CP violation in the semileptonic B_{l4} decays ($B^{\pm}\to\pi^{+}\pi^{-}l^{\pm}\nu$)

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(Received 29 April 1999; published 11 October 1999)

Direct CP violations in B_{l4} decays ($B^{\pm}\to\pi^{+}\pi^{-}l^{\pm}\nu_l$) are investigated within the standard model (SM) and also in its extensions. In the decay processes, we include various excited states as intermediate states decaying to the final hadrons $\pi^{+}+\pi^{-}$. The CP violation within the SM is induced by the interferences between intermediate resonances with different quark flavors. As extensions of the SM, we consider CP violations implemented through complex scalar-fermion couplings in the multi-Higgs doublet model and the scalar-leptoquark models. We calculate the CP-odd rate asymmetry and the optimal asymmetry. We find that the optimal asymmetry can be measured at the 1σ level with about 10^9 B-meson pairs in the SM case and 10^3–10^7 pairs in the extended model case, for maximally allowed values of CP-odd parameters in each case. [S0556-2821(99)06719-3]

PACS number(s): 13.20.He, 11.30.Er

I. INTRODUCTION

Semileptonic four-body decays of B mesons with emission of a single pion have been studied in detail by many authors [1–3]. Recently we investigated the possibility of probing direct CP violation in the decay $B^{\pm}\to D\pi l^{\pm}\nu$ [3] in extensions of the standard model (SM), where we extended the weak charged current by including a scalar-exchange interaction with a complex coupling, and considered as specific models the multi-Higgs doublet (MHD) model and the scalar-leptoquark (SLQ) models. In the present work, we investigate the same possibility in the decay of $B^{\pm}\to(\pi^{+}\pi^{-})l^{\pm}\nu$. In this case we find there may be direct CP violation even within the SM.

As is well known, in order to observe direct CP violation effects, there should exist interferences not only through weak CP-violating phases but also with different CP-conserving strong phases. In the decay of $B^{\pm}\to\pi\pi l^{\pm}\nu$, we consider it as a two-stage process: $B\to(\Sigma_i M_i\to\pi\pi)l\nu$, where M_i stands for an intermediate state which is decaying to $\pi^{+}+\pi^{-}$. In this picture the CP-conserving phases may come from the absorptive parts of the intermediate resonances. Here we try to include as many intermediate states decaying to $\pi^{+}+\pi^{-}$ as possible so that they can represent a pseudocomplete set of the relevant decay. The candidates in $b\to u$ transition are ρ, f_0, and f_2 mesons, which decay dominantly to 2π mode (see Table I). Furthermore, we find that even in $b\to c$ transition a D^0 meson can decay to $\pi^{+}+\pi^{-}$, although its branching fraction is very small compared to those of $u\bar{u}$ states. However, we can find that the contribution through an intermediate D meson is not negligible because of Cabibbo-Kobayashi-Maskawa (CKM) favored nature of $b\to c$ transition compared to the $b\to u$ one of $u\bar{u}$ states, ρ, f_0, and f_2 mesons. If we include D meson as an intermediate state as well as the $u\bar{u}$ states, direct CP violation may arise even within the SM through their relative weak phases of the different CKM matrix elements (V_{cb} and V_{ub}). Therefore, we first consider CP violation within the SM by including ρ, f_0, f_2, and D mesons[1] as intermediate states decaying to $\pi^{+}\pi^{-}$. Next we also consider CP violations in extensions of the SM, in which we use a cutoff to the final state $\pi\pi$ invariant mass so that the effects of D meson cannot enter, thus ensuring that the result is solely from new physics.

In Sec. II, we present our formalism dealing with $B\to\pi\pi l\nu$ decays within the SM and in its extensions, and the observable asymmetries are considered in Sec. III. Section IV contains our numerical results and conclusions. All the relevant formulas we use here are presented in the Appendix.

II. THEORETICAL DETAILS OF DECAY AMPLITUDES

A. Within the standard model

The decay amplitudes for the processes of Fig. 1, with M_i listed in Table I,

$$B^{-}(p_B)\to M_i(p_i,\lambda_i)+W^*(q)$$
$$\to\pi^{+}(p_{+})+\pi^{-}(p_{-})+l^{-}(p_l,\lambda_l)+\bar{\nu}(p_\nu) \quad (1)$$

are expressed as

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[1]Here we are including fully known resonances only, and neglecting possible non-resonant 2π decays. A significant experimental enhancement can be made, if we use the reduced kinematic region around 1.4 GeV $\leqslant\sqrt{(p_\pi+p_\pi)^2}\leqslant 1.9$ GeV (see Table I).

TABLE I. Properties and branching ratios of $\pi^+\pi^-$ resonances.

Label i	M_i	J^P	m_i (MeV)	Γ_i (MeV)	$\mathcal{BR}_i(M_i\to\pi^+\pi^-)$
0	$M_0=f_0(980)$	0^+	980	40~100	0.52
0′	$M_{0'}=f_0(1500)$	0^+	1500	112	0.3
1	$M_1=\rho(770)$	1^-	770	151	1
1′	$M_{1'}=\rho(1700)$	1^-	1700	240	0.3
2	$M_2=f_2(1270)$	2^+	1275	186	0.56
3	$M_3=D^0$	0^-	1865	1.59×10^{-9}	1.53×10^{-3}

$$\mathcal{A}^{\lambda_l}=-\frac{G_F}{\sqrt{2}}\sum_i\sum_{\lambda_i}V_ic_i\langle l^-(p_l,\lambda_l)\overline{\nu}(p_\nu)|j^{\mu\dagger}|0\rangle \times\langle M_i(p_i,\lambda_i)|J_{i\mu}|B^-(p_B)\rangle\Pi_i(s_M) \times\langle\pi^+(p_+)\pi^-(p_-)\|M_i(p_i,\lambda_i)\rangle, \tag{2}$$

where $\lambda_i=0$ for spin 0 states (f_0 and D), $\lambda_i=\pm1,0$ for spin 1 states (ρ), $\lambda_i=\pm2,\pm1,0$ for spin 2 states (f_2), and λ_l is the lepton helicity $\pm\frac{1}{2}$.

The leptonic current is

$$j^\mu=\overline{\psi}_\nu\gamma^\mu(1-\gamma_5)\psi_l, \tag{3}$$

and for the hadronic currents we have

$$J_i^\mu=\overline{\psi}_u\gamma^\mu(1-\gamma_5)\psi_b,\quad V_i=V_{ub},$$

$$c_i=\frac{1}{\sqrt{2}}\ \text{for label}\ \ i=0^{(\prime)},1^{(\prime)},2 \tag{4}$$

and

$$J_3^\mu=\overline{\psi}_c\gamma^\mu(1-\gamma_5)\psi_b,\quad V_3=V_{cb},\quad c_3=1\ \ \text{for label}\ \ i=3, \tag{5}$$

where c_i stands for the isospin factor especially due to $u\overline{u}$ mesons. We assume that the resonance contributions of the intermediate states can be treated by the Breit-Wigner form, which is written in the narrow width approximation as

$$\Pi_i(s_M)=\frac{\sqrt{m_i\Gamma_i/\pi}}{s_M-m_i^2+im_i\Gamma_i}, \tag{6}$$

where $s_M=(p_++p_-)^2$ and the m_i's and Γ_i's are the masses and widths of the resonances respectively (see Table I). For the decay parts of the resonances we use [4]

$$\langle\pi^+(p_+)\pi^-(p_-)\|M_i(p_i,\lambda_i)\rangle=\sqrt{\mathcal{BR}_i}Y^{\lambda_i}_{\lambda_i\ \max}(\theta^*,\phi^*), \tag{7}$$

where $Y_l^m(\theta,\phi)$ are the $J=l$ spherical harmonics listed in Appendix B, and the angles θ^* and ϕ^* are those of the final state π^- specified in the M_i rest frame [see Fig. 2(c)]. The couplings of M_i to $\pi\pi$ are effectively taken into account by the branching fractions $\mathcal{BR}_i(M_i\to\pi^+\pi^-)$.

In order to obtain the full helicity amplitude of the $B\to\pi\pi l\nu$ decay, we first consider the amplitude of $B\to M_il\overline{\nu_l}$ [5], denoted as $\mathcal{M}^{\lambda_l}_{\lambda_i}$:

$$\mathcal{M}^{\lambda_l}_{\lambda_i}=-\frac{G_F}{\sqrt{2}}V_ic_i\langle l^-(p_l,\lambda_l)\overline{\nu}(p_\nu)|j^{\mu\dagger}|0\rangle \times\langle M_i(p_i,\lambda_i)|J_{i\mu}|B^-(p_B)\rangle. \tag{8}$$

We express the matrix elements $\mathcal{M}^{\lambda_l}_{\lambda_i}$ into the following form:

$$\mathcal{M}^{\lambda_l}_{\lambda_i}=\frac{G_F}{\sqrt{2}}V_ic_i\sum_{\lambda_W}\eta_{\lambda_W}L^{\lambda_l}_{\lambda_W}H^{\lambda_i}_{\lambda_W}, \tag{9}$$

where for the decays $B\to M_iW^*$ and $W^*\to l\overline{\nu}$, respectively,

$$H^{\lambda_i}_{\lambda_W}=\epsilon^*_{W\mu}\langle M_i(p_i,\lambda_i)|J_i^\mu|B^-(p_B)\rangle,$$

$$L^{\lambda_l}_{\lambda_W}=\epsilon_{W\mu}\langle l^-(p_l,\lambda_l)\overline{\nu}(p_\nu)|j^{\mu\dagger}|0\rangle, \tag{10}$$

in terms of the polarization vectors $\epsilon_W\equiv\epsilon(q,\lambda_W)$ of the virtual W. These ϵ_W's satisfy the relation

$$-g^{\mu\nu}=\sum_{\lambda_W}\eta_{\lambda_W}\epsilon^\mu_W\epsilon^{*\nu}_W, \tag{11}$$

where the summation is over the helicities $\lambda_W=\pm1,0,s$ of the virtual W, with the metric $\eta_\pm=\eta_0=-\eta_s=1$.

