}$, and hence $u^t = F/v$. Obviously the heat flow is orthogonal to u_i (i.e. $q^i u_i = 0$). As equation of state for the fluid we choose $p_f = (\gamma_f - 1)\rho_f$ where γ_f is a function of t and r. Taking into account the additivity of the stress-energy tensor it makes sense to consider an effective perfect fluid description with equation of state $p = (\gamma - 1)\rho$ where $p = p_f + p_\phi$, $\rho = \rho_f + \rho_\phi$ and

$$\gamma = \frac{\gamma_f \rho_f + \gamma_\phi \rho_\phi}{\rho_f + \rho_\phi}, \quad (5)$$

is the overall (i.e., effective) adiabatic index.

The requirement that the cosmological term Λ is just a function of t leads to the restriction that γ also depends only on t to render the system of Einstein equations integrable. The nice result we are seeking is a solution that has an asymptotic FLRW stage, with Λ evolving towards a constant, and the heat flow vanishing in that limit.

For the metric (1) the Einstein-Klein-Gordon (EKG) equations are

$$12x\,F'^2 - 12FF' - 8xFF'' - 3\frac{\dot{F}^2}{v^2} + \rho + \Lambda = 0, \quad (6)$$

$$12x\,F'^2 - 8FF' - 3\frac{\dot{F}^2}{v^2} - 2\frac{F\dot{F}\dot{v}}{v^3} + 2\frac{F\ddot{F}}{v^2} + 4\frac{F^2 v'}{v}$$
$$-8\frac{xFF'v'}{v} - p + \Lambda = 0, \quad (7)$$

$$12x\,F'^2 - 8FF' - 8xFF'' - 3\frac{\dot{F}^2}{v^2} - 2\frac{F\dot{F}\dot{v}}{v^3} + 2\frac{F\ddot{F}}{v^2}$$
$$-8\frac{xFF'v'}{v} + 4\frac{F^2 v'}{v} + 4\frac{xF^2 v''}{v} - p + \Lambda = 0, \quad (8)$$

$$-8\frac{xF^2\dot{F}v'}{v^2} + 8\frac{xF^2\dot{F}'}{v} + q_r = 0, \quad (9)$$

$$\ddot{\phi} - 2\frac{\dot{F}}{F}\dot{\phi} + \frac{1}{F^2}\frac{dV}{d\phi} = 0, \quad (10)$$

where $x = r^2$, an overdot means $\partial/\partial t$, and a prime means $\partial/\partial x$. We thus have five equations and seven unknowns (F, v, ρ, p, ϕ, q_r, and Λ). Hence two more equations are needed to render the system determinate. These are the above equation of state relating ρ and p and the condition $\partial\Lambda/\partial x = 0$.

From Eqs. (7) and (8) we get

$$\frac{v''}{v} = 2\frac{F''}{F}, \quad (11)$$

then using the ansatz $F = [a(t) + b(t)x]^k$ and $v = [c(t) + d(t)x]^n$ in Eq. (11) we find that the constraints $n(n-1) = 2k(k-1)$ and $a(t)d(t) = b(t)c(t)$ must be satisfied. Next we study the simplest case, namely $n = k = 1$. This is the most interesting instance because the functions $a(t)$, $b(t)$, $c(t)$, and $d(t)$ can be chosen freely. Another set of solutions can be obtained if the relationship $v_{xx}/v = 2[\nu(t)]^2$ is assumed in Eq. (11). These solutions are presently under study and we shall report on them in a future paper.

Einstein equations with time-dependent Λ and γ. When $n = k = 1$ we obtain a set of solutions that contains those of Modak ($b = 0$) [18], Bergmann ($c = a, d = b$) [19] and Maiti ($b = d = ka/4$, with $k = 0, \pm 1$) [20]. Another possibility arises when $d = 0$. This solution can be also obtained from the metric (1) by imposing this metric be conformal to Minkowski's. Namely, the integrability of the transformation $-v\,dt + dr = d\eta$ and $v\,dt + dr = d\sigma$ leads to $v = v(t)$ and $n = k = 1$. This allows us to redefine the time by $v\,dt \to dt$. Then the metric (1) becomes

$$ds^2 = \frac{1}{[a(t) + b(t)r^2]^2}(-d\,t^2 + d\,r^2 + r^2 d\Omega^2), \quad (12)$$

