

Gauge invariant and infrared finite theory of nonleptonic heavy meson decays

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We show that the controversies on the gauge dependence and the infrared singularity which have emerged in the generalized factorization approach for nonleptonic heavy meson decays within the framework of the operator product expansion can be resolved by the perturbative QCD factorization theorem. The gauge invariance of the decay amplitude is maintained under radiative corrections by assuming on-shell external quarks. For on-shell external quarks, infrared poles in radiative corrections have to be extracted using dimensional regularization. These poles, signifying nonperturbative dynamics of a decay process, are absorbed into bound-state wave functions. Various large logarithms produced in radiative corrections are summed to all orders into the Wilson and Sudakov evolution factors. The remaining finite part gives a hard subamplitude. A decay rate is then factorized into a convolution of the hard subamplitude, the Wilson coefficient, and the Sudakov factor with the bound-state wave functions, all of which are well defined and gauge invariant.

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I. INTRODUCTION

The effective Hamiltonian is the standard starting point for describing the nonleptonic weak decays of hadrons. Consider the decay $\bar{B}^0 \rightarrow D^+ \pi^-$ as an example. The relevant effective $\Delta B=1$ weak Hamiltonian is

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [c_1(\mu) O_1(\mu) + c_2(\mu) O_2(\mu)], \quad (1)$$

where

$$O_1 = (\bar{c}b)_{V-A} (\bar{d}u)_{V-A}, \quad O_2 = (\bar{d}b)_{V-A} (\bar{c}u)_{V-A}, \quad (2)$$

with $(\bar{q}_1 q_2)_{V \pm A} \equiv \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$. In order to ensure the renormalization-scale and -scheme independence for the physical amplitude, the matrix elements of 4-quark operators have to be evaluated in the same renormalization scheme as that for Wilson coefficients and renormalized at the same scale μ .

Although the hadronic matrix element $\langle O(\mu) \rangle$ can be directly calculated in the lattice framework, it is conventionally evaluated under the factorization hypothesis so that $\langle O(\mu) \rangle$ is factorized into the product of two matrix elements of single currents, governed by decay constants and form factors. In spite of its tremendous simplicity, the naive factorization approach encounters two principal difficulties. First, it fails to describe the color-suppressed weak decay modes. For example, the predicted decay rate of $D^0 \rightarrow \bar{K}^0 \pi^0$ by naive factorization is too small by two orders of magnitude compared to experiment. Second, the hadronic matrix element under factorization is renormalization-scale μ indepen-

dent as the vector or axial-vector current is partially conserved. Consequently, the amplitude $c_i(\mu) \langle O \rangle_{\text{fact}}$ is not truly physical as the scale dependence of Wilson coefficients does not get compensation from the matrix elements.

A plausible solution to the aforementioned scale problem is to extract the μ dependence from the matrix element $\langle O(\mu) \rangle$, and combine it with the μ -dependent Wilson coefficients to form μ -independent effective Wilson coefficients. After making a physical amplitude explicitly μ -independent, the factorization hypothesis is applied to the hadronic matrix elements. However, the μ -evolution factor extracted from $\langle O(\mu) \rangle$ depends on an infrared cutoff, which is originally implicit in $\langle O(\mu) \rangle$. Since an off-shell external quark momentum is usually chosen as the infrared cutoff, the μ -evolution factor also contains a gauge dependent term accompanied by off-shell external quarks. Therefore, this solution, though it removes the scale and scheme dependence of a physical amplitude in the framework of the factorization hypothesis, often introduces the infrared cutoff and gauge dependence.

In this paper we shall show that the above controversies can be resolved by the perturbative QCD (PQCD) factorization theorem. In this formalism, partons, i.e., external quarks, are assumed to be on shell, and both ultraviolet and infrared divergences in radiative corrections are isolated using the dimensional regularization. Because external quarks are on shell, gauge invariance of the decay amplitude is maintained under radiative corrections. The obtained ultraviolet poles are subtracted in a renormalization scheme, while the infrared poles are absorbed into nonperturbative bound-state wave functions. Various large logarithms produced in radiative corrections are summed to all orders into the Wilson and Sudakov evolution factors. The remaining finite piece is

grouped into a hard decay subamplitude. The decay rate is then factorized into the convolution of the hard subamplitude, the Wilson coefficient, and the Sudakov factor with the bound-state wave functions, all of which are well-defined and gauge invariant. The partition of the nonperturbative and perturbative contributions is quite arbitrary. Different partitions correspond to different factorization schemes. However, the decay rate, as the convolution of the above factors, is independent of factorization schemes as it should be.

In Sec. II we review the conventional solutions to the scale and scheme dependence present in the factorization hypothesis, and their problems. Gauge invariance of radiative corrections is explicitly justified to all orders in Sec. III. The PQCD approach is introduced in Sec. IV. Explicit calculations of the evolution factor $g_1(\mu)$ to be defined below are shown in Sec. V. Section VI is the conclusion.

