Pair production of light pseudoscalar particles in strong inhomogeneous fields by the Schwinger mechanism

J. A. Grifols and Eduard Massó
Grup de Física Teòrica and IFAE, Universitat Autònoma de Barcelona, Bellaterra, Barcelona, Spain

Subhendra Mohanty and K. V. Shajesh
Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India

We calculate the rate for pair production of light pseudoscalars by strong inhomogeneous and static electric (magnetic) fields. We show that, in the case of axions, the stability of atoms over cosmic lifetimes is jeopardized unless the Peccei-Quinn symmetry breaking scale f_a is larger than $O(10^{10}$ GeV). [S0556-2821(99)06219-0]

The Schwinger mechanism is a non-perturbative process by which an infinite number of zero frequency photons can produce an electron-positron pair. For example a constant electric field larger than $O(10^{11}$ V/cm) will decay into electron-positron pairs, according to the Schwinger pair production probability formula [1]. In this paper we consider the pseudoscalar–two-photon coupling. We find that the pseudoscalar pair production probability is given by an expression $w \sim g^2 \exp\{-[O(m^2/g)]\}$ in terms of the pseudoscalar–two-photon coupling g and the pseudoscalar mass m. This allows us to constrain the parameter space of $\{m,g\}$ for any pseudoscalar model, based on the stability of electromagnetic fields. In axion models [2,3] the effective axion–two-photon coupling arises from axion couplings to charged fermions in a one loop diagram. This coupling is inversely proportional to the Peccei-Quinn symmetry breaking scale f_a, and is independent of the mass of the fermion in the loop. The axion mass is inversely proportional to the square of f_a. The pair production formula therefore can be expressed in terms of the single parameter f_a.

The generic electromagnetic interaction of a pseudoscalar ϕ can be written as (see Fig. 1)

$$\mathcal{L}_I = \frac{1}{4} g \phi F_{\mu\nu} \widetilde{F}^{\mu\nu} \qquad (1)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\widetilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$.

We need to evaluate the loop diagram of the type shown in Fig. 2 with infinite number of zero-frequency photon external legs. The imaginary part of this diagram gives the probability for the decay of the external electromagnetic field.

We evaluate this diagram in two stages. First we find the effective vertex for the two axion–two photon vertex which arises when we contract the photon legs between two vertices like Eq. (1). This two axion–two photon vertex is of the form

$$4\left(\frac{1}{2}g\phi\right)^2 k_\mu \widetilde{F}^{\mu\nu} \frac{-ig_{\nu\nu'}}{k^2}(-k_{\mu'})\widetilde{F}^{\mu'\nu'}. \qquad (2)$$

The factor of 4 in Eq. (2) is for the four possible ways of joining the photon legs. Due to the presence of the k^2 term in the denominator, Eq. (2) is non-local. However, momentum k is integrated over when we calculate the effective action for the external electromagnetic field. One can therefore make use of the identity

$$\int d^4k f(k_\mu k_{\mu'}) = \int d^4k f\left(\frac{g_{\mu\mu'}}{4}k^2\right) \qquad (3)$$

to simplify Eq. (2) and obtain

$$\mathcal{L}'_I = -\frac{1}{4}g^2\phi^2 F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}g^2\phi^2(\mathbf{E}^2 - \mathbf{B}^2). \qquad (4)$$

When the momentum k is integrated over, the effective two axion–two photon interaction reduces to a local interaction vertex (Fig. 3) given by Eq. (4).

