Survival probability of large rapidity gaps in a three channel model

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The values and energy dependence for the survival probability $\langle|S|^2\rangle$ of large rapidity gaps (LRGs) are calculated in a three channel model. This model includes single and double diffractive production, as well as elastic rescattering. It is shown that $\langle|S|^2\rangle$ decreases with increasing energy, in line with recent results for LRG dijet production at the Fermilab Tevatron, this in spite of the weak dependence on energy of the ratio $(\sigma_{el}+\sigma_{SD})/\sigma_{tot}$. [S0556-2821(99)06919-2]

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I. INTRODUCTION

A large rapidity gap (LRG) process is defined as a gap where no hadrons are produced in a sufficiently large rapidity region. Historically, both Dokshitzer *et al.* [1] and Bjorken [2] suggested utilizing LRGs as a signature for Higgs production in a W-W fusion process in hadron-hadron collisions. It turns out that the LRG processes give a unique opportunity to measure the high energy asymptotic behavior of the amplitudes at short distances, where one can use the methods of perturbative QCD (PQCD) developed to calculate the amplitudes. Consider a typical LRG process—the production of two jets with large transverse momenta $\vec{p}_{t1} \approx -\vec{p}_{t2} \gg \mu$, with a LRG between the two jets. μ is a typical mass scale of ''soft'' interactions. We have the reaction

$$\begin{aligned} p(1)+p(2) \to & M_1[\text{hadrons}+\text{jet}_1(y_1,p_{t1})] \\ & +\text{LRG}[\Delta y=|y_1-y_2|] \\ & +M_2[\text{hadrons}+\text{jet}_2(y_2,p_{t2})], \end{aligned} \tag{1}$$

where y_1 and y_2 are the rapidities of the jets and $\Delta y = |y_1-y_2| \gg 1$. The production of two jets with LRGs between them can occur because of the following.

(1) A fluctuation in the rapidity distribution of a typical inelastic event. However, the probability for such a fluctuation is proportional to $e^{-\Delta y/L}$, where L denotes the value of the correlation length. We can evaluate $L \approx 1/(dn/dy)$, where dn/dy is the number of particles per unit in rapidity. A LRG means that $\Delta y \gg L$ and so the probability is small.

(2) The exchange of a colorless state in QCD. This exchange is given by the amplitude of the high energy interaction at short distances. We denote it as a ''hard'' Pomeron in Fig. 1. We denote by F_s the ratio of the cross section due to this Pomeron exchange, to the typical inelastic event cross section generated by gluon exchange (see Fig. 1). In QCD we do not expect this ratio to decrease as a function of the rapidity gap $\Delta y = y_1 - y_2$. For a Balitskii-Fadin-Kuraev-Lipatov (BFKL) Pomeron [3], we expect an increase once $\Delta y \gg 1$. Using a simple QCD model for the Pomeron, namely, one in which it can be approximated by two gluon exchange [4], Bjorken [2] gave the first estimate for $F_s \approx 0.15$, which is unexpectantly large.

As noted by Bjorken, we are not able to measure F_s directly in a LRG experiment. The experimentally measured ratio of the number of events with a LRG to the number of events without a LRG (see Fig. 1) is not equal to F_s, but has to be modified by an extra factor which is called the survival probability of LRGs:

$$f_{gap} = \langle|S|^2\rangle F_s. \tag{2}$$

The appearance of $\langle|S|^2\rangle$ in Eq. (2) has a very simple physical interpretation. It is the probability that the rapidity gap due to Pomeron exchange will not be filled by the produced particles (partons and/or hadrons) from the rescattering of spectator partons [see Fig. 2(a)], or from the emission of bremsstrahlung gluons from partons taking part in the ''hard'' interaction, or from the ''hard'' Pomeron [see Fig. 2(b)]:

$$\langle|S|^2\rangle = \langle|S_{bremsstrahlung}(\Delta y=|y_1-y_2|)|^2\rangle \langle|S_{spectator}(s)|^2\rangle, \tag{3}$$

where s denotes the total c.m. energy squared.

(i) $\langle|S_{bremsstrahlung}(\Delta y)|^2\rangle$ can be calculated in PQCD [5]; it depends on the kinematics of each specific process and on the value of the LRG.

(ii) To calculate $\langle|S_{spectator}(s)|^2\rangle$ we need to find the probability that all partons with rapidity $y_i > y_1$ in the first hadron [see Fig. 2(a)] and all partons with $y_j < y_2$ in the second hadron do not interact inelastically and, hence, do not produce additional hadrons in the LRG. This is a difficult problem, since not only do partons at short distances contrib-

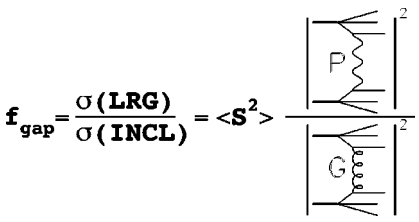

FIG. 1. Pictorial definition of f_{gap}, where P and G represent, respectively, the exchange of a color singlet and a color octet.

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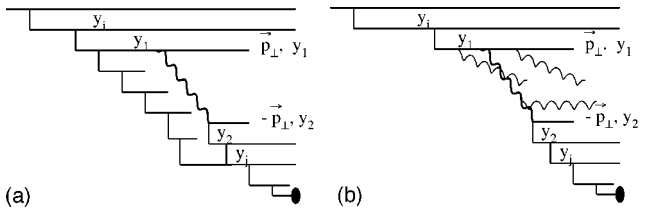

FIG. 2. Rescatterings of the spectator partons and emission of the bremsstrahlung gluons that fill the LRG.

ute to such a calculation, but also partons at long distances for which the PQCD approach is not valid. Many attempts have been made to estimate $\langle|S_{spectator}(s)|^2\rangle$ [2,6–11] but this problem still awaits a solution.

On the other hand, experimental studies of LRG processes have progressed, and have yielded interesting results both at the Tevatron [12,13], and at HERA [14].

The main results of the experimental data pertaining to the survival probability are the following.

(i) The value of $\langle|S|^2\rangle$ is rather small. Indeed, the experimental values for $f_{gap}(\sqrt{s}=630\ \text{GeV})=1.6\pm0.2\%$ (DØ [13]) $=2.7\pm0.9\%$ [Collider Detector at Fermilab (CDF) [12]] and $f_{gap}(\sqrt{s}=1800\ \text{GeV})=0.6\pm0.2\%$ (DØ [13]) $=1.13\pm0.16\%$ (CDF [12]) can be understood only if $\langle|S|^2\rangle\approx1-10\%$.

(ii) The energy dependence of f_{gap}, namely, the obsevation that [12,13]

$$R_{gap}^{CDF}=\frac{f_{gap}(\sqrt{s}=630\ \text{GeV})}{f_{gap}(\sqrt{s}=1800\ \text{GeV})}=2.4\pm0.9,$$

$$R_{gap}^{D\emptyset}=\frac{f_{gap}(\sqrt{s}=630\ \text{GeV})}{f_{gap}(\sqrt{s}=1800\ \text{GeV})}=2.67\pm0.38,$$

leads to $\langle|S|^2\rangle$ which decreases by a factor of 2 in the above range of energy. So we expect that

$$R_S=\frac{\langle|S|^2\rangle_{\sqrt{s}=630}}{\langle|S|^2\rangle_{\sqrt{s}=1800}}\approx2.$$

A recent DØ estimate [13] of the above ratio is 2.2 ± 0.8.