II. GAUGE DEPENDENCE AND INFRARED SINGULARITY

The aforementioned scale problem with naive factorization can be circumvented in two different approaches. In the first approach, one incorporates nonfactorizable effects into the effective coefficients [1–3]:

$$\begin{aligned} a_1^{\text{eff}} &= c_1(\mu) + c_2(\mu) \left(\frac{1}{N_c} + \chi_1(\mu) \right), \\ a_2^{\text{eff}} &= c_2(\mu) + c_1(\mu) \left(\frac{1}{N_c} + \chi_2(\mu) \right), \end{aligned} \quad (3)$$

where nonfactorizable terms are characterized by the parameters χ_i . For the decay $\bar{B}^0 \rightarrow D^+ \pi^-$, χ_1 is given by

$$\begin{aligned} c_2(\mu) \chi_1(\mu) &= \left(c_1(\mu) + \frac{c_2(\mu)}{N_c} \right) \varepsilon_1^{(BD, \pi)}(\mu) \\ &\quad + c_2(\mu) \varepsilon_8^{(BD, \pi)}(\mu), \end{aligned} \quad (4)$$

where

$$\begin{aligned} \varepsilon_1^{(BD, \pi)} &= \frac{\langle D^+ \pi^- | (\bar{c}b)_{V-A} (\bar{d}u)_{V-A} | \bar{B}^0 \rangle}{\langle D^+ | (\bar{c}b)_{V-A} | \bar{B}^0 \rangle \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle} - 1, \\ \varepsilon_8^{(BD, \pi)} &= \frac{\langle D^+ \pi^- | \frac{1}{2} (\bar{c}\lambda^a b)_{V-A} (\bar{d}\lambda^a u)_{V-A} | \bar{B}^0 \rangle}{\langle D^+ | (\bar{c}b)_{V-A} | \bar{B}^0 \rangle \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle} \end{aligned} \quad (5)$$

are nonfactorizable terms originated from color-singlet and color-octet currents, respectively, $(\bar{q}_1 \lambda^a q_2)_{V-A} \equiv \bar{q}_1 \lambda^a \gamma_\mu (1 - \gamma_5) q_2$. The μ dependence of Wilson coefficients is assumed to be exactly compensated by that of $\chi_i(\mu)$ [4]. That is, the correct μ dependence of the matrix elements is restored by the nonfactorized parameters $\chi_i(\mu)$. However, there are two potential problems with this approach. First, the renormalized 4-quark operator by itself still depends on μ , though the scale dependence of $\langle O(\mu) \rangle$ is lost in the factorization approximation. Second, to the next-to-leading

order (NLO), the Wilson coefficients depend on the choice of the renormalization scheme. It is not clear if $\chi_i(\mu)$ can restore the scheme dependence of the matrix element.

In the second approach, it is postulated that $\langle O(\mu) \rangle$ is related to the tree-level hadronic matrix element via the relation $\langle O(\mu) \rangle = g(\mu) \langle O \rangle_{\text{tree}}$ and that $g(\mu)$ is independent of the external hadron states. Then schematically we can write

$$c(\mu) \langle O(\mu) \rangle = c(\mu) g(\mu) \langle O \rangle_{\text{tree}} \equiv c^{\text{eff}} \langle O \rangle_{\text{tree}}. \quad (6)$$

The factorization approximation is applied afterwards to the hadronic matrix element of the operator O at the tree level. Since the tree-level matrix element $\langle O \rangle_{\text{tree}}$ is renormalization-scheme and -scale independent, so are the effective Wilson coefficients c_i^{eff} and the effective parameters a_i^{eff} expressed by [5,6]

$$\begin{aligned} a_1^{\text{eff}} &= c_1^{\text{eff}} + c_2^{\text{eff}} \left(\frac{1}{N_c} + \chi_1 \right), \\ a_2^{\text{eff}} &= c_2^{\text{eff}} + c_1^{\text{eff}} \left(\frac{1}{N_c} + \chi_2 \right). \end{aligned} \quad (7)$$

However, the problem is that we do not know how to carry out first-principles calculations of $\langle O(\mu) \rangle$ and hence $g(\mu)$. It is natural to ask the question: Can $g(\mu)$ be calculated at the quark level in the same way as the Wilson coefficient $c(\mu)$? One of the salient features of the operator product expansion (OPE) is that the determination of the short-distance $c(\mu)$ is independent of the choice of external states. Consequently, we can choose quarks as external states in order to extract $c(\mu)$. For simplicity, we consider a single multiplicatively renormalizable 4-quark operator O (say, O_+ or O_-) and assume massless quarks. The QCD-corrected weak amplitude induced by O in full theory is

$$A_{\text{full}} = \left[1 + \frac{\alpha_s}{4\pi} \left(-\frac{\gamma}{2} \ln \frac{M_W^2}{-p^2} + a \right) \right] \langle O \rangle_q, \quad (8)$$

where γ is an anomalous dimension, p is an off-shell momentum of the external quark lines, which is introduced as an infrared cutoff, and the nonlogarithmic constant term a in general depends on the gauge chosen for the gluon propagator. The subscript q in Eq. (8) emphasizes the fact that the matrix element is evaluated between external quark states. In effective theory, the renormalized $\langle O(\mu) \rangle_q$ is related to $\langle O \rangle_q$ in full theory via

$$\begin{aligned} \langle O(\mu) \rangle_q &= \left[1 + \frac{\alpha_s}{4\pi} \left(-\frac{\gamma}{2} \ln \frac{\mu^2}{-p^2} + r \right) \right] \langle O \rangle_q \\ &\equiv g'(\mu, -p^2, \lambda) \langle O \rangle_q, \end{aligned} \quad (9)$$