and Eqs. (6), (7), and (9) turn into

$$\rho + \Lambda = 12ab + 3\dot{a}^2 + 6\dot{a}\dot{b}x + 3\dot{b}^2x^2, \quad (13)$$

$$p-\Lambda=(2b\ddot{b}-3\dot{b}^2)x^2+2(2b^2-3\dot{a}\dot{b}+a\ddot{b}+\ddot{a}b)x-8ab$$
$$-3\dot{a}^2+2a\ddot{a}, \tag{14}$$

and

$$q_r=-4\sqrt{x}\dot{b}(a+bx)^2, \tag{15}$$

respectively. We next impose that Λ depends solely on time, so it must have the form

$$\Lambda(t)=12ab+3\dot{a}^2+f(t), \tag{16}$$

where $f(t)$ is a function to be determined. Then, Eqs. (13) and (14) imply

$$\rho(x,t)=6\dot{a}bx+3\dot{b}^2x^2-f(t), \tag{17}$$

$$p(x,t)=(2b\ddot{b}-3\dot{b}^2)x^2+2(2b^2-3\dot{a}\dot{b}+a\ddot{b}+\ddot{a}b)x+4ab$$
$$+2a\ddot{a}+f(t). \tag{18}$$

We further impose the equation of state $p=(\gamma-1)\rho$. Taking into account that a and b depend only on time, and equating the coefficients of same power of x we obtain a set of equations to determine a, b, and f

$$b\ddot{b}-\frac{3}{2}\gamma\dot{b}^2=0, \tag{19}$$

$$\ddot{a}-3\gamma\frac{\dot{b}}{b}\dot{a}+\frac{3\gamma}{2}\frac{\dot{b}^2}{b^2}a=-2b, \tag{20}$$

$$f=-\frac{2a}{\gamma}(2b+\ddot{a}). \tag{21}$$

To show the conditions that lead asymptotically to a FLRW metric it is expedient to introduce the time coordinate $d\tau=dt/a$. Thus Eq. (1) becomes

$$ds^2=\frac{1}{(1+Mr^2)^2}[-d\tau^2+R^2(dr^2+r^2\,d\Omega^2)], \tag{22}$$

where $M=b/a$ and $R=1/|a|$. This metric is conformal to FLRW, and the conformal factor approaches unity when $M\to0$.

III. CONSTANT ADIABATIC INDEX

When γ is a constant different from 2/3, the general solution of Eqs. (19) and (20) becomes

$$b(t)=K\Delta t^{2/(2-3\gamma)}, \tag{23}$$

$$a(t)=C_1\Delta t^{-2/(3\gamma-2)}+C_2\Delta t^{-3\gamma/(3\gamma-2)}$$
$$-\frac{1}{3}K\Delta t^{6(\gamma-1)/(3\gamma-2)}, \tag{24}$$

thereby

$$M(t)=K\left(C_1+C_2\Delta t^{-1}-\frac{1}{3}K\Delta t^2\right)^{-1}. \tag{25}$$

Whereas for $\gamma=2/3$, we get

$$b(t)=Ke^{C\Delta t}, \tag{26}$$

$$a(t)=(C_1+C_2\Delta t-K\Delta t^2)e^{C\Delta t}, \tag{27}$$

and

$$M(t)=K(C_1+C_2\Delta t-K\Delta t^2)^{-1}, \tag{28}$$

where K, C, C_1, and C_2 are arbitrary integration constants, and $\Delta t=t-t_0$ with t_0 some initial time. Inserting Eqs. (24), (23), (26), and (27) in Eqs. (21), (15), and (16) we get

$$\Lambda(t)=-\frac{8}{3}K^2\Delta t^{2(3\gamma-4)/(3\gamma-2)}+12C_1K\Delta t^{-4/(3\gamma-2)}$$
$$+16C_2K\Delta t^{-(3\gamma+2)/(3\gamma-2)}+3C_2^2\Delta t^{-4(3\gamma-1)/(3\gamma-2)}, \tag{29}$$

and

$$q_r(r,t)=\frac{8Kr\Delta t^{-3\gamma/(3\gamma-2)}}{9(3\gamma-2)}[3C_2\Delta t^{-3\gamma/(3\gamma-2)}$$
$$+3(C_1+Kr^2)\Delta t^{-2/(3\gamma-2)}-K\Delta t^{6(\gamma-1)/(3\gamma-2)}]^2, \tag{30}$$

for $\gamma\neq2/3$, and

$$\Lambda(t)=(3C_2^2+12KC_1)e^{2C\Delta t}, \tag{31}$$

$$q_r(r,t)=-4Kr[C_1+C_2\Delta t+K(r^2-\Delta t^2)]^2e^{3C\Delta t}, \tag{32}$$

for $\gamma=2/3$. In the following we analyze the asymptotic behavior of these solutions in the limits for t such that $M\to0$. We note by passing that in the $M\to0$ limit the time coordinate τ becomes the cosmological time, as can be seen from Eq. (22). Two alternatives of asymptotically expanding universes appear depending on the map between t and τ equations (34), (35) and (43), (48) below.

