Bell inequalities for $K^0\overline{K^0}$ pairs from Φ-resonance decays

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(Received 2 December 1998; published 4 October 1999)

We analyze the premises of recent propositions to test local realism via the Bell inequalities using neutral kaons from Φ resonance decays as entangled Einstein-Podolsky-Rosen pairs. We pay special attention to the derivation of the Bell inequalities, or related expressions, for unstable and oscillating kaon ''quasispin'' states and to the possibility of the actual identification of these states through their associated decay modes. We discuss an indirect method to extract probabilities to find these states by combining experimental information with theoretical input. However, we still find inconsistencies in previous derivations of the Bell inequalities. We show that the identification of the quasispin states via their associated decay mode does not allow the free choice to perform different tests on them, a property which is crucial to establish the validity of any Bell inequality in the context of local realism. In view of this we propose a different kind of Bell inequality in which the free choice or adjustability of the experimental setup is guaranteed. We also show that the proposed inequalities are violated by quantum mechanics. [S0556-2821(99)08219-3]

PACS number(s): 03.65.Bz, 13.65.+i

I. INTRODUCTION

The quantum entanglement shown by the separate parts of a nonfactorizable composite system is an extremely peculiar feature of quantum mechanics and has recently become a powerful resource of new developments such as quantum teleportation and communication [1]. On the other hand, ever since the paper by Einstein, Podolsky, and Rosen (EPR) [2] quantum entanglement has been also a continuous source of food for thought and speculation on the ''spooky action at a distance,'' better characterized as nonlocality in the correlations of an EPR pair [3]. A useful tool to probe into this non-locality has been given to us by Bell in form of his Bell inequalities [4]. Related versions which a local realistic theory should satisfy are also known, namely, the Clauser–Horne [5] and Wigner [6] inequalities to which very often we will simply and more generically refer to as Bell inequalities. For a general review on this subject we refer the reader to Ref. [7].

The Bell inequalities have been subjected to experimental tests with the general outcome that they are violated [8], i.e., local realistic theories should be discarded and nature is indeed nonlocal. However, loopholes in the tests have been pointed out [9]. It is then understandable that there is a continuous interest to test Bell inequalities in different experiments and, more importantly, in different branches of physics. One such place which offers the opportunity to do exactly this is the Φ resonance, a $C=-1$ neutral vector meson decaying into $K^0\overline{K^0}$ pairs. An e^+e^- machine which is expected to produce a large amount of EPR-entangled $K^0\overline{K^0}$ pairs through the reaction $e^+e^-\to\Phi\to K^0\overline{K^0}$ will be soon operating in Frascati [10]. Because of the $C=-1$ nature of the Φ meson the EPR entanglement of the neutral kaon pair can be explicitly written as

$$|\Phi\rangle=\frac{1}{\sqrt{2}}[|K^0\rangle\otimes|\overline{K^0}\rangle-|\overline{K^0}\rangle\otimes|K^0\rangle] \tag{1.1}$$

directly at the Φ-decay point into the $K^0\overline{K^0}$ initial state. The neutral kaons then fly apart allowing the definition by collimation of a left and a right hand beam. Along these two beams kaon propagation takes place, including both K^0-$\overline{K^0}$ oscillations and $K_{L,S}$ weak decays.

It would certainly add to our knowledge if we could examine the nature of nonlocality using unstable, oscillating states such as these neutral kaons. Clearly, because of the nontrivial time development of the states involved here, this scenario is quite distinct from the usually considered Bohm reformulation of EPR, even if the singlet-spin initial state

$$|0,0\rangle=\frac{1}{\sqrt{2}}[|+\rangle\otimes|-\rangle-|-\rangle\otimes|+\rangle], \tag{1.2}$$

is formally identical to the initial $K^0\overline{K^0}$ state (1.1) and, moreover, in both cases one deals with an antisymmetric system consisting of two two-dimensional components.

However, as will be evident below, the derivation of the Bell-like inequalities for unstable, oscillating states requires more care and a too close analogy to the spin case can be misleading.

Early attempts to check local realistic theories in kaonic Φ decays used Bell inequalities involving probabilities of K^0 and/or $\overline{K^0}$ detection defined at different times [11–13]. The identification of K^0 versus $\overline{K^0}$ is not problematic and can be performed exploiting their distinct strong interactions on nucleons. Moreover, the use of different detection times allows to fulfill a crucial prerequisite needed to derive Bell inequalities from local realism, namely, that different measurements corresponding to alternative experimental setups could be alternatively performed over the measured system.

Unfortunately, it was found that this kind of firmly derived Bell inequalities cannot be violated by quantum mechanics due to the specific values of kaon parameters such as their masses and decays widths (see Ref. [12] for similar investigations in the *B*-meson system).

Recently, there has been a renewed interest in this subject [14–21]. The idea in Refs. [14,15,17] has been to drop the ''different time'' Bell inequalities in favor of the identification of what are called ''quasispin'' kaon states. These are essentially arbitrary superpositions of the two K^0 and $\overline{K^0}$ basis states, $|K_\alpha\rangle = \alpha_1|K^0\rangle + \alpha_2|\overline{K^0}\rangle$ with $|\alpha_1|^2 + |\alpha_2|^2 = 1$, defined in close analogy to the spinors in the spin case. The results look also quite encouraging in the sense that one could show that these Bell inequalities for kaons are violated by quantum mechanics. However, as we will show in this paper, there are several drawbacks in using this quasispin analogy. First of all the identification of such quasispin kaon states is problematic (except for K^0 vs $\overline{K^0}$, as just mentioned) and has not been addressed in full satisfaction so far. Indeed, it is not possible to observe such states directly. An indirect method, using a theoretical input and direct observable experimental information, seems to be the only way to extract the probabilities to find specific quasispin states. We believe that we have found such a method, which is an interesting result in its own respect. However, this is still not sufficient to derive Bell inequalities for these states in the context of local realistic theories. The reason is that the indirect identification method mentioned above is only possible by a prior identification (this is the experimental information) of decay products of the unstable kaons. As previously stated, the theoretical derivation of any Bell inequality requires, as starting assumption, that the experimentalist has the *free choice* among several different tests (such as adjusting the different directions of spin analyzers in the case of spin or the correspondingly different detection times of K^0 and/or $\overline{K^0}$ in Refs. [11–13]), the other inherent properties of the states being fixed by the assumptions of local realism. This is crucial to derive Bell inequalities and is often forgotten in the formalism. This possibility of experimental *intervention* is not guaranteed if the quasispin is determined by identifying the decays products. We have no possibilities of choice or intervention in this situation neither on the decay time nor on the decay channel. Therefore several of the suggested Bell inequalities for neutral kaon pairs fail in their derivation from local realism which they are supposed to test versus quantum mechanics.

In view of this unsatisfactory situation we propose a new type of Bell inequalities for entangled kaons produced in Φ-resonance decays where the free choice is indeed guaranteed in terms of the possibility of installing different ''regenerator'' slabs along the kaon flight paths. These thin slabs—which are essentially those used in neutral-kaon regeneration experiments—are characterized by adjustable parameters (such as their thickness and nucleonic density) thus mimicking the different orientations of the analyzer in the analogous spin case. Using such an experimental setup, we can then show that quantum mechanics predicts a violation of these Bell inequalities.

The paper is organized as follows. In Sec. II we discuss the possible kaon-state observables in Φ-resonance decays. We show what condition has to be put on the kaonic quasispin states in order to be able to extract the probabilities to identify such states. A formula which determines such probabilities from observable quantities is given. In Sec. III we discuss several Bell inequalities previously suggested by other authors for entangled kaon pairs. We argue that in the analyses already performed reporting violations of the inequalities by quantum mechanics, the experimental verification of this violation would not necessarily exclude local realistic theories as the proposed inequalities are not a strict consequence of this kind of theories. In Sec. IV we derive our new Bell inequalities which do follow from local realism and show that they are violated by quantum mechanics. Section V summarizes our results.