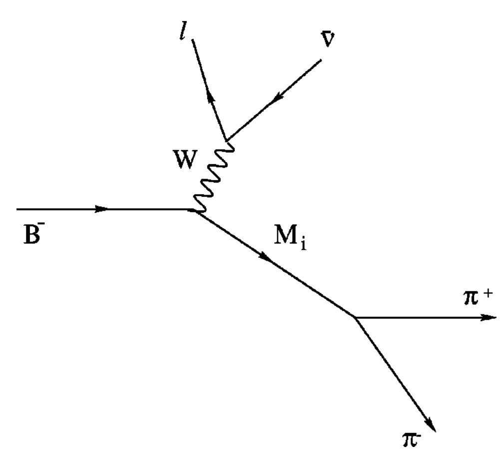

FIG. 1. Diagrams for $B\to M_iW^*\to\pi^+\pi^-l^-\overline{\nu_l}$ decays.

We evaluate the leptonic amplitude $L_{\lambda_W}^{\lambda_l}$ in the rest frame of the virtual W [see Fig. 2(b)] with the z axis chosen along the M_i direction, and the x-z plane chosen as the virtual W decay plane, with $(p_l)_x > 0$. Using the two-component spinor technique [6] and polarization vectors given in Appendix B, we find

$$L_\pm^- = 2\sqrt{q^2}\, v d_\pm, \quad L_0^- = -2\sqrt{q^2}\, v d_0, \quad L_s^- = 0,$$

$$L_\pm^+ = \pm 2 m_l v d_0, \quad L_0^+ = \sqrt{2} m_l v (d_+ - d_-), \quad L_s^+ = -2 m_l v, \tag{12}$$

where

$$v = \sqrt{1 - \frac{m_l^2}{q^2}}, \quad d_\pm = \frac{1 \pm \cos\theta_l}{\sqrt{2}}, \text{ and } d_0 = \sin\theta_l. \tag{13}$$

Here we show only the sign of λ_l as a superscript on L. Note that the L^+ amplitudes are proportional to the lepton mass m_l, and the scalar amplitude L_s^- vanishes due to angular momentum conservation.

For $B \to M_i$ transitions through the weak charged current

$$J_i^\mu = V_i^\mu - A_i^\mu, \tag{14}$$

the most general forms of matrix elements are as follows. For $f_0(0^+)$ states,

$$\langle f_0(p_i)|V_\mu|B(p_B)\rangle = 0,$$

$$\langle f_0(p_i)|A_\mu|B(p_B)\rangle = u_+(q^2)(p_B + p_i)_\mu + u_-(q^2)(p_B - p_i)_\mu,$$

for $\rho(1^-)$ states,

$$\langle \rho(p_i,\epsilon_1)|V_\mu|B(p_B)\rangle = i g(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon_1^{*\nu}(p_B + p_i)^\rho (p_B - p_i)^\sigma,$$

$$\langle \rho(p_i,\epsilon_1)|A_\mu|B(p_B)\rangle = f(q^2)\epsilon_{1\mu}^* + a_+(q^2)(\epsilon_1^*\cdot p_B)(p_B + p_i)_\mu + a_-(q^2) \times (\epsilon_1^*\cdot p_B)(p_B - p_i)_\mu,$$

for $f_2(2^+)$ states,

$$\langle f_2(p_i,\epsilon_2)|V_\mu|B(p_B)\rangle = i h(q^2)\epsilon_{\mu\nu\lambda\rho}\epsilon_2^{*\nu\alpha} p_{B\alpha}(p_B + p_i)^\lambda \times (p_B - p_i)^\rho,$$

$$\langle f_2(p_i,\epsilon_2)|A_\mu|B(p_B)\rangle = k(q^2)\epsilon_{2\mu\nu}^* p_B^\nu + b_+(q^2)(\epsilon_{2\alpha\beta}^* p_B^\alpha p_B^\beta) \times (p_B + p_i)_\mu + b_-(q^2)(\epsilon_{2\alpha\beta}^* p_B^\alpha p_B^\beta) \times (p_B - p_i)_\mu,$$

for $D(0^-)$ states,

$$\langle D(p_i)|V_\mu|B(p_B)\rangle = f_+(q^2)(p_B + p_i)_\mu + f_-(q^2)(p_B - p_i)_\mu,$$

$$\langle D(p_i)|A_\mu|B(p_B)\rangle = 0, \tag{15}$$

where ϵ_1 and ϵ_2 are the polarization vectors of the spin 1 and spin 2 states, respectively. Using the above expressions and the polarization vectors given in Appendix B, we find non-zero $B \to M_i W^*$ amplitudes are as follows. For $i = 0, H_{\lambda_W}^0 \equiv S_{\lambda_W}^0$,

$$S_0^0 = -u_+(q^2)\frac{\sqrt{Q_+ Q_-}}{\sqrt{q^2}},$$

$$S_s^0 = -\left(u_+(q^2)\frac{(m_B^2 - s_M)}{\sqrt{q^2}} + u_-(q^2)\sqrt{q^2} \right), \tag{16}$$

for $i = 1^{(\prime)}, H_{\lambda_W}^{\lambda_1} \equiv V_{\lambda_W}^{\lambda_1}$,

$$V_0^0 = -\frac{1}{2\sqrt{s_M q^2}}[f(q^2)(m_B^2 - s_M - q^2) + a_+(q^2) Q_+ Q_-],$$

$$V_{\pm 1}^{\pm 1} = f(q^2) \mp g(q^2)\sqrt{Q_+ Q_-},$$

$$V_s^0 = -\frac{\sqrt{Q_+ Q_-}}{2\sqrt{s_M q^2}}[f(q^2) + a_+(q^2)(m_B^2 - s_M) + a_-(q^2) q^2], \tag{17}$$

for $i = 2, H_{\lambda_W}^{\lambda_2} \equiv T_{\lambda_W}^{\lambda_2}$,

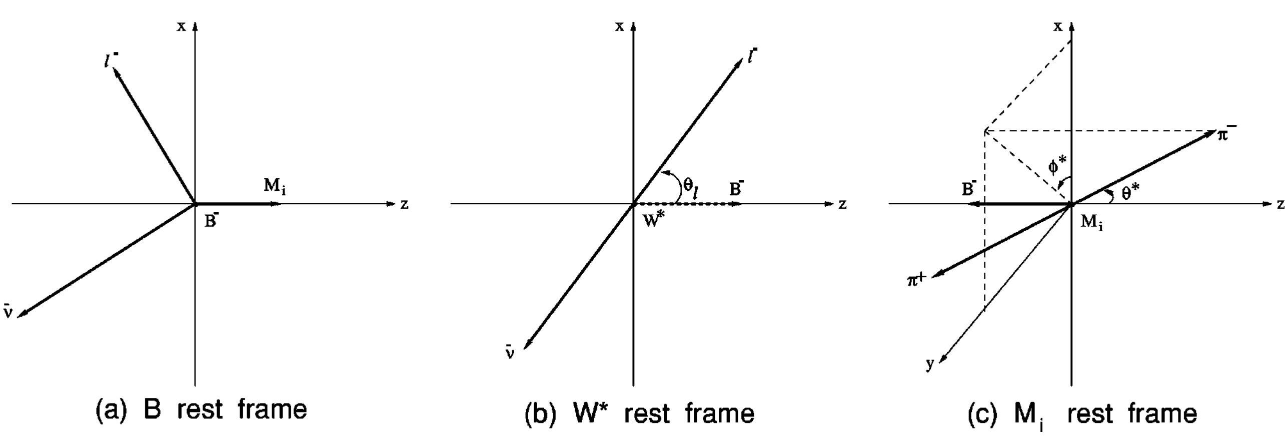

FIG. 2. The decay $B \to M_i W^* \to (\pi^+\pi^-)(l\overline{\nu})$ viewed from the (a) B^-, (b) W^*, and (c) M_i rest frames.