It was shown in Ref. [11] that the eikonal model for the ''soft'' interaction at high energies is able to describe the features of the experimental data. However, we may question the reliability of this approach. Especially worrying is the energy dependence of $\langle|S|^2\rangle$, since a natural parameter related to the parton rescatterings is the ratio

$$R_D=\frac{\sigma_D}{\sigma_{tot}}\equiv\frac{\sigma_{el}+\sigma_{SD}+\sigma_{DD}}{\sigma_{tot}},\tag{4}$$

where σ_{SD} and σ_{DD} are the cross sections of single and double diffraction. Experimentally, R_D is approximately constant over a wide range of energy. In the eikonal model we consider $\sigma_{SD}\ll\sigma_{el}$ and $\sigma_{DD}\ll\sigma_{el}$ and, therefore, we model $R_D\to R_{el}=\sigma_{el}/\sigma_{tot}$. Experimentally, R_{el} depends on energy, which gives rise to a considerable decrease of the survival probability. Indeed, the first attempt to take into account the parameter R_D in the calculation of $\langle|S|^2\rangle$ given in Ref. [9] leads to

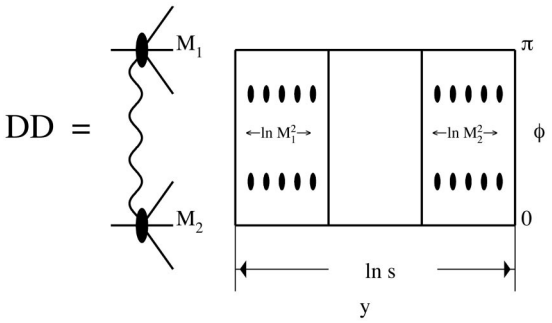

FIG. 3. Lego plot for double diffractive dissociation.

$$\langle|S|^2\rangle=[1-2R_D]^2,\tag{5}$$

which yields a reasonable value of the survival probability, but cannot account for its substantial dependence on energy (see Ref. [11]).

The main goal of this paper is to develop a simple model for ''soft'' high energy interactions, which correctly includes the processes of diffractive dissociation, and to study the value and energy dependence of $\langle|S_{spectator}(s)|^2\rangle$. In Sec. II we develop our approach elaborating in Sec. II A on a general treatment of diffraction dissociation in a multichannel representation. The role of shadowing corrections and eikonalization are discussed in Sec. II B. The formulation of the eikonal model with our simplified assumptions is briefly reviewed in Sec. II C and our three channel model is specified in Sec. II D. The relevant expressions of the output observables derived from our model are given in Secs. II E and II F. Section III is devoted to a numerical evaluation of the spectator survival probability. We show that our model gives the experimentally observed decrease of the survival probability as a function of energy. A summary of our results and a short discussion are presented in Sec. IV.

II. THREE CHANNEL MODEL

A. Diffractive dissociation (general approach)

Diffractive dissociation is the simplest process with a LRG in which no hadrons are produced in the central rapidity region. In these processes we have production of one [single diffraction (SD)] or two [double diffraction (DD)] groups of hadrons with masses (M_1 and M_2 in Fig. 3) much less than the total energy ($M_1\ll\sqrt{s}$ and $M_2\ll\sqrt{s}$).

From a theoretical point of view, as was suggested by Feinberg [15] and Good and Walker [16], diffractive dissociation can be viewed as *a typical quantum mechanical process which occurs since the hadron states are not diagonal with respect to the strong interaction scattering matrix.*

We consider this point in more detail, and denote the wave functions which are diagonal with respect to the strong interaction by Ψ_n. The quantum numbers n are called the correct degrees of freedom at high energy. The amplitude of the high energy interaction is, therefore, given by

$$A_{n_1 n_2}=\langle \Psi_{n_1}\Psi_{n_2}|T|\Psi_{n_1'}\Psi_{n_2'}\rangle = A_{n_1,n_2}\delta_{n_1,n_1'}\delta_{n_2,n_2'}, \quad (6)$$

where the angular brackets denote all needed integrations and T is the scattering matrix.

The wave function of a hadron is

$$\Psi_{hadron}=\sum_{n=1}^{\infty} C_n \Psi_n . \quad (7)$$

For a hadron-hadron interaction we have a wave function $\Psi_{hadron}\times\Psi_{hadron}$ before the collision, while after the collision the scattering matrix T gives a new wave function

$$\begin{aligned}\Psi_{final}=&\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_1'=1}^{\infty}\sum_{n_2'=1}^{\infty}\\ &\times C_{n_1}C_{n_2}\langle \Psi_{n_1}\Psi_{n_2}|T|\Psi_{n_1'}\Psi_{n_2'}\rangle \Psi_{n_1'}\Psi_{n_2'}\\ =&\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}C_{n_2}A_{n_1,n_2}\Psi_{n_1}\Psi_{n_2}. \end{aligned} \quad (8)$$

Equation (8) leads to the elastic amplitude

$$a_{el}=\langle \Psi_{final}|\Psi_{hadron}\times\Psi_{hadron}\rangle = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 A_{n_1,n_2}, \quad (9)$$

and to another process, namely, to the production of other hadron states, since Ψ_{final} may be different from $\Psi_{hadron}\times\Psi_{hadron}$:

$$\begin{aligned}\sigma_D(s,b)=&\langle \Psi_{final}|\Psi_{final}\rangle - \langle \Psi_{final}|\Psi_{hadron}\times\Psi_{hadron}\rangle^2\\ =&\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_1}^2 A_{n_1,n_2}^2\\ &-\left(\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 A_{n_1,n_2}\right)^2. \end{aligned} \quad (10)$$

Using the normalization condition for the hadron wave function ($\Sigma_n C_n^2=1$), the cross section of the diffractive dissociation processes, Eq. (10), can be reduced to the form [16,17]

$$\sigma_D(s,b)=\langle |\sigma^2(s,b)|\rangle - \langle |\sigma(s,b)|\rangle^2, \quad (11)$$

where $\langle |f| \rangle \equiv \Sigma_{n_1}\Sigma_{n_2} C_{n_1}^2 C_{n_2}^2 f_{n_1,n_2}$ and we have returned to our original variables: energy (s) and impact parameter (b).

B. Diffractive dissociation and shadowing corrections

For $A_{n_1,n_2}(s,b)$, and only for $A_{n_1,n_2}(s,b)$, we have the unitarity constraint

$$2\,\mathrm{Im}\,A_{n_1,n_2}^{el}(s,b)=|A_{n_1,n_2}^{el}(s,b)|^2+G_{n_1,n_2}^{in}(s,b). \quad (12)$$

Assuming that the amplitude at high energy is predominantly imaginary, we obtain the solution of Eq. (12):

$$A_{n_1,n_2}^{el}(s,b)=i(1-e^{-\Omega_{n_1,n_2}(s,b)/2}), \quad (13)$$

$$G_{n_1,n_2}^{in}(s,b)=1-e^{-\Omega_{n_1,n_2}(s,b)}. \quad (14)$$

To find a relation between the processes of diffraction dissociation and the value of the shadowing corrections (SCs), we assume that $\Omega_{n_1,n_2}(s,b)\ll 1$ and expand Eqs. (12)–(14) with respect to Ω_{n_1,n_2}:

$$A_{n_1,n_2}^{el}(s,b)=\frac{\Omega_{n_1,n_2}(s,b)}{2}-\frac{\Omega_{n_1,n_2}^2(s,b)}{8}+O(\Omega_{n_1,n_2}^3), \quad (15)$$

$$G_{n_1,n_2}^{in}(s,b)=\Omega_{n_1,n_2}(s,b)-\frac{\Omega_{n_1,n_2}^2(s,b)}{2}+O(\Omega_{n_1,n_2}^3). \quad (16)$$