II. QUASISPIN OBSERVABLES IN NEUTRAL-KAON DECAYS

We start our discussion by quoting some elementary definitions regarding neutral kaons and the properties and time evolution of K^0-$\overline{K^0}$ pairs from Φ-resonance decays. The $CP = \pm 1$ eigenstates $K_{1/2}$ are defined by

$$|K_{1/2}\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle \pm |\overline{K^0}\rangle] \tag{2.1}$$

and the mass eigenstates $|K_{S/L}\rangle$ in terms of $K_{1/2}$ and the CP-violation parameter ϵ are

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}[|K_1\rangle + \epsilon|K_2\rangle],$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}[|K_2\rangle + \epsilon|K_1\rangle]. \tag{2.2}$$

The proper-time (τ) development of these nonoscillating mass eigenstates is given by

$$|K_{S/L}(\tau)\rangle = e^{-i\lambda_{S/L}\tau}|K_{S/L}\rangle,$$

$$\lambda_{S/L} = m_{S/L} - \frac{i}{2}\Gamma_{S/L}, \tag{2.3}$$

with $m_{S/L}$ and $\Gamma_{S/L}$ being the mass and width of K_S and K_L, respectively. This reverts into the corresponding time evolution for the initial two-component kaonic system (1.1)

$$|\Phi(0)\rangle \to |\Phi(\tau_1,\tau_2)\rangle = \frac{N}{\sqrt{2}}[|K_S(\tau_1)\rangle \otimes |K_L(\tau_2)\rangle - |K_L(\tau_1)\rangle \otimes |K_S(\tau_2)\rangle], \tag{2.4}$$

with $|N| = (1+|\epsilon|^2)/|1-\epsilon^2| \simeq 1$. The arguments τ_1 and τ_2 refer to the proper times of the time evolution on the left and right hand sides, respectively. For simplicity we restrict ourselves to the *CPT*-conserving case as the arguments we put forward are independent of any *CPT* violation.

Using Eq. (2.4) one can immediately construct a double decay amplitude for weak kaon decays of the form [22]

$$\mathcal{A}(f_1,\tau_1;f_2,\tau_2)=\frac{1}{\sqrt{2}}[\langle f_1|T|K_S(\tau_1)\rangle\langle f_2|T|K_L(\tau_2)\rangle - \langle f_1|T|K_L(\tau_1)\rangle\langle f_2|T|K_S(\tau_2)\rangle], \tag{2.5}$$

where T is the transition operator and f_i denotes the various K_S and K_L decay modes. The normalization of this doubly time dependent decay amplitude is such that

$$\int_0^\infty d\tau_1\int_0^\infty d\tau_2\sum_{f_1 f_2}|\mathcal{A}(f_1,\tau_1;f_2,\tau_2)|^2=1, \tag{2.6}$$

where the summation $\Sigma_{f_1 f_2}$ includes also the phase space integrals $\int dph(f_1),\int dph(f_2)$ which we define by

$$\Gamma(K_{S/L}\to f)=\int dph(f)|\langle f|T|K_{S/L}\rangle|^2. \tag{2.7}$$

Note that $\mathcal{A}$ in Eq. (2.5) has different dimensions for different *n*-body final states. It is then certainly useful to construct the joint decay rate $\Gamma(f_1,\tau_1;f_2,\tau_2)$ given by [22]

$$\Gamma(f_1,\tau_1;f_2,\tau_2)\equiv\frac{d^2\mathcal{P}}{d\tau_1 d\tau_2}(f_1,\tau_1;f_2,\tau_2) \equiv\int dph(f_1)\int dph(f_2)|\mathcal{A}(f_1,\tau_1;f_2,\tau_2)|^2. \tag{2.8}$$

This is a doubly time-dependent decay rate into both the specific decay mode f_1 on the left beam at the time τ_1 and f_2 on the right one at the time τ_2. The way this joint decay rate—or *joint decay probability density* (in two times)—is constructed makes it independent of the momenta of the decay products and, much more important for our discussion, $\Gamma(f_1,\tau_1;f_2,\tau_2)$ is a standard, fully measurable quantity at a Φ factory.

Had we asked, at least formally at this stage, for an observable to find a given kaon quasispin state $|K_\alpha\rangle\equiv\alpha_1|K^0\rangle+\alpha_2|\overline{K}^0\rangle$ on the left and $|K_\beta\rangle$ (defined correspondingly) on the right, we would have to calculate the *joint probability*

$$P(K_\alpha,\tau_1;K_\beta,\tau_2)\equiv\left|\frac{1}{\sqrt{2}}[\langle K_\alpha|K_S(\tau_1)\rangle\langle K_\beta|K_L(\tau_2)\rangle - \langle K_\alpha|K_L(\tau_1)\rangle\langle K_\beta|K_S(\tau_2)\rangle]\right|^2. \tag{2.9}$$

More precisely, $P(K_\alpha,\tau_1;K_\beta,\tau_2)$ is the probability of finding both an *undecayed* K_α on the left at τ_1 and an *undecayed* K_β on the right at τ_2 in a hypothetical experiment being also able to distinguish between K_α and its orthogonal state $\tilde{K}_\alpha$ and between K_β and $\tilde{K}_\beta$. It is a well-defined probability which can be computed exactly in the same way as the corresponding one in the spin-singlet case. Indeed, this latter too is obtained by simply projecting Eq. (1.2) on the basis states defined by the spin-analyzer orientation. There are however three important differences: (i) our $P(K_\alpha,\tau_1;K_\beta,\tau_2)$'s are not constant but (doubly) time-dependent, (ii) because of the kaon instability, the probability normalizations are also different and a unifying ''renormalization prescription'' will be proposed at the end of this section, and, more importantly, (iii) in the spin case the probabilities are directly measurable whereas in our kaon quasispin case they are not (except for $K^0-\overline{K}^0$), the directly measurable quantities being the joint decay rates (2.8).

The convenience of working with these just defined joint probabilities $P(K_\alpha,\tau_1;K_\beta,\tau_2)$ has already been noticed by other authors [14,17]. Benatti and Floreanini [17], for instance, based their recent analysis on what they call ''the 'double decay probabilities' $P(f_1,\tau_1;f_2,\tau_2)$, i.e., the probabilities that one kaon decays into a final state f_1 at proper time τ_1, while the other kaon decays into the final state f_2 at proper time τ_2.'' In their formalism (see below, Refs. [17] and [23]) one has

$$P(f_1,\tau_1;f_2,\tau_2)=\mathrm{Tr}[(\mathcal{O}_{f_1}\otimes\mathcal{O}_{f_2})\rho_\Phi(\tau_1,\tau_2)], \tag{2.10}$$

where $\rho_\Phi(\tau_1,\tau_2)$ is the density operator corresponding to the two-kaon state in Eq. (2.4) and $\mathcal{O}_{f_1}$ and $\mathcal{O}_{f_2}$ are projector matrices describing each single kaon decay into f_1 and f_2 normalized by $\mathrm{Tr}\,\mathcal{O}_{f_1}=\mathrm{Tr}\,\mathcal{O}_{f_2}=1$. The same authors correctly stress that their $P(f_1,\tau_1;f_2,\tau_2)$ are *not* decay rates and, in spite of calling them ''double decay probabilities,'' one can easily convince oneself [see also our analysis below leading to Eq. (2.17)] that the $P(f_1,\tau_1;f_2,\tau_2)$'s in Ref. [17] defined by Eq. (2.10) coincide with our $P(K_{f_1},\tau_1;K_{f_2},\tau_2)$'s defined by Eq. (2.9) once the kaon quasispin states K_α and K_β are associated to the specific decay modes f_1 and f_2, respectively. The essential problem—a problem which is too naively addressed in Ref. [17] and not satisfactorily solved [24]—is then that these theoretically well-defined probabilities $P(K_{f_1},\tau_1;K_{f_2},\tau_2)=P(f_1,\tau_1;f_2,\tau_2)$ are not directly measurable, as we have already discussed. A relation between the latter probabilities and the truly measurable decay rates $\Gamma(f_1,\tau_1;f_2,\tau_2)$ defined in Eq. (2.8) is therefore highly desirable. A first attempt along this direction has been proposed and briefly discussed by Di Domenico working in a similar context [14]. However, some improvements are required to definitely establish such a relation as we proceed to discuss in the following paragraphs.

