Solar neutrinos and the violation of the equivalence principle

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(Received 28 October 1998; published 7 October 1999)

In this Brief Report, a non-standard solution to the solar neutrino problem is reexamined. This solution assumes that neutrino flavors could have different couplings to gravity; hence, the equivalence principle is violated in this mechanism. The gravity induced mixing has the potential of accounting for the current solar neutrino data from several experiments even for massless neutrinos. We fit this solution to the total rate of neutrino events in the SuperKamiokande detector (first 504 days) together with the total rate from other detectors and also with the SuperKamiokande results for the recoil-electron spectrum.
[S0556-2821(99)05019-5]

PACS number(s): 13.15.+g, 04.80.Cc, 14.60.Lm, 14.60.Pq

I. INTRODUCTION

At present several neutrino detectors indicate that there is a major discrepancy between the standard solar model (SSM) predictions for the solar neutrino flux [1] and its observed value. Recently, five neutrino experiments (GALLEX [2], SAGE [3], Homestake [4], Kamiokande [5] and SuperKamiokande [6]) reported that the observed neutrino flux is less than the values predicted by the SSM for different neutrino energies. The first three experiments detect the solar electron neutrinos, ν_e, via absorption processes, so they do not give any information about the direction and energy spectrum of the observed neutrino events. The last two water Cherenkov detectors employ ν_e-e scattering, so the direction and energy distribution of the detected neutrino can be fairly determined. This would allow us to test theoretical models not only against the measured total rate but also against the energy spectrum of the neutrino flux. The analysis of the first 504 days data from SuperKamiokande [7] (SK), when combined with the data from earlier experiments, provide us with important constraints on the famous Mikheyev-Smirnov-Wolfenstein (MSW) effect [8] and vacuum oscillation solutions of the solar neutrino problem. The MSW effect involves the standard neutrino interactions with the matter background in the Sun, leading to an enhanced ν_e-ν_x resonant transition. The ''just so'' vacuum oscillation solution explains the suppression using only vacuum oscillations of the neutrinos on their way to Earth. This solution is in fact in slightly better agreement with the recent data from SK than the matter-induced MSW effect [9]. However, it requires some ''fine-tuning'' of the Sun-Earth distance to account for the apparent deficit. Both of these solutions require that neutrinos have non-degenerate masses; they will be referred to as the mass-induced (MI) scenario.

In this Brief Report, we explore yet another possible solution of the solar neutrino problem. However, it requires a rather unorthodox assumption, that the neutrino flavors couple differently to gravity. This is a statement of the violation of the weak equivalence principle which stipulates that all matter couples equally to gravity. This solution has been suggested by different authors [10,11]; here, we consider the violation of equivalence principle (VEP) scenario in the light of the first 504 days results of the total rate and recoil-electron spectral shape from the SK detector. It should be noted that the pure VEP mechanism does not require the introduction of neutrino masses and thus can be consistent with a zero or degenerate mass scheme for light neutrinos (ν_e, ν_μ). We assume a two-flavor oscillation scenario in our analysis for simplicity. A complete VEP three-flavor analysis treatment was studied [12]; however, the present experimental results are not enough to constrain the parameter space for a three-flavor analysis. So the assumption made here is that the VEP mechanism, if it were to be the cause of the solar neutrino problem, is due to only two neutrino flavors. A mixed VEP+MI scenario where neutrinos are assumed to be massive was studied by the authors of [13] assuming that the MI effect is the main cause of the neutrino flux deficit. We do not cover this possibility here.

In addition to the VEP scenario considered here, there are other scenarios which are related to the violation of equivalence principle. The first involves the neutrino coupling to the massless dilaton field which arises from string theory [14,15]. This scenario allows for a vacuum oscillation solution of the solar neutrino problem, but there is no clear distinction to be made in the present neutrino experiments between it and the conventional mass mixing. The second one proposed by Glashow *et al.* [16] assumes that Lorentz invariance is violated during neutrino propagation. The analysis is equivalent to the VEP scenario with a redefinition of parameters. In this scenario, different neutrino flavors are allowed to have different velocities under the assumption that the gravitational potential $\Phi(r)$ does not change appreciably over the distance of interest.

The article is organized as follows. Section II presents a χ^2 analysis of the solar neutrino data which is used to constrain the relevant VEP parameters. Some concluding remarks are given in Sec. III.