Case $\Delta t\to0$. In this limit we obtain

$$a\simeq C_2\Delta t^{3\gamma/(2-3\gamma)}, \tag{33}$$

$$\Delta\tau\simeq\frac{2-3\gamma}{2C_2(1-3\gamma)}\Delta t^{2(1-3\gamma)/(2-3\gamma)} \quad (\gamma\neq1/3), \tag{34}$$

$$\Delta\tau\simeq\frac{1}{C_2}\ln\Delta t \quad (\gamma=1/3), \tag{35}$$

$$R(\tau)\simeq\frac{1}{|C_2|}\left[\frac{2C_2(1-3\gamma)}{2-3\gamma}\Delta\tau\right]^{3\gamma/2(3\gamma-1)} \quad (\gamma\neq1/3), \tag{36}$$

$$R(\tau)\simeq\frac{1}{|C_2|}e^{-C_2\Delta\tau} \quad (\gamma=1/3), \tag{37}$$

$$\Lambda(\tau) \simeq \frac{3(2-3\gamma)^2}{4(1-3\gamma)^2 \Delta\tau^2} \quad (\gamma \neq 1/3), \qquad (38)$$

$$\Lambda \simeq 3C_2^2 \quad (\gamma = 1/3), \qquad (39)$$

$$q_r(r,\tau) \simeq \frac{8KC_2}{3\gamma-2} r \left[\frac{2C_2(1-3\gamma)}{2-3\gamma} \Delta\tau \right]^{9\gamma/2(1-3\gamma)} \quad (\gamma \neq 1/3), \qquad (40)$$

$$q_r(r,\tau) \simeq -8KC_2^2 r e^{3C_2\Delta\tau} \quad (\gamma = 1/3). \qquad (41)$$

When $1/3 < \gamma < 2/3$ we have, for large cosmological time τ, an accelerating universe that homogenizes with vanishing cosmological term and heat flow. In this stage we may define an asymptotic adiabatic index by equating the exponent in Eq. (36) to $2/(3\gamma_{asy})$, i.e., $\gamma = 4/3 \ (4 - 3\gamma_{asy})$. In this range one has $1 < \gamma_{asy} < \infty$ leading to a final power-law expansion era. For $\gamma = 1/3$ the map between t and τ changes and we have an asymptotically de Sitter universe with finite limit cosmological term. For the remaining values of γ the universe begins at a homogeneous singularity with a divergent cosmological term. When $\gamma < 1/3$, the heat flux asymptotically vanishes near the singularity, while for $\gamma > 2/3$ it diverges.

Case $\Delta t \to \infty$. In this limit we obtain

$$a \simeq -\frac{1}{3} K \Delta t^{6(\gamma-1)/(3\gamma-2)}, \qquad (42)$$

$$\Delta\tau \simeq \frac{3}{K} \frac{2-3\gamma}{4-3\gamma} \Delta t^{(-4+3\gamma)/(2-3\gamma)}, \qquad (43)$$

$$R(\tau) \simeq -\frac{3}{K} \left[\frac{K(4-3\gamma)}{3(2-3\gamma)} \Delta\tau \right]^{6(1-\gamma)/(4-3\gamma)}, \qquad (44)$$

$$\Lambda(\tau) \simeq -\frac{24(2-3\gamma)^2}{(4-3\gamma)^2 \Delta\tau^2}, \qquad (45)$$

$$q_r(r,\tau) \simeq -24 \frac{(2-3\gamma)^2}{(4-3\gamma)^3} \frac{r}{\Delta\tau^3}, \qquad (46)$$

for $\gamma \neq 2/3$, and

$$a \simeq -K \Delta t^2 e^{C\Delta t}, \qquad (47)$$

$$\Delta\tau \simeq e^{-C\Delta t}, \qquad (48)$$

$$R(\tau) \simeq -C\Delta\tau, \qquad (49)$$

$$\Lambda(\tau) \simeq \frac{(3C_2^2 + 12KC_1)}{\Delta\tau^2}, \qquad (50)$$

$$q_r(r,\tau) \simeq -\frac{4K^3 r}{C^3 \Delta\tau^3} (\ln\Delta\tau)^4, \qquad (51)$$

for $\gamma = 2/3$.