Next, to calculate the diagram with infinite insertions of the two-photon vertices (4) in an axion loop we use the Schwinger method. Starting with the total action for the axion-photon system we integrate out the axion fields to determine the effective action for just the electromagnetic fields

$$e^{iS_{eff}[\mathbf{E},\mathbf{B}]} = \int \mathcal{D}\phi\, e^{i\int d^4x\{(1/2)\phi(-\partial^2 - m^2)\phi + (1/2)g^2\phi^2(\mathbf{E}^2 - \mathbf{B}^2)\}}.$$

To proceed we shall evaluate $\mathcal{L}_{eff}$ using a technique developed in [4]. It amounts to expanding the classical background fields about a reference point, to compute the (interacting) Green's function in momentum space, to revert it to configuration space, and finally obtaining $\mathcal{L}_{eff}$ by a parametric integration. We shall be interested in stationary, inhomo-
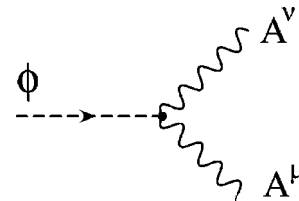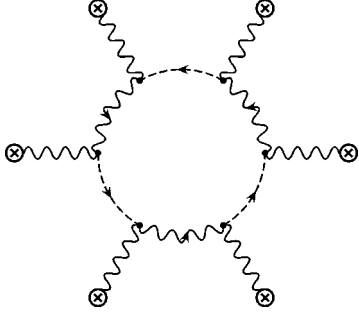

FIG. 1. Axion-two-photon interaction.

FIG. 2. The effective photon-photon interaction diagram on integrating out the axion and a photon field.

geneous (on the scale of the pseudoscalar Compton wavelength) external fields (electric and/or magnetic fields). The Green's function of the axion field in a background electromagnetic field is given by

$$[\partial_x^2 + m^2 - \alpha - \beta_i(x-x')^i - \gamma_{ij}^2(x-x')^i(x-x')^j]G(x,x')$$
$$= \delta^4(x-x') \tag{5}$$

$(i,j=1,2,3)$ where, as advertised, we have expanded the electromagnetic fields to second order about the reference point x'. In momentum space this equation reads

$$\left[-p_0^2 + p_i^2 + m^2 - \alpha + i\beta_i \frac{\partial}{\partial p_i} + \gamma_{ij}^2 \frac{\partial^2}{\partial p_i \partial p_j}\right] G(p) = 1. \tag{6}$$

This equation is solved by making the ansatz

$$G(p) = i \int_0^\infty ds\, e^{is(p_0^2 - m^2)} e^{-i\mathbf{p}\cdot\mathbf{A}\cdot\mathbf{p} + \mathbf{B}\cdot\mathbf{p} + C} \tag{7}$$

and solving for matrix $\mathbf{A} \equiv A^{ij}$, vector $\mathbf{B} \equiv B^i$, and C. The solution is given by

$$\mathbf{A} = \frac{1}{2}\boldsymbol{\gamma}^{-1} \cdot \tanh 2\boldsymbol{\gamma}s$$

$$\mathbf{B} = -\frac{i}{2}\boldsymbol{\gamma}^{-2} \cdot (1 - \operatorname{sech} 2\boldsymbol{\gamma}s) \cdot \boldsymbol{\beta} \tag{8}$$

$$C = i\alpha s - \frac{1}{2}\operatorname{tr}\ln\cosh 2\boldsymbol{\gamma}s$$

$$+ \frac{i}{8}\boldsymbol{\beta} \cdot \boldsymbol{\gamma}^{-3} \cdot (\tanh 2\boldsymbol{\gamma}s - 2\boldsymbol{\gamma}s) \cdot \boldsymbol{\beta}$$
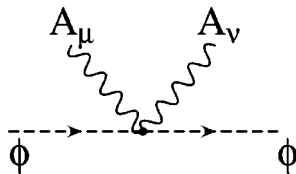
where tr is a trace over indices i,j.

FIG. 3. The effective two axion–two photon interaction in the loop.