II. NUMERICAL ANALYSIS

The violation of the equivalence principle has been tested in several famous experiments by Eötvös, Dicke *et al.*, etc. [17]. The current bounds on the extent of the violation still allow an appreciable parameter space for the mechanism to have an effect on the neutrino propagation in regions where we have large gravitational fields. As stated before, we consider here the simple case of a pure VEP mechanism where the neutrinos are assumed to be massless.

In the presence of ordinary matter, the propagation equation for *massless* neutrinos in a gravitational field can be written as[1] ($\nu_x = \nu_\mu, \nu_\tau$)

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix} = 2E\,\delta\gamma\begin{pmatrix} A & \frac{1}{2}\sin 2\theta_G \\ \frac{1}{2}\sin 2\theta_G & \cos 2\theta_G\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}, \quad (1)$$

where E is the neutrino energy and $\delta\gamma \equiv |\Phi|\delta f$, δf being the mismatch between the gravitational couplings of neutrinos. We shall use $\delta\gamma$ in our analysis instead of δf due to the ambiguity in choosing the potential $\Phi(r)$ [11].[2] $A\delta\gamma \equiv \sqrt{2}G_F N_e/2E$ represents the charged weak interaction term between the neutrinos and the electrons in matter with a number density N_e. In matter, the survival probability is given by

$$P(\nu_e \to \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P_c\right)\cos 2\theta_G \cos 2\theta_{Gm} \quad (2)$$

where the matter mixing angle θ_{Gm} is defined as $\tan 2\theta_{Gm} = \sin 2\theta_G/(\cos 2\theta_G - A)$ and P_c is the level-crossing probability given by its standard form for an exponentially varying density (see last reference of [8]). The survival probability is plotted versus $E\delta\gamma$ for different values of $\sin^2 2\theta_G$ in Fig. 1.

In the following discussion, we present a χ^2 analysis of the first 504 days solar neutrino data in the light of the VEP mechanism. In Sec. II A, a χ^2 analysis on the total rate of the neutrino flux is first performed to obtain the constraints on the VEP parameters ($\delta\gamma$, $\sin^2 2\theta_G$). The following section, Sec. II B, deals with the analysis of the recoil-electron spectrum from the SK detector. Both analyses are then combined to constrain the relevant VEP parameters.

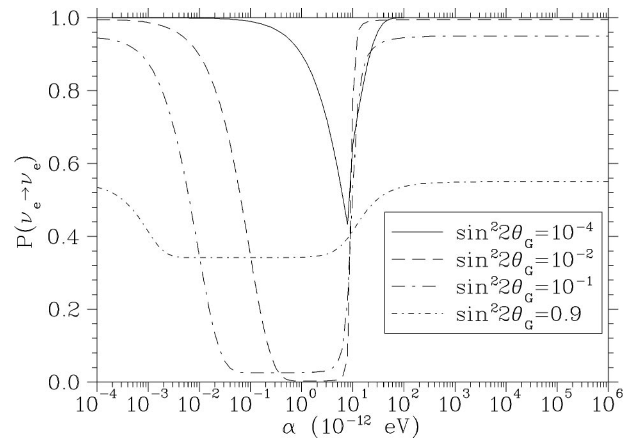

FIG. 1. The survival probability $P(\nu_e \to \nu_e)$ in the VEP scenario. It is plotted against the product of the neutrino energy E and the measure of the equivalence principle violation $\delta\gamma$ ($\alpha = E\delta\gamma$ in units of 10^{-12} eV).

A. Total rates

The experimental results of the total rates for the four neutrino experiments used in this article are shown in Table I. We use the predictions of the Bahcall-Pinsonneault standard solar model of Ref. [18] (BP98) assuming the Institute of Nuclear Theory (INT) estimate [19] for the ^{8}B production cross section. Our procedure takes into account the uncertainties in the theoretical estimates of the neutrino flux. The χ^2 function is given by [9]

$$\chi^2_{\rm rates} = \sum_{i,j=1,\ldots,4} (R_i^{\rm th} - R_i^{\rm expt}) V_{ij}^{-1} (R_j^{\rm th} - R_j^{\rm expt}), \quad (3)$$

where $R_i^{\rm th(expt)}$ represents the VEP theoretical predictions (experimental values) of the neutrino detection rate divided by the SSM predictions for the ith experiment. V_{ij} is the error matrix which contains the theoretical uncertainties as

TABLE I. The current experimental results and SSM predictions for the total rate in four neutrino detection experiments. The rate is measured in SNU for all of the experiments except the SuperKamiokande detector. The SSM predicted rates are based on the Bahcall-Pinsonneault standard solar model (BP98) of Ref. [18]. The theoretical errors are at the 1σ level.