In this case the asymptotic adiabatic index is given by $\gamma = (9\gamma_{asy}-4)/(9\gamma_{asy}-3)$ and we have two alternatives for asymptotically expanding universes. When $2/3 \lesssim \gamma \lesssim 1$ the

universe homogenizes for large cosmological time with vanishing cosmological term and heat flow. We note that even though an asymptotic negative cosmological term occurs, the universe ends in a power-law evolution $R \propto \Delta\tau^\alpha$ with $0 < \alpha < 1$. When $\gamma = 1$, the late time evolution changes to an asymptotically Minkowski stage. For $1 < \gamma < 4/3$ the universe starts homogeneously in the remote past with a vanishing scale factor, cosmological term and heat flow. For the remaining values of γ the universe begins at a homogeneous singularity with a divergent cosmological term.

An exact solution with explicit dependence on the asymptotic cosmological time τ can be found when the integration constants C_1 and C_2 vanish. In such a case each approximate expression (42)–(46) becomes an equality and the metric reduces to

$$ds^2 = \frac{1}{(1 + m\Delta\tau^{2(2-3\gamma)/(4-3\gamma)} r^2)^2} \times [-d\tau^2 + \Delta\tau^{12(1-\gamma)/(4-3\gamma)}(dr^2 + r^2 d\Omega^2)], \qquad (52)$$

where m is a redefinition of the old integration constant K, the adiabatic index γ and r_0. The last constant was introduced by scaling the radial coordinate $r \to r_0 r$.

Proposed varieties of soft matter with $\gamma < 1$ include cosmic string networks [21], ''K matter'' [22], and quantum zero-point field [23], all with $\gamma = 2/3$, as well as ''QCDM'' (for ''unknown cold dark matter'') with $\simeq 0.4$ [1], and the quintessence (or Q component) with $0 < \gamma < 1$ [2]. Fluids with values of γ less than $2/3$ may be termed ''inflationary matter.'' Equation (36) shows an evolution that corresponds to this kind of matter.

These results illustrate how homogenization of a universe dominated by matter that has negative pressure in the present era may have occurred. A smooth unclustered dark matter component with negative presure could reconcile a flat, or nearly flat, universe with a density in clustered matter well below the critical value, and moreover explain the recent high redshift supernovae data suggesting that the universe is currently under an accelerated expansion [24,25]. For a perfect fluid negative presure leads to instabilities that are most severe on the shortest scales. However, if instead the dark matter is a solid, with an elastic resistance to pure shear deformations, an equation of state with negative presure can avoid these short wavelength instabilities. Such a solid may arise as the result of different kinds of microphysics. Two possible candidates for a solid dark matter component are a frustrated network of non-Abelian cosmic strings or a frustrated network of domain walls. If these networks settle down to an equilibrium configuration that gets carried along and stretched by the Hubble flow, equations of state result with $\gamma = 1/3$ and $\gamma = 2/3$, respectively. One expects the sound speeds for the solid dark matter component to comprise an appreciable fraction of the light speed. Therefore, the solid dark matter does not cluster, except on the very largest scales, accessible only through observing the cosmic background radiation (CBR) anisotropy at large angles [4].

IV. ASYMPTOTIC EVOLUTION
TO A QUINTESSENCE-DOMINATED ERA

As a first stage of towards more general scenarios with a slowly time varying γ, we will explore a model that evolves towards an asymptotic FLRW regime dominated by Q matter (i.e., the scalar field). We will show that this system approaches to the constant γ solutions for large times found above. In this regime Eqs. (6) and (10) become

$$3H^2 \simeq \rho_f + \frac{1}{2}\dot{\phi}^2 + V(\phi) + \Lambda, \tag{53}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} \simeq 0, \tag{54}$$

where $H = \dot{R}/R$ and a dot means $d/d\tau$ in this section. From these equations and Eq. (3) it follows that