where g' indicates the perturbative corrections to the 4-quark operator renormalized at the scale μ . The constant term r is in general renormalization-scheme, gauge and external momentum dependent, which has the general expression [7]:

$$r = r^{\text{NDR, HV}} + \lambda r^\lambda, \quad (10)$$

where NDR and HV stand for the naive dimension regularization and 't Hooft–Veltman renormalization schemes, respectively, and λ is a gauge parameter with $\lambda=0$ corresponding to Landau gauge. Matching the effective theory with full theory, $A_{\text{full}}=A_{\text{eff}}=c(\mu)\langle O(\mu)\rangle_q$, leads to

$$c(\mu) = 1 + \frac{\alpha_s}{4\pi} \left(-\frac{\gamma}{2} \ln \frac{M_W^2}{\mu^2} + d \right), \quad (11)$$

where $d=a-r$. Evidently, the Wilson coefficient is independent of the infrared cutoff and it is gauge invariant as the gauge dependence is compensated between a and r . Of course, $c(\mu)$ is still renormalization-scheme and -scale dependent.

Since A_{eff} in full theory [Eq. (8)] is μ and scheme independent, it is obvious that

$$c'^{\text{eff}} = c(\mu)g'(\mu, -p^2, \lambda) \quad (12)$$

is also independent of the choice of the scheme and scale. Unfortunately, c'^{eff} is subject to the ambiguities of the infrared cutoff and gauge dependence, which come along with g' extracted from $\langle O(\mu)\rangle_q$. As stressed in [7], the gauge and infrared dependence always appears as long as the matrix elements of operators are calculated between quark states. Therefore, it is unreliable to define the effective Wilson coefficients by applying the existing calculations in the literature. The reason has been implicitly pointed out in [8] that ‘‘off-shell renormalized vertices of gauge-invariant operators are in general gauge dependent.’’

The existing problems associated with the off-shell regularization scheme are as follows:

(1) When working with off-shell fermions, there exists the so-called P operator [8], e.g., $\not{p}(1-\gamma_5)\otimes\not{p}(1-\gamma_5)$ which cannot be removed by the equation of motion.

(2) The finite terms are external momentum dependent (see Fig. 3 of [9]) and they are obtained in some specific condition. For example, two incoming fermion legs 1,2 and two outgoing legs 3,4 with external momentum p are chosen in Figs. 3a and 3c of [9,10], while legs 1,3 incoming and 2,4 outgoing with p in Fig. 3b.

Hence we cannot avoid the gauge problems if adopting off-shell fermions and the finite parts of c'^{eff} are not well-defined. To circumvent this difficulty, we should work in a physical on-shell scheme and employ the dimensional regularization for infrared divergences. Gauge invariance of the decay amplitude is maintained under radiative corrections, and the infrared poles are absorbed into the hadronic matrix element as stated in the Introduction. Consequently, the effective coefficient $c^{\text{eff}}=c(\mu)g(\mu)$ does not suffer from the gauge ambiguity.

III. GAUGE INVARIANCE IN ON-SHELL REGULARIZATION

In this section we show that gluon exchanges among the on-shell quarks involved in heavy meson decays, including the spectator quarks, indeed give gauge-invariant contributions. We present the proof in the covariant gauge $\partial\cdot A=0$,

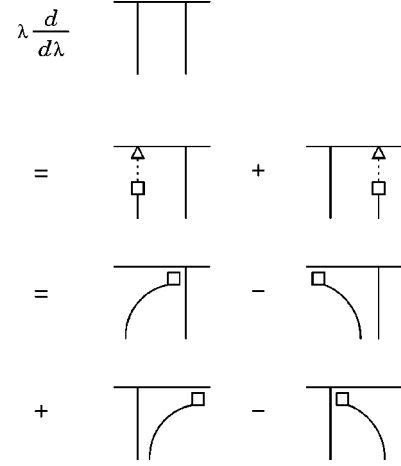


FIG. 1. Graphical representation for the proof of Eq. (14).

in which the gluon propagator is given by $(-i/l^2)N^{\mu\nu}(l)$ with

$$N^{\mu\nu}(l) = g^{\mu\nu} - (1-\lambda) \frac{l^\mu l^\nu}{l^2}, \quad (13)$$

where λ is the gauge parameter. We shall show that the quark amplitude A_{full} with the spectator quarks included is independent of λ to all orders, namely,

$$\lambda \frac{dA_{\text{full}}}{d\lambda} = 0. \quad (14)$$

The differential operator applies only to gluon propagators, leading to

$$\lambda \frac{d}{d\lambda} N^{\mu\nu} = \lambda \frac{l^\mu l^\nu}{l^2} = v_\alpha [l^\mu N^{\alpha\nu} + l^\nu N^{\alpha\mu}], \quad (15)$$

with the special vertex $v_\alpha = l_\alpha / (2l^2)$. The loop momentum l^μ (l^ν) carried by the differentiated gluon contracts with a vertex in A_{full} , which is then replaced by the special vertex v_α . Equation (15) is graphically described by the first expression in Fig. 1, where the arrow represents l^μ (l^ν) contracting with the gluon vertex, and the square represents v_α .