Experiment	Experimental rate	SSM predicted rate	Threshold energy
Homestake (Cl) [4]	2.55 ± 0.25	$7.7^{+1.2}_{-1.0}$	0.81 MeV
GALLEX (Ga) [2]	$77.5 \pm 6.2^{+4.3}_{-4.7}$	129^{+8}_{-6}	0.24 MeV
SAGE (Ga) [3]	$66.6^{+7.8}_{-8.1}$	129^{+8}_{-6}	0.24 MeV
SuperKamiokande [a] [6]	$2.44 \pm +0.05^{+0.09}_{-0.07}$	$5.15^{+1.0}_{-0.7}$	6.5 MeV

[a]The SuperKamiokande result is given in terms of the measured ^{8}B flux in 10^6 cm^{-2} s^{-1}.

[1]See Refs. [10,11] for a detailed discussion on the theory of the VEP mechanism.

[2]Many authors suggest that the correct choice of the potential lies in choosing the potential of the Great Attractor, $\Phi \sim 10^{-5}$, not the solar potential, $\Phi \sim 10^{-6}$. We will assume the former potential to be the dominant in our analysis, especially in that it has no considerable variation over the relevant distance scale.

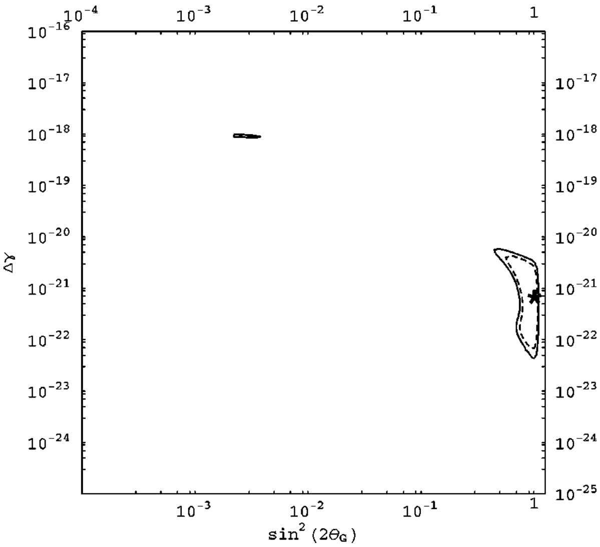

FIG. 2. Contour plot of the allowed regions in the $(\delta\gamma,\sin^2 2\theta)$ plane. These are found from the results of the total rate for the four neutrino experiments (HomeStake, GALLEX, SAGE and SuperKamiokande). The GALLEX and SAGE results have been combined in the analysis. The solid curve represents the 95% C.L. region while the dashed one indicates the 90% C.L. region. The star shows the position of the best fit solution at χ^2_{min}.

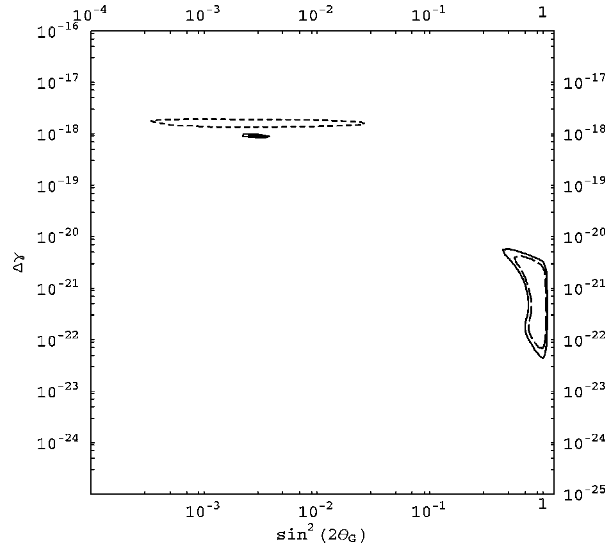

FIG. 4. Contour plot of the exclusion region in the $(\delta\gamma,\sin^2 2\theta)$ plane. The dotted curve shown is an isocontour at $\chi^2=25.0$ for 13 degrees of freedom with $\chi^2_{min}=14$. This has been based on the recent results of the recoil-electron spectral shape from the 504 day SK data run. The regions allowed from the rate analysis are also shown. As in Fig. 2, the dashed curve represents the 90% C.L. while the solid curve represents the 95% C.L.