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\gamma_f\rho_f + \frac{\dot{\Lambda}}{6H} \tag{55}$$

and

$$\gamma_\phi = \frac{\dot{\phi}^2}{(1/2)\dot{\phi}^2 + V(\phi)}. \tag{56}$$

In the last section we found that the general asymptotic solution for the scale factor $R(\tau) \propto \Delta\tau^\alpha$ has the power-law behaviors (36), (44) for any value of the effective adiabatic index γ. Then, using these expressions and Eqs. (38) and (45) together with Eqs. (53) and (55) in Eq. (56), the latter becomes

$$\gamma_\phi = \frac{2}{3\alpha} \frac{1 - 3\alpha\gamma_f\rho_f\Delta\tau^2/2(3\alpha^2 - \beta)}{1 - \rho_f\Delta\tau^2/(3\alpha^2 - \beta)}, \tag{57}$$

where

$$\alpha = \frac{3\gamma}{2(3\gamma - 1)}, \quad \beta = \frac{3(2 - 3\gamma)^2}{4(1 - 3\gamma)^2}, \quad \left(\frac{1}{3} < \gamma < \frac{2}{3}\right), \tag{58}$$

$$\alpha = \frac{6(1 - \gamma)}{4 - 3\gamma}, \quad \beta = -\frac{24(2 - 3\gamma)^2}{(4 - 3\gamma)^2}, \quad (\tfrac{2}{3} < \gamma < 1). \tag{59}$$

To investigate the asymptotic limit in which the energy of the scalar field dominates over the contribution of the perfect fluid we assume that $3\alpha\gamma_f > 2$. In this regime the two terms in Eq. (57) proportional to the energy density ρ_f are positive and neglegible. The adiabatic scalar field index can be approximated by

$$\gamma_\phi \simeq \frac{2}{3\alpha}\left[1 + \left(1 - \frac{3\gamma_f}{2}\sigma\right)\right], \tag{60}$$

where $\sigma = \rho_f/\rho_\phi \ll 1$. Inserting these equations in Eq. (5) we obtain the first correction to the effective adiabatic index for the solutions (58)

$$\gamma \simeq \frac{2}{3}[1 \pm \sqrt{(3\gamma_f - 2)\sigma}] \tag{61}$$

and $\gamma \simeq 2/3 + \gamma_f\sigma/2$ for the solution (59). The negative branch of Eq. (61) yields a consistent asymptotic solution for the range $\frac{1}{3} < \gamma < \frac{2}{3}$. We note that this solution describes a deflationary stage with a limiting exponent $\alpha = 1$.

Often, the power-law evolution of the scale factor is associated with logarithmic dependence of the scalar field on proper time [26]. Thus, assuming that $\phi(\tau) \simeq C \ln \tau$ with the constant C to be determined by the system of Eqs. (53) and (54), and using these expressions together with Eqs. (38) and (45) in Eqs. (53) and (54) it follows that

$$\frac{3\alpha^2}{\tau^2} \simeq \frac{C^2}{2\tau^2} + V + \frac{\beta}{\tau^2}, \tag{62}$$

$$\frac{(3\alpha - 1)C}{\tau^2} + \frac{dV}{d\phi} \simeq 0. \tag{63}$$

From Eqs. (62), (63) we obtain the leading term of $V(\phi)$ for large ϕ

$$V(\phi) \simeq V_0 e^{-A\phi} \tag{64}$$

and

$$A^2 = 3\gamma, \quad V_0 = \frac{3\gamma + 2}{3\gamma(3\gamma - 1)}, \tag{65}$$

for $\frac{1}{3} < \gamma < \frac{2}{3}$, while

$$A^2 = 3\frac{(-4 + 3\gamma)(-1 + \gamma)}{17 - 42\gamma + 27\gamma^2},$$

$$V_0 = \frac{2(-14 + 15\gamma)(17 - 42\gamma + 27\gamma^2)}{3(-1 + \gamma)(-4 + 3\gamma)^2}, \tag{66}$$

for $\frac{2}{3} < \gamma < 1$. Inserting the dominant value of the effective adiabatic index in Eqs. (65), (66) we find $A^2 = 2$, $V_0 = 2$ and $C = 1/\sqrt{2}$.