The contraction of l^μ (l^ν) leads to the Ward identity [11] shown in the second expression of Fig. 1, where the solid lines may represent quarks or gluons. Summing all the diagrams with various differentiated gluons, those embedding the special vertices cancel by pairs. For example, the pair cancellation occurs between the first and last diagrams in the second expression of Fig. 1. Only the diagram, in which the special vertex moves to the outer end of the quark line, is left. This diagram comes from the second term in the following expression,

$$\begin{aligned} & \frac{i(\mathbf{k}+l+M)}{(k+l)^2 - M^2} (-il)u(k) \\ & = u(k) - \frac{\mathbf{k}+l+M}{(k+l)^2 - M^2} (\mathbf{k}-M)u(k), \end{aligned} \quad (16)$$

where u is the fermion spinor associated with an external quark. The first term is canceled by the term from the contraction of l with the adjacent vertex. If all the external quarks are on shell, the second term vanishes because of the equation of motion $(\not{k}-M)u(k)=0$. Then we arrive at the desired result (14).

We take one-loop corrections as an example to elucidate the above proof. We consider only the gauge-dependent part of the gluon propagator [see Eq. (13)],

$$\frac{-i}{l^2} \left[-(1-\lambda) \frac{l^\mu l^\nu}{l^2} \right], \quad (17)$$

in the loop calculations and demonstrate that the result vanishes after summing all the diagrams. The gauge-dependent part of Fig. 2(a) reads

$$I_a^{\text{gauge}} = i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} (1-\lambda) g_s^2 \mu^\epsilon \int \frac{d^D k}{(2\pi)^D} \frac{\bar{u}_3 t^a \gamma_\mu (1-\gamma_5) (\not{p}_1 + \not{k} + M_1) \not{k} u_1 \bar{u}_4 t^a \gamma^\mu (1-\gamma_5) (\not{p}_2 - \not{k} + M_2) \not{k} u_2}{k^4 [(k+p_1)^2 - M_1^2] [(k-p_2)^2 - M_2^2]}, \quad (18)$$

where V_{CKM} is the relevant Cabibbo-Kobayashi-Maskawa matrix elements.

To proceed, we replace the \not{k} which is adjacent to u_1 as

$$\not{k} = \not{p}_1 + \not{k} - M_1 - (\not{p}_1 - M_1), \quad (19)$$

and the \not{k} adjacent to u_2 as

$$\not{k} = -\not{p}_2 + \not{k} + M_2 + (\not{p}_2 - M_2). \quad (20)$$

Applying Eq. (16) and the equation of motion, $(\not{p}-M)u=0$, Eq. (18) becomes

$$I_a^{\text{gauge}} = i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} (1-\lambda) g_s^2 \mu^\epsilon \int \frac{d^D k}{(2\pi)^D} \frac{\bar{u}_3 t^a \gamma_\mu (1-\gamma_5) u_1 \bar{u}_4 t^a \gamma^\mu (1-\gamma_5) u_2}{k^4}. \quad (21)$$

Likewise, one can apply the same trick to the calculations of Figs. 2(b)–2(f) and obtain $I_b^{\text{gauge}} = -I_c^{\text{gauge}} = -I_d^{\text{gauge}} = I_a^{\text{gauge}}$, and

$$\begin{aligned} I_e^{\text{gauge}} = I_f^{\text{gauge}} &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} (1-\lambda) g_s^2 \mu^\epsilon C_F \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^4} \bar{u}_3 \gamma_\mu (1-\gamma_5) u_1 \bar{u}_4 \gamma^\mu (1-\gamma_5) u_2 \\ &= -\frac{G_F}{\sqrt{2}} V_{\text{CKM}} (1-\lambda) \frac{\alpha_s}{4\pi} C_F \left(\frac{2}{\epsilon_{\text{UV}}} - \frac{2}{\epsilon_{\text{IR}}} \right) \bar{u}_3 \gamma_\mu (1-\gamma_5) u_1 \bar{u}_4 \gamma^\mu (1-\gamma_5) u_2, \end{aligned} \quad (22)$$

where $\epsilon_{\text{UV(IR)}} = 4-D$ is the ultraviolet (infrared) pole, and $C_F = (N_c^2 - 1)/(2N_c)$ with N_c being the number of colors. After lengthy but straightforward calculation, the renormalization constant of a fermion with mass M is found to be

$$Z_2 = 1 - \frac{\alpha_s}{4\pi} C_F \left(\frac{2}{\epsilon_{\text{UV}}} - 3\gamma_E + 4 - 3 \ln \frac{M^2}{4\pi^2 \mu^2} - \frac{4}{\epsilon_{\text{IR}}} \right) + (1-\lambda) \frac{\alpha_s}{4\pi} C_F \left(\frac{2}{\epsilon_{\text{UV}}} - \frac{2}{\epsilon_{\text{IR}}} \right). \quad (23)$$

We see that, contrary to the gauge-independent part of $Z_2 - 1$, the gauge-dependent contribution due to the fermion wave function renormalization

$$\frac{G_F}{\sqrt{2}} V_{\text{CKM}} (1-\lambda) \frac{\alpha_s}{4\pi} C_F \left(\frac{2}{\epsilon_{\text{UV}}} - \frac{2}{\epsilon_{\text{IR}}} \right) \bar{u}_3 \gamma_\mu (1-\gamma_5) u_1 \bar{u}_4 \gamma^\mu (1-\gamma_5) u_2 \quad (24)$$

is free of mass singularity. Summing over all the contributions, it is obvious that the final result indeed vanishes as it should be.

