Cosmological constraints on theories with large extra dimensions

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(Received 26 April 1999; published 27 September 1999)

In theories with large extra dimensions, constraints from cosmology lead to nontrivial lower bounds on the gravitational scale M, corresponding to upper bounds on the radii of the compact extra dimensions. These constraints are especially relevant to the case of two extra dimensions, since only if M is 10 TeV or less do deviations from the standard gravitational force law become evident at distances accessible to planned submillimeter gravity experiments. By examining the graviton decay contribution to the cosmic diffuse gamma radiation, we derive, for the case of two extra dimensions, a conservative bound $M>110$ TeV, corresponding to $r_2<5.1\times10^{-5}$ mm, well beyond the reach of these experiments. We also consider the constraint coming from graviton overclosure of the universe and derive an independent bound $M>6.5/\sqrt{h}$ TeV or $r_2<0.015h$ mm. [S0556-2821(99)05918-4]

PACS number(s): 11.10.Kk, 98.80.Cq

I. INTRODUCTION

It was recently proposed that the large hierarchy between the weak and Planck scales arises because there exist n extra compact spatial dimensions, within which only gravity, and not standard model particles and interactions, can propagate [1,2,3]. In this framework, the Planck scale M_P is not a fundamental scale of nature, but is rather an effective coupling related to M, the scale of $(4+n)$-dimensional gravity, by

$$M_P^2=4\pi r_n^n M^{2+n}, \tag{1}$$

where r_n is the radius of compactification of the n extra dimensions.[1] Setting $M\sim1$ TeV transforms the hierarchy problem into the question of why the radii are large. The approximate values for r_n obtained for $M\sim1$ TeV indicate that $n=1$ is ruled out immediately, while for the $n=2$ case, deviations from the standard force law may easily be detected by planned experiments sensitive to gravitational forces at distances of tens of micrometers [6], depending on the precise value of M. The cases of higher n can be tested instead at high energy colliders.[2]

Bringing the fundamental scale of gravity down near 1 TeV dramatically alters our view of the universe, and it is not a trivial matter that this picture is allowed experimentally. In [3], a diverse range of collider, astrophysical, and cosmological phenomena are examined to verify that the framework is in fact safe for all $n>1$. However, lower bounds on M_F from rough estimates of both energy loss in stellar objects and cosmological constraints, described in Sec. II, cast uncertainty on whether these theories can be probed in future submillimeter gravity experiments, even for the $n=2$ case. In this paper we perform a detailed calculation of the most stringent cosmological constraints and derive an upper bound on r_2 that is far below the anticipated range of these experiments.

II. COSMOLOGY IN THEORIES WITH LARGE EXTRA DIMENSIONS

In standard cosmology, big bang nucleosynthesis (BBN) provides a detailed and accurate understanding of the observed light element abundances [7]. In order not to lose this understanding in the context of theories with large extra dimensions, we must require that, before the onset of BBN, the influence of the extra dimensions on the expansion of our 4D wall will somehow become negligible. In particular, we must imagine that starting at some ''normalcy temperature'' T_*, the extra dimensions are virtually empty of energy density and their radii are fixed. In [3] it is suggested that the emptiness of the bulk can be explained if T_* is the reheat temperature following inflation and if the inflation is localized on our 4D wall and decays only into wall states.

What is the allowed range for T_*? We need $T_*>1$ MeV in order for ordinary BBN to be recovered. On the other hand, if T_* is too large, then copious production of bulk gravitons by standard model particles can alter cosmology in unacceptable ways. The authors of [3] perform rough estimates of several such effects and find that the most serious constraints come from overclosure of the universe by gravitons and contributions to the cosmic diffuse gamma (CDG) radiation from graviton decay. They estimate that

[1] In this work we assume that the extra dimensions are compactified on an n-dimensional torus with a single radius. The scale M defined in Eq. (1) is related to Newton's constant in $(4+n)$ dimensions according to $M^{2+n}=(2\pi)^n/S_{2+n}G_{(4+n)}^{-1}$, where S_k is the surface area of a unit radius sphere in $k-1$ dimensions. This is the same definition of the gravitational scale used in several recent phenomenological studies [3,4,5].