Using Eqs. (7)–(10) we obtain, for the observables,

$$\sigma_{el}(s,b)=\tfrac{1}{4}\left(\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 \Omega_{n_1,n_2}(s,b)\right)^2, \quad (17)$$

$$\begin{aligned}\sigma_{tot}=&\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 \Omega_{n_1,n_2}(s,b)\\ &-\tfrac{1}{4}\left(\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 \Omega_{n_1,n_2}^2(s,b)\right), \end{aligned} \quad (18)$$

$$\begin{aligned}\sigma_{in}=&\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 \Omega_{n_1,n_2}(s,b)\\ &-\tfrac{1}{2}\left(\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 \Omega_{n_1,n_2}^2(s,b)\right), \end{aligned} \quad (19)$$

$$\begin{aligned}\sigma_{diff}=&\tfrac{1}{4}\left\{\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 \Omega_{n_1,n_2}^2(s,b)\right.\\ &\left.-\left(\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} C_{n_1}^2 C_{n_2}^2 \Omega_{n_1,n_2}(s,b)\right)^2\right\}. \end{aligned} \quad (20)$$

In the parton model, one Pomeron exchange corresponds to a typical inelastic event with the production of a large number of particles. In this case, we can associate this exchange with Ω, and consequently $\sigma_{tot}^P(s,b)=\sigma_{in}^P(s,b)\propto\Omega$. All terms which are proportional to Ω^2 describe two Pomeron exchange, and they induce SCs.

We can evaluate the scale of the SCs using experimental data on σ_{tot}, σ_{el}, and σ_{SD}. Indeed, we can write the expression for the total cross section in the form

$$\sigma_{tot}=\sigma_{tot}^P-\Delta\sigma_{tot}^{SC}, \quad (21)$$

where σ_{tot}^P is the contribution of Pomeron exchange to the total cross section. Summing Eq. (17) and Eq. (20) we derive that

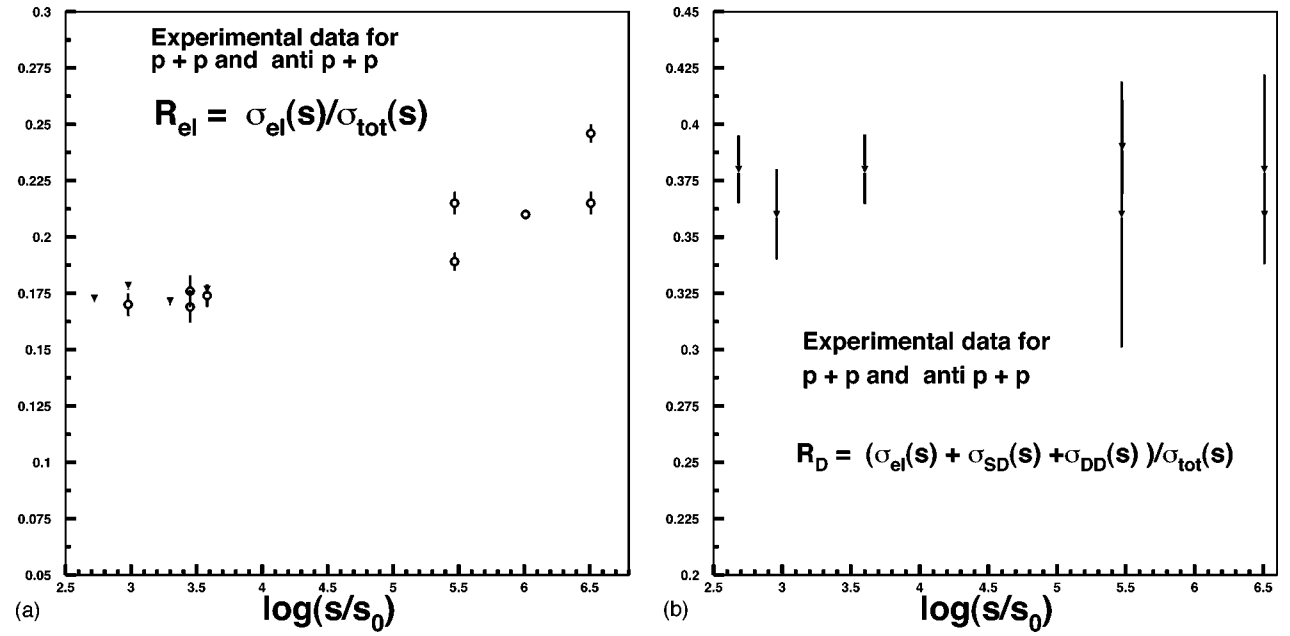

FIG. 4. The experimental data on the ratio $R_{el}=\sigma_{el}/\sigma_{tot}$ (a) and on the ratio $R_D=(\sigma_{el}+\sigma_{SD}+\sigma_{DD})/\sigma_{tot}$ (b) versus $\log(s/s_0)$ with $s_0=1$ GeV^2.

$$\Delta\sigma_{tot}^{SC}=\sigma_{el}+\sigma_{diff}=\sigma_{el}+\sigma_{SD}+\sigma_{DD}=\sigma_D. \tag{22}$$

The ratio R_D [see Eq. (5)] gives the scale of the SCs in the situation when these are sufficiently weak ($\Omega_{n_1,n_2}\ll 1$), since Eq. (22) can be rewritten in the form

$$R_D=\frac{\Delta\sigma_{tot}^{SC}}{\sigma_{tot}}. \tag{23}$$

Figure 4(b) shows that over a wide range of energy $R_D\approx 0.34$, and it appears to be independent of energy. The large value of R_D implies that SCs should be taken into account, and that SCs lead to a small value of the survival probability. The almost constant R_D suggests that the SCs cannot induce the observed strong energy dependence of the survival probability. However, the value of R_D is so large that we have to develop an approach to calculate SCs for $\Omega_{n_1,n_2}\approx 1$.

C. Eikonal model

We first discuss the eikonal model. This is an approximation which has been widely used to estimate the value of the SCs, in a situation when they are not small. The main assumption of this model is that hadrons are the correct degrees of freedom at high energy. In other words, we assume that the interaction matrix is diagonal with respect to hadrons. From Eqs. (7)–(11) one can see that this model does not include diffractive dissociation processes. Therefore, the accuracy of our estimates in the eikonal model will be given by the ratio $(\sigma_{SD}+\sigma_{DD})/\sigma_{el}$. From Fig. 4 one can see that this ratio is about 1 at $\sqrt{s}\approx 20$ GeV and decreases, reaching a value of about 0.4–0.5 at the Tevatron energies. Thus, we cannot expect the eikonal approach to yield a reasonable estimate. However, this model has the advantage of being simple, and it provides a good illustration, given below, of the main elements and approximations used in previous calculations.

1. Assumptions

(i) Hadrons are the correct degrees of freedom at high energies.

(ii) $\sigma_{SD}\ll\sigma_{el}$ and $\sigma_{DD}\ll\sigma_{el}$.

(iii) At high energy the scattering amplitude is almost pure imaginary, $\mathrm{Re}\,a_{el}\ll \mathrm{Im}\,a_{el}$.

(iv) Only the fastest partons can interact with each other.

The last assumption is the most restrictive, and clearly indicates how far from reality the eikonal model estimates could be.