Since

$$\frac{\partial \mathcal{L}_{eff}}{\partial \alpha} = \frac{1}{2i}G(x,x) = \frac{1}{2i}\int \frac{d^4p}{(2\pi)^4}G(p) \tag{9}$$

subject to the boundary condition $\mathcal{L}_{eff}=0$ when $\mathbf{E}=\mathbf{B}=0$, we can find $\mathcal{L}_{eff}$ by Gaussian integration to obtain

$$\mathcal{L}_{eff} = -\frac{1}{32\pi^2}\int_0^\infty \frac{ds}{s^3}$$

$$\times \left[e^{-is(m^2-\alpha)}\left(\det\frac{2\boldsymbol{\gamma}s}{\sinh 2\boldsymbol{\gamma}s}\right)^{1/2} e^{il(s)} - e^{-ism^2} \right] \tag{10}$$

where

$$l(s) = \frac{1}{4}\boldsymbol{\beta}\cdot\boldsymbol{\gamma}^{-3}\cdot(\tanh\boldsymbol{\gamma}s - \boldsymbol{\gamma}s)\cdot\boldsymbol{\beta}.$$

We can now apply this formula to the special case of spherically symmetric electric fields, like the ones in atoms. In this case $g^2 E^2$ appearing in the interaction Lagrangian is written as

$$g^2 E^2 = g^2 E_0^2 + g^2 E_0^{2\prime}(r-r_0) + \frac{1}{2}g^2 E_0^{2\prime\prime}(r-r_0)^2, \tag{11}$$

when expanded about r_0. Clearly, $\alpha = g^2 E_0^2$. Also, vector β^i above reads now $\boldsymbol{\beta} = (g^2 E_0^{2\prime}/r_0)\mathbf{r}_0$ and matrix $\boldsymbol{\gamma}^2$ is, after diagonalization, $\boldsymbol{\gamma}^2 = \operatorname{diag}(\gamma_1^2, \gamma_2^2, \gamma_2^2)$ where $\gamma_1^2 = \frac{1}{2}g^2 E_0^{2\prime\prime}$ and $\gamma_2^2 = g^2 E_0^{2\prime}/2r_0$. With this input the function $l(s)$ given in Eq. (12) is explicitly

$$l(s) = r_0^2\gamma_2^4\gamma_1^{-3}(\tanh\gamma_1 s - \gamma_1 s). \tag{12}$$

So, finally,

$$\mathcal{L}_{eff} = -\frac{1}{32\pi^2}\int_0^\infty \frac{ds}{s^3}\left[e^{-is(m^2-\alpha)}\frac{2\gamma_2 s}{\sinh 2\gamma_2 s} \right.$$

$$\left. \times \sqrt{\frac{2\gamma_1 s}{\sinh 2\gamma_1 s}} e^{il(s)} - e^{-ism^2} \right] \tag{13}$$

Now, when we specialize to the field created by an atomic nucleus $E^2 = Z^2\alpha/4\pi r^4$, we have $\gamma_2^2 = -g^2 Z^2\alpha/2\pi r_0^6$ and $\gamma_1^2 = 5g^2 Z^2\alpha/2\pi r_0^6$. The probability w per unit volume and unit time for creation of pseudoscalar pairs is $2\operatorname{Im}\mathcal{L}_{eff}$ and can be obtained from the previous equation by complex integration. We obtain

$$w = \frac{\tilde{\gamma}_2\gamma_1}{4\pi^2}\sum_{n=0}^\infty (-1)^n C_n e^{-\kappa(2n+1)\pi}. \tag{14}$$

Here, $\tilde{\gamma}_2 \equiv +(-\gamma_2^2)^{1/2}$, and $\kappa \equiv \frac{1}{2}(m^2/\gamma_1 - 3\lambda/2)$, with $\lambda \equiv \tilde{\gamma}_2^4\gamma_1^{-3}r_0^2$. The coefficients C_n in the asymptotic series above are given by

$$C_n \equiv \int_0^\pi du \frac{e^{-[\kappa u + \lambda \cot(u/2)]}}{[u+(2n+1)\pi]^{3/2}(\sin u)^{1/2}\sinh(\tilde{\gamma}_2/\gamma_1)[u+(2n+1)\pi]}. \tag{15}$$