[2] In [3] it is shown that if there exist gauge fields that propagate in the bulk, they can mediate long range forces relevant to submillimeter experiments, regardless of the number of extra dimensions. In theories with approximately unbroken supersymmetry (SUSY) in the bulk, light scalar fields such as the radius modulus can also mediate long range forces [2]. In this paper we restrict our attention to gravitational forces.

these constraints require, for $n=2$, $M \gtrsim 10\,\mathrm{TeV}$, even if the normalcy temperature is pushed down to $T_* \sim 1\,\mathrm{MeV}$.[3] As M is raised to this level, it becomes unclear whether experiments probing macroscopic gravity at small distances will be sensitive to the extra dimensions, even if $n=2$.

In light of the potential implications of cosmological constraints on planned experiments, it is worthwhile to calculate them more carefully. Detailed studies [9,10] show that, in the early universe, the electron neutrinos decouple at 1.25 MeV, while the other flavors of neutrinos decouple at 2.15 MeV. From the results of [9], one can deduce that at $T=1\,\mathrm{MeV}$, the relaxation time for muon and tau neutrinos is 10 times longer than the inverse Hubble rate of expansion. If the reheat temperature were less than 1 MeV, the weak interactions would thus be unable to produce the thermal distribution of neutrinos required as an initial condition for standard BBN. For this reason we believe that by taking $T_* = 1\,\mathrm{MeV}$, we suppress the cosmological effects of the extra dimensions as much as is conceivably allowable, so that bounds we derive on M by requiring $T_* > 1\,\mathrm{MeV}$ should be robust. We also present bounds obtained using the less conservative choice $T_* = 2.15\,\mathrm{MeV}$, which, given that this it is the decoupling temperature for two of the three neutrino species, may in fact be a more realistic value. We find that the strongest bounds on M come from the CDG radiation, to which we dedicate the bulk of our analysis.

III. CALCULATION OF THE DIFFUSE GAMMA RAY BACKGROUND

To calculate the CDG background, we imagine that at the normalcy temperature T_*, the bulk is entirely empty, while standard model particles on our 4D wall assume thermal distributions. The Kaluza-Klein (KK) excitations of the graviton are produced through the process $\nu\bar{\nu} \to G$, for example. The spin-summed amplitude squared for this process is[4]

$$\sum |\mathcal{M}|^2 = \frac{s^2}{4\bar{M}_P^2}, \tag{2}$$

where $\bar{M}_P^2$ is the reduced Planck mass. The number density of mass m KK states is then governed by the Boltzmann equation:

$$\dot{n}_m + 3n_m H = \int \frac{d^3\mathbf{p}_\nu}{(2\pi)^3 2|\mathbf{p}_\nu|} \frac{d^3\mathbf{p}_{\bar{\nu}}}{(2\pi)^3 2|\mathbf{p}_{\bar{\nu}}|} \times \frac{d^3\mathbf{p}_m}{(2\pi)^3 2\sqrt{|\mathbf{p}_m|^2 + m^2}} (2\pi)^4 \delta^4(p_m - p_\nu - p_{\bar{\nu}}) \times \sum |\mathcal{M}|^2 e^{-|p_\nu|/T} e^{-|p_{\bar{\nu}}|/T},$$

and the integrations can be performed analytically to obtain

$$s\dot{Y}_m = \dot{n}_m + 3n_m H = \frac{m^5 T}{128\pi^3 \bar{M}_p^2} \mathcal{K}_1\left(\frac{m}{T}\right), \tag{3}$$

where $\mathcal{K}_1$ is a Bessel function of the second kind. We have applied entropy conservation to express the evolution in terms of the scaled number density $Y_m = n_m/s$, where s is the entropy density. We will be interested in KK states that decay to photons in the MeV range, and from Eq. (3) we see that essentially all of the graviton production occurs at temperatures near m and thus at times well within the radiation-dominated era. The neutrino temperature T is therefore related to the time by [13]

$$t = 1.5 g_*^{-1/2} \bar{M}_p T^{-2}, \tag{4}$$

where, since we will be considering temperatures of order 1 MeV and lower, $g_* = 10.75$. Applying $s \propto T^3$ then leads to a present-day graviton density (neglecting decay) of

$$n_0^{(m)} = (2.3 \times 10^{-4}) \frac{m T_0^3}{\bar{M}_p} \int_{m/T_*}^{\infty} dx\, x^3 \mathcal{K}_1(x), \tag{5}$$

where the present day neutrino temperature is $T_0 = 1.96\,\mathrm{K}$.