Our first step is to define the orthogonal basis containing a specific kaon state associated to the physical (i.e., really occurring) *f*-decay mode

$$|K_f\rangle\equiv\frac{1}{\sqrt{|a_f|^2+|b_f|^2}}[a_f|K_1\rangle+b_f|K_2\rangle] \tag{2.11}$$

and its orthogonal counterpart

$$|\tilde{K}_f\rangle \equiv \frac{1}{\sqrt{|\tilde{a}_f|^2+|\tilde{b}_f|^2}}[\tilde{a}_f|K_1\rangle+\tilde{b}_f|K_2\rangle], \tag{2.12}$$

with $\langle K_f|\tilde{K}_f\rangle=0$. We fix the coefficients $\tilde{a}_f$ and $\tilde{b}_f$ by demanding [25]

$$\langle f|T|\tilde{K}_f\rangle=0. \tag{2.13}$$

The unique solution (up to a phase) reads

$$|\tilde{K}_f\rangle=\frac{\tilde{b}_f}{|\tilde{b}_f|}\frac{1}{\sqrt{1+|\tilde{r}_f|^2}}[\tilde{r}_f|K_1\rangle+|K_2\rangle],$$

$$|K_f\rangle=\frac{a_f}{|a_f|}\frac{1}{\sqrt{1+|\tilde{r}_f|^2}}[|K_1\rangle-\tilde{r}_f^*|K_2\rangle], \tag{2.14}$$

with

$$\tilde{r}_f=\frac{\tilde{a}_f}{\tilde{b}_f}=-\frac{\langle f|T|K_2\rangle}{\langle f|T|K_1\rangle}. \tag{2.15}$$

It is now easy to check the following identities:

$$|K_f\rangle\langle K_f|=\rho_{K_f}=\mathcal{O}_f,$$

$$|\tilde{K}_f\rangle\langle\tilde{K}_f|=\rho_{\tilde{K}_f}=\mathcal{O}_{\tilde{f}},$$

$$|K_f\rangle\langle K_f|+|\tilde{K}_f\rangle\langle\tilde{K}_f|=\mathcal{O}_f+\mathcal{O}_{\tilde{f}}=1. \tag{2.16}$$

From these equations and Eqs. (2.9) and (2.10) one immediately obtains

$$P(f_1,\tau_1;f_2,\tau_2)=\mathrm{Tr}[\mathcal{O}_{f_1}\otimes\mathcal{O}_{f_2})\rho_\Phi(\tau_1,\tau_2)]$$
$$=P(K_{f_1},\tau_1;K_{f_2},\tau_2), \tag{2.17}$$

thus justifying the previously announced identification of $P(f_1,\tau_1;f_2,\tau_2)$ from Ref. [17] with our $P(K_{f_1},\tau_1;K_{f_2},\tau_2)$ in Eq. (2.9). Note also that the last equation in Eq. (2.16) is the correct unitarity sum for undecayed kaon states.

Our second step consists in expanding the physically decaying K_S and K_L states in two of the orthogonal bases just introduced: K_{f_i} and $\tilde{K}_{f_i}$ with $f_i=f_1$ and f_2. One then has

$$|K_{S/L}\rangle=\frac{1}{\sqrt{|a_{S1/L1}|^2+|\tilde{a}_{S1/L1}|^2}}[a_{S1/L1}|K_{f_1}\rangle+\tilde{a}_{S1/L1}|\tilde{K}_{f_1}\rangle],$$

$$|K_{S/L}\rangle=\frac{1}{\sqrt{|a_{S2/L2}|^2+|\tilde{a}_{S2/L2}|^2}}[a_{S2/L2}|K_{f_2}\rangle+\tilde{a}_{S2/L2}|\tilde{K}_{f_2}\rangle]. \tag{2.18}$$

Using Eq. (2.13), we can now rewrite the double decay amplitude (2.5) as follows:

$$\mathcal{A}(f_1,\tau_1;f_2,\tau_2)=\frac{1}{\sqrt{2}}[a_{S1}a_{L2}e^{-i\lambda_S\tau_1}e^{-i\lambda_L\tau_2}\langle f_1|T|K_{f_1}\rangle \times\langle f_2|T|K_{f_2}\rangle - a_{L1}a_{S2}e^{-i\lambda_L\tau_1}e^{-i\lambda_S\tau_2}\langle f_1|T|K_{f_1}\rangle \times\langle f_2|T|K_{f_2}\rangle]. \tag{2.19}$$

Then, using Eqs. (2.8), (2.9), and (2.19) one can easily conclude that

$$\Gamma(f_1,\tau_1;f_2,\tau_2)=\frac{1}{2}|a_{S1}a_{L2}e^{-i\lambda_S\tau_1}e^{-i\lambda_L\tau_2} - a_{L1}a_{S2}e^{-i\lambda_L\tau_1}e^{-i\lambda_S\tau_2}|^2\Gamma(K_{f_1}\to f_1) \times\Gamma(K_{f_2}\to f_2)$$
$$=P(K_{f_1},\tau_1;K_{f_2},\tau_2)\Gamma(K_{f_1}\to f_1) \times\Gamma(K_{f_2}\to f_2), \tag{2.20}$$

where

$$\Gamma(K_{f_1}\to f_1)=\int dph(f_1)|\langle K_{f_1}|T|f_1\rangle|^2$$
$$=\int dph(f_1)|a_{S1}^*\langle K_S|T|f_1\rangle + a_{L1}^*\langle K_L|T|f_1\rangle|^2 \tag{2.21}$$

and the smallness of the mass difference $\Delta m=m_S-m_L$ makes possible the use of the same phase-space factor for the two terms in the integrand of the latter expression.

As a result of the algebraic manipulations in the last two paragraphs, we can now take the joint decay rate $\Gamma(f_1,\tau_1;f_2,\tau_2)$ from experiment and, quite independently, we can also calculate $\Gamma(K_{f_{1/2}}\to f_{1/2})$ via Eq. (2.21). As announced before, one thus obtains the *joint probability*

$$P(K_{f_1},\tau_1;K_{f_2},\tau_2)=\frac{\Gamma(f_1,\tau_1;f_2,\tau_2)}{\Gamma(K_{f_1}\to f_1)\Gamma(K_{f_2}\to f_2)}. \tag{2.22}$$

This equation is the desired connection between the simple and easily interpretable *joint probability* $P(K_{f_1},\tau_1;K_{f_2},\tau_2)$ and the measurable *joint decay rate* $\Gamma(f_1,\tau_1;f_2,\tau_2)$. A formally identical equation can be found in the analysis on the same topic performed by Di Domenico [14], but our expressions (2.21) for $\Gamma(K_{f_1}\to f_1)$ and the corresponding ones in [14] are, unfortunately, not the same. Notice also that our procedure to extract the probability $P(K_{f_1},\tau_1;K_{f_2},\tau_2)$ strongly relies on a theoretical input in form of the condition (2.13), where f refers exclusively to physical, realistic $K_{S/L}$ decay modes. In other words, the same procedure would not work for an *arbitrary* superposition of K^0 and $\overline{K^0}$, because a relation, as established in Eq. (2.22), between Eqs. (2.8) and (2.9) does not hold in general.

Strictly speaking, this means also that probabilities such as $P(\tilde{K}_{f_1},\tau_1;K_{f_2},\tau_2)$ or $P(\tilde{K}_{f_1},\tau_1;\tilde{K}_{f_2},\tau_2)$, involving one or two kaon states $\tilde{K}_f$, cannot be extracted by the same method. However, we can do that in a different way using a certain approximation. Let us first introduce the notion of ''any,'' i.e., the probability to detect any of the two basis states on one of the two sides and a specific state on the other. We have

$$P(-,\tau_1;K_{f_2};\tau_2)\equiv P(K_{f_1},\tau_1;K_{f_2},\tau_2)+P(\tilde{K}_{f_1},\tau_1;K_{f_2},\tau_2)$$
$$=P(K^0,\tau_1;K_{f_2},\tau_2)+P(\overline{K^0},\tau_1;K_{f_2},\tau_2), \tag{2.23}$$

where the bar denotes that we have summed over the two possible orthogonal outcomes on the left hand side. Similar definitions hold of course for the right hand side and for both sides, i.e., $P(-,\tau_1;-\tau_2)$. It should be clear that Eq. (2.23) does not depend on the choice of f_1 on the left hand beam and therefore we can replace K_{f_1} by K^0, as done in the second line of Eq. (2.23). In the excellent approximation of the $\Delta Q=\Delta S$ rule, we have $\langle\pi^+l^-\overline{\nu}|T|K^0\rangle=0$ and $\langle\pi^-l^+\nu|T|\overline{K^0}\rangle=0$ thus fulfilling in both cases a condition like that in Eq. (2.13). Thanks to this, both probabilities in the second line in Eq. (2.23) can be extracted via Eq. (2.22). This obviously allows the subsequent computation of $P(\tilde{K}_{f_1},\tau_1;K_{f_2},\tau_2)$ through Eq. (2.23). In other words, the basis consisting of the two strangeness eigenstates K^0 and $\overline{K^0}$ is exceptional not only in that these two states can be unambiguously detected using their distinct strong interactions in nucleonic matter but also in that Eq. (2.22) can be used to measure the K^0- or $\overline{K^0}$-detection probabilities through their associated semileptonic decay modes $\pi^-l^+\nu$ or $\pi^+l^-\overline{\nu}$, respectively. For another basis, such as that consisting of K_f and $\tilde{K}_f$ associated to the *f*-decay mode, probabilities involving K_f detection can be similarly measured via Eq. (2.22) but those for $\tilde{K}_f$-detection require the use of Eq. (2.23). Finally, for bases consisting of ''quasispin'' states not linked to an specific, realistic decay mode none of the probabilities seems to be measurable.