The models considered in this section are based on the notion of ''late time dominating field'' (LTDF), a form of quintessence in which the field ϕ rolls down a potential $V(\phi)$ according to an attractor solution to the equations of motion. This solution is an attractor since for a very wide range of initial values for M, ϕ and $\dot{\phi}$ it rapidly approaches a common evolutionary path, i.e., the late behavior is insensitive to the initial conditions. This model has an advantage similar to inflation in that for a wide range of initial conditions the universe is driven to the same final evolution. The ratio σ of the background fluid to the field energy changes steadily as ϕ proceeds down its path. This is desirable because in that way the Q matter ultimately dominates the energy density and drives the universe toward an accelerated expansion.

Recently Ferreira and Joyce [27] proposed a model based on an exponential potential. Their self-adjusting solutions are attractors and Ω_ϕ remains constant for a constant back-

ground equation of state as $\gamma_\phi = \gamma_f$. (Ω_ϕ changes slightly when the universe shifts from radiation-dominated to matter-dominated expansion). This means, for example, that Ω_ϕ is constant throughout the matter-dominated epoch. For a constant Ω_ϕ to satisfy the structure formation constraints requires $\Omega_\phi < 0.2$ and $\Omega_f > 0.8$. This, however, runs into conflict with the current best estimates of Ω_f and produces a decelerating universe at variance with recent supernovae observations [24,25].

By contrast, our LTDF model only requires that the potential has an asymptotic exponential shape for large ϕ. So, the interesting and significant features of our model are (a) like the self-adjusting case, a wide range of initial conditions are drawn towards a common evolution; however, (b) the LTDF solutions do not ''self-adjust'' to the background equation of state but rather, maintain some finite difference in the equation of state such that the field energy eventually dominates and the universe enters a period of acceleration. Compared to the self-adjusting model, ours does not require any additional parameter and allows a much wider range of potentials, provided they have an exponential tail. LTDF solutions exist for a very wide class of potentials. The energy density of the field decreases as R^{-2}, and this power law remains unchanged in every epoch of the universe when the background expansion turns from radiation- to matter- to quintessence-dominated. The value of γ_ϕ differs from the background equation of state such that the value of Ω_ϕ increases as the universe ages. Hence, Ω_ϕ grows to order unity late with time.

V. GENERAL SOLUTION FOR A TIME-DEPENDENT ADIABATIC INDEX

A time-varying γ is very natural because different matter components redshift at various rates. Then the question arises whether the conditions leading to homogenization we have found with constant γ also hold when γ varies with time. The rationale behind this approach is the following. When different components enter the stress-energy tensor of the cosmic medium, it is natural to expect that each epoch is dominated by the energy density of just a single component with a constant, or nearly constant, adiabatic index, say γ_{01}. The other components can be regarded as small perturbations. As the universe expands, sooner or later, the component with the adiabatic index inmediately lower than γ_{01} (say γ_{02}) takes over, and a new epoch begins.

We start by giving the general solution to Eqs. (19) and (20) when γ is a function of time

$$b = \exp\left(\int dt\, w \right), \tag{67}$$

$$a = -2 \exp\left(\int dt\, w \right) \int dt\, w^2 \int \frac{dt}{w^2}, \tag{68}$$

where

$$w = \frac{2}{\int dt(2 - 3\gamma)}, \tag{69}$$

provided $\gamma \neq 2/3$. Inserting Eqs. (68) and (67) in Eqs. (21), (15), and (16) it follows that

$$\Lambda = 4K^2 \exp\left(2\int dt\, w \right) \left[3w^4 \left(\int \frac{dt}{w^2} \right)^2 \right.$$

$$+ \frac{4w^3}{\gamma} \left(\int dt\, w^2 \int \frac{dt}{w^2} \right) \int \frac{dt}{w^2} + \frac{2w^2}{\gamma} \left(\int dt\, w^2 \int \frac{dt}{w^2} \right)^2$$

$$\left. - 6 \int dt\, w^2 \int \frac{dt}{w^2} \right], \tag{70}$$

$$q_r(r,t) = -4K^3 r w \exp\left(3\int dt\, w \right)$$

$$\times \left(-2 \int dt\, w^2 \int \frac{dt}{w^2} + r^2 \right)^2. \tag{71}$$

Though we have reduced the general coupled system of equations to quadratures, it is very involved to obtain the solution in closed form except for constant γ. This is why the next subsection focus on approximated solutions assuming that $\gamma(t)$ admits an analytic expansion.

Homogenization with a varying adiabatic index. The transition previously described from one epoch to the next, assumed to be gentle, may be modeled by a homographic function $\gamma(t) = (At + B)/(Ct + D)$ where A, B, C, and D are constants [28]. With this in mind, we next investigate two kind of behaviors for the fluid components that lead to a final homogeneous stage characterized by $M(t) \to 0$. These can be associated with two asymptotic evolutions of $\gamma(t)$.