$$T_0^0 = -\frac{1}{2\sqrt{6}} \frac{\sqrt{Q_+Q_-}}{s_M\sqrt{q^2}}[k(q^2)(m_B^2 - s_M - q^2) + b_+(q^2)Q_+Q_-],$$

$$T_{\pm 1}^{\pm 1} = \frac{1}{2\sqrt{2}}\sqrt{\frac{Q_+Q_-}{s_M}}[k(q^2) \mp h(q^2)\sqrt{Q_+Q_-}],$$

$$T_s^0 = -\frac{1}{2\sqrt{6}}\frac{Q_+Q_-}{s_M\sqrt{q^2}}[k(q^2) + b_+(q^2)(m_B^2 - s_M) + b_-(q^2)q^2], \tag{18}$$

for $i=3, H_{\lambda_W}^0 \equiv P_{\lambda_W}^0$,

$$P_0^0 = f_+(q^2)\frac{\sqrt{Q_+Q_-}}{\sqrt{q^2}},$$

$$P_s^0 = f_+(q^2)\frac{(m_B^2 - s_M)}{\sqrt{q^2}} + f_-(q^2)\sqrt{q^2}, \tag{19}$$

where

$$Q_\pm = (m_B \pm \sqrt{s_M})^2 - q^2. \tag{20}$$

Here

$$Q_+Q_- = \lambda(m_B^2, s_M, q^2) \tag{21}$$

gives the triangle function $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$. Combining all the formulas, we can write the full helicity amplitudes of $B^- \to \pi^+\pi^- l^- \overline{\nu}$ decays as

$$\mathcal{A}^{\lambda_l} = V_{ub}\frac{G_F}{\sqrt{2}}\Bigg[\sum_{\lambda=0,s} \eta_\lambda L_\lambda^{\lambda_l}(\Pi_{f_0}S_\lambda^0 Y_0^0 + \xi \Pi_D P_\lambda^0 \tilde{Y}_0^0 + \Pi_\rho V_\lambda^0 Y_1^0 + \Pi_{f_2}T_\lambda^0 Y_2^0) + \sum_{\lambda=\pm 1} L_\lambda^{\lambda_l}(\Pi_\rho V_\lambda^\lambda Y_1^\lambda + \Pi_{f_2}T_\lambda^\lambda Y_2^\lambda)\Bigg], \tag{22}$$

where

$$\Pi_{f_0} \equiv \frac{1}{\sqrt{2}}(\sqrt{\mathcal{B}R_0}\Pi_0 + \sqrt{\mathcal{B}R_{0'}}\Pi_{0'}),$$

$$\Pi_\rho \equiv \frac{1}{\sqrt{2}}(\sqrt{\mathcal{B}R_1}\Pi_1 + \sqrt{\mathcal{B}R_{1'}}\Pi_{1'}),$$

$$\Pi_{f_2} \equiv \frac{1}{\sqrt{2}}\sqrt{\mathcal{B}R_2}\Pi_2,$$

$$\Pi_D \equiv \sqrt{\mathcal{B}R_3}\Pi_3, \tag{23}$$

and

$$\xi = \frac{V_{cb}}{V_{ub}}. \tag{24}$$

Note that we use $\tilde{Y}_0^0$ for the pseudoscalar meson D, which is actually the same quantity as $Y_0^0 = 1/\sqrt{4\pi}$ for the scalar meson f_0 except that it changes sign under the parity transformation. Concerning the parametrization of ξ, other CKM factors, such as V_{cd}^* from $D^0 \to \pi^+\pi^-$ decay, are already included in its branching fraction calculation. And because we use implicitly Wolfenstein parametrization [7] for CKM matrix, in which the complex phases are approximately in the elements V_{td} and V_{ub} only, the imaginary part of ξ here comes only from the element V_{ub}.

The differential partial width of interest can be expressed as

$$d\Gamma(B^- \to \pi^+\pi^- l^- \overline{\nu_l}) = \frac{1}{2m_B}\sum_{\lambda_l}|\mathcal{A}^{\lambda_l}|^2 \frac{(q^2 - m_l^2)\sqrt{Q_+Q_-}}{256\pi^3 m_B^2 q^2} d\Phi_4, \tag{25}$$

where the four-body phase space $d\Phi_4$ is

$$d\Phi_4 \equiv ds_M \cdot dq^2 \cdot d\cos\theta^* \cdot d\cos\theta_l \cdot d\phi^*. \tag{26}$$

Kinematically allowed regions of the variables are

$$4m_\pi^2 < s_M < (m_B - m_l)^2,$$

$$m_l^2 < q^2 < (m_B - \sqrt{s_M})^2,$$

$$-1 < \cos\theta^*, \cos\theta_l < 1,$$

$$0 < \phi^* < 2\pi. \tag{27}$$

Since the initial B^- system is not CP self-conjugate, any genuine CP-odd observable can be constructed only by considering both the B^- decay and its charge-conjugated B^+ decay, and by identifying the CP relations of their kinematic distributions. Before constructing possible CP-odd asymmetries explicitly, we calculate the decay amplitudes for the charge-conjugated process $B^+ \to \pi^+\pi^- l^+\nu_l$. For the charge-conjugated B^+ decays, the amplitudes can be written as

$$\overline{\mathcal{A}}^{\lambda_l} = -\frac{G_F}{\sqrt{2}}\sum_i \sum_{\lambda_i} V_i^* c_i \langle l^+(p_l,\lambda_l)\nu(p_\nu)|j^\mu|0\rangle \times \langle \overline{M}_i(p_i,\lambda_i)|J_{i\mu}^\dagger|B^+(p_B)\rangle \Pi_i(s_M) \times \langle \pi^+(p_+)\pi^-(p_-)\|\overline{M}_i(p_i,\lambda_i)\rangle. \tag{28}$$

The leptonic amplitudes $\overline{L}_{\lambda_W}^{\lambda_l}$ are

$$\overline{L}_\pm^+ = -2\sqrt{q^2}v d_\mp, \quad \overline{L}_0^+ = -2\sqrt{q^2}v d_0, \quad \overline{L}_s^+ = 0,$$

$$\overline{L}_\pm^- = \pm 2m_l v d_0, \quad \overline{L}_0^- = \sqrt{2}m_l v(d_+ - d_-), \quad \overline{L}_s^- = -2m_l v. \tag{29}$$

The transition amplitudes $\overline{H}^{\lambda_i}_{\lambda_W}$ for $B^+\to\overline{M}_iW^*$ are given by a simple modification of the amplitudes $H^{\lambda_i}_{\lambda_W}$ of the B^- decays:

$$\overline{H}^{\lambda_i}_{\lambda_W}=H^{\lambda_i}_{\lambda_W}\{g\to -g,h\to -h,f_\pm\to -f_\pm\}. \tag{30}$$

Then, we find the full amplitude for $B^+\to\pi^+\pi^-l^+\nu$:

$$\overline{\mathcal{A}}^{\lambda_l}=V_{ub}^*\frac{G_F}{\sqrt{2}}\Bigg[\sum_{\lambda=0,s}\eta_\lambda\overline{L}^{\lambda_l}_\lambda(\Pi_{f_0}\overline{S}^0_\lambda Y^0_0+\xi^*\Pi_D\overline{P}^0_\lambda\tilde{Y}^0_0+\Pi_\rho\overline{V}^0_\lambda Y^0_1+\Pi_{f_2}\overline{T}^0_\lambda Y^0_2)+\sum_{\lambda=\pm 1}\overline{L}^{\lambda_l}_\lambda(\Pi_\rho\overline{V}^\lambda_\lambda Y^\lambda_1+\Pi_{f_2}\overline{T}^\lambda_\lambda Y^\lambda_2)\Bigg], \tag{31}$$

where

$$\overline{S}^0_{\lambda_W}=S^0_{\lambda_W},$$
$$\overline{P}^0_{\lambda_W}=-P^0_{\lambda_W},$$
$$\overline{V}^0_{0,s}=V^0_{0,s}, \quad \overline{V}^{\pm 1}_{\pm 1}=V^{\mp 1}_{\mp 1},$$
$$\overline{T}^0_{0,s}=T^0_{0,s}, \quad \overline{T}^{\pm 1}_{\pm 1}=T^{\mp 1}_{\mp 1}. \tag{32}$$

It is easy to see that if V_{ub} and V_{cb} are real, the amplitude (22) of the B^- decay and Eq. (31) of the B^+ decay satisfy the *CP* relation

$$\mathcal{A}^\pm(\theta^*,\phi^*,\theta_l)=\eta_{CP}\overline{\mathcal{A}}^\mp(\theta^*,-\phi^*,\theta_l;\tilde{Y}^0_0\to -\tilde{Y}^0_0), \tag{33}$$

where θ^* and ϕ^* in $\overline{\mathcal{A}}^{\lambda_l}$ are the angles of the final state π^+, while those in $\mathcal{A}^{\lambda_l}$ are for π^-. Then, with a complex weak phase ξ, $d\Gamma/d\Phi_4$ can be decomposed into a *CP*-even part $\mathcal{S}$ and a *CP*-odd part $\mathcal{D}$:

$$\frac{d\Gamma}{d\Phi_4}=\frac{1}{2}(\mathcal{S}+\mathcal{D}). \tag{34}$$

The *CP*-even part $\mathcal{S}$ and the *CP*-odd part $\mathcal{D}$ can be easily identified by making use of the *CP* relation (33) between B^- and B^+ decay amplitudes, and they are expressed as

$$\mathcal{S}=\frac{d(\Gamma+\overline{\Gamma})}{d\Phi_4}, \quad \mathcal{D}=\frac{d(\Gamma-\overline{\Gamma})}{d\Phi_4}, \tag{35}$$

where Γ and $\overline{\Gamma}$ are the decay rates for B^- and B^+, respectively, and we have used the same kinematic variables $\{s_M,q^2,\theta^*,\theta_l\}$ for the $d\overline{\Gamma}/d\Phi_4$ except for the replacements of $\phi^*\to-\phi^*$ and $\tilde{Y}^0_0\to-\tilde{Y}^0_0$, as shown in Eq. (33). The *CP*-even $\mathcal{S}$ term and the *CP*-odd $\mathcal{D}$ term can be obtained from $B^\mp$ decay probabilities, and their explicit form is listed in Appendix C. Note that the *CP*-odd term is proportional to the imaginary part of the parameter ξ in Eq. (24).