IV. PQCD FACTORIZATION THEOREM

We have shown that radiative corrections to a decay amplitude of on-shell external quarks are gauge invariant to all orders. The one-loop diagrams have been evaluated explicitly, whose results confirm our proof. Next we shall explain how to treat the infrared poles in the PQCD factorization theorem. The one-loop contributions in full theory are ultraviolet finite because of the current conservation stated above. The existence of the infrared poles simply signifies the non-perturbative dynamics, which demands the inclusion of bound-state effects into the formalism of heavy meson decays. The standard treatment of infrared poles is to absorb them into a universal meson wave function. To absorb the infrared poles associated with the b quark, such as those from the self-energy corrections, it is necessary to introduce a B meson wave function. That is, we must take into account the spectator quark of the B meson in order to develop a complete theory of heavy meson decays.

Accordingly, the decay amplitude A_{full} to one-loop in full theory can be rewritten as

$$\begin{aligned} A_{\text{full}} &= 1 + \frac{\alpha_s}{4\pi} \left(\frac{c}{\epsilon_{\text{IR}}} + \gamma \ln \frac{M_W}{M_b} + \gamma' \ln \frac{M_b}{\mu_f} + a \right) \\ &= \left[1 + \frac{\alpha_s}{4\pi} \left(\gamma \ln \frac{M_W}{M_b} + \gamma' \ln \frac{M_b}{\mu_f} + a \right) \right] \\ &\quad \times \left(1 + \frac{\alpha_s}{4\pi} \frac{c}{\epsilon_{\text{IR}}} \right) + O(\alpha_s^2), \end{aligned} \quad (25)$$

where the factorization scale μ_f arises from the dimensional regularization of infrared divergences, and the factorization of the infrared pole is performed in the minimal subtraction scheme. The anomalous dimensions of the logarithms $\ln(M_W/M_b)$ and $\ln(M_b/\mu_f)$, γ and γ' , respectively, are different, since the latter involves an extra contribution related to the spectator quark. The factor containing the infrared pole can be formulated as a matrix element of a nonlocal operator, which is the definition of a meson wave function $\phi(\mu_f)$. A wave function, describing the amplitude that a parton carries a fraction of the meson momentum, cannot be derived in perturbation theory. It must be parametrized as a function of parton momentum fraction.

We further factorize the infrared finite part into

$$\begin{aligned} A_{\text{full}} &= \left[1 + \frac{\alpha_s}{4\pi} \left(\gamma \ln \frac{M_W}{\mu} + a' \right) \right] \\ &\quad \times \left[1 + \frac{\alpha_s}{4\pi} \left(\gamma \ln \frac{\mu}{M_b} + \gamma' \ln \frac{M_b}{\mu_f} + a - a' \right) \right] \\ &\quad \times \left(1 + \frac{\alpha_s}{4\pi} \frac{c}{\epsilon_{\text{IR}}} \right) + O(\alpha_s^2). \end{aligned} \quad (26)$$

The first factor, characterized by the matching scale M_W , is identified as the Wilson coefficient $c(\mu)$ after summing $\ln(M_W/\mu)$ to all orders using renormalization group (RG) equations. The second factor, characterized by the b quark mass M_b , is the hard subamplitude which will be denoted by $H(M_b, \mu, \mu_f)$ below. Extending the above procedures to all orders, we obtain the factorization formula for B meson (not b quark) decays,

$$A_{\text{full}} = c(\mu) H(M_b, \mu, \mu_f) \phi(\mu_f), \quad (27)$$

which is exactly the three-scale factorization formula for exclusive nonleptonic decays derived in [12]. Note that Eq. (27) in fact denotes a convolution relation, because the momentum fractions should be integrated out.

Compared to Eq. (6), the matrix element $\langle O(\mu) \rangle$ corresponds to

$$\langle O(\mu) \rangle = H(M_b, \mu, \mu_f) \phi(\mu_f). \quad (28)$$

Summing $\ln(\mu/M_b)$ in H to all orders using renormalization group (RG) equations, we obtain an evolution factor $g_1(\mu)$, whose behavior from μ to M_b is governed by the same anomalous dimension as that of $c(\mu)$. Summing $\ln(M_b/\mu_f)$ in H to all orders, we obtain another factor $g_2(\mu_f)$ describing the evolution from M_b to μ_f , whose anomalous dimension differs from that of $c(\mu)$ because of the inclusion of the dynamics associated with spectator quarks. Note that g_2 is part of the Sudakov evolution obtained in [12]. Hence, the μ dependence of H is extracted as

$$H(M_b, \mu, \mu_f) = g_1(\mu) g_2(\mu_f) H(M_b, M_b, M_b). \quad (29)$$

The combination of c , g_1 , and g_2 leads to the effective coefficient

$$c^{\text{eff}} = c(\mu) g_1(\mu) g_2(\mu_f), \quad (30)$$

which is not only μ and scheme independent but also gauge invariant. The factor $g(\mu)$ in Eq. (6) can be identified as $g_1(\mu) g_2(\mu_f)$, which describes the evolution down to the factorization scale. However, $g_1(\mu) g_2(\mu_f)$ contains a matching condition at the scale M_b between the Wilson and Sudakov evolutions with different anomalous dimensions. Therefore, there is an ambiguity of the matching condition: the two evolutions can also match at rM_b with r a constant of order unity. Obviously, Eq. (30) is subtler than the naive definition of c^{eff} in Eq. (6).