Let us also mention that in order to formulate Bell inequalities for unstable two-component systems such as kaons, $P(K_{f_1},\tau_1;K_{f_2},\tau_2)$ is not, strictly speaking, the most suitable quantity. The reason is exactly the instability of the components under consideration which superimposes an irrelevant time evolution (due to weak decays) to the relevant one (due to quasispin oscillations). The suitable observable for unstable and oscillating states is not $P(K_{f_1},\tau_1;K_{f_2},\tau_2)$, but rather

$$p(K_{f_1},\tau_1;K_{f_2},\tau_2)\equiv P(K_{f_1},\tau_1;K_{f_2},\tau_2)/P(-,\tau_1;-,\tau_2), \tag{2.24}$$

where

$$P(-,\tau_1;-,\tau_2)=P(K^0,\tau_1;K^0,\tau_2)+P(K^0,\tau_1;\overline{K^0},\tau_2)$$
$$+P(\overline{K^0},\tau_1;K^0,\tau_2)+P(\overline{K^0},\tau_1;\overline{K^0},\tau_2) \tag{2.25}$$

is an obvious generalization of Eq. (2.23). Equation (2.24) means that we have ''renormalized'' the probabilities not to the total number of decay events, but to the restricted set of decays happening at the times τ_1 and τ_2 and covering the four possible outcomes associated to any given pair of dimension–2 orthogonal bases, as exemplified in Eq. (2.25). From Eq. (2.25) one can easily compute

$$P(-,\tau_1;-,\tau_2)\simeq e^{-1/2(\Gamma_L+\Gamma_S)(\tau_1+\tau_2)}$$
$$\times\cosh\left[\frac{1}{2}(\Gamma_L-\Gamma_S)(\tau_1-\tau_2)\right], \tag{2.26}$$

where small terms of order $|\epsilon|^2$ and higher have been safely neglected. This allows us to cancel the spurious time evolution induced by decays in the $P(K_{f_1},\tau_1;K_{f_2},\tau_2)$'s defined by Eq. (2.9) and the new $p(K_{f_1},\tau_1;K_{f_2},\tau_2)$'s turn out to be simply normalized by

$$p(-,\tau_1;-,\tau_2)=p(K_{f_1},\tau_1;-,\tau_2)+p(\tilde{K}_{f_1},\tau_1;-,\tau_2)=1$$

in such a way that the similarities between these $p(K_{f_1},\tau_1;K_{f_2},\tau_2)$'s and the corresponding ones in the conventional spin case cannot be increased any further. This renormalization is not an essential point in most applications of Bell inequalities for unstable systems [26], but exceptions, which without insisting on this point lead to contradictions, can be shown to exist.

III. BELL INEQUALITIES FOR K^0-$\overline{K^0}$ SYSTEMS IN Φ DECAYS

In the last section we derived a formula [Eq. (2.22)] and a ''renormalization prescription'' which yields the probability $p(K_{f_1},\tau_1;K_{f_2},\tau_2)$ provided we have experimental information on the direct measurable quantity $\Gamma(f_1,\tau_1;f_2,\tau_2)$ defined in Eq. (2.8) and we impose on the kaon states the crucial condition (2.13) for the physical $f_{1,2}$ decay modes. It should be noted that quite a lot of a theoretical input is required to arrive at the probabilities $p(K_{f_1},\tau_1;K_{f_2},\tau_2)$. But, apart from that, it might appear that these probabilities—so close to those appearing in the spin case—could be sufficient to establish well-defined Bell inequalities for Φ-resonance decays into neutral kaons. This is, unfortunately, not the case. To understand this point, we best compare the Bell inequalities for the usually considered singlet-spin state with the ones suggested in Refs. [14] and [17] for entangled kaon pairs.

Let $A=\boldsymbol{a},\boldsymbol{a}',\ldots$ $(B=\boldsymbol{b},\boldsymbol{b}',\ldots)$ be the set of the various directions among which we can *choose* to measure the polarization of the spin one-half subsystem coming from the initial spin-singlet state (1.2) and propagating along the left (right) hand beam. Let s_i, with $s_i=\pm$ and $i=a,a',b,b'\ldots$, be the various possible outcomes of these measurements in units of $\hbar/2$. Following Redhead [27], any (i.e., deterministic or nondeterministic) local realistic theory can be shown to satisfy the following equation

$$p(s_a,s_b,\lambda)_{\boldsymbol{a},\boldsymbol{b}}=p(s_a|\lambda)_{\boldsymbol{a}}p(s_b|\lambda)_{\boldsymbol{b}}\rho(\lambda), \tag{3.1}$$

where $p(s_a,s_b,\lambda)_{\boldsymbol{a},\boldsymbol{b}}$ refers to the joint probability for the singlet (1.2) to be emitted in a given state fully characterized by the set of hidden variables λ [28] and to produce the outcomes s_a and s_b when measuring the spin one-half projections along $\boldsymbol{a}$ and $\boldsymbol{b}$. Obviously one also has $p(s_a,s_b,\lambda)_{\boldsymbol{a},\boldsymbol{b}}=p(s_a;s_b|\lambda)_{\boldsymbol{a},\boldsymbol{b}}\rho(\lambda)$. Here and in the right hand side of Eq. (3.1), the notation $p(X|Y)$ is reserved for *conditional* probabilities, the first and second arguments s_a and s_b refer to the left and right hand beams, respectively, and $\rho(\lambda)$ is the probability distribution for the two-component system being emitted in the state λ with the obvious normalization $\int d\lambda\rho(\lambda)=1$. Equation (3.1)—often referred to as the ''factorizability'' rather than ''locality'' condition, as discussed in detail in Refs. [27] and [29]—is also equivalent to the locality condition used by Clauser and Horne in Ref. [5] to derive their general class of Bell inequalities.

The derivation of these Bell inequalities proceeds by requiring that the observed probabilities correspond to an average of the λ-dependent probabilities via

$$p(s_a;s_b)_{\boldsymbol{a},\boldsymbol{b}}=\int d\lambda\rho(\lambda)p(s_a|\lambda)_{\boldsymbol{a}}p(s_b|\lambda)_{\boldsymbol{b}},$$

$$p(s_a)_{\boldsymbol{a}}=\int d\lambda\rho(\lambda)p(s_a|\lambda)_{\boldsymbol{a}},$$

$$p(s_b)_{\boldsymbol{b}}=\int d\lambda\rho(\lambda)p(s_b|\lambda)_{\boldsymbol{b}}. \tag{3.2}$$

A general Bell-type inequality follows then from the simple mathematical theorem stating that

$$x_1x_2-x_1x_4+x_2x_3+x_3x_4\leqslant x_3+x_2, \tag{3.3}$$

provided that $0\leqslant x_i\leqslant 1$ [5]. Translating x_ix_j into product of probabilities $p(s_a|\lambda)_{\boldsymbol{a}}p(s_b|\lambda)_{\boldsymbol{b}}$, using then the factorizability condition (3.1) and finally integrating over λ one gets

$$p(s_a;s_b)_{\boldsymbol{a},\boldsymbol{b}}-p(s_a;s_d)_{\boldsymbol{a},\boldsymbol{d}}+p(s_c;s_b)_{\boldsymbol{c},\boldsymbol{b}}+p(s_c;s_d)_{\boldsymbol{c},\boldsymbol{d}} \leqslant p(s_c)_{\boldsymbol{c}}+p(s_b)_{\boldsymbol{b}}, \tag{3.4}$$

which is the well-known Clauser Horne version of Bell inequalities.

Alternative versions of Bell–type inequalities can also be obtained. Possibly the most simple and best known is due to Wigner [6]:

$$p(s_a;s_b)_{\boldsymbol{a},\boldsymbol{b}}\leqslant p(s_a;s_c)_{\boldsymbol{a},\boldsymbol{c}}+p(s_c;s_b)_{\boldsymbol{c},\boldsymbol{b}}, \tag{3.5}$$

which follows from identifying two of the four orientations in Eq. (3.4) and requiring the perfect anticorrelation, $p(s_a;s_a)_{\boldsymbol{a},\boldsymbol{a}}=0$, for the singlet state which is not only the obvious quantum mechanical prediction but also a well-tested experimental fact. This requirement, however, restricts the derivability of Wigner inequalities [6] only to deterministic theories; indeed, if perfect anticorrelation is imposed in expressions analogous to the first one in Eq. (3.2) the various conditional probabilities $p(s_a|\lambda)_{\boldsymbol{a}}$ and $p(s_b|\lambda)_{\boldsymbol{b}}$ turn out to be either zero or one and, therefore, any stochastic local realistic theory collapses into a deterministic one (see Ref. [27] for details).