Before we go further on to beyond the SM analyses, we note that in addition to the resonant tree diagram contributions there are other SM contributions through annihilation diagrams and electroweak penguin diagrams, which are relevant for nonresonant case. As written in Sec. I, we consider only resonant contributions by assuming nonresonant contributions can be separated through data analyses.

B. With complex scalar couplings

Next we consider *CP* violation effects in extensions of the SM, where we extend the virtual *W*-exchange part in Fig. 1 by including an additional scalar interaction with complex couplings. First we describe the formalism in a model independent way, but later we consider specific models such as multi-Higgs doublet models and scalar-leptoquark models. In this case *CP*-violating phases can be generated through the interference between *W*-exchange diagrams and scalar exchange diagrams with complex couplings.

The decay amplitudes for $B^-\to\pi^+\pi^-l^-\overline{\nu}_l$ are expressed as

$$\mathcal{A}^{\lambda_l}=-V_{ub}\frac{G_F}{\sqrt{2}}\frac{1}{\sqrt{2}}\sum_i\sum_{\lambda_i}[\langle l^-(p_l,\lambda_l)\overline{\nu}(p_\nu)|j^{\mu\dagger}|0\rangle\times\langle M_i(p_i,\lambda_i)|J_\mu|B^-(p_B)\rangle+\zeta\langle l^-(p_l,\lambda_l)\overline{\nu}(p_\nu)|j^\dagger_s|0\rangle\times\langle M_i(p_i,\lambda_i)|J_s|B^-(p_B)\rangle]\Pi_i(s_M)\times\langle\pi^+(p_+)\pi^-(p_-)\|M_i(p_i,\lambda_i)\rangle, \tag{36}$$

where

$$j^\mu=\overline{\psi}_\nu\gamma^\mu(1-\gamma_5)\psi_l,$$
$$J^\mu=\overline{\psi}_u\gamma^\mu(1-\gamma_5)\psi_b\equiv V^\mu-A^\mu, \tag{37}$$

and their corresponding scalar currents are

$$j_s=\overline{\psi}_\nu(1-\gamma_5)\psi_l, \quad J_s=\overline{\psi}_u(1-\gamma_5)\psi_b, \tag{38}$$

the additional factor $1/\sqrt{2}$ comes from the isospin factor as mentioned earlier. Here the parameter ζ, which parametrizes contributions from physics beyond the SM, is in general a complex number. And as explained earlier, in order to exclude any possible *CP* violation effects induced within the SM, we only include the lowest three states $\rho(770)$, $f_0(980)$, and $f_2(1270)$ as intermediate states. By using the Dirac equation for the leptonic current, $q_\mu j^\mu=m_l j_s$, the amplitude can be written as

$$\mathcal{A}^{\lambda_l}=-V_{ub}\frac{G_F}{2}\sum_i\sum_{\lambda_i}\langle l^-(p_l,\lambda_l)\overline{\nu}(p_\nu)|j^{\mu\dagger}|0\rangle\times\langle M_i(p_i,\lambda_i)|\Omega_\mu|B^-(p_B)\rangle\Pi_i(s_M)\times\langle\pi^+(p_+)\pi^-(p_-)\|M_i(p_i,\lambda_i)\rangle, \tag{39}$$

where the effective hadronic current Ω_μ is defined as

$$\Omega_\mu \equiv J_\mu + \zeta \frac{q_\mu}{m_l} J_s . \tag{40}$$

In this case the amplitudes $\mathcal{M}^{\lambda_l}_{\lambda_i}$ of $B \to M_i l \bar{\nu}$ have the same form as the previous SM case (9) except for the modification in the hadronic current part due to the additional scalar current:

$$\mathcal{M}^{\lambda_l}_{\lambda_i} = \frac{G_F}{2} V_{ub} \sum_{\lambda_W} \eta_{\lambda_W} L^{\lambda_l}_{\lambda_W} \mathcal{H}^{\lambda_i}_{\lambda_W}, \tag{41}$$

where $\mathcal{H}^{\lambda_i}_{\lambda_W}$ stands for the hadronic amplitudes modified by the scalar current J_s. Using the equation of motion for u and b quarks, we get within the on-shell approximation

$$J_s = (p_b^\mu - p_u^\mu) \left[\frac{V_\mu}{m_b - m_u} + \frac{A_\mu}{m_b + m_u} \right]. \tag{42}$$

Later for numerical calculations, we use the approximation, $(p_b^\mu - p_u^\mu) \approx (p_B^\mu - p_{M_i}^\mu) \equiv q^\mu$, which is assumed in quark model calculations of form factors [17]. After explicit calculation, we find that the additional scalar current modifies only the scalar component of $\mathcal{H}^{\lambda_i}_{\lambda_W}$: i.e.,

$$\mathcal{H}^0_s = (1 - \zeta') H^0_s$$

and

$$\mathcal{H}^{\lambda_i}_{\lambda_W} = H^{\lambda_i}_{\lambda_W} \quad \text{for} \quad \lambda_W = 0, \pm 1, \tag{43}$$

where

$$\zeta' = \frac{q^2}{m_l (m_b + m_u)} \zeta. \tag{44}$$

In this case, $d\Gamma / d\Phi_4$ also can be decomposed into a CP-even part $\mathcal{S}$ and a CP-odd part $\mathcal{D}$:

$$\frac{d\Gamma}{d\Phi_4} = \frac{1}{2} (\mathcal{S} + \mathcal{D}). \tag{45}$$

Their explicit form is listed in Appendix C. Note that the CP-odd term is proportional to the imaginary part of the parameter ζ and the lepton mass m_l. Therefore, we have to consider massive leptonic (μ or τ) decays.

As specific extensions of the SM, we consider four types of scalar-exchange models which preserve the symmetries of the SM [8]: One of them is the multi-Higgs-doublet (MHD) model [9] and the other three models are scalar-leptoquark (SLQ) models [10,11]. The authors of Ref. [12] investigated CP violations in τ decay processes with these extended models. We follow their description and make it to be appropriate for our analysis.

In the MHD model CP violation can arise in the charged Higgs sector with more than two Higgs doublets [13] and when not all the charged scalars are degenerate. As in most previous phenomenological analyses, we assume that all but the lightest of the charged scalars effectively decouple from fermions. The effective Lagrangian of the MHD model contributing to the decay $B \to \pi\pi l \bar{\nu}_l$ is then given at energies considerably low compared to M_H by

$$\mathcal{L}_{\rm MHD} = 2\sqrt{2} G_F V_{ub} \frac{m_l}{M_H^2} [m_b X Z^* (\bar{u}_L b_R) + m_u Y Z^* (\bar{u}_R b_L)] \times (\bar{l}_R \nu_L), \tag{46}$$

where X, Y, and Z are complex coupling constants which can be expressed in terms of the charged Higgs mixing matrix elements. From the effective Lagrangian, we obtain for the MHD CP-violation parameter ${\rm Im}(\zeta_{\rm MHD})$,

$${\rm Im}(\zeta_{\rm MHD}) = \frac{m_l m_b}{M_H^2} \left\{ {\rm Im}(XZ^*) - \left(\frac{m_u}{m_b} \right) {\rm Im}(YZ^*) \right\}. \tag{47}$$

The constraints on the CP-violation parameter (47) depend upon the values chosen for the u and b quark masses. In the present work, we use (see Appendix A)

$$m_u = 0.33 \ \text{GeV}, \quad m_b = 5.12 \ \text{GeV}. \tag{48}$$

In the MHD model the strongest constraint [9] on ${\rm Im}(XZ^*)$ comes from the measurement of the branching ratio $\mathcal{B}(b \to X \tau \nu_\tau)$, which actually gives a constraint on $|XZ|$. For $M_H < 440$ GeV, the bound on ${\rm Im}(XZ^*)$ is given by

$${\rm Im}(XZ^*) < |XZ| < 0.23 M_H^2 \ \text{GeV}^{-2}. \tag{49}$$

On the other hand, the bound on ${\rm Im}(YZ^*)$ is mainly given by $K^+ \to \pi^+ \nu \bar{\nu}$. The present bound [9] is

$${\rm Im}(YZ^*) < |YZ| < 110. \tag{50}$$

Combining the above bounds, we obtain the following bounds on ${\rm Im}(\zeta_{\rm MHD})$ as

$$|{\rm Im}(\zeta_{\rm MHD})| < 2.06 \quad \text{for} \quad \tau \quad \text{family},$$

$$|{\rm Im}(\zeta_{\rm MHD})| < 0.12 \quad \text{for} \quad \mu \quad \text{family}. \tag{51}$$