2. Unitarity

In the eikonal model we only have one amplitude, since the scattering matrix is diagonal in the hadronic wave functions. Therefore, the unitarity constraints of Eq. (12) simplify to

$$2\,\mathrm{Im}\,a_{el}(s,b)=|a_{el}(s,b)|^2+G^{in}(s,b). \tag{24}$$

Equation (24) has the solution

$$a_{el}(s,b)=i(1-e^{-\Omega(s,b)/2}), \tag{25}$$

$$G^{in}(s,b)=1-e^{-\Omega(s,b)}. \tag{26}$$

3. Pomeron hypothesis

The main assumption of the eikonal model is the identification of the opacity $\Omega(s,b)$ with a single Pomeron exchange, namely,

$$\Omega(s,b)=\Omega^P(s,b), \tag{27}$$

$$\Omega^P(s,b)=\sigma_0\Gamma(b)=\frac{\sigma_0}{\pi R^2(s)}\left(\frac{s}{s_0}\right)^{\Delta_P}e^{-b^2/R^2(s)}$$

$$=\nu(s)e^{-b^2/R^2(s)}, \tag{28}$$

$$R^2(s)=4R_0^2+4\alpha'_P\ln(s/s_0), \tag{29}$$

$$\nu(s)=\frac{\sigma_0}{\pi R^2(s)}\left(\frac{s}{s_0}\right)^{\Delta_P}=\Omega^P \quad (s,b=0). \tag{30}$$

We assume a Pomeron trajectory $\alpha_P(t)=\alpha_P(0)+\alpha'_P t$. Equation (27) is a reasonable approximation in the kinematic region where Ω is small, i.e. either at low energies or at high energies when b is large. Therefore, the eikonal approach is the natural generalization of the single Pomeron exchange satisfying s-channel unitarity. Equation (27) is an explicit analytic expression for the well-known partonic picture for the Pomeron structure; namely, single Pomeron exchange is responsible for the inelastic production of particles which are uniformly distributed in rapidity.

4. Exponential parametrization

In Eq. (28) a Gaussian form is explicitly assumed for the profile function $\Gamma(b)$:

$$\Gamma(b)=\frac{1}{\pi R^2(s)}e^{-b^2/R^2(s)}. \tag{31}$$

This form is assumed due to its simplicity; the resulting integrals can be done analytically and we can write an explicit answer for the physical observables. The above Gaussian profile corresponds to an exponential form in t space, compatible with a convention Regge description [18]. Clearly, this is an oversimplification in as much as it only gives a good reproduction of the forward data with $|t|$, 0.3–0.5 GeV^2 (depending on s). Its main deficiency is that it produces a diffraction dip in $d\sigma/dt$ which is positioned at smaller $|t|$ values than experimentally observed. We note [18,25] that this deficiency, which is important in the study of medium and high t data, is not very significant in the study of the total and integrated elastic and diffractive cross sections. Consequently, we consider our approximation to be reasonably adequate in the present context, since our estimates of the survival probability are not sensitive to the details of medium and high $|t|$ properties. The proper definition of the profile function $\Gamma(b)$ is given by the following Fourier transform:

$$\Gamma(b)=\frac{1}{2\pi}\int d^2b e^{-i\vec{\mathbf{q}}\cdot\vec{\mathbf{b}}}G_P^2(q^2), \tag{32}$$

where $G_P(q^2)$ is the Pomeron form factor. For example, one can take for $G_P(q^2)$ the prediction of the additive quark model (see Ref. [19]) in which $G_P(q^2)\propto G_{em}$, where G_{em} is the electromagnetic form factor of a hadron. In general, our basic approach can be utilized numerically as long as the opacity Ω^P can be factorized into an energy dependent function $\nu(s)$ and b dependent profile $\Gamma(b)$ which can tolerate a weak (logarithmic) s dependence.

5. R_D

Using Eqs. (27)–(30) one can obtain a closed expression for the ''soft'' observables:

$$\sigma_{tot}=2\pi R^2(s)\{\ln(\nu(s)/2)+C-\mathrm{Ei}(-\nu/2)\}, \tag{33}$$

$$\sigma_{in}=\pi R^2(s)\{\ln(\nu(s))+C-\mathrm{Ei}(-\nu)\}; \tag{34}$$

$$\sigma_{el}(s)=\sigma_{tot}(s)-\sigma_{in}(s), \tag{35}$$

where $\mathrm{Ei}(x)=\int_{-\infty}^{x}(e^t/t)dt$ and $C=0.5773$.

We define

$$R_{el}=\frac{\sigma_{el}}{\sigma_{tot}}=\frac{\ln\left[\frac{\nu}{4}\right]+C+\mathrm{Ei}(-\nu)-2\,\mathrm{Ei}\left(-\frac{\nu}{2}\right)}{2\left\{\ln\left[\frac{\nu}{2}\right]+C-\mathrm{Ei}\left(-\frac{\nu}{2}\right)\right\}}. \tag{36}$$

Note that the ratio R_{el} depends only on ν and does not depend on the value of radius. Using Eq. (36) one can find the value of ν from the experimental data on R_{el} [see Fig. 4(a)]. This was done in Ref. [11]. With the value of ν determined in this way we can calculate the survival probability.

6. Survival probability

From Eq. (26) one can conclude that the factor

$$P(s,b)=e^{-\Omega(s,b)}, \tag{37}$$

is the probability that the two initial hadrons do not interact inelastically. In a QCD approach, this means that the fastest parton from one hadron does not interact with the fastest parton from another. Therefore, in the eikonal model the survival probability can be easily calculated in the following way [2,6]:

$$\langle|S_{spectator}(s)|^2\rangle=\frac{\int d^2bP(s,b)A_{HP}(\Delta y,b)}{\int d^2bA_{HP}(\Delta y,b)}, \tag{38}$$

where $A_{HP}(\Delta y,b)$ is the cross section for a two parton jet production with a LRG due to single ''hard'' Pomeron exchange (see Fig. 1). It has been proved that for a ''hard'' cross section, the b dependence can be factorized out [7,20,21]. If we assume $A_{HP}(\Delta y,b)$ to be Gaussian, we have

$$A_{HP}(\Delta y,b)=\sigma_{HP}(\Delta y)\Gamma_H(b)=\frac{\sigma_{HP}(\Delta y)}{\pi R_H^2}e^{-b^2/R_H^2}, \tag{39}$$

where R_H^2 is the radius of the ''hard'' interaction.

Based on this assumption we obtain, for the survival probability,

$$\langle |S_{spectator}(s)|^2 \rangle = \frac{a\gamma[a,\nu]}{\nu^a}, \tag{40}$$

where the incomplete gamma function $\gamma(a,x) = \int_0^x z^{a-1} e^{-z} dz$ and $a = R^2(s)/R_H^2$.

In Ref. [11], Eq. (36) and Eq. (40) were used to calculate the value and energy dependence of the survival probability. It was shown that both the value and the energy dependence are sensitive to the value of the ''hard'' radius, which was extracted in Ref. [11] from the experimental data on (i) vector meson diffractive dissociation in deep inelastic scattering (DIS) at HERA [26] and on (ii) the CDF double parton cross section at the Tevatron [22].

Even though the Eikonal model, as used in Ref. [11], can reproduce both the experimentally measured value and its energy behavior, the reliability of such an approach is questionable. In the following we attempt to construct a more realistic model.

D. Three channel model: Assumptions and general formulas

We want to construct a model which takes into account the processes of diffractive dissociation for the case when these are not small ($\sigma_{SD} \approx \sigma_{el}$).