There exists a more general derivation of the inequalities (3.4) and (3.5) which fully takes into account ''new'' possible hidden variables associated, in principle, with the measuring apparatus [30]. We note here that this general situation leads to the same Bell inequalities (3.4) and (3.5). Since later we will use Bell inequalities with kaons traveling in absorbers, we will in Sec. IV give a derivation similar to that in Ref. [30] considering the general possibility of extra hidden variables in connection with these absorbers.

Let us also note here that a more detailed notation for $p(s_a)_{\boldsymbol{a}}$ [and, similarly, for $p(s_b)_{\boldsymbol{b}}$] would be $p(s_a,\cdot)_{\boldsymbol{a},\boldsymbol{b}}$ where the center dots denotes, as the bar in Sec. II did, that the two possible outcomes s_b on the right hand side have been integrated out. We can then write $p(s_a,\cdot)_{\boldsymbol{a},\boldsymbol{b}}\equiv p(s_a,+)_{\boldsymbol{a},\boldsymbol{b}}+p(s_a,-)_{\boldsymbol{a},\boldsymbol{b}}$. This makes then contact with the notation of Sec. II which we will also continue to use. The locality condition establishes that $p(s_a)_{\boldsymbol{a}}=p(s_a,\cdot)_{\boldsymbol{a},\boldsymbol{b}}=p(s_a,\cdot)_{\boldsymbol{a},\boldsymbol{b}'}$ is independent from the distant orientation $\boldsymbol{b},\boldsymbol{b}'\cdots$.

In *purely formal* analogy to Eqs. (3.4) and (3.5) we can now derive Bell inequalities involving our previously discussed kaon identification probabilities $p(K_{f_1},\tau_1;K_{f_2},\tau_2)$ (2.24). Each probability $p(s_a;s_b)_{\boldsymbol{a},\boldsymbol{b}}$ in expressions (3.4) and (3.5) can be substituted by a corresponding $p(k_{f_1};k_{f_2})_{K_{f_1},\tau_1;K_{f_2},\tau_2}$, where the two dichotomic arguments k_{f_i} are assumed to take the values $k_{f_i}=+$ or $-$ according to the identification of the ''quasispin'' state as K_{f_i} or its orthogonal state $\tilde{K}_{f_i}$. Reverting to the notation introduced in Sec. II, one thus has $p(K_{f_1},\tau_1;K_{f_2},\tau_2)\equiv p(+;+)_{K_{f_1},\tau_1;K_{f_2},\tau_2}$, $p(K_{f_1},\tau_1;\tilde{K}_{f_2},\tau_2)\equiv p(+;-)_{K_{f_1},\tau_1;K_{f_2},\tau_2}$, $\ldots$. Using the shortest notation (quite in line with that in Refs. [14] and [17]), the Clauser-Horne inequality (3.4) can be rewritten as

$$p(K_{f_1},\tau_1;K_{f_2},\tau_2)-p(K_{f_1},\tau_1;K_{f_4},\tau_2)+p(K_{f_3},\tau_1;K_{f_2},\tau_2) +p(K_{f_3},\tau_1;K_{f_4},\tau_2) \leqslant p(K_{f_3},\tau_1;-,\tau_2)+p(-,\tau_1;K_{f_2},\tau_2). \tag{3.6}$$

and a series of equivalent expressions obtained by replacing one or several K_{f_i} by $\tilde{K}_{f_i}$ consistently everywhere. The Wigner version of Bell inequalities corresponding to Eq. (3.5) restricts now to the equal time case, $\tau_1=\tau_1\equiv\tau$, and can be immediately written as

$$p(K_{f_3},\tau;K_{f_2},\tau)\leqslant p(K_{f_3},\tau;K_{f_1},\tau)+p(K_{f_1},\tau;K_{f_2},\tau). \tag{3.7}$$

As discussed in Eq. [17], this simple expression follows also from the most general one (3.6) by making the replacement $K_{f_1}\rightarrow\tilde{K}_{f_1}$ after having identified $f_1=f_4$, $\tau_1=\tau_1\equiv\tau$ and imposing $p(K_{f_1},\tau;K_{f_4},\tau)=0$. But the previous considerations concerning this last requirement of perfect anticorrelation reduce the derivability of Eq. (3.7) only to deterministic local realistic theories [27].

One could argue that the *purely formal* analogy between the singlet-spin and the kaonic Φ-decay cases discussed in the previous paragraph is broken by the different role played by the time parameter(s). This is only partially true. In both cases time plays a fundamental role because the real riddle of the ''spooky action at a distance'' in quantum mechanical entanglement is the apparent possibility of causally connecting *spacelike* separated events. To make sure that kaon decay events on the left are causally disconnected from the events on the right we have to impose the condition $x_1+x_2/|t_2-t_1|\geqslant 1$, where x_1 and x_2 are the distances traveled along the left and right sides, respectively. Using the semiclassical relation $x=\beta ct$, which makes full sense to use for kaons from Φ decays [31] where $\beta\simeq 0.2$, we get

$$\frac{1-\beta}{1+\beta}\leqslant\frac{t_1}{t_2}\leqslant\frac{1+\beta}{1-\beta} \tag{3.8}$$

which is symmetric in t_1/t_2. For t_1 and t_2 obeying Eq. (3.8) and, in particular, for $t_1=t_2$ there cannot be any classical communication between the two events. Equal times are then the most convenient choice and the two Wigner inequalities (3.5), where equal times are tacitly assumed, and Eq. (3.7), where equal times are explicitly stated, are in perfect analogy. The situation is different for the Clauser-Horne inequalities (3.4) and (3.6), where the explicit time dependence of the latter allows for the *a priori* interesting possibilities first explored in Refs. [11–13] (see, however, our comments below).

Having noticed some *purely formal* analogies we now turn to analyze a profound difference between the two cases we are considering. Whereas in the singlet-spin case the directions of the spin–analyzers can be adjusted at *free will* by the experimentalist who can choose among $A=\boldsymbol{a},\boldsymbol{a}',\ldots$ and $B=\boldsymbol{b},\boldsymbol{b}',\ldots$, the decay mode in the kaonic case is not an observable we have any freedom to choose or adjust. It is important to insist that this freedom to choose among different tests to be eventually performed on the physical system is crucial in the context of local realistic theories (see, e.g., Refs. [16,27,32], and our discussion above). In these theories, the behavior of the physical system is contained in the set of its hidden variables λ. Whatever one *chooses* (provided a choice exists) to measure produces an outcome which was somehow ''inherent'' in these hidden variables ''instructions'' telling the state how to react under each possible choice. If alternative experimental measurements on a single system are admissible, the corresponding probabilities for these alternative choices with their different possible outcomes are assumed to exist and a Bell-like inequality can in principle be established in terms of these probabilities. However, this is not possible if there is no free choice on the side of the experimentalist, as happens when dealing with decay modes and decay times of freely propagating unstable particles. In other words, a particular kaon decay mode or decay instant is ''contained'' already in the ''set of instructions'' parametrized by λ and, in general, there is no possibility for a real choice allowing to establish Bell inequalities.

We are now in the position to pursue our discussion on the analyses recently performed by several authors trying to establish Bell inequalities for entangled neutral-kaon pairs. We have reconsidered most of the arguments put forward by Benatti and Floreanini [17] and, formally speaking, we have reached their same generic Bell inequalities (3.4) and (3.5). These authors then concentrate on the Wigner inequalities (3.5) written also at equal times and specified to kaon quasispin states associated to the $\pi^+\pi^-$, $\pi^0\pi^0$, and $\pi^-l^+\nu$ (or $\pi^+l^-\overline{\nu}$, in a second inequality) decay modes. Since the difference between the charged and neutral two-pion decay amplitudes is proportional to the phenomenological parameter ϵ' (which is a measure for direct CP violation), one obtains the Bell inequality $|\mathrm{Re}(\epsilon')|\leqslant 3|\epsilon'|^2$. This inequality can clearly be violated by small (but not vanishing) values of ϵ', which are quite compatible with present day experimental data. Formally, we fully agree with all these results found in Ref. [17] (see also Ref. [33]). In our opinion, however, the inequalities in Ref. [17] do not follow strictly from local realistic theories: the required possibility of intervention by the experimentalist allowing a choice among different measurements is not there if one simply detects decay events, as discussed in the previous paragraph. The same remark applies to the detailed paper by Di Domenico [14], where similar Wigner inequalities (not necessarily at equal times here) are also derived. The three binary alternatives proposed in this case, consist in identifying K^0 vs $\overline{K}^0$ (assuming the $\Delta S=\Delta Q$ rule for semileptonic decays), K_1 vs K_2 (via two-pion decays in the limit $\epsilon'=0$) and a third quasispin state $\tilde{K}_S$ vs its orthogonal counterpart. The latter identification is achieved through a clever trick based on regeneration phenomena which has inspired our present treatment of the subject (see next section), but the required possibility of choice is not contemplated.