The matrix element $\langle O \rangle_{\text{tree}}$ in Eq. (6) is then identified as

$$\langle O \rangle_{\text{tree}} = H_{\text{tree}}(M_b, M_b, M_b) \phi(\mu_f), \quad (31)$$

where the hard subamplitude is evaluated to lowest order with one hard gluon exchange, since all large logarithms have been organized by RG equations. We emphasize that the factorization hypothesis for $\langle O \rangle_{\text{tree}}$ in the conventional approach is not necessary in the PQCD formalism. The purpose of the factorization hypothesis is to simplify the decay amplitude into products of decay constants and form factors, which are then parametrized as various models. To have a better fit to experimental data, nonfactorizable contributions,

parametrized as χ [see Eq. (7)], are included. Note that H_{tree} in the PQCD approach includes both factorizable contributions (form factors), when the hard gluon attaches to the two quarks in a meson, and nonfactorizable contributions (octet amplitudes), when the hard gluon attaches to the quarks in different mesons. Therefore, we may compute all possible diagrams for H_{tree} [15] and convolute them with the same meson wave functions ϕ . That is, we use the single parametrization, i.e., the meson wave functions, for both factorizable and nonfactorizable contributions based on Eq. (31). In this sense the PQCD formalism is more systematic.

At last, we explain how to handle the nonperturbative meson wave functions with the dependence of the factorization scale μ_f . It can be shown that these wave functions are universal for all decay processes involving the same mesons. For example, the B meson wave function for the nonleptonic decays $B \rightarrow D^{(*)} \pi(\rho)$ and for the radiative decay $B \rightarrow K^* \gamma$ is the same. This universality can be easily understood, since a wave function collects long-distance (infrared) dynamics, which should be insensitive to short-distance dynamics involved in the decay of the b quark into light quarks with large energy release. Based on the universality of wave functions, the application of factorization formulas is as follows [13]. We evaluate the Wilson and Sudakov evolutions down to a factorization scale μ_f and the hard subamplitude for a decay mode, say, $B \rightarrow K^* \gamma$, in perturbation theory. These calculations are simply performed at the quark level with infrared poles dropped (in the minimal subtraction scheme). Adjust the B meson wave function such that the predictions from the relevant factorization formula match the experimental data. At this stage, we determine the B meson wave function defined at the scale μ_f . Then evaluate the Wilson and Sudakov evolutions down to the same scale μ_f and the hard subamplitude for another decay, say, $B \rightarrow D \pi$. Convolute them with the same B meson wave function and make predictions. At this stage, there are no free parameters in the formalism. With the above strategy, the PQCD factorization theorem possesses predictive power.

The main uncertainties in the PQCD factorization theorem come from higher-order corrections to the hard subamplitude and higher-twist corrections from the Fock states other than the leading one with only valence quarks which we are considering here. According to Eq. (29), the argument of the running coupling constant in the hard subamplitude $H(M_b, M_b, M_b)$ should be set to the b quark mass M_b , implying that the next-to-leading-order diagrams give about $\alpha_s(M_b)/\pi \sim 10\%$ corrections. Since H is characterized by M_b , the next-to-leading-twist correction from the Fock state with one more parton entering the hard subamplitude is about $\mu_f/M_b \sim 10\%$. Note that meson wave functions are usually defined at the factorization scale $\mu_f \sim 0.5$ GeV [14]. We believe that other nonperturbative corrections, such as final-state interactions, should play a minor role because of the large energy release involved in two-body B meson decays. If the hadronic matrix elements are evaluated using the factorization approximation (i.e., vacuum insertion approximation), the related uncertainties have been discussed at length in [17].

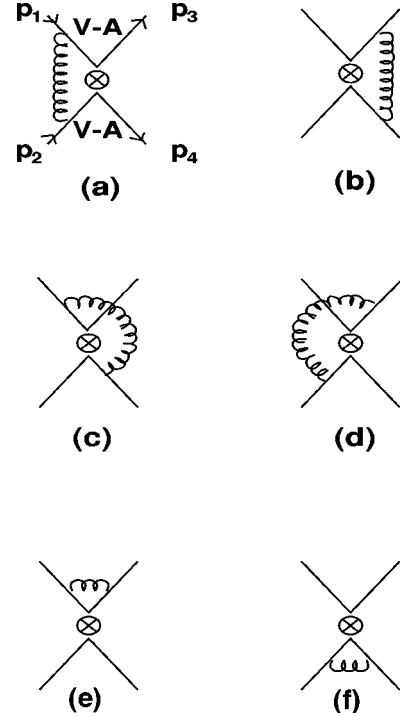


FIG. 2. Vertex corrections to the 4-quark operators O_1 and O_2 .

In conclusion, all the factors in the PQCD formalism are well-defined (including the nonperturbative meson wave functions) and gauge invariant. Physical quantities obtained in this formalism are scale and scheme independent. We have applied this approach to exclusive semileptonic, nonleptonic, and radiative B meson decays and the results are very successful. Nonfactorizable contributions have been calculated, and found to play an important role in the decays $B \rightarrow J/\psi K^{(*)}$ [15]. The opposite signs of a_2/a_1 in bottom and charm decays have been explained by the effects of the Wilson evolution [15]. The mechanism for the sign change of the nonfactorizable contributions in bottom and charm decays has also been explored [16], which is closely related to the success and failure of the large- N_c limit in charm and bottom decays, respectively.