For the case of axions, the mass and coupling are related to the global symmetry breaking scale f_a [2,3]

$$g \simeq 0.75 \frac{\alpha}{2\pi f_a}$$

$$m_a \simeq 0.6 \text{ eV} \frac{10^7 \text{ GeV}}{f_a} \tag{16}$$

and one can estimate the magnitude of the pair-creation phenomenon in terms of the single parameter f_a. The asymptotic expansion in the equation for w only makes sense for $\kappa \gtrsim 1$ and w is vanishingly small for $\kappa \gg 1$, i.e. for second derivatives of the fields small on the scale of m_a. Only when fields approach critical values for which $\kappa \sim O(1)$, pair creation could eventually reach catastrophic rates and produce vacuum breakdown and disruption of the external field. Pair creation, which is driven by the second derivative of the field, becomes critical (i.e. $\kappa = 1$) in a Z-atom for $r_{crit} = (Zf_a[\text{GeV}]/34)^{1/3}$ fm. Since r_{crit} must be larger than a few fm, our arguments apply only for

$$f_a > O(10^2 \text{ GeV}). \tag{17}$$

Let us now constrain f_a by realizing that the observed cosmic abundance of atomic helium agrees with nucleosynthesis up to a few percent and therefore we can use the stability over the lifetime of the Universe of He-atoms. We consider a thin shell of thickness $\delta r_0 = 0.1 r_0$ at critical radius r_{crit} inside the He-atom. The atom will become ionized due to breakdown of the electric field in this shell at some time τ when the product $Vw\tau \sim 1$ where V is the volume of the shell, and w is as given in Eq. (14).

Imposing $\tau > 1.5 \times 10^{10}$ years, and taking into account Eq. (17), we get the following exclusion range for f_a:

$$10^2 \text{ GeV} < f_a < 0.6 \times 10^{10} \text{ GeV}. \tag{18}$$

We note that, for the whole of the exclusion region, the critical radius $r_{crit} \ll 1$ Å, i.e. inside the atom, and the Compton wavelength of the axion exceeds 10^3 fm, much larger than the distances over which the field is inhomogeneous.

We can improve the constraint on f_a by recognizing that there are many heavy atoms which have remained un-ionized over the lifetime of the universe. We know from astronomical observations that there are many stars with lifetimes larger than $\tau \sim 10^{10}$ years. This is determined by comparing the abundance of the heavy element ratio (Th/Eu) in the stars with that in the solar system. Using the fact that the lifetime of thorium is 14×10^9 years it has been estimated that there are stars which have ages in excess of 12×10^9 years [5]. Since these atomic abundances in the stars are measured from the intensity of their spectral lines we know that they have not been ionized due to axion-pair production over times larger than $\tau \sim 10^{10}$ years. However, because r_{crit} grows with Z, we cannot use too large a Z for then r_{crit} would penetrate the atomic electron cloud and screening would distort the field. We estimate that we can go up to Z ~ 20 and then the bound can be pushed up only to somewhat above 10^{10} GeV. Anyway, we are not aiming at a very elaborate limit here [6]. We just wish to point out that to avoid the breakdown of electric fields in atoms by catastrophic axion pair-creation requires f_a to lie about or beyond the 10^{10} GeV scale, which is in the ballpark or already closing the presently still allowed axion-photon coupling window.

[1] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[2] J. E. Kim, Phys. Rep. **150**, 1 (1987); astro-ph/9812257.
[3] A.P. Zhitnitskii, Yad. Fiz. **31**, 497 (1980) [Sov. J. Nucl. Phys. **31**, 260 (1980)]; M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).
[4] M.R. Brown and M.J. Duff, Phys. Rev. D **11**, 2124 (1975).
[5] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, NJ, 1993).
[6] Further work along these and related lines is in progress. J. A. Grifols, E. Massó, and S. Mohanty (in preparation).