On the other hand, the effective Lagrangians for the three SLQ models [8,10] contributing to the decay $B \to \pi\pi l \nu$ are written in the form, after a few Fierz rearrangements:

$$\mathcal{L}^{\rm I}_{\rm SLQ} = -\frac{x_{3j} x'^*_{1j}}{2M^2_{\phi_1}} \left[(\bar{b}_L u_R)(\bar{\nu}_{lL} l_R) + \frac{1}{4} (\bar{b}_L \sigma^{\mu\nu} u_R) \times (\bar{\nu}_{lL} \sigma_{\mu\nu} l_R) \right] + \text{H.c.},$$

$$\mathcal{L}^{\rm II}_{\rm SLQ} = -\frac{y_{3j} y'^*_{1j}}{2M^2_{\phi_2}} \left[(\bar{b}_L u_R)(\bar{l}^c_R \nu^c_{lL}) + \frac{1}{4} (\bar{b}_L \sigma^{\mu\nu} u_R) \times (\bar{l}^c_R \sigma_{\mu\nu} \nu^c_{lL}) \right] + \frac{y_{3j} y^*_{1j}}{2M^2_{\phi_2}} (\bar{b}_L \gamma_\mu u_L)(\bar{l}^c_L \gamma^\mu \nu^c_{lL}) + \text{H.c.},$$

$$\mathcal{L}^{\rm III}_{\rm SLQ} = -\frac{z_{3j} z^*_{1j}}{2M^2_{\phi_3}} (\bar{b}_L \gamma_\mu u_L)(\bar{l}^c_L \gamma^\mu \nu^c_{lL}) + \text{H.c.}, \tag{52}$$

TABLE II. The CP-violating rate asymmetry A and the optimal asymmetry $\varepsilon_{\rm opt}$, determined within the SM, and the number of charged B meson pairs N_B needed for detection at 1σ level, at reference value $\mathrm{Im}(\xi)=12.5$.

Mode	$B\to\pi^+\pi^- l\overline{\nu_l}$					
	$l=e$		$l=\mu$		$l=\tau$	
Asym.	Size(%)	N_B	Size(%)	N_B	Size(%)	N_B
A	0.94×10^{-6}	1.37×10^{18}	1.71×10^{-6}	4.16×10^{17}	1.14×10^{-6}	1.46×10^{18}
$\varepsilon_{\rm opt}$	1.45×10^{-2}	5.75×10^{9}	1.44×10^{-2}	5.79×10^{9}	1.11×10^{-2}	1.56×10^{10}

where $j=2,3$ for $l=\mu,\tau$, respectively and the coupling constants $x_{ij}^{(\prime)}$, $y_{ij}^{(\prime)}$, and z_{ij} are in general complex so that CP is violated in the scalar-fermion Yukawa interaction terms. The superscript c in the Lagrangians $\mathcal{L}_{\rm SLQ}^{\rm II}$ and $\mathcal{L}_{\rm SLQ}^{\rm III}$ denotes charge conjugation, i.e., $\psi_{R,L}^c=i\gamma^0\gamma^2\overline{\psi}_{R,L}^T$ in the chiral representation. Then we find that the size of the SLQ model CP-violation effects is dictated by the CP-odd parameters

$$\mathrm{Im}(\zeta_{\rm SLQ}^{\rm I})=-\frac{\mathrm{Im}[x_{3j}x'^{*}_{1j}]}{4\sqrt{2}G_FV_{ub}M_{\phi_1}^2},$$

$$\mathrm{Im}(\zeta_{\rm SLQ}^{\rm II})=-\frac{\mathrm{Im}[y_{3j}y'^{*}_{1j}]}{4\sqrt{2}G_FV_{ub}M_{\phi_2}^2},$$

$$\mathrm{Im}(\zeta_{\rm SLQ}^{\rm III})=0. \tag{53}$$

Although there are at present no direct constraints on the SLQ model CP-odd parameters in Eq. (53), a rough constraint to the parameters can be provided by the assumption [14] that $|x'_{1j}|\sim|x_{1j}|$ and $|y'_{1j}|\sim|y_{1j}|$, that is to say, the leptoquark couplings to quarks and leptons belonging to the same generation are of a similar size; then the experimental upper bounds from $B\overline{B}$ mixing for τ family, $B\to\mu\overline{\mu}X$ decay for μ in model I, and $B\to l\nu X$ for τ together with the V_{ub} measurement for μ in model II yield [14]

$$|\mathrm{Im}(\zeta_{\rm SLQ}^{\rm I})|<2.76,\quad |\mathrm{Im}(\zeta_{\rm SLQ}^{\rm II})|<18.4\ \text{ for }\ \tau\ \text{ family},$$

$$|\mathrm{Im}(\zeta_{\rm SLQ}^{\rm I})|<0.37,\quad |\mathrm{Im}(\zeta_{\rm SLQ}^{\rm II})|<1.84\ \text{ for }\ \mu\ \text{ family}. \tag{54}$$

Based on the constraints (51) and (54) to the CP-odd parameters, we quantitatively estimate the number of $B^-\to\pi^+\pi^-l^-\overline{\nu_l}$ decays to detect CP violation for the maximally allowed values of the CP-odd parameters.

III. OBSERVABLE CP ASYMMETRIES

An easily constructed CP-odd asymmetry is the rate asymmetry

$$A\equiv\frac{\Gamma-\overline{\Gamma}}{\Gamma+\overline{\Gamma}}, \tag{55}$$

which has been used as a probe of CP violation in Higgs and top quark sectors [15]. Here Γ and $\overline{\Gamma}$ are the decay rates for B^- and B^+, respectively. The statistical significance of the asymmetry can be computed as

$$N_{\rm SD}=\frac{N_--N_+}{\sqrt{N_-+N_+}}=\frac{N_--N_+}{\sqrt{N\cdot Br}}, \tag{56}$$

where $N_{\rm SD}$ is the number of standard deviations, $N_\pm$ is the number of events predicted in B_{l4} decay for $B^\pm$ meson, N is the number of B-mesons produced, and Br is the branching fraction of the relevant B decay mode. For a realistic detection efficiency ϵ, we have to rescale the number of events by this parameter, $N_-+N_+\to\epsilon(N_-+N_+)$. Taking $N_{\rm SD}=1$, we obtain the number N_B of the B mesons needed to observe CP violation at 1-σ level:

$$N_B=\frac{1}{Br\cdot A^2}. \tag{57}$$

Next, we consider the so-called optimal observable. An appropriate real weight function $w(s_M,q^2;\theta^*,\theta_l,\phi^*)$ is usually employed to separate the CP-odd $\mathcal{D}$ contribution and to enhance its analysis power for the CP-odd parameter through the CP-odd quantity

$$\langle w\mathcal{D}\rangle\equiv\int[w\mathcal{D}]d\Phi_4, \tag{58}$$

and the analysis power is determined by the parameter

$$\varepsilon=\frac{\langle w\mathcal{D}\rangle}{\sqrt{\langle\mathcal{S}\rangle\langle w^2\mathcal{S}\rangle}}. \tag{59}$$

For the analysis power ε, the number N_B of the B mesons needed to observe CP violation at the 1 σ level is

$$N_B=\frac{1}{Br\cdot\varepsilon^2}. \tag{60}$$

Certainly, it is desirable to find the optimal weight function with the largest analysis power. It is known [16] that when the CP-odd contribution to the total rate is relatively small, the optimal weight function is approximately given as

$$w_{\rm opt}(s_M,q^2;\theta^*,\theta_l,\phi^*)=\frac{\mathcal{D}}{\mathcal{S}}\Rightarrow\varepsilon_{\rm opt}=\sqrt{\frac{\langle\mathcal{D}^2/\mathcal{S}\rangle}{\langle\mathcal{S}\rangle}}. \tag{61}$$

TABLE III. The CP-violating rate asymmetry A and the optimal asymmetry $\varepsilon_{\rm opt}$, determined in the extended models, and the number of charged B meson pairs N_B needed for detection at 1σ level, at reference values (a) $\mathrm{Im}(\zeta_{\rm MHD})=2.06$, $\mathrm{Im}(\zeta^{\rm I}_{\rm SLQ})=2.76$, and $\mathrm{Im}(\zeta^{\rm II}_{\rm SLQ})=18.4$ for the $B_{\tau 4}$ decays and (b) $\mathrm{Im}(\zeta_{\rm MHD})=0.12$, $\mathrm{Im}(\zeta^{\rm I}_{\rm SLQ})=0.37$, and $\mathrm{Im}(\zeta^{\rm II}_{\rm SLQ})=1.84$ for the $B_{\mu 4}$ decays.

(a) $B^-\to\pi^+\pi^-\tau\overline{\nu_\tau}$ mode

Model	MHD		SLQ I		SLQ II	
Asym.	Size(%)	N_B	Size(%)	N_B	Size(%)	N_B
A	1.47×10^{-3}	7.63×10^{11}	2.67×10^{-3}	1.99×10^{11}	3.62×10^{-3}	7.39×10^{9}
$\varepsilon_{\rm opt}$	16.2	6.23×10^{3}	18.2	4.27×10^{3}	9.67	1.04×10^{3}

(b) $B^-\to\pi^+\pi^-\mu\overline{\nu_\mu}$ mode

Model	MHD		SLQ I		SLQ II	
Asym.	Size(%)	N_B	Size(%)	N_B	Size(%)	N_B
A	2.61×10^{-5}	1.93×10^{15}	0.90×10^{-4}	1.59×10^{14}	3.43×10^{-4}	8.89×10^{12}
$\varepsilon_{\rm opt}$	0.18	3.89×10^{7}	0.50	5.13×10^{6}	1.48	4.76×10^{5}

We adopt this optimal weight function in the following numerical analyses.