(1) In the limit $t \to 0$ we assume that $\gamma(t)$ has a Taylor expansion

$$\gamma(t) = \gamma_{01} + \gamma_{11} t + \mathcal{O}(t^2), \quad \gamma_{01} = \frac{B}{D}, \quad \gamma_{11} = \frac{A}{D} - \frac{B}{D^2}, \tag{72}$$

with $\gamma_{01} = B/D$ and $\gamma_{11} = (AD^{-1} - BD^{-2})$. Then, for $\gamma_{01} \neq 2/3$ we find that

$$M(t) = \frac{Kt}{C_2} \left[1 - \frac{3\gamma_{11} t}{2 - 3\gamma_{01}} \ln t + \mathcal{O}(t^2) \right], \tag{73}$$

and that the first corrections to Eqs. (33) and (40) vanish as $t \ln t$, while the first correction to Eq. (38) is of higher order.

(2) In the limit $t \to \infty$ we assume for $\gamma(t)$ the expansion

$$\gamma(t) = \gamma_{02} + \frac{\gamma_{12}}{t} + \mathcal{O}(t^{-2}), \quad \gamma_{02} = A/C, \quad \gamma_{12} = \frac{B}{C} - \frac{A}{C^2}, \tag{74}$$

obtaining for $\gamma_{02} \neq 2/3$

$$M(t) = -\frac{3}{t^2} \left[1 + \frac{6\gamma_{12}}{2 - 3\gamma_{02}} \frac{\ln t}{t} + \mathcal{O}(t^{-1}) \right]. \tag{75}$$

In this case the first corrections to Eqs. (42), (45), and (46) vanish as $\ln t/t$.

These results show that homogenization occurs under the same conditions for both γ_0 as it does for constant γ for a wide range of evolutions of the adiabatic index, provided it has a constant limit, γ_{01} and γ_{02} for $t \to 0$ and $t \to \infty$, respectively, and is analytic about these points.

VI. CONCLUDING REMARKS

We have investigated a class of solutions of the Einstein field equations with a variable cosmological term, heat flow and a fluid with variable adiabatic index that includes those of Modak, Bergmann, and Maiti and contains a new exact conformally flat solution. We have also considered the contribution of a homogeneous minimally coupled scalar field to the stress-energy tensor. The solutions of the EKG system that lead to an asymptotic FLRW stage were analyzed when the adiabatic index remains constant. We have found that asymptotically expanding universes occur when $1/3 < \gamma < 1$ that homogenizes for large cosmological time with vanishing cosmological term and heat flow. For $1/3 < \gamma < 2/3$ the evolution is given by Eq. (36) and corresponds to a power-law accelerated expansion for large cosmological time τ, as follows from Eq. (34). On the other hand, when $2/3 \leq \gamma < 1$ even though an asymptotic negative cosmological term occurs, the universe evolves toward a decelerated expansion. The particular case $\gamma = 1/3$ leads asymptotically to a de Sitter universe with a finite limit for Λ. We have shown that these results also apply to a time dependent adiabatic index.

We have carried out a detailed analysis of a model in which Q matter dominates over cold dark matter. This LTDF solution is an attractor because, even for large initial inhomogeneities and a wide range of initial values for ϕ and $\dot{\phi}$, the evolution approaches a common path. It was shown that this model can be realized for a wide range of potentials provided they have an exponential tail. This is quite interesting because recently, there has been increasing activity related to scalar fields with the Liouville form (exponential) potential. It arises as an effective potential in many theories such as Jordan-Brans-Dicke theory, Salam-Sezgin theories and superstring theories. Indeed, most theories undergoing dimensional reduction to an effective four-dimensional theory, result in a linear combination of exponential potentials, and one of these will eventually dominate.