To further convince the reader that there is real trouble in the Bell inequalities proposed in these two papers let us also quote a previous analysis by Bigi [16] in which the necessity of active choice or intervention by the experimenter is explicitly emphasized. Indeed, the inequality proposed in Ref. [16] involving also three binary alternatives is essentially the same as in the previous two analyses. However, the possibility of identifying the ''third'' quasispin direction, a possibility attempted only latter by Di Domenico [14] and improved in the present paper, is simply not contemplated. Because of this, rather pessimistic conclusions were reached in Ref. [16]. Similar comments apply to the recent analysis by Uchiyama [15]. Again, the requirement of intervention by the experimenter (choosing two measurements among three possible options) is stressed, but a new problem appears: the need to discriminate between the two mass eigenstates K_S vs K_L. From the theoretical point of view, it is not obvious how to compute the corresponding K_S and K_L detection probabilities since these two states are not orthogonal, $\langle K_S|K_L\rangle \neq 0$, due to CP violation [34–36]. Indeed, naively computing these probabilities by the usual quantum mechanical projections over K_S or K_L states can lead to paradoxa [35–37] and to curious effects [38]. Experimentally, discriminating K_S from K_L seems also not feasible and the possibility of deciding that we have a pure K_L beam by waiting long enough until the ''short'' component died out [14,18] would not work either in our case. Indeed, comparably large times, imposed by the spacelike separation condition (3.8), should be used also on the other side beam thus producing an almost complete depletion of coincident counts.

We have repeatedly argued above that the experimental violation of a Bell inequality would not necessarily signal a breakdown of local realistic theories unless the possibility of active intervention in the corresponding experimental setup is guaranteed. However, as explicitly indicated by the arguments of Eq. (3.6), each side of our neutral kaon EPR configuration is characterized by (i) a kaon quasispin state K_f (or its associated decay product f) and (ii) a time variable t (or proper time τ). Therefore the observables entering these inequalities can be varied in another way. Indeed, in Refs. [11–13] different *times* instead of different states were used. Note that now the freedom of choice, quite independent from the hidden variables themselves, is indeed given in terms of the possibility of having different K^0 vs $\bar{K}^0$ detection times. The Clauser-Horne inequalities following from the locality condition can be derived in the same way we reached at Eq. (3.4). For instance, and with an obvious and simplified notation, they read [11]

$$\begin{aligned} &p(K^0,\tau_1;\bar{K}^0,\tau_2) - p(K^0,\tau_1;\bar{K}^0,\tau_4) + p(K^0,\tau_2;\bar{K}^0,\tau_3) \\ &\quad + p(K^0,\tau_2;\bar{K}^0,\tau_4) \\ &\quad \leq p(-,\tau_1;\bar{K}^0,\tau_2) + p(K^0,\tau_1;-,\tau_2) \end{aligned} \tag{3.9}$$

and remain valid when replacing $K^0 \rightarrow \bar{K}^0$, or $\bar{K}^0 \rightarrow K^0$, or both $K^0 \leftrightarrow \bar{K}^0$ [see also Ref. [12] for a generalization of Eq. (3.9)]. It is worth pointing out again that the states $K^0/\bar{K}^0$ are directly detectable through their different strong interactions on nucleonic matter too. High-density detectors could then be placed at conveniently chosen (time-of-flight) distances from the production point. Unfortunately it is then found in Refs. [11] and [12] that quantum mechanics does not violate these firmly established inequalities (3.9) involving directly measurable probabilities of finding K^0-$\bar{K}^0$ states.

IV. NEW BELL INEQUALITIES FOR K^0-$\bar{K}^0$ SYSTEMS IN Φ DECAYS

In view of the discussions in Secs. II and III, the situation of testing quantum mechanics versus local realistic theories using K^0-$\bar{K}^0$ pairs from Φ-resonance decays is quite unsatisfying. The inequality (3.9) is a correct derivation of local realism, but as shown in Refs. [11–13], quantum mechanics will not violate this inequality due to the specific values of the neutral kaon parameters. Hence, performing a discriminating test is not possible. Suggestions such as those in Refs. [14] and [17] have, in principle, two drawbacks. One is the extraction from experiment of the probabilities entering the inequalities, the second one is the impossibility of having the required free choice to perform different tests aiming to identify the different quasispin states K_f. Although the first point has already been clarified in Sec. II and found that the relevant probabilities can be extracted in an indirect way, the second criticism still remains a defect of the suggested tests. Other related suggestions use in their computations of the quantum mechanical probabilities the projection method over $K_{S/L}$ states which, on account of $\langle K_S|K_L\rangle \neq 0$, is not without ambiguities [36]. An asymmetric Φ factory is needed for other tests, as proposed in Ref. [18], but unfortunately such a factory will not be available in the near future.

It is worth noting in this context that the K^0-$\bar{K}^0$ system from Φ decays is one of the most interesting entangled systems presently available to test quantum mechanics. We have here unstable and oscillating states. In addition, this system is up to now the only system to display CP violation; indeed, the results in Refs. [15] and [17] are seemingly related to the CP-violating parameters ϵ and ϵ'. It is therefore an interesting challenge to search for a Bell-type inequality which on the one hand is a clear consequence of local realism and on the other hand could be violated by quantum mechanical predictions. Below we will present such an inequality.

Instead of using different quasispin states K_f in the probabilities, as in Eqs. (3.6) and (3.7), or different times, as in Eq. (3.9), we propose to exploit the possibilities that one has to modify by free choice the propagation conditions along one (or both) kaon flight path(s). This can be done by introducing appropriate kaon ''regenerators'' or ''absorbers,'' i.e., thin slabs of nucleonic matter with adjustable characteristics, which produce ''quasispin rotations'' in the state of the neutral kaons passing through. Such an ''active rotation'' of the states has the same effects as changing the spin-analyzer orientation from, say, $\boldsymbol{a}$ to $\boldsymbol{a}'$ or counting f rather than f' decay modes. Over these modified states we then need to detect kaon eigenstates only in a *single* quasispin direction, the most convenient one being obviously that distinguishing K^0 from $\bar{K}^0$. Indeed, these strangeness eigenstates can be identified both by their distinct decay modes, as explained in Sec. II, or by their different strong interactions on nucleons in a detector, as indicated above and explicitly emphasized in Ref. [13]. As we can guarantee now a clearly free intervention of the experimentalist—who can adjust the parameters for different propagation conditions translating into different ''quasispin rotations''—the resulting Bell inequalities reflect clearly the requirements and consequences of local realistic theories. However, the question whether quantum mechanics violates these inequalities remains to be investigated.

In order to do this, we will restrict ourselves to the equal time situation $\tau_1 = \tau_2 \equiv \tau$ which ensures that the spacelike separation of events is automatically fulfilled. We can establish a complete analogy to the singlet-spin case in form of the inequality (3.4)—or, equivalently, Eq. (3.6)—by writing

$$p(\kappa_1;\kappa_2)_{\nu_1,\nu_2}-p(\kappa_1;\kappa_4)_{\nu_1,\nu_4}+p(\kappa_3;\kappa_2)_{\nu_3,\nu_2}+p(\kappa_3;\kappa_4)_{\nu_3,\nu_4}\le p(\kappa_3;-)_{\nu_3}+p(-;\kappa_2)_{\nu_2}, \tag{4.1}$$

where p is again a λ averaged probability as before, κ_i stands for either K^0 or $\overline{K}^0$ detection and ν_i refers to the physical characteristics of the different absorbers that the experimentalist can introduce (or not, $\nu_i=0$) along the path(s). The Wigner version of Bell inequalities can be obtained as before (see also Ref. [39])

$$p(\kappa_1;\kappa_2)_{\nu_1,\nu_2}\le p(\kappa_1;\kappa_3)_{\nu_1,\nu_3}+p(\kappa_3;\kappa_2)_{\nu_3,\nu_2}, \tag{4.2}$$

which is simpler and less general than Eq. (4.1), as previously discussed, but it is also the most convenient for our elementary present purposes.