well as the experimental errors for each experiment.[3] Figure 2 shows the 90% and 95% C.L. regions for the VEP parameters $\delta\gamma$ and $\sin^2 2\theta_G$ with $\chi^2_{\rm min}=0.46$. This is to be compared to our estimate of the MI best fit corresponding to the MSW small angle solution to the solar neutrino problem, where $\chi^2_{\rm min}=0.62$. Our $\chi^2_{\rm min}$ value for the MI solution is not in good agreement with the value obtained in Ref. [9] due to the simplified assumptions we took in our analysis, especially neglecting the profile of the neutrino production region. However, these simplifications do not change our conclusions since the purpose of this paper is to see whether or not the VEP solution is allowed at the same level of confidence or less compared to the MI solution. In Fig. 2, we see that there are two allowed regions: the first is at $\delta\gamma\sim 10^{-18}$ and $\sin^2 2\theta_G\sim 10^{-3}$ (small angle solution), while the second region appears at a smaller value of $\delta\gamma\sim 10^{-21}$ and maximal mixing where $\theta_G\lesssim\pi/4$ (large angle solution).

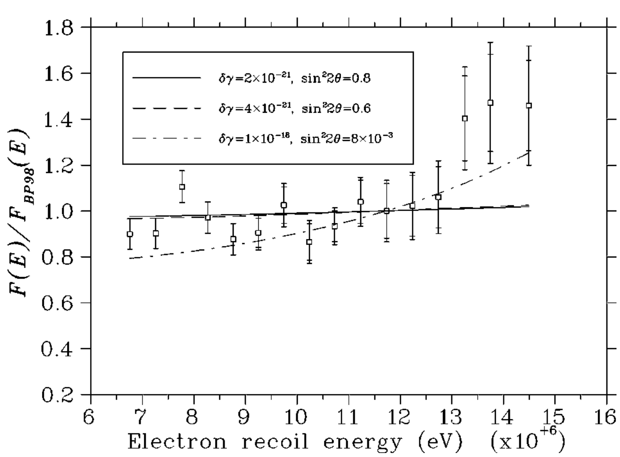

FIG. 3. The ratio of the VEP predicted recoil-electron spectrum $F(E_e)$ to the SSM spectrum $F_{SSM}(E_e)$ for different values of $\delta\gamma$ and θ_G. The ratio is normalized to 1 at $E_e=11.7$ MeV. The error bars indicate the statistical and systematic errors of the experimental data.

B. Recoil electron spectrum

We next consider the analysis of the SK recoil electron spectrum using the following χ^2:

$$\chi^2_{\rm spectra}=\sum_{i,j=1,\ldots,16}(\alpha\phi_i^{\rm th}-\phi_i^{\rm expt})V'^{-1}_{ij}(\alpha\phi_j^{\rm th}-\phi_j^{\rm expt}). \tag{4}$$

Here $\phi_i^{\rm th(expt)}$ is the ith energy bin theoretical (experimental) value for the measured flux divided by the SSM prediction. The error matrix used here is given as $V'_{ij}\equiv\sigma_i^{\rm stat}\sigma_j^{\rm stat}\delta_{ij}+\sigma_i^{\rm syst}\sigma_j^{\rm syst}$ [9]. α is the flux normalization parameter which is allowed to vary independently of $\delta\gamma$ and $\sin^2 2\theta_G$. This variation allow for the testing of measured spectrum and not the overall rate of SK. Figure 3 shows the ratio of the VEP predicted flux to the SSM predictions for the ^{8}B neutrino flux plotted against the recoil electron energy. The predicted ratio and the experimental results are normalized at[4] 11.7

[3]See Ref. [20] for a detailed discussion on the construction of the error matrix in solar neutrino analyses.

[4]The plots shown are normalized to one at this value since we are allowing the flux normalization to vary and we have to fix the ratio at a certain energy.

MeV. Figure 4 presents the spectral shape analysis exclusion region at the $\chi^2=25.0$ level ($\chi^2_{\rm min}=14$) for 13 degrees of freedom (16 energy bins minus three free parameters) together with the allowed regions from the rate analysis. We note that both the small angle solution and the large angle solution from the rate analysis are not affected.

In the spectral shape analysis, we allow the hep flux contribution to vary as suggested in Refs. [21] to see whether we will get any effect on the exclusion region or not. It is found that a hep flux enhancement of about 10^3 is needed to have any appreciable effect on the exclusion region. In any case, the exclusion region does not intersect with any of our allowed regions from the rate analysis. Thus, the spectral shape analysis based on the recent SK data yields no additional constraints on the VEP allowed parameter space.