This work can be generalized in different directions. The more obvious one is to allow Λ and γ to depend on position too, but this seems analytically impracticable. A less hard possibility is to consider nonflat spatial sections. This is interesting since although the location of the first acoustic peak in the angular power spectrum of the CBR suggests a flat universe, its exact position is still uncertain [29]. Therefore it may well happen that the universe is open or even closed. Again the difficulty with the corresponding analysis is basically mathematic since it is doubtful that in this more general case the EKG field equations admit analytical solutions. A seemingly less involved generalization is to include a bulk dissipative pressure in the stress-energy tensor (4), something rather natural [30]. Its effect should tend to further accelerate the expansion, as it lowers the total pressure of the cosmic fluid. The problem with this is the lack of fully realiable expressions for the coefficient of bulk viscosity of the fluids involved in the hydrodynamic description. However, in this regard the situation is no worse than that encountered in inflationary scenarios in which potentials for the scalar field are frequently proposed with no obvious physical ground.

[1] M.S. Turner and M. White, Phys. Rev. D **56**, R4439 (1997).

[2] R.R. Caldwell, R. Dave, and P.J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1995).

[3] I. Zlatev, L. Wang, and P.J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999).

[4] M. Bucher and D. Spergel, Phys. Rev. D **60**, 043505 (1999).

[5] P.J.E. Peebles and A. Vilenkin, Phys. Rev. D **59**, 063505 (1999).

[6] M.S. Turner, ''Dark matter and energy in the Universe,'' astro-ph/9901109.

[7] N. Mustapha, C. Hellaby, and G.F.R. Ellis, Mon. Not. R. Astron. Soc. **292**, 817 (1997).

[8] Reports of J.D. Barrow, G.F.R. Ellis, and M.A.H. MacCallum, in *The Renaissance of General Relativity*, edited by G.F.R. Ellis, A. Lanza, and J. Miller (Cambridge University Press, Cambridge, England, 1993).

[9] A. Krasiński, *Inhomogeneous Cosmological Models* (Cambridge University Press, Cambridge, England, 1997).

[10] R. Maartens, M. Govender, and S.D. Maharaj, Gen. Relativ. Gravit. **31**, 815 (1999).

[11] N. Dadhich, J. Astrophys. Astron. **18**, 343 (1997).

[12] G. Efstathiou, in *Physics of the Early Universe*, Proceedings of the Thirty Sixth Scottish Universities Summer School in Physics, edited by J.A. Peacock, A.F. Heavens, and A.T. Davies (IOP, Bristol, 1990); T. Padmanabhan, *Formation of Structures in the Universe* (Cambridge University Press, Cambridge, England 1995); P.K.S. Dunsby, Class. Quantum Grav. **8**, 1785 (1991).

[13] A. Aragoneses, D. Pavón, and W. Zimdahl, Gen. Relativ. Gravit. **30**, 299 (1997).

[14] C. Baccigalupi and F. Perrotta, ''Perturbations in Qintessential inflation,'' astro-ph/9811385.

[15] H. Nariai, Prog. Theor. Phys. **40**, 1013 (1968).

[16] W. Zimdahl and D. Pavón, Mon. Not. R. Astron. Soc. **266**, 872 (1994).

[17] W. Zimdahl, D. Pavón, and R. Maartens, Phys. Rev. D **55**, 4681 (1997).

[18] B. Modak, J. Astrophys. Astron. **5**, 317 (1984).

[19] O. Bergmann, Phys. Lett. **82A**, 383 (1981).

[20] S.R. Maiti, Phys. Rev. D **25**, 2518 (1982).

[21] J.R. Gott III and M.J. Rees, Mon. Not. R. Astron. Soc. **227**, 453 (1987).

[22] E.W. Kolb, Astrophys. J. **344**, 543 (1989).

[23] P.S. Wesson, Phys. Essays **5**, 561 (1998).

[24] S. Perlmutter *et al.*, Nature (London) **391**, 51 (1998).

[25] A.G. Riess *et al.*, Astron. J. **116**, 1009 (1998).

[26] L.P. Chimento, Class. Quantum Grav. **15**, 965 (1998).

[27] P. G. Ferreira and M. Joyce, Phys. Rev. Lett. **79**, 4740 (1997); see also C. Wetterich, Nucl. Phys. **B302**, 668 (1988); and E. J. Copeland, A.R. Liddle, and D. Wands, Phys. Rev. D **57**, 4686 (1998).

[28] V. Méndez and D. Pavón, Mon. Not. R. Astron. Soc. **282**, 753 (1996).

[29] J. Silk, ''Seven paradigms in structure formation,'' astro-ph/9903402; K. Coble *et al.*, ''Anisotropy in the cosmic microwave background at degree angular scales: Python V results,'' astro-ph/9902195.

[30] D. Pavón and W. Zimdahl, Phys. Lett. A **179**, 261 (1993).