1. Assumptions

The main idea behind the model which is presented here is to replace the many final states of the diffractively produced hadrons by one state (effective hadron). Doing so, we assume that we have two wave functions which are diagonal with respect to the strong interactions: Ψ_1 and Ψ_2. In this case the general equation (7) can be reduced to the form

$$\Psi_{hadron} = \alpha\Psi_1 + \beta\Psi_2, \tag{41}$$

with the condition $\alpha^2 + \beta^2 = 1$, which follows from the normalization of the wave function. The wave function of the diffractively produced bunch of hadrons should be orthogonal to Ψ_{hadron} and has the form

$$\Psi_D = -\beta\Psi_1 + \alpha\Psi_2. \tag{42}$$

Equation (42) is the explicit form of our assumption, which we replace the complicated final state of diffractively produced systems of hadrons by one wave function Ψ_D.

2. General formulas

Substituting Eq. (41) and Eq. (42) into Eq. (8) we can obtain

$$\Psi_{final} = \alpha^2 A_{1,1} \Psi_1 \times \Psi_1 + \alpha\beta A_{1,2} [\Psi_1 \times \Psi_2 + \Psi_2 \times \Psi_1] + \beta^2 A_{2,2} \Psi_2 \times \Psi_2. \tag{43}$$

The amplitudes $A_{i,k}$ $(i,k = 1,2)$ can be written in the form of Eq. (13) and Eq. (14) (see Fig. 5):

$$A^{el}_{i,k}(s,b) = i\{1 - e^{-\Omega_{i,k}(s,b)/2}\}, \tag{44}$$

$$G^{in}_{i,k}(s,b) = 1 - e^{-\Omega_{i,k}(s,b)}. \tag{45}$$

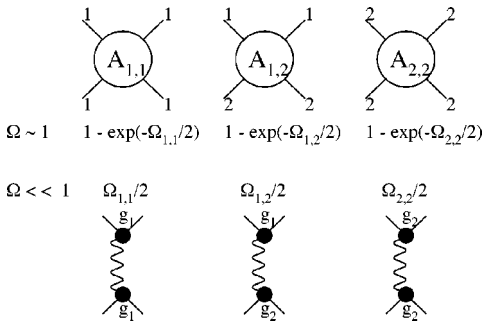

FIG. 5. Solution of the unitarity constraints and the Regge parametrization for the correct degrees of freedom in the three channel model. The wavy lines denote the exchange of the Pomeron.

The elastic amplitude is equal to

$$\begin{aligned} a_{el}(s,b) &= \langle \Psi_{hadron} \times \Psi_{hadron} | T | \Psi_{hadron} \times \Psi_{hadron} \rangle \\ &= \langle \Psi_{hadron} \times \Psi_{hadron} | \Psi_{final} \rangle \\ &= \alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}. \end{aligned} \tag{46}$$

For single diffraction we have the amplitude

$$\begin{aligned} a_{SD}(s,b) &= \langle \Psi_{hadron} \times \Psi_{hadron} | T | \Psi_D \times \Psi_{hadron} \rangle \\ &= \langle \Psi_{hadron} \times \Psi_D | \Psi_{final} \rangle \\ &= \alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2 - \beta^2) A_{1,2} + \beta^2 A_{2,2}\}, \end{aligned} \tag{47}$$

while the amplitude for double diffractive production is

$$\begin{aligned} a_{DD}(s,b) &= \langle \Psi_{hadron} \times \Psi_{hadron} | T | \Psi_D \times \Psi_D \rangle \\ &= \langle \Psi_D \times \Psi_D | \Psi_{final} \rangle \\ &= \alpha^2\beta^2\{A_{1,1} - 2A_{1,2} + A_{2,2}\}. \end{aligned} \tag{48}$$

Equations (46)–(48) together with Eq. (44) and Eq. (45) give the general formulas for our model.

In general this three channel model is an attempt to sum all rescatterings shown in Fig. 6. All of them are eikonal type rescatterings, but contrary to the eikonal model, the quasielastic rescatterings with production and rescattering of an effective diffractive state have been taken into account.

We call this model a three channel one because three physical processes—elastic scattering, and single and double diffraction—are included in it. All general formulas of Eqs. (6)–(11) were known long ago (see Ref. [23], for example). In Ref. [8] this general formalism was applied to obtain estimates of the value of the survival probability. We now develop a systematic analysis to obtain the value of $\langle |S^2| \rangle$ in the framework of the three channel model, utilizing the experimental data pertaining to the ''soft'' processes that have been measured.

E. Three channel model: Physical observables

For the opacities $\Omega_{i,k}$ we use the same approach as in the eikonal model [see Eqs. (27)–(30) and Fig. 5]:

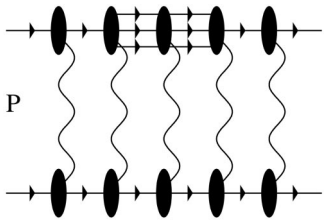

FIG. 6. The Pomeron interaction in the three channel model.

$$\Omega_{i,k}=\nu_{i,k}(s)e^{-b^2/R^2_{i,k}(s)}=\frac{g_k g_i}{\pi R^2_{i,k}(s)}\left(\frac{s}{s_0}\right)^{\Delta_P}e^{-b^2/R^2_{i,k}(s)}, \tag{49}$$

where we have used Reggeon factorization as is shown in Fig. 5. In this paper, we do not use the exact Reggeon dependence on energy, but we utilize the factorization properties which appear in Eq. (49). In addition we need to take into account the factorization properties of the radii

$$R^2_{i,k}=2R^2_{i0}+2R^2_{k0}+4\alpha'_P\ln(s/s_0), \tag{50}$$

where R^2_{i0} is a radius which describes the t dependence of the Pomeron-hadron i vertex. It is obvious from Eq. (49) and Eq. (50) that $\Omega_{i,k}$ can be written in the form

$$\Omega_{1,1}=\nu_1 X, \tag{51}$$

$$\Omega_{2.2}=\nu_2 X^{r/(2-r)}, \tag{52}$$

$$\Omega_{1,2}=\sqrt{\nu_1\nu_2 r(2-r)}X^r, \tag{53}$$

where $X=e^{-b^2/R^2_{1,1}(s)}$ and $r=R^2_{1,1}(s)/R^2_{1,2}(s)$.

Equations (51)–(53) together with Eqs. (45)–(48) allow us to express all physical observables through the variables ν_1, ν_2, r, and β. The first three variables depend on energy squared (s) [see Eq. (49) and Eq. (50)] while β is a constant in our model. From Eq. (49) we expect ν_2 to be proportional to ν_1 and r to be only weakly (logarithmically) dependent on energy.

From Eqs. (51)–(53) we have the following expressions for the amplitudes $A_{i,k}$ in terms of our variables:

$$A_{1,1}(\nu_1,X)=i(1-e^{-\nu_1 X/2}); \tag{54}$$

$$A_{2,2}(\nu_2,r,X)=i(1-e^{-\nu_2 X^{r/(2-r)}/2}), \tag{55}$$

$$A_{1,2}(\nu_1,\nu_2,r,X)=i(1-e^{-\sqrt{\nu_1\nu_2 r(2-r)}X^r/2}). \tag{56}$$

After some simple calculations we have

$$\sigma_{tot}=2\pi R^2_{1,1}\int_0^1\frac{dX}{X}\{(1-\beta^2)^2A_{1,1}+2(1-\beta^2)\beta^2A_{1,2}+\beta^4A_{2,2}\}; \tag{57}$$

$$\sigma_{el}=\pi R^2_{1,1}\int_0^1\frac{dX}{X}\{(1-\beta^2)^2A_{1,1}+2(1-\beta^2)\beta^2A_{1,2}+\beta^4A_{2,2}\}^2, \tag{58}$$

$$\sigma_{SD}=\pi R^2_{1,1}(1-\beta^2)\beta^2\int_0^1\frac{dX}{X}\{-(1-\beta^2)A_{1,1}+(1-2\beta^2)A_{1,2}+\beta^2A_{2,2}\}^2, \tag{59}$$

$$\sigma_{DD}=\pi R^2_{1,1}(1-\beta^2)^2\beta^4\int_0^1\frac{dX}{X}\{A_{1,1}-2A_{1,2}+A_{2,2}\}^2. \tag{60}$$

One can see that the ratio R_D, as well as R_{el}, does not depend on $R^2_{1,1}$. We will use these ratios to fix our variables from the experimental data.