III. DISCUSSION

We have shown in this Brief Report that the VEP scenario is still a viable solution to the solar neutrino problem. From just the rate analysis, we obtain two allowed regions in the VEP parameter space: the first is at $\delta\gamma\sim 10^{-21}$ and maximal mixing, while the second is at a smaller value of $\delta\gamma$ ($\sim 10^{-18}$) and $\sin^2 2\theta\sim 10^{-3}$. The spectral shape analysis has no effect on these allowed regions. We should also note that the combined best fit regions are still allowed by the current experimental bounds on the violation of the equivalence principle. Although the allowed parameter regions for the scenario are quite small and may seem unlikely, they are by no means excluded by the available data. Thus, to obtain the final word about either the MI or VEP explanations to the solar neutrino problem, more data are needed, especially on the solar neutrino spectral shape from the SK and SNO detectors, so that we can eventually accept or reject one of these scenarios.

ACKNOWLEDGMENTS

We are grateful to Jim Pantaleone and Terry Leung for useful discussions. S.M. is supported by a research grant from the Purdue Research Foundation. T.K. is supported by Department of Energy Grant No. DE-FG02-91ER40681B.

[1] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, England, 1989).
[2] GALLEX Collaboration, P. Anselmann *et al.*, Phys. Lett. B **342**, 440 (1995); GALLEX Collaboration, W. Hampel *et al.*, *ibid.* **388**, 384 (1996).
[3] SAGE Collaboration, V. Gavrin *et al.*, in *Neutrino 98*, Proceedings of the XVIII International Conference on Neutrino Physics and Astrophysics, Takayama, Japan, 1998, edited by Y. Suzuki and Y. Totsuka [Nucl. Phys. B (Proc. Suppl.) **77**, 20 (1999)].
[4] R. Davis, D. S. Harmer, and K. C. Hoffman, Phys. Rev. Lett. **20**, 1205 (1968); B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998); B. T. Cleveland *et al.*, Nucl. Phys. B (Proc. Suppl.) (Proc. Suppl.) **38**, 47 (1995); R. Davis, Prog. Part. Nucl. Phys. **32**, 13 (1994).
[5] Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996).
[6] SuperKamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1158 (1998).
[7] SuperKamiokande Collaboration, Y. Fukuda *et al.*, in *Neutrino 98* [3].
[8] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; for a general review on matter-induced oscillations see T. K. Kuo and J. Pantaleone, Rev. Mod. Phys. **61**, 937 (1989).
[9] J. N. Bahcall, P. I. Krastev, and A. Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998).
[10] M. Gasperini, Phys. Rev. D **38**, 2635 (1988); **39**, 3606 (1989); A. Halprin and C. N. Leung, Phys. Rev. Lett. **67**, 1833 (1991).
[11] A. Halprin, C. N. Leung, and J. Pantaleone, Phys. Rev. D **53**, 5365 (1996); J. N. Bahcall, P. I. Krastev, and C. N. Leung, *ibid.* **52**, 1770 (1995).
[12] J. R. Mureika and R. B. Mann, Phys. Rev. D **54**, 2761 (1996).
[13] H. Minakata and H. Nunokawa, Phys. Rev. D **51**, 6625 (1995).
[14] T. Damour and A. Polyakov, Gen. Relativ. Gravit. **26**, 1171 (1994); Nucl. Phys. **B423**, 532 (1994).
[15] A. Halprin and C. N. Leung, Phys. Lett. B **416**, 361 (1998).
[16] S. Coleman and S. L. Glashow, Phys. Lett. B **405**, 249 (1997); S. L. Glashow *et al.*, Phys. Rev. D **56**, 2433 (1997).
[17] E. Adelberger *et al.*, Phys. Rev. D **42**, 3267 (1990); for a review on the different tests of the equivalence principle see Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998), p. 113.
[18] J. N. Bahcall, S. Basu, and M. H. Pinsonneault, Phys. Lett. B **433**, 1 (1998).
[19] E. Adelberger *et al.*, Rev. Mod. Phys. **70**, 1265 (1998).
[20] G. L. Fogli and E. Lisi, Astropart. Phys. **3**, 185 (1995); E. Gates, L. Krauss, and M. White, Phys. Rev. D **51**, 2631 (1995).
[21] R. Escribano *et al.*, Phys. Lett. B **444**, 397 (1998); J. N. Bahcall and P. I. Krastev, *ibid.* **436**, 243 (1998).