We have mentioned in Sec. III that possible extra hidden variables associated with the measuring apparatus do not alter the form of the Bell inequalities [30]. To be complete we should pose a similar question here with regards to the regenerators which, in principle, could also introduce this kind of extra hidden variables. Let us therefore consider the situation where the probabilities depend not only on λ (the *usual* hidden variable values specifying the kaon system) but also on $\lambda_i=\lambda_{1,2}$ (the extra hidden variable values associated with the two regenerators, $i=1,2$). In this case Eqs. (3.2) from the spin paradigm generalize to

$$p(\kappa_i)_{\nu_i}=\int d\lambda d\lambda_i\rho(\lambda,\lambda_i)p(\kappa_i,\nu_i,\lambda,\lambda_i),\quad i=1,2,$$

$$p(\kappa_1,\kappa_2)_{\nu_1,\nu_2}=\int d\lambda d\lambda_1 d\lambda_2\rho(\lambda,\lambda_1,\lambda_2)\times p(\kappa_1,\nu_1,\lambda,\lambda_1)p(\kappa_2,\nu_2,\lambda,\lambda_2). \tag{4.3}$$

The joint probability $\rho(\lambda,\lambda_1,\lambda_2)$ in the second equation in Eq. (4.3) can be expressed as

$$\rho(\lambda,\lambda_1,\lambda_2)=\rho(\lambda)p(\lambda_1,\lambda_2|\lambda)=\rho(\lambda)p(\lambda_1|\lambda)p(\lambda_2|\lambda), \tag{4.4}$$

where the first equality is obvious and the second one comes from *locality* or λ_1,λ_2 independence due to their spacelike separation; this is indeed the same argument used in Eqs. (4.3) to express $p(\kappa_1,\nu_1,\lambda,\lambda_1)$ and $p(\kappa_2,\nu_2,\lambda,\lambda_2)$ with no dependence on λ_2 and λ_1, respectively. Similarly one has $\rho(\lambda,\lambda_i)=\rho(\lambda)p(\lambda_i|\lambda)$ in the first equation in Eq. (4.3). In our local realistic context, Eqs. (4.3) can therefore be rewritten as

$$p(\kappa_i)_{\nu_i}=\int d\lambda\rho(\lambda)\left[\int d\lambda_i p(\lambda_i|\lambda)p(\kappa_i,\nu_i,\lambda,\lambda_i)\right],\quad i=1,2,$$

$$p(\kappa_1,\kappa_2)_{\nu_1,\nu_2}=\int d\lambda\rho(\lambda)\left[\int d\lambda_1 p(\lambda_1|\lambda)p(\kappa_1,\nu_1,\lambda,\lambda_1)\right]\times\left[\int d\lambda_2 p(\lambda_2|\lambda)p(\kappa_2,\nu_2,\lambda,\lambda_2)\right]. \tag{4.5}$$

The two ‘‘bracketed’’ factors $[\int d\lambda_i\cdots]$ in the second equation in Eq. (4.5) and the single one from the first line in Eq. (4.5) play the role of the two and single x factors in the left and right hand side of the usual x inequality (3.3) from Sec. III. One then multiplies by $\rho(\lambda)$ and integrates over $d\lambda$ to obtain Eq. (4.1). Equation (4.2) follows by simply restricting to equal times and perfect anticorrelations. Hence we arrive at the very same inequality as in the standard treatment (see Ref. [28]) with Bell inequalities in form of Eqs. (4.1) and (4.2) arising in the context of very general local realistic theories.

Before exploiting the inequality (4.2), we have to examine briefly the regeneration of neutral kaons in homogeneous nucleonic media. We follow here Refs. [14], [34], and [35], where further details can be found. The eigenstates of the mass matrix inside nucleonic matter are

$$|K'_S\rangle\simeq|K_S\rangle-\varrho|K_L\rangle,$$
$$|K'_L\rangle\simeq|K_L\rangle+\varrho|K_S\rangle, \tag{4.6}$$

where we have neglected (small) corrections of order ϱ^2 and higher. This crucial regeneration parameter, ϱ, is defined as

$$\varrho=\frac{\pi\nu}{m_K}\frac{f-\overline{f}}{\lambda_S-\lambda_L}, \tag{4.7}$$

where $m_K=(m_S+m_L)/2$, $f(\overline{f})$ is the forward scattering amplitude for $K^0(\overline{K}^0)$ on nucleons and ν is the nucleonic density (for numerical values and a detailed discussion, see Ref. [14]). The latter is probably the easiest parameter to adjust in an experimental setup, hence its explicit appearance in our notation for the inequalities (4.1) and (4.2). The time evolution inside matter for the eigenstates $|K'_{S/L}\rangle$ follows the standard exponential and nonoscillating form

$$|K'_{S/L}(\tau)\rangle=e^{-i\lambda'_{S/L}\tau}|K'_{S/L}\rangle, \tag{4.8}$$

where

$$\lambda'_{S/L}\simeq\lambda_{S/L}-\Delta\lambda+\mathcal{O}(\varrho^2),$$
$$\Delta\lambda=\frac{\pi\nu}{m_K}(f+\overline{f}). \tag{4.9}$$

This allows us to compute the net effect of a thin absorber over the entering $|K_{S/L}\rangle$ states in three steps: (i) using Eq. (4.6) the entering $|K_{S/L}\rangle$ states are projected into the $|K'_{S/L}\rangle$—basis which is the appropriate to account for inside matter propagation, (ii) the inside matter time evolution of the latter is then taken into account as dictated by Eq. (4.8) and, finally, (iii) one reverts to the original $|K_{S/L}\rangle$ basis using again Eq. (4.6). One thus finds (see, for instance, Refs. [14] and [34])

$$|K_S\rangle \to e^{-i\lambda_S'\Delta\tau}(|K_S\rangle + i\varrho(\lambda_S'-\lambda_L')\Delta\tau|K_L\rangle) \simeq |K_S\rangle + \eta(\varrho)|K_L\rangle,$$

$$|K_L\rangle \to e^{-i\lambda_L'\Delta\tau}(|K_L\rangle + i\varrho(\lambda_S'-\lambda_L')\Delta\tau|K_S\rangle) \simeq |K_L\rangle + \eta(\varrho)|K_S\rangle, \quad (4.10)$$

where $\Delta\tau$ is the time-of-flight inside matter (short enough to justify the use of first order approximations) and $\eta(\varrho) \equiv i\varrho(m_S-m_L)\Delta\tau + (1/2)\varrho(\Gamma_S-\Gamma_L)\Delta\tau$.

To calculate the probabilities appearing in Eq. (4.2) we need also the time development of the initial entangled pair in Eq. (1.1) or, more precisely, in Eq. (2.4) referring to the $|K_{S/L}\rangle$ free-propagating states. Let us consider a symmetric situation in which the kaons move in vacuum up to a proper time τ_1 on both sides. At this time τ_1, one kaon enters the absorber we put on the left hand side (the parameters of this absorber will be distinguished by a prime) and simultaneously the other kaon enters a right hand side absorber (with parameters denoted by a double prime). If we follow now the time evolution of the entangled kaon pair up to the total exit time, $\tau=\tau_1+\Delta\tau$, we get in our usual thin absorber approximation

$$\begin{aligned}|\Phi(\tau,\varrho';\tau,\varrho'')\rangle &\simeq \frac{N(\tau)}{\sqrt{2}}[|K_L\rangle\otimes|K_S\rangle - |K_S\rangle\otimes|K_L\rangle \\ &\quad + \eta(\varrho',\varrho'')(|K_L\rangle\otimes|K_L\rangle - |K_S\rangle\otimes|K_S\rangle)] \\ &\simeq \frac{N(\tau)}{\sqrt{2}}[|\overline{K^0}\rangle\otimes|K^0\rangle - |K^0\rangle\otimes|\overline{K^0}\rangle \\ &\quad + \eta(\varrho',\varrho'')(|K^0\rangle\otimes|\overline{K^0}\rangle + |\overline{K^0}\rangle\otimes|K^0\rangle)],\end{aligned} \quad (4.11)$$

where, apart from a global phase, $|N(\tau)| \equiv (1+|\epsilon|^2)e^{-1/2(\Gamma_S+\Gamma_L)\tau}/|1-\epsilon^2|$ and

$$\eta(\varrho',\varrho'') \equiv -i(\varrho''-\varrho')(\lambda_L-\lambda_S)\Delta\tau. \quad (4.12)$$

The cases with only one absorber on one of the two sides can be recovered from Eq. (4.11) by letting one of the ϱ' or ϱ'' go to zero.