V. EFFECTIVE WILSON COEFFICIENTS

In this section we present the results for the evolution factor $g_1(\mu)$ which describes the evolution from the scale μ to M_b for the current-current operators¹ O_1 and O_2 . Setting $\mu_f = M_b$, the effective Wilson coefficients obtained from the one-loop vertex diagrams Figs. 2(a)–2(f) for the operators O_i have the form

$$c_1^{\text{eff}}|_{\mu_f=M_b} = c_1(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{M_b}{\mu} + r^T \right)_{1i} c_i(\mu), \quad (32)$$

¹The complete results of $g_1(\mu)$ for $\Delta B=1$ transition current-current operators O_1, O_2 , QCD-penguin operators O_3, \dots, O_6 and electroweak penguin operators O_7, \dots, O_{10} are given in [17].

$$c_2^{\text{eff}}|_{\mu_f=M_b} = c_2(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{M_b}{\mu} + r^T \right) c_i(\mu),$$

where the superscript T denotes a transpose of the matrix, and the anomalous dimension matrix $\gamma^{(0)}$ due to the one-loop vertex corrections has the well-known expression

$$\gamma^{(0)} = \begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix}. \quad (33)$$

The matrix r gives momentum-independent constant terms which depend on the treatment of γ_5 . Working in the (massless) on-shell scheme and assuming zero momentum transfer squared between color-singlet currents, i.e., $(p_1 - p_3)^2 = 0$ as well as $(p_1 + p_2)^2 = (-p_2 + p_3)^2 \approx m_b^2$ for O_1 operators and $(p_1 - p_4)^2 = 0$, $(p_1 + p_2)^2 = (-p_2 + p_4)^2 \approx m_b^2$ for O_2 operators (see Fig. 2 for momentum notation), we obtain

$$r_{\text{NDR}} = \begin{pmatrix} 3 & -9 \\ -9 & 3 \end{pmatrix}, \quad r_{\text{HV}} = \begin{pmatrix} \frac{7}{3} & -7 \\ -7 & \frac{7}{3} \end{pmatrix} \quad (34)$$

in NDR and HV schemes, respectively. It should be accentuated that, contrary to the previous work [5]

$$r_{\text{NDR}}^{\lambda=0} = \begin{pmatrix} \frac{7}{3} & -7 \\ -7 & \frac{7}{3} \end{pmatrix}, \quad r_{\text{HV}}^{\lambda=0} = \begin{pmatrix} 7 & -5 \\ -5 & 7 \end{pmatrix}, \quad (35)$$

obtained in Landau gauge and off-shell regularization, the matrix r given in Eq. (34) is gauge invariant.

Two remarks are in order. First, there are infrared double poles, i.e., $1/\epsilon_{\text{IR}}^2$, in the amplitudes of Figs. 2(a)–2(d), but they are canceled out when adding all amplitudes together. Second, care must be taken when applying the projection method to reduce the tensor products of Dirac matrices to the form $\Gamma \otimes \Gamma$ with $\Gamma = \gamma_\mu(1 - \gamma_5)$. For example, a direct evaluation of the tensor product $\gamma_\alpha \not{p}_1 \Gamma \otimes \Gamma \not{p}_2 \gamma^\alpha$ yields ($\epsilon = 4 - D$)

$$\gamma_\alpha \not{p}_1 \Gamma \otimes \Gamma \not{p}_2 \gamma^\alpha = -\epsilon(p_1 \cdot p_2) \Gamma \otimes \Gamma \quad (36)$$

in the NDR scheme with the on-shell condition being applied first to the massless quarks followed by Fierz transformation, whereas the projection method of [18,19,9] leads to

$$\gamma_\alpha \not{p}_1 \Gamma \otimes \Gamma \not{p}_2 \gamma^\alpha = (p_1 \cdot p_2) \Gamma \otimes \Gamma + E, \quad (37)$$

where E stands for the evanescent operator (EO). This means that it is incorrect to take the coefficient of $\Gamma \otimes \Gamma$ in Eq. (37) directly without taking into account the effect of EOs. Note that we have applied Eq. (36) to show the absence of infrared double poles in the total amplitude.

In order to check the scheme and scale independence of c_i^{eff} , it is convenient to work in the diagonal basis in which the operators $O_\pm = \frac{1}{2}(O_1 \pm O_2)$ do not mix under renormalization. Then [see, e.g., [20] for the general expression of $c(\mu)$]

$$c_\pm^{\text{eff}}|_{\mu_f=M_b} = c_\pm(\mu) g_1^\pm(\mu) = \left[1 + \frac{\alpha_s(\mu)}{4\pi} (r_\pm^T + J_\pm) \right] \left[1 + \frac{\alpha_s(M_W)}{4\pi} (B_\pm - J_\pm) \right] \left(\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{\gamma_\pm^{(0)}/(2\beta_0)} + \frac{\alpha_s}{4\pi} \frac{\gamma_\pm^{(0)}}{2} \ln \frac{M_b^2}{\mu^2} \right), \quad (38)$$