F. Three channel model: Survival probability

To calculate the survival probability of the LRG in our three channel model we recall that the physical meaning of the factor

$$P_{i,k}=e^{-\Omega_{i,k}(s,b)}\equiv\{1-A_{i,k}\}^2 \tag{61}$$

is the probability that two hadronic states with quantum numbers i and k scatter, without any inelastic interaction at given energy s and impact parameter b. Therefore, we have to multiply the ''hard'' cross section of their interaction by $P_{i,k}$, and sum over all possible i and k for hadron-hadron collisions to obtain the survival probability. The cross section for two jet production with large transverse momenta and LRGs, can be reduced to the form

$$\sigma_H(s,b)=(1-\beta^2)^2\nu_1\frac{R^2_{1,1}}{\tilde{R}^2_{1,1}}e^{-b^2/\tilde{R}^2_{1,1}}+2(1-\beta^2)\beta^2\sqrt{\nu_1\nu_2 a(2-a)}\times\frac{R^2_{1,2}}{\tilde{R}^2_{1,2}}e^{-b^2/\tilde{R}^2_{1,2}}+\beta^4\nu_2\frac{R^2_{2,2}}{\tilde{R}^2_{2,2}}e^{-b^2/\tilde{R}^2_{2,2}}, \tag{62}$$

where $\tilde{R}^2_{i,k}$ denotes the ''hard'' interaction radius of two states i and k. Strictly speaking, $\tilde{R}^2_{i,k}=R^2_{i,k}$ $(s=s_0)$ but, really, we do not know the value of s_0. However, we will show in the next section that we are able to find the value of $\tilde{R}^2_{i,k}$ directly from experimental data.

Finally, the survival probability of the LRG is

$$\langle|S_{spectator}(s)|^2\rangle=\frac{N(s)}{D(s)}, \tag{63}$$

where

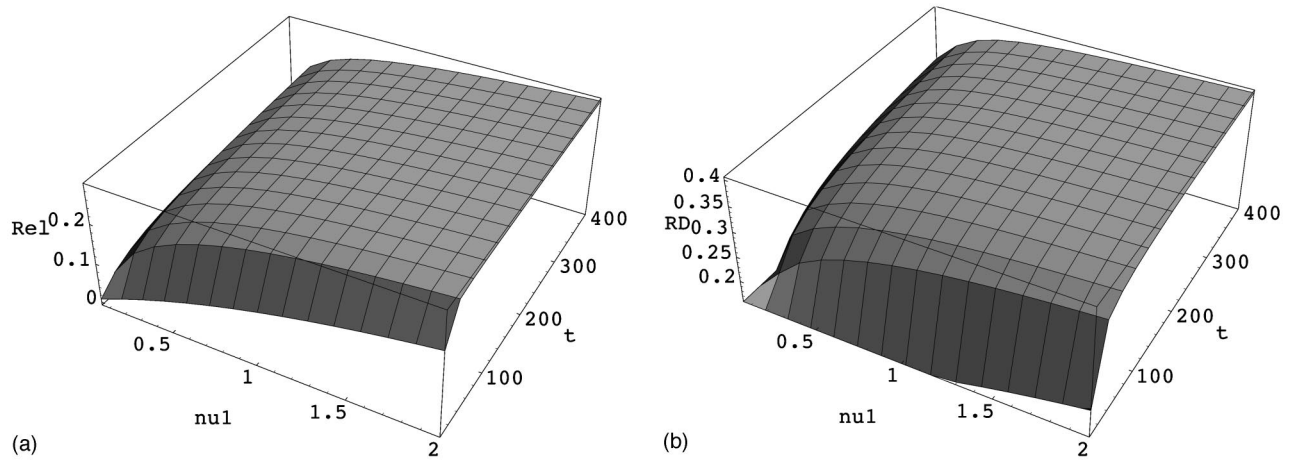

FIG. 7. Ratios $R_{el}=\sigma_{el}/\sigma_{tot}$ and $R_D=(\sigma_{el}+\sigma_{SD}+\sigma_{DD})/\sigma_{tot}$ versus ν_1 and t, where t is defined as $\nu_2=t\nu_1$.

$$N(s)=\int_0^1\frac{dX}{X}\{(1-\beta^2)^2P_{1,1}\nu_1 a_{1,1}X^{a_{1,1}} + 2(1-\beta^2)\beta^2P_{1,2}\nu_{1,2}a_{1,2}X^{a_{1,2}r} + \beta^4P_{2,2}\nu_2 a_{2,2}X^{a_{2,2}r/(2-r)}\} \tag{64}$$

and

$$D(s)=\int_0^1\frac{dX}{X}\{(1-\beta^2)^2\nu_1 a_{1,1}X^{a_{1,1}} + 2(1-\beta^2)\nu_{1,2}a_{1,2}X^{a_{1,2}r}+\beta^4\nu_2 a_{2,2}X^{a_{2,2}r/(2-r)}\}, \tag{65}$$

where we denote $\nu_{1,2}=\sqrt{\nu_1\nu_2 r(2-r)}$ and $a_{i,k}=R^2_{i,k}/\tilde{R}^2_{i,k}$. In the next section we will determine all parameters from the experimental data, and will find the value and the energy dependence which are typical for the survival probability in our model.

III. NUMERICAL EVALUATION OF $\langle|S_{spectator}|^2\rangle$ FROM THE EXPERIMENTAL DATA

A. Fixing the parameters of the model

Following the ideas of Ref. [11] we determine all the parameters of the model directly from the experimental data.

(1) The most striking experimental fact is that R_D is almost independent of energy. R_D is rather big ($R_D\approx 0.4$) [see Fig. 4(b)]. We found that this energy behavior, as well as the value of R_D, allowed us to find the coefficient t in the equation $\nu_2=t\nu_1$.

(2) The energy behavior and the value of R_{el} [see Fig. 4(a)] is used to determine the value of ν_2 at different energies, as has been done in Ref. [11].

(3) Unfortunately, no reliable measurement is available for the double diffractive cross section. We used the estimates of Ref. [24] for the value of this cross section to check that our choice of ν_1 and ν_2 is not in a contradiction with the value of the double diffraction cross section.

(4) We used the experimental data on hard diffraction in DIS to fix the value of the ''hard'' radii in Eq. (63).

(5) Our goal is not to fix all parameters, but rather to find out how sensitive the value and the energy behavior of the survival probability is to uncertainties in the values of the model parameters. We also evaluate the range of typical values of $\langle|S^2_{spectator}\rangle$.