IV. NUMERICAL RESULTS AND CONCLUSIONS

Now we show our numerical results. We use the so-called ISGW predictions [17] for all the form factors in $B\to M_i$ transition amplitudes of Eq. (15). One can find in Ref. [17] the detailed description of the general formalism and relevant form factors for $B\to Xe\overline{\nu_e}$ after neglecting lepton masses. In Appendix A, we give explicit expressions of form factors needed for semileptonic decays with non-zero lepton masses.

We first consider the case within the SM. Total branching ratio of the $B^-\to(\Sigma_i M_i\to\pi^+\pi^-)e\overline{\nu_e}, M_i=\rho, f_{0,2}, D$ is about 0.8%. It depends on the chosen value for $|V_{ub}|$, and here we adopt the result by CLEO [18]

$$|V_{ub}|=3.3\pm0.4\pm0.7\times10^{-3}. \tag{62}$$

In Table II, we show the results of $B\to\pi\pi l\nu$ decays for the two CP-violating asymmetries; the rate asymmetry A and the optimal asymmetry $\varepsilon_{\rm opt}$. We estimated the number of B-meson pairs, N_B, needed for detection at 1σ level for maximally-allowed values of CP-odd parameters $\mathrm{Im}(\xi)$ in Eq. (24). We use the current experimental bound [19]

$$\left|\frac{V_{ub}}{V_{cb}}\right|=0.08\pm0.02, \tag{63}$$

which means

$$|\xi|=12.5\pm3.13. \tag{64}$$

The results in Table II are for the maximal case with $\mathrm{Im}(\xi)=|\xi|=12.5$. Due to the large cancelations in the simple rate asymmetry [4] when we integrated over the phase space, the optimal observable gives much better result. For example, using the optimal observable, we need $\sim10^9$ B-meson pairs to detect the maximal CP-odd effect in electron mode. CP violation effects in $B\to\pi^+\pi^-l\overline{\nu_l}$ decays within the SM are not likely to be detected, with $\mathcal{O}(10^8)B$-meson pairs to be produced at the asymmetric B factories. One may rely on hadronic B factories of BTeV and LHC-B.

Next we consider the extended model case. In this case, CP violation effects are proportional to the lepton mass, and we consider only massive lepton (μ or τ) cases. In Table III, we show the results of $B_{\tau4}$ and $B_{\mu4}$ decays. Here in order to distinguish new physics effect from the SM one, we use a cutoff for the invariant mass of the final state $\pi^+\pi^-$ as

$$\sqrt{s_M}\leqslant1.4\ \ \mathrm{GeV}. \tag{65}$$

We consider only the lowest three $u\overline{u}$ states, $\rho(770)$, $f_0(980)$, and $f_2(1275)$ as intermediate resonances in Table I so that the effects of D meson cannot enter, and we can thus ensure that the result is solely from new physics. Similarly as in the SM case, we estimate the number of B-meson pairs N_B needed for detection at 1σ level for *maximally allowed* values of CP-odd parameters $\mathrm{Im}(\zeta)$ of Eq. (51) and (54). We again find the optimal observable gives much better results than the simple rate asymmetry.

The results in Table III show that CP violation effects from new physics are readily observed in the forthcoming asymmetric B factories, by using optimal observables. As expected, $B_{\tau4}$ decay modes give better results than $B_{\mu4}$ cases for the MHD model, where the CP-odd parameter itself is proportional to the lepton mass. For example, the current bounds in the MHD model

$$\mathrm{Im}(\zeta_{\rm MHD})=2.06(0.12)\quad\text{for}\quad\tau(\mu)$$

directly result from the lepton mass dependence. But there is no such dependence in the SLQ models. The current numerical values of CP-odd parameters in the SLQ models,

$$\mathrm{Im}(\zeta^{\rm I}_{\rm SLQ})=2.76(0.37),$$

$$\mathrm{Im}(\zeta^{\rm II}_{\rm SLQ})=18.4(1.84)\quad\text{for}\quad\tau(\mu),$$

are just from different experimental bounds. Therefore, the smaller CP-odd value for μ family is a consequence of the fact that the current experimental constraints on the muon mode are more stringent. $B_{\tau 4}$ decay modes would provide more stringent constraints to all the extended models that we have considered.

In conclusion, we have investigated direct CP violations from physics beyond the SM as well as within the SM through semileptonic B_{l4} decays: $B^{\pm}\to\pi^{+}\pi^{-}l^{\pm}\nu_l$. Within the SM, CP violation could be generated through interference between resonances with different quark flavors, that is, with different CKM matrix elements. We included $u\bar{u}$ state mesons (ρ, f_0 and f_2) and D meson as intermediate resonances which decay to $\pi^{+}\pi^{-}$. Using optimal observables, we found $\mathcal{O}(10^9)B$-meson pairs are needed to probe CP violation effects at 1σ level for the current maximal value of $\mathrm{Im}(\xi)=|V_{cb}/V_{ub}|=12.5$. We have also investigated CP violation effects in extensions of the SM. We considered multi-Higgs doublet model and scalar-leptoquark models. Here CP violation is implemented through interference between W-exchange diagrams and scalar-exchange diagrams with complex couplings in the extended models. We calculated the CP-odd rate asymmetry and the optimal asymmetry for $B_{\tau 4}$ and $B_{\mu 4}$ decay modes. We found that the optimal asymmetries for both modes are sizable and can be detected at 1σ level with about 10^3–10^7 B-meson pairs, for maximally allowed values of CP-odd parameters. Since $\sim 10^8$ B-meson pairs are expected to be produced yearly at the asymmetric B factories, one could easily investigate CP-violation effects in these decay modes B_{l4} to extract much more stringent constraints on CP-odd parameters, $\mathrm{Im}(\zeta_{\mathrm{MHD}})$ and $\mathrm{Im}(\zeta^{\mathrm{I,II}}_{\mathrm{SLQ}})$.

ACKNOWLEDGMENTS

We thank G. Cvetic for carefully reading of the manuscript and his valuable comments. The work of C.S.K. was supported in part by KRF Non-Directed-Research-Fund, Project No. 1997-001-D00111, in part by the BSRI Program, Ministry of Education, Project No. 98-015-D00061, in part by the KOSEF-DFG large collaboration project, Project No. 96-0702-01-01-2. J.L. and W.N. wish to acknowledge the financial support of 1997-sughak program of Korean Research Foundation.

APPENDIX A: FORM FACTORS

Form factors in Eq. (15) within ISGW model [17] are

$$u_{+}(q^2)=-F_5(q^2;f_0)\frac{m_u m_b m_q}{\sqrt{6}\beta_B\tilde{m}_{f_0}\mu_-},\quad u_{+}(q^2)=F_5(q^2;f_0)\frac{m_u(\tilde{m}_B+\tilde{m}_{f_0})}{\sqrt{6}\beta_B\tilde{m}_{f_0}},\tag{A1}$$

$$g(q^2)=\frac{F_3(q^2;\rho)}{2}\left[\frac{1}{m_q}-\frac{m_u\beta_B^2}{2\mu_-\tilde{m}_\rho\beta_{B\rho}^2}\right],\quad f(q^2)=2\tilde{m}_B F_3(q^2;\rho),$$

$$a_{+}(q^2)=-\frac{F_3(q^2;\rho)}{2\tilde{m}_\rho}\left[1+\frac{m_u}{m_b}\left(\frac{\beta_B^2-\beta_\rho^2}{\beta_B^2+\beta_\rho^2}\right)-\frac{m_u^2\beta_\rho^2}{4\mu_-\tilde{m}_B\beta_{B\rho}^4}\right],$$

$$a_{-}(q^2)=\frac{F_3(q^2;\rho)}{2\tilde{m}_\rho}\left[1+\frac{m_u}{m_b}\left(1+\frac{m_u\beta_\rho^2}{m_q\beta_{B\rho}^2}\right)-\frac{m_u^2\beta_\rho^2}{4\mu_+\tilde{m}_B\beta_{B\rho}^4}\right],\tag{A2}$$

$$h(q^2)=F_5(q^2;f_2)\frac{m_u}{2\sqrt{2}\tilde{m}_B\beta_B}\left[\frac{1}{m_q}-\frac{m_u\beta_B^2}{2\mu_-\tilde{m}_{f_2}\beta_{Bf_2}^2}\right],\quad k(q^2)=\sqrt{2}\,\frac{m_u}{\beta_B}F_5(q^2;f_2),$$

$$b_{+}(q^2)=-\frac{F_5(q^2;f_2)m_u}{2\sqrt{2}\beta_B\tilde{m}_{f_2}m_b}\left[1-\frac{m_u\beta_{f_2}^2}{\tilde{m}_B\beta_{Bf_2}^2}-\frac{m_u^2 m_b\beta_{f_2}^4}{4\mu_-\tilde{m}_B^2\beta_{Bf_2}^4}\right],$$