where $c_\pm = c_1 \pm c_2$, $\beta_0 = 11 - \frac{2}{3}n_f$ with n_f being the number of flavors between M_W and μ scales, B_\pm specifies the initial condition of $c(m_W)$: $c(m_W) = 1 + [\alpha_s(m_W)/4\pi] B_\pm$ and it is γ_5 -scheme dependent, and $J_\pm = \gamma_\pm^{(0)} \beta_1 / (2\beta_0^2) - \tilde{\gamma}_\pm^{(1)}/(2\beta_0)$, with $\beta_1 = 102 - 38n_f/3$. The scheme-dependent anomalous dimensions $\tilde{\gamma}_\pm^{(1)}$ are given by [20,7]

$$\tilde{\gamma}_\pm^{(1)} = \gamma_\pm^{(1)} - 2\gamma_J = \frac{3\mp 1}{6} \left(-21 \pm \frac{4}{3}n_f - 2\beta_0\kappa_\pm \right), \quad (39)$$

where $\gamma_\pm^{(1)}$ are the two-loop anomalous dimensions of O_\pm , γ_J is the anomalous dimension of the weak current in full theory, and the parameter κ_\pm distinguishes various renormalization schemes: $\kappa_\pm = 0$ in the NDR scheme and $\kappa_\pm = \mp 4$ in the HV scheme. As shown in [20], $B_\pm - J_\pm$ is γ_5 -scheme

independent. Therefore, the effective Wilson coefficients c_\pm^{eff} are scheme independent if we are able to show that $(r^T + J)$ is independent of the choice of renormalization scheme. Since the short-distance Wilson coefficients are independent of the choice of external states, one can show the independence of $\gamma_\pm^{(1)} - 2\gamma_J$ from external states [20]. In the on-shell scheme, γ_J vanishes up to the two-loop level. It follows from Eq. (34) that $r_+ = -6, -7 + 7/3$ and $r_- = 12, 7 + 7/3$ in NDR and HV schemes, respectively, with fermions being on-shell. Then it is easily seen that $r^T + J$ is indeed renormalization-scheme independent. To the leading logarithmic approximation,

$$\left(\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right)^{\gamma_\pm^{(0)}/(2\beta_0)} \approx 1 - \frac{\alpha_s}{4\pi} \frac{\gamma_\pm^{(0)}}{2} \ln \frac{M_W^2}{\mu^2}. \quad (40)$$

Hence, the scale independence of the effective Wilson coefficients follows.

Since the weak current is partially conserved, its anomalous dimension γ_J is zero. However, if fermions are off-shell, γ_J is nonvanishing at the two-loop level in the HV scheme. To maintain the requirement that $\gamma_J=0$, one can force a vanishing γ_J in this case by applying a finite renormalization term to the weak current. (Note that in this new choice, $\gamma_{\pm}^{(1)'} = \gamma_{\pm}^{(1)}$, $B'_{\pm} = B_{\pm} - \gamma_J/\beta_0$, and $B'_{\pm} - J'_{\pm}$ is still scheme independent.) Using the identity $\gamma^{(1)}(\text{on-shell}) = \gamma^{(1)}(\text{off-shell}) - 2\gamma_J$, we find that $\gamma^{(1)}$ in the off-shell fermion scheme is given by [19]:

$$\gamma_{\text{NDR}}^{(1)} = \begin{pmatrix} -\frac{21}{2} - \frac{2}{9}n_f & \frac{7}{2} + \frac{2}{3}n_f \\ \frac{7}{2} + \frac{2}{3}n_f & -\frac{21}{2} - \frac{2}{9}n_f \end{pmatrix}, \quad (41)$$

$$\gamma_{\text{HV}}^{(1)} = \begin{pmatrix} \frac{553}{6} - \frac{58}{9}n_f & \frac{95}{2} - 2n_f \\ \frac{95}{2} - 2n_f & \frac{553}{6} - \frac{58}{9}n_f \end{pmatrix}.$$

From Eqs. (35) and (41), it is straightforward to show that $r_{\pm} + J_{\pm}$ is γ_s -scheme independent in the off-shell regularization. As a result, c^{eff} is renormalization-scheme independent, irrespective of the fermion state, on-shell or off-shell.

VI. CONCLUSION

In this paper we have shown how to construct a gauge-invariant and infrared finite theory of exclusive nonleptonic B meson decays based on the PQCD factorization theorem. Gauge invariance is maintained under radiative corrections by working in the physical on-shell scheme. The infrared divergences in radiative corrections should be then isolated using the dimensional regularization. The resultant infrared poles are absorbed into the universal meson wave functions, which can be determined once for all from experimental data. The absorption of the poles associated with the b quark requires the inclusion of the spectator quark into the theory. The remaining finite contributions form a hard subamplitude. Applying RG analyses to sum various large logarithms in the above factorization formula, the scale and scheme dependences are removed. Hence, in the PQCD formalism physical quantities are guaranteed to be gauge invariant, infrared finite, scale and scheme independent. By working out the evolution factor $g_1(\mu)$ explicitly, we have constructed gauge-invariant, scale and scheme independent effective Wilson coefficients $c(\mu)g_1(\mu)$ at the factorization scale $\mu_f = M_b$. We have shown explicitly that c^{eff} are renormalization-scheme and -scale independent.

We shall take one of the exclusive nonleptonic B meson decay modes as an example to demonstrate how to construct a factorization formula explicitly. This work will be published elsewhere.

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