Let us now concentrate on two specific versions of the inequality (4.2), namely,

$$p(K^0;\overline{K^0})_{0,\nu} \leq p(K^0;\overline{K^0})_{0,2\nu} + p(\overline{K^0};\overline{K^0})_{2\nu,\nu},$$

$$p(K^0;\overline{K^0})_{0,\nu} \leq p(K^0;K^0)_{0,2\nu} + p(K^0;\overline{K^0})_{2\nu,\nu}, \quad (4.13)$$

where the two arguments refer to the particles detected and the subindices correspond to the absence of an absorber ($\nu_i = 0$), to its presence ($\nu_i=\nu$) and to the presence of a double density absorber ($\nu_i=2\nu$). Using Eq. (4.11), the first inequality in Eq. (4.13), leads to

$$2\,\mathrm{Re}\{\eta[0,\varrho(\nu)]\} \leq 0, \quad (4.14)$$

whereas the second one gives

$$0 \leq 4\,\mathrm{Re}\{\eta[0,\varrho(\nu)]\}. \quad (4.15)$$

Clearly we have achieved our objectives, at least at the most simple level. The Bell inequalities (4.2) follow from deterministic local realism and one of their two possible versions is predicted to be violated by quantum mechanics. Note that this eventual violation should be there for any absorber and, less importantly, also independent of the small CP-violating parameters. Of course, it remains to analyze how such a tiny violation of Eq. (4.2) can be increased to a finite, observable level and to check whether it is confirmed by the experiment or not.

V. CONCLUSIONS

In this paper we have investigated possible tests of local realism through Bell inequalities using Φ resonance decays into entangled neutral kaon pairs. For previously suggested Bell inequalities, one finds that either they are not violated by quantum mechanics (which renders any test impossible) or the inequality itself could not be considered as a strict consequence of local realism.

As far as the latter is concerned, we could clarify how to extract the probabilities entering these inequalities from experiments performable with Φ-resonance decays. It turned out that this is not possible for arbitrary quasispin kaon states, but only for specially defined K_f associated to physically occurring decay modes *f*. This in its own right is an interesting observation which might have some consequences in considering future test using Φ decays into two kaons. However, the impossibility of submitting the kaon states to different identification tests and the necessity of having to identify the kaonic states K_f by its associate decays mode (on which one has no possibility of intervention or choice) excludes these inequalities to be considered a ''true'' Bell inequality in the local realistic sense.

To improve this situation, we therefore suggest a new experimental configuration based on the possibility of installing different and adjustable regenerator slabs along the kaon flight paths. Bell inequalities can then be obtained having virtues such as (i) being strictly derived from local realism and (ii) being violated by quantum mechanics regardless the parameters of the system.

ACKNOWLEDGMENTS

This work has been partly supported by the Spanish Ministerio de Educación y Ciencia, by the EURODAPHNE EEC-TMR program CT98-0169 and by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Programa de Apoio a Núcleos de Excelência (PRONEX).

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$$|\mathcal{A}(f_1,\tau_1;f_2,\tau_2)|^2=\mathrm{Tr}[(\tilde{\mathcal{O}}_{f_1}\otimes\tilde{\mathcal{O}}_{f_2})\rho_\Phi(\tau_1,\tau_2)]$$

with $\rho_{S/L}=|K_{S/L}\rangle\langle K_{S/L}|$. For more details see, e.g., P. Huet and M. E. Peskin, Nucl. Phys. **B434**, 3 (1995). Note that this and some other papers go at some stage beyond quantum mechanics and need the density matrix formalism. Within conventional quantum mechanics, computing $|\mathcal{A}|^2$ via Eq. (2.5) or the density matrix formalism is the same, however.

[24] In Ref. [17] one clearly states that the probability $P(f_1,\tau_1;f_2,\tau_2)$ can be experimentally obtained as the ratio of partial f_1,f_2 counts (at τ_1,τ_2) over the total number of counts. This interpretation of the $P(f_1,\tau_1;f_2,\tau_2)$'s defined by Eq. (2.10) as a sort of double decay branching ratio is however untenable. Indeed, the obviously analog interpretation of $\mathrm{Tr}[\mathcal{O}_f\rho_{S/L}]$ as the probability that $K_{S/L}$ decays into f would then hold for a single kaon beam of K_S or K_L. The easiest way to check the inconsistency of this point is to go to the CP-conserving limit and keep the exact $\Delta S=\Delta Q$ rule. One obtains then, for instance, $\mathrm{Tr}[\mathcal{O}_{\pi\pi}\rho_S]=1$ and $\mathrm{Tr}[\mathcal{O}_{\pi e^+\nu}\rho_S] =\mathrm{Tr}[\mathcal{O}_{\pi e^-\nu}\rho_S]=1/2$ thus violating the correct normalization of the probabilities (2.10). Stated otherwise, the proposal to measure probabilities involving K_f's by counting f decay events does not define a one-to-one mapping between K_f and f to be used automatically in a quantitative way. Counting, for instance, the $\pi\pi$ decay events in an initial K_S beam will not yield a probability equal to one. The point is of course that there are other decay modes such as the semileptonic one into which the same kaon can decay. Obviously, these comments do not exclude a *proportionality* relation between the *probabilities* (2.10) and the *joint decay rate* (2.8) as we will give in Eq. (2.22).

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[28] To clarify the notion of λ we quote here from Ref. [7] p. 1888: ''We denote this state by the single symbol λ, although it may well have many dimensions, discrete and/or continuous parts, and *different parts of it interacting with either apparatus*, etc.''

[29] Redhead discusses these issues in the context of general hidden variable theories using a more detailed notation for the different probabilities. In his notation $p(s_a|\lambda)^A_{\boldsymbol{a}}$ refers, for instance, to the conditional probability that s_a is the value possessed by the observable $\sigma\cdot\boldsymbol{a}$ for a state fully specified by λ. The superindex $A=\boldsymbol{a},\boldsymbol{a}',\ldots,$ reminds us that different choices are possible. The central point in a realistic approach is the assumption that joint probabilities such as $p(s_a,s_b,\lambda)^{A,B}_{\boldsymbol{a},\boldsymbol{b}}$ are defined for all $A=\boldsymbol{a},\boldsymbol{a}',\ldots,$ and $B=\boldsymbol{b},\boldsymbol{b}',\ldots$. If one chooses to measure along the directions $\boldsymbol{a}$ and $\boldsymbol{b}$, the probabilities will be simply denoted as in the main text by $p(s_a,s_b,\lambda)_{\boldsymbol{a},\boldsymbol{b}}$ and correspond to a given, feasible experiment. If, instead, any other choice is considered, the corresponding probabilities cannot be immediately translated into experimental results but their values are assumed to exist as well. The locality condition can be then most conveniently expressed in this notation by writing $p(s_a|\lambda)^{A,B}_{\boldsymbol{a}}=p(s_a|\lambda)^A_{\boldsymbol{a}}$, $p(s_b|\lambda)^{A,B}_{\boldsymbol{b}} =p(s_b|\lambda)^B_{\boldsymbol{b}}$ and $\rho(\lambda)^{A,B}=\rho(\lambda)$, which explicitly establish the independence of the various probabilities from the alternative settings of distant experimental setups. In the context of local realistic theories one obviously has $p(s_a,s_b,\lambda)^{A,B}_{\boldsymbol{a},\boldsymbol{b}} =p(s_a|\lambda)^{A,B}_{\boldsymbol{a}}p(s_b|\lambda)^{A,B}_{\boldsymbol{b}}\rho(\lambda)^{A,B}$ from which one can deduce Eq. (3.1) by specifying to the measurable probabilities associated to the choices $\boldsymbol{a}$ and $\boldsymbol{b}$ among those offered by the sets A and B. See also M. Ardehali, Phys. Rev. A **57**, 114 (1998).

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[33] Although the following is only of minor importance for our present analysis, we would also like to point out that Bell inequalities are meant to test *experimentally* local realistic theories versus quantum mechanics. A gedanken experiment [15], [17] cannot decide this issue. In Ref. [17] it was found that quantum mechanical predictions for the probability $p(K_{f_1},\tau;K_{f_2},\tau)$ violates Eq. (3.7) provided the parameter ϵ' which signals direct CP violation in the K^0-$\overline{K}^0$ system is nonzero (see Ref. [10] for a definition of ϵ'). It was then concluded that any measure of $\epsilon' \neq 0$ is a test of the Bell inequalities with no need of a direct check. There is, however, no automatism ''$\epsilon' \neq 0$, then local realism is ruled out.'' Or, not every measure of ϵ' is a test of local realism and quantum mechanics. It is known otherwise also that the quantum mechanical predictions violate the Bell inequalities. This does not spare us the experiment to confirm or reject this result. At the end Bell inequalities and related expressions for kaons address the question, how *entangled* kaons behave.

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