$$b_{-}(q^2)=\frac{F_5(q^2;f_2)m_u}{2\sqrt{2}\beta_B\tilde{m}_{f_2}m_b}\left[1+\frac{m_u^2\beta_{f_2}^2}{m_q\tilde{m}_B\beta_{Bf_2}^2}-\frac{m_u^2 m_b\beta_{f_2}^4}{4\mu_+\tilde{m}_B^2\beta_{Bf_2}^4}\right],\tag{A3}$$

$$f_{+}(q^2)=F_3(q^2;D)\left[1+\frac{m_b}{2\mu_-}-\frac{m_b m_q m_u\beta_B^2}{4\mu_+\mu_-\tilde{m}_D\beta_{BD}^2}\right],$$

$$f_-(q^2)=F_3(q^2;D)\left[1-(\tilde{m}_B+\tilde{m}_D)\left(\frac{1}{2m_q}-\frac{m_u\beta_B^2}{4\mu_+\tilde{m}_D\beta_{BD}^2}\right)\right], \tag{A4}$$

where

$$\beta_{BX}^2=\frac{1}{2}(\beta_B^2+\beta_X^2), \quad \mu_\pm=\left(\frac{1}{m_q}\pm\frac{1}{m_b}\right)^{-1} \tag{A5}$$

and

$$F_n(q^2;X)=\left(\frac{\tilde{m}_X}{\tilde{m}_B}\right)^{1/2}\left(\frac{\beta_B\beta_X}{\beta_{BX}^2}\right)^{n/2}\exp\left[-\left(\frac{m_u^2}{4\tilde{m}_B\tilde{m}_X}\right)\frac{q_m-q^2}{\kappa^2\beta_{BX}^2}\right], \tag{A6}$$

where relativistic compensation factor $\kappa=0.7$, and q_m is the maximum value of q^2:

$$q_m=(m_B-\sqrt{s_M})^2, \tag{A7}$$

and m_q is m_u for $u\bar{u}$ state mesons and m_c for D mesons. The numerical values of β_X in GeV unit are

$$\beta_B=0.41, \quad \beta_D=0.39, \quad \beta_{f_0}=0.27, \quad \beta_\rho=0.31,$$

$$\beta_{f_2}=0.27, \tag{A8}$$

and quark masses in GeV unit are

$$m_u=0.33, \quad m_c=1.82, \quad m_b=5.12. \tag{A9}$$

The so-called mock meson masses $\tilde{m}_X$ are defined as

$$\tilde{m}_B=m_b+m_u, \quad \tilde{m}_D=m_c+m_u, \quad \tilde{m}_{\rho,f_0,f_2}=2m_u. \tag{A10}$$

APPENDIX B: KINEMATICS

Spherical harmonics:

$$Y_0^0=\tilde{Y}_0^0=\frac{1}{\sqrt{4\pi}},$$

$$Y_1^0=\sqrt{\frac{3}{4\pi}}\cos\theta, \quad Y_1^{\pm1}=\mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi},$$

$$Y_2^0=\sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right), \quad Y_2^{\pm1}=\mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\phi}, \tag{B1}$$

Polarization vectors. In the B rest frame, where the coordinates are chosen such that the z axis is along the M_i momentum and the charged lepton momentum is in the x-z plane with positive x component [cf. Fig. 2(a)], the polarization vectors for the virtual W are

$$\epsilon(q,\pm)^\mu=\mp\frac{1}{\sqrt{2}}(0,1,\mp i,0),$$

$$\epsilon(q,0)^\mu=\frac{1}{\sqrt{q^2}}(p_M,0,0,-q^0),$$

$$\epsilon(q,s)^\mu=\frac{1}{\sqrt{q^2}}q^\mu, \tag{B2}$$

and the polarization states of the spin 1 mesons are

$$\epsilon(\pm1)^\mu=\mp\frac{1}{\sqrt{2}}(0,1,\pm i,0), \quad \epsilon(0)^\mu=\frac{1}{\sqrt{s_M}}(p_M,0,0,E_M), \tag{B3}$$

where $p_M=\sqrt{Q_+Q_-}/2m_B$ with $Q_\pm$ defined in Eq. (20), and $E_M=(m_B^2+s_M-q^2)/2m_B$. For the spin 2 meson we get

$$\epsilon(\pm2)^{\mu\nu}=\epsilon^\mu(\pm1)\epsilon^\nu(\pm1),$$

$$\epsilon(\pm1)^{\mu\nu}=\frac{1}{\sqrt{2}}[\epsilon^\mu(\pm1)\epsilon^\nu(0)+\epsilon^\mu(0)\epsilon^\nu(\pm1)],$$

$$\epsilon(0)^{\mu\nu}=\frac{1}{\sqrt{6}}[\epsilon^\mu(+1)\epsilon^\nu(-1)+\epsilon^\mu(-1)\epsilon^\nu(+1)]+\sqrt{\frac{2}{3}}\epsilon^\mu(0)\epsilon^\nu(0). \tag{B4}$$

In the W rest frame the polarization states of the virtual W are

$$\epsilon(q,\pm)^\mu=\mp\frac{1}{\sqrt{2}}(0,1,\mp i,0),$$

$$\epsilon(q,0)^\mu=(0,0,0,-1),$$

$$\epsilon(q,s)^\mu=\frac{1}{\sqrt{q^2}}q^\mu=(1,0,0,0). \tag{B5}$$

APPENDIX C: CP-EVEN AND CP-ODD QUANTITIES

1. Within the SM

The CP-even quantity $\mathcal{S}$ is

$$\mathcal{S}=2C(q^2,s_M)\Sigma, \tag{C1}$$

with

$$\begin{aligned}\Sigma=&(L_0^- S_0^0 Y_0^0)^2|\Pi_{f_0}|^2+(L_0^- P_0^0 Y_0^0)^2|\xi|^2|\Pi_D|^2+|\langle V^-\rangle\Pi_\rho|^2+|\langle T^-\rangle\Pi_{f_2}|^2+2(L_0^- S_0^0 Y_0^0)\mathrm{Re}(\Pi_{f_0}\Pi_\rho^*\langle V^-\rangle^*+\Pi_{f_0}\Pi_{f_2}\langle T^-\rangle^*)\\&+2\ \mathrm{Re}(\Pi_\rho\Pi_{f_2}^*\langle V^-\rangle\langle T^-\rangle^*)+2(L_0^- P_0^0 Y_0^0)\mathrm{Re}(\xi)[(L_0^- S_0^0 Y_0^0)\mathrm{Re}(\Pi_D\Pi_{f_0}^*)+\mathrm{Re}(\Pi_D\Pi_\rho^*\langle V^-\rangle^*+\Pi_D\Pi_{f_2}^*\langle T^-\rangle^*)]\\&+(L_0^+ S_0^0 Y_0^0-L_s^+ S_s^0 Y_0^0)^2|\Pi_{f_0}|^2+(L_0^+ P_0^0 Y_0^0-L_s^+ P_s^0 Y_0^0)^2|\xi|^2|\Pi_D|^2+|\Pi_\rho|^2|\langle V^+\rangle-L_s^+ V_s^0 Y_1^0|^2+|\Pi_{f_2}|^2|\langle T^+\rangle-L_s^+ T_s^0 Y_2^0|^2\\&+2(L_0^+ S_0^0-L_s^+ S_s^0)Y_0^0[-(L_s^+ V_s^0 Y_1^0)\mathrm{Re}(\Pi_{f_0}\Pi_\rho^*)+\mathrm{Re}(\Pi_{f_0}\Pi_\rho^*\langle V^+\rangle^*)-(L_s^+ T_s^0 Y_2^0)\mathrm{Re}(\Pi_{f_0}\Pi_{f_2}^*)+\mathrm{Re}(\Pi_{f_0}\Pi_{f_2}^*\langle T^+\rangle^*)]\\&+2\ \mathrm{Re}(\Pi_\rho\Pi_{f_2}^*\langle V^+\rangle\langle T^+\rangle^*)+2(L_s^+ V_s^0 Y_1^0)(L_s^+ T_s^0 Y_2^0)\mathrm{Re}(\Pi_\rho\Pi_{f_2}^*)-2(L_s^+ T_s^0 Y_2^0)\mathrm{Re}(\Pi_\rho\Pi_{f_2}^*\langle V^+\rangle)\\&-2(L_s^+ V_s^0 Y_1^0)\mathrm{Re}(\Pi_\rho\Pi_{f_2}^*\langle T^+\rangle^*)+2(L_0^+ P_0^0-L_s^+ P_s^0)Y_0^0\ \mathrm{Re}(\xi)[(L_0^+ S_0^0-L_s^+ S_s^0)Y_0^0\ \mathrm{Re}(\Pi_D\Pi_{f_0}^*)-(L_s^+ V_s^0 Y_1^0)\mathrm{Re}(\Pi_D\Pi_\rho^*)\\&+\mathrm{Re}(\Pi_D\Pi_\rho^*\langle V^+\rangle^*)-(L_s^+ T_s^0 Y_2^0)\mathrm{Re}(\Pi_D\Pi_{f_2}^*)+\mathrm{Re}(\Pi_D\Pi_{f_2}^*\langle T^+\rangle^*)],\end{aligned} \tag{C2}$$