B. R_D and R_{el}

We start by fixing the model parameters for the case of very high energies for which $r\to 1$. In Fig. 7 we plot R_D and R_{el} at fixed $\beta=0.65$ versus ν_1 and t, where t is defined $\nu_2=t\nu_1$. We have argued that in the Regge approach we expect that ν_2 is proportional to ν_1. One can see from Fig. 7 that R_D is about 0.4, only for large values of t. Note that $\nu_{1,2}=\sqrt{t}\nu_1$ at high energies. Therefore, for $t>100$ we have $\nu_{1,2}>10\nu_1$ in accordance with the global fit of the experimental data on ''soft'' processes [25].

In Fig. 8 we take $t=300$ and plot R_D and R_{el} versus ν_1. One can see that R_D is a very smooth function, while R_{el} depends substantially on ν_1. Such a behavior reflects the experimental situation shown in Fig. 4. We use the experimental data for R_{el} given in Fig. 4 to assign a definite value of ν_1 for a definite value of energy. In particular, we find $\nu_1=0.25$ for $\sqrt{s}=640$ GeV and $\nu_1=0.5$ for $\sqrt{s}=1800$ GeV to give $R_{el}=0.187$ and $R_{el}=0.237$, respectively.

Comparing Figs. 8(a) and 8(b) we can see that a β dependence for $\beta>0.5$ is not essential.

C. ''Hard'' radii

Before calculating the value of the survival probability we discuss the values of the ''hard'' radii $\tilde{R}^2_{i,k}$. Fortunately, these can be determined directly from the experimental data on diffractive production of vector mesons in DIS [26] at HERA. The data show that this production depends differently on the momentum transfer for elastic (see, for example, the process $\gamma^*+p\to\Psi+p$ in Fig. 9) and inelastic ($\gamma^*+p\to\Psi+X$ in Fig. 9) production. In the exponential parametrization, the t dependence is characterized by the slopes B_{el} and B_{in}.

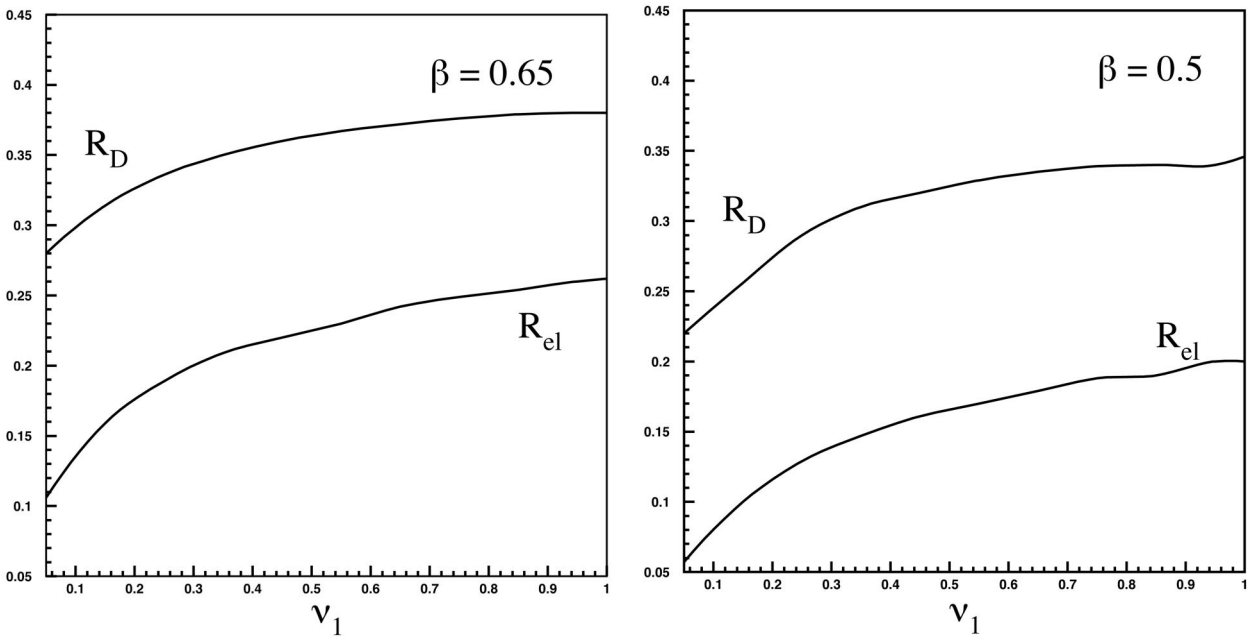

FIG. 8. Ratios $R_{el}=\sigma_{el}/\sigma_{tot}$ and $R_D=(\sigma_{el}+\sigma_{SD}+\sigma_{DD})/\sigma_{tot}$ versus ν_1 for two values of β [$\beta=0.65$ (a) and $\beta=0.5$ (b)].

Experimentally, these slopes are $B_{el}=4$ GeV^{-2} and $B_{in}=1.66$ GeV^{-2}. Since the vertex $\gamma^*\to\Psi$ does not depend on t at large values of the photon virtuality Q^2, we can view such an experiment as a way of measuring the t dependence of the Pomeron-hadron form factor and/or the transition form factor of a hadron to a diffractive state, due to Pomeron exchange. Incorporating the experimental data on slopes in the expressions of the ''hard'' radii we find

$$\tilde{R}^2_{1,1}=16\ \text{GeV}^{-2},$$
$$\tilde{R}^2_{1,2}=11.32\ \text{GeV}^{-2} \tag{66}$$
$$\tilde{R}^2_{2,2}=6.64\ \text{GeV}^{-2}.$$

One can see from Eq. (63) that the value of the survival probability depends on the ratios $r_{i,k}$. To calculate these ratios we have to specify the values of ''soft'' radii $R^2_{i,k}$. We assumed the following values for the ''soft'' radii:

$$R^2_{1,1}=12+4\alpha'_P(0)\ln(s/s_0)\ \text{GeV}^{-2},$$

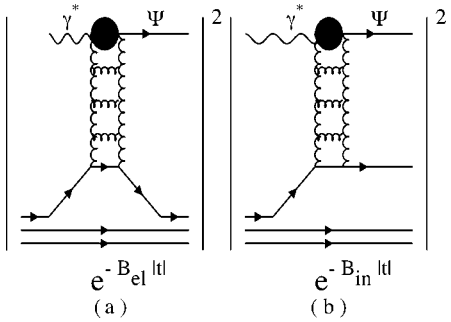

FIG. 9. Two slopes in diffractive J/Ψ production in DIS in the additive quark model.

$$R^2_{1,2}=6+4\alpha'_P(0)\ln(s/s_0)\ \text{GeV}^{-2}, \tag{67}$$
$$R^2_{2,2}=4\alpha'_P\ln(s/s_0)\ \text{GeV}^{-2}.$$

It should be stressed that these radii are taken from our parametrization of the ''soft'' data, but they also describe the data on the elastic slope, and agree with known information on the t dependence of single diffraction dissociation [24].

D. $\langle|S_{spectator}(s)|^2\rangle$

In Fig. 10 (solid curve) we show the value of the survival probability at $\sqrt{s}=1800$ GeV as a function of β. This value turns out to be rather small, in agreement with the experimental data [13].

Figure 11 (solid curve) shows the energy dependence of $\langle|S^2_{spectator}(s)|^2\rangle$, namely, the ratio

$$R=\frac{\langle|S_{spectator}(\sqrt{s}=640\ \text{GeV})|^2\rangle}{\langle|S_{spectator}(\sqrt{s}=1800\ \text{GeV})|^2\rangle}, \tag{68}$$

versus β.