and the CP-odd quantity $\mathcal{D}$ is

$$\mathcal{D}=2\ \mathrm{Im}(\xi)C(q^2,s_M)\Delta, \tag{C3}$$

with

$$\begin{aligned}\Delta=&-2(L_0^- P_0^0 Y_0^0)[(L_0^- S_0^0 Y_0^0)\mathrm{Im}(\Pi_D\Pi_{f_0}^*)+\mathrm{Im}(\Pi_D\Pi_\rho^*\langle V^-\rangle^*)+\mathrm{Im}(\Pi_D\Pi_{f_2}^*\langle T^-\rangle^*)]\\&-2(L_0^+ P_0^0-L_s^+ P_s^0)Y_0^0[(L_0^+ S_0^0-L_s^+ S_s^0)Y_0^0\ \mathrm{Im}(\Pi_D\Pi_{f_0}^*)-(L_s^+ V_s^0 Y_1^0)\mathrm{Im}(\Pi_D\Pi_\rho^*)+\mathrm{Im}(\Pi_D\Pi_\rho^*\langle V^+\rangle^*)\\&-(L_s^+ T_s^0 Y_2^0)\mathrm{Im}(\Pi_D\Pi_{f_2}^*)+\mathrm{Im}(\Pi_D\Pi_{f_2}^*\langle T^+\rangle^*)],\end{aligned} \tag{C4}$$

where

$$\langle V^\pm\rangle\equiv\sum_{i=0,\pm1} L_\lambda^\pm V_\lambda^\lambda Y_1^\lambda,\quad \langle T^\pm\rangle\equiv\sum_{i=0,\pm1} L_\lambda^\pm T_\lambda^\lambda Y_2^\lambda, \tag{C5}$$

and the overall function $C(q^2,s_M)$ is given by

$$C(q^2,s_M)=|V_{ub}|^2\frac{G_F^2}{2}\frac{1}{2m_B}\frac{(q^2-m_l^2)\sqrt{Q_+Q_-}}{256\pi^3 m_B^2 q^2}. \tag{C6}$$

2. With a complex scalar coupling

The CP-even quantity $\mathcal{S}$ is

$$\mathcal{S}=2C(q^2,s_M)\Sigma, \tag{C7}$$

with

$$\Sigma=(L_0^- S_0^0 Y_0^0)^2|\Pi_{f_0}|^2+|\langle V^-\rangle\Pi_\rho|^2+|\langle T^-\rangle\Pi_{f_2}|^2+2(L_0^- S_0^0 Y_0^0)\mathrm{Re}(\Pi_{f_0}\Pi_\rho^*\langle V^-\rangle^*+\Pi_{f_0}\Pi_{f_2}^*\langle T^-\rangle^*)+2\,\mathrm{Re}(\Pi_\rho\Pi_{f_2}^*\langle V^-\rangle$$
$$\times\langle T^-\rangle^*)+|\Pi_{f_0}|^2|L_0^+S_0^0Y_0^0-(1-\zeta')L_s^+S_s^0Y_0^0|^2+|\Pi_\rho|^2[|\langle V^+\rangle|^2+(L_s^+V_s^0Y_1^0)^2|1-\zeta'|^2-2(L_s^+V_s^0Y_1^0)\mathrm{Re}(\langle V^+\rangle)$$
$$\times\mathrm{Re}(1-\zeta')]+|\Pi_{f_2}|^2[|\langle T^+\rangle|^2+(L_s^+T_s^0Y_2^0)^2|1-\zeta'|^2-2(L_s^+T_s^0Y_2^0)\mathrm{Re}(\langle T^+\rangle)\mathrm{Re}(1-\zeta')]+2\,\mathrm{Re}(\Pi_{f_0}\Pi_\rho^*)$$
$$\times[(L_0^+S_0^0-L_s^+S_s^0)Y_0^0\,\mathrm{Re}(\langle V^+\rangle)-(L_0^+S_0^0Y_0^0)(L_s^+V_s^0Y_1^0)\mathrm{Re}(1-\zeta')+(L_s^+S_s^0Y_0^0)\mathrm{Re}(\langle V^+\rangle)\mathrm{Re}(\zeta')+(L_s^+S_s^0Y_0^0)$$
$$\times(L_s^+V_s^0Y_1^0)|1-\zeta'|^2]+2\,\mathrm{Im}(\Pi_{f_0}\Pi_\rho^*)\mathrm{Im}(\langle V^+\rangle)[(L_0^+S_0^0-L_s^+S_s^0)Y_0^0+(L_s^+S_s^0Y_0^0)\mathrm{Re}(\zeta')]+2\,\mathrm{Re}(\Pi_{f_0}\Pi_{f_2}^*)$$
$$\times[(L_0^+S_0^0-L_s^+S_s^0)Y_0^0\,\mathrm{Re}(\langle T^+\rangle)-(L_0^+S_0^0Y_0^0)(L_s^+T_s^0Y_2^0)\mathrm{Re}(1-\zeta')+(L_s^+S_s^0Y_0^0)\mathrm{Re}(\langle T^+\rangle)\mathrm{Re}(\zeta')+(L_s^+S_s^0Y_0^0)$$
$$\times(L_s^+T_s^0Y_2^0)|1-\zeta'|^2]+2\,\mathrm{Im}(\Pi_{f_0}\Pi_{f_2}^*)\mathrm{Im}(\langle T^+\rangle)[(L_0^+S_0^0-L_s^+S_s^0)Y_0^0+(L_s^+S_s^0Y_0^0)\mathrm{Re}(\zeta')]+2\,\mathrm{Re}(\Pi_\rho\Pi_{f_2}^*)$$
$$\times[\mathrm{Re}(\langle V^+\rangle\langle T^+\rangle^*)-(L_s^+T_s^0Y_2^0)\mathrm{Re}(\langle V^+\rangle)+(L_s^+T_s^0Y_2^0)\mathrm{Re}(\langle V^+\rangle)\mathrm{Re}(\zeta')-(L_s^+V_s^0Y_1^0)\mathrm{Re}(\langle T^+\rangle)\mathrm{Re}(1-\zeta')$$
$$+(L_s^+V_s^0Y_1^0)(L_s^+T_s^0Y_2^0)|1-\zeta'|^2]-2\,\mathrm{Im}(\Pi_\rho\Pi_{f_2}^*)[\mathrm{Im}(\langle V^+\rangle\langle T^+\rangle^*)-(L_s^+T_s^0Y_2^0)\mathrm{Im}(\langle V^+\rangle)$$
$$+(L_s^+T_s^0Y_2^0)\mathrm{Im}(\langle V^+\rangle)\mathrm{Re}(\zeta')+(L_s^+V_s^0Y_1^0)\mathrm{Im}(\langle T^+\rangle)\mathrm{Re}(1-\zeta')], \tag{C8}$$

and the CP-odd quantity $\mathcal{D}$ is

$$\mathcal{D}=2\,\mathrm{Im}(\zeta')C(q^2,s_M)\Delta, \tag{C9}$$

with

$$\Delta=2[\mathrm{Im}(\langle V^+\rangle)\{(L_s^+V_s^0Y_1^0)|\Pi_\rho|^2+(L_s^+S_s^0Y_0^0)\mathrm{Re}(\Pi_{f_0}\Pi_\rho^*)+(L_s^+T_s^0Y_2^0)\mathrm{Re}(\Pi_\rho\Pi_{f_2}^*)\}+\mathrm{Im}(\langle T^+\rangle)\{(L_s^+T_s^0Y_2^0)|\Pi_{f_2}|^2$$
$$+(L_s^+S_s^0Y_0^0)\mathrm{Re}(\Pi_{f_0}\Pi_{f_2}^*)+(L_s^+V_s^0Y_1^0)\mathrm{Re}(\Pi_\rho\Pi_{f_2}^*)\}+\mathrm{Re}(\langle V^+\rangle)\{(L_s^+T_s^0Y_2^0)\mathrm{Im}(\Pi_\rho\Pi_{f_2}^*)-(L_s^+S_s^0Y_0^0)\mathrm{Im}(\Pi_{f_0}\Pi_\rho^*)\}$$
$$-\mathrm{Re}(\langle T^+\rangle)\{(L_s^+V_s^0Y_1^0)\mathrm{Im}(\Pi_\rho\Pi_{f_2}^*)+(L_s^+S_s^0Y_0^0)\mathrm{Im}(\Pi_{f_0}\Pi_{f_2}^*)\}+(L_0^+S_0^0Y_0^0)(L_s^+V_s^0Y_1^0)\mathrm{Im}(\Pi_{f_0}\Pi_\rho^*)+(L_0^+S_0^0Y_0^0)$$
$$\times(L_s^+T_s^0Y_2^0)\mathrm{Im}(\Pi_{f_0}\Pi_{f_2}^*)]. \tag{C10}$$

Note that since every term in Δ of Eq. (C10) contains square terms of L_i^+ which are proportional to m_l [see Eq. (12)], the CP-odd quantity $\mathcal{D}$ of Eq. (C9) is proportional to lepton mass due to the definition of ζ' [see Eq. (44)].

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