In both these figures, at each fixed β, we found the values of ν_1 for $\sqrt{s}=640$ GeV and for $\sqrt{s}=1800$ GeV from the values of R_{el} ($R_{el}=0.187$ and $R_{el}=0.237$, respectively; see Fig. 4(a)). The value of the survival probability was calculated using Eq. (63).

We would like to stress that the accuracy of our estimates is not high. This is mostly because of the large dispersion of the experimental data for R_{el} [see Fig. 4(a)]. To illustrate this point we plot the value of $\langle|S^2|\rangle$ in Fig. 10 (dotted line) taking $R_{el}=0.215$ at $\sqrt{s}=1800$ GeV. One can see that the difference between these two curves is about a factor of 2. The spread of the experimental data influences dramatically

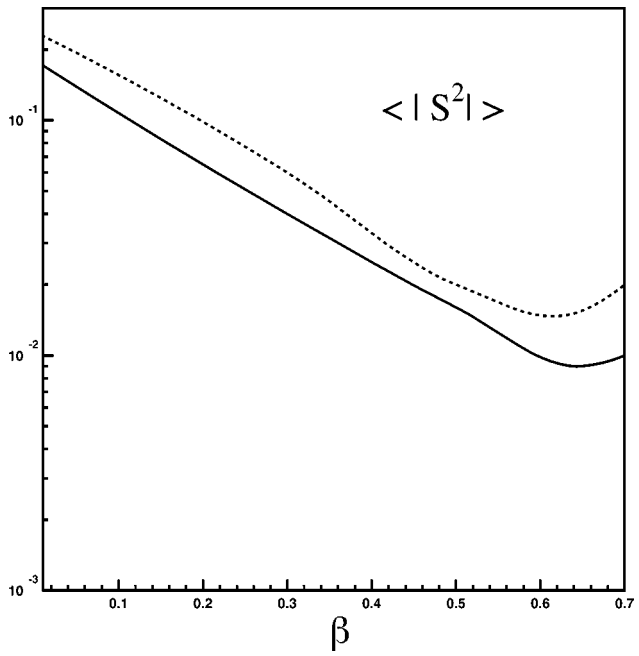

FIG. 10. The value of $\langle |S|^2 \rangle$ at $\sqrt{s} = W = 1800$ GeV versus β in the three channel model. The solid and dashed lines correspond to the parameters of Eq. (69) and Eq. (70), respectively.

the ratio of Eq. (68) (see Fig. 11). To illustrate it we plot in Fig. 11 several lines that correspond to different choices of R_{el}:

$$R_{el}(\sqrt{s} = 640 \ \text{GeV}) = 0.187$$

and (69)

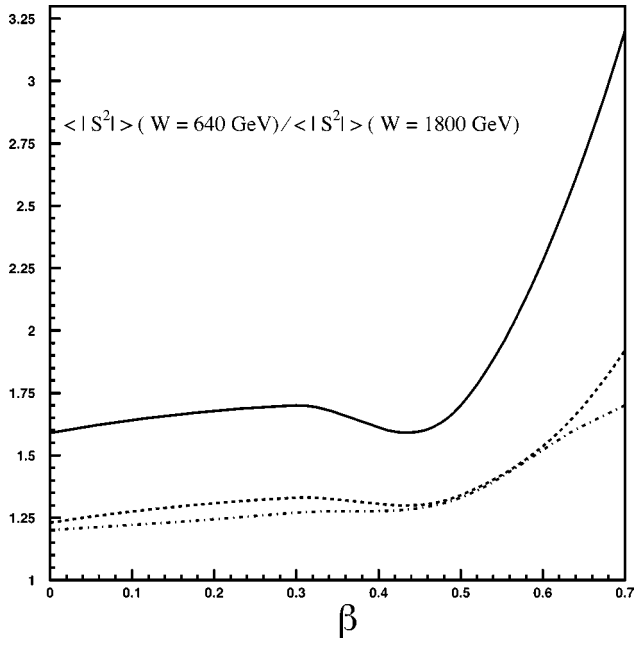

FIG. 11. The value of ratio $\langle |S|^2 \rangle (W = 640 \text{ GeV}) / \langle |S^2| \rangle (W = 1800 \text{ GeV})$ versus β in the three channel model. $W = \sqrt{s}$. The solid, dashed, and dash-dotted lines correspond to the parameters of Eq. (69), Eq. (70), and Eq. (69), respectively

$$R_{el}(\sqrt{s} = 1800 \ \text{GeV}) = 0.237,$$

$$R_{el}(\sqrt{s} = 640 \ \text{GeV}) = 0.212$$

and (70)

$$R_{el}(\sqrt{s} = 1800 \ \text{GeV}) = 0.237,$$

$$R_{el}(\sqrt{s} = 640 \ \text{GeV}) = 0.187$$

and (71)

$$R_{el}(\sqrt{s} = 1800 \ \text{GeV}) = 0.215.$$

The solid line in Fig. 11 corresponds to the parameters of Eq. (69), the dashed line to the parameters of Eq. (70), and the dash-dotted line to the parameters of Eq. (71).

A second source of ambiguity in our calculation is introduced through the dependence on β, which is the fitted parameter. It should be stressed that, realistically, the values of β are confined to the range between 0.5 and 0.7.

One can see that the ratio R_S reaches the value of 2.2 for $\beta = 0.6$, which we consider a typical value which fits the experimental data. It should be stressed that for $\beta < 0.5$ the value of R_D is smaller than 0.3. This fact is in contradiction with the experimental data shown in Fig. 4(b). At $\beta = 0$ we recover the usual eikonal model and obtain a ratio R_S which is about 1.6. In Ref. [11] we found a larger value as we used an average value for the ''hard'' radius, while here we have introduced different radii for the different processes.

IV. DISCUSSION AND SUMMARY

In this paper we give an example of a model which has the following characteristics.

(1) The processes of elastic and diffractive rescatterings have been taken into account, their cross sections being of the same order of magnitude ($\sigma_{SD} \approx \sigma_{el}$ and $\sigma_{DD} \approx \sigma_{el}$).

(2) It was shown that the scale of the SCs is not given by the ratio R_D, but rather by the separate ratios $\sigma_{el}/\sigma_{\text{tot}}$, σ_{SD}/σ_{tot}, and σ_{DD}/σ_{tot}. Since each of these ratios shows considerable energy dependence, we do not expect a constant survival probability, contrary to the simpler model of Ref. [9].

(3) It was demonstrated that the small value of the survival probability, as well as its strong energy dependence, appears naturally in our approach.

(4) The rather large value of $\nu_2 \approx 300 \nu_1$ reflects (i) a smaller value of $R^2_{2,2}$ in comparison with $R^2_{1,1}$ observed experimentally and (ii) the fact that this value takes into account the integration over the mass of the produced hadrons in our oversimplified model.

(5) The parameters that have been used are in agreement with the more detailed fit of the experimental data on ''soft'' processes (see Ref. [25]).

Theoretical predictions for the value of the survival probability are still not very reliable. However, developing different models enables us to learn and assess which class of models provides natural predictions for both the value and the energy behavior of the survival probability. We want to

draw the reader's attention to the fact that our estimates for the value of the survival probability given in Figs. 10 and 11 are very close to the estimates obtained in the eikonal model [11], in spite of the fact that the three channel model is quite different from the single channel eikonal one.

New measurements both on LRG processes and on the cross sections of diffraction dissociation (in particular, on double diffraction) would be very useful for a deeper understanding of ''soft'' interactions at high energy.

ACKNOWLEDGMENTS

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