A photon produced in the decay of a KK graviton of mass m will have a detected energy that depends on the redshift or, equivalently, the time at which the decay occurred. Thus the energy spectrum of photons produced in the decays of mass m KK gravitons can be calculated using

$$\frac{dn_\gamma^{(m)}}{dE} = \frac{dn_\gamma^{(m)}}{dt} \frac{dt}{dz} \frac{dz}{dE}. \tag{6}$$

The derivatives are evaluated by applying $E = (m/2)(1+z)^{-1}$, $t = t_0(1+z)^{-3/2}$, and $n_\gamma^{(m)} = 2n_0^{(m)} \Gamma_\gamma / \Gamma_T (1 - e^{-\Gamma_T t})$, where Γ_γ is the decay width of the graviton into two photons and Γ_T is its total decay width. We use the time-redshift relation that holds for the matter-dominated era, because for KK gravitons that are produced near $T_* \sim 1\,\mathrm{MeV}$ and which decay into photons during the radiation-dominated era, the redshifted photon energies are far below the MeV range that interests us. The spectrum is evaluated to be

$$\frac{dn_\gamma^{(m)}}{dE} = 3 n_0^{(m)} \Gamma_\gamma t_0 (2/m)^{3/2} E^{1/2} e^{-\Gamma_T t_0 (2E/m)^{3/2}}. \tag{7}$$

To calculate the full photon spectrum, all that remains is to sum over KK modes. This is accomplished using the measure

$$dN = 2 S_{n-1} \frac{\bar{M}_P^2}{M^{2+n}} m^{n-1} dm, \tag{8}$$

where $S_{n-1} = 2\pi^{n/2}/\Gamma(n/2)$ is the surface area of a unit-radius sphere in n dimensions. Using Eqs. (5) and (7), and Eq. (8), and the calculated width

[3] The authors of [8] estimate $M > 100\,\mathrm{TeV}$, by requiring $T_* \gtrsim 3\,\mathrm{MeV}$.

[4] Feynman rules for the coupling of gravity to matter are derived in [11,12].

TABLE I. Values of the parameters $\alpha_n(E)$ and $f_n(E,T_*=1\ \mathrm{MeV})$ defined in Eqs. (11) and (13).

E(MeV)	$\frac{f_2(E,T_*=1\ \mathrm{MeV})}{\mathrm{MeV}^{9/2}}$	$\alpha_2(E)$	$\frac{f_3(E,T_*=1\ \mathrm{MeV})}{(\mathrm{MeV}^{11/2})}$	$\alpha_3(E)$
1	1456	4.2×10^4	9570	0.56
2	1228	5.0×10^4	8835	0.72
3	778	4.0×10^4	6571	0.66
4	379	2.2×10^4	3801	0.44
5	150	9.8×10^3	1773	0.22
6	51.1	3.6×10^3	698	1.0×10^{-1}
7	15.5	1.2×10^3	240	3.6×10^{-2}
8	4.27	3.6×10^2	74	1.2×10^{-2}
9	1.09	94	21.2	3.6×10^{-3}
10	.263	24	5.6	1.0×10^{-3}

$$\Gamma(G\to\gamma\gamma)=\frac{m^3}{80\pi\bar{M}_P^2}, \tag{9}$$

we obtain the spectrum

$$\frac{dn_\gamma}{dE}=(1.6\times10^{-5})S_{n-1}\frac{t_0T_0^3}{M^{2+n}\bar{M}_P}E^{1/2}f_n(E,T_*), \tag{10}$$

where the function $f_n(E,T_*)$ is given by

$$f_n(E,T_*)=\int_{2E}^{\infty}dm\,m^{n+3/2}e^{-\Gamma_T t_0(2E/m)^{3/2}}\int_{m/T_*}^{\infty}dx\,x^3\mathcal{K}_1(x). \tag{11}$$

Numerically, one finds $\Gamma(G\to\gamma\gamma)t_0\sim3\times10^{-7}(m/\mathrm{MeV})^3$, so that for the KK excitations that interest us, the graviton lifetime will be much longer that the lifetime of the universe. Even after considering other decay channels, we find that Γ_T is so small that setting the exponential factor in Eq. (11) to unity does not significantly change the values of $f_n(E,T_*)$.

Taking $t_0=10^{10}$ yr, we find that for $T_*=1$ MeV, the spectrum can be written as

$$\left.\frac{dn_\gamma}{dE}\right|_{T_*=1\ \mathrm{MeV}}=4.6\times10^{-6(n-2)}S_{n-1}\frac{f_n(E,T_*=1\ \mathrm{MeV})}{\mathrm{MeV}^{(n+5/2)}}\times\left(\frac{E}{\mathrm{MeV}}\right)^{1/2}\left(\frac{M}{\mathrm{TeV}}\right)^{-(n+2)}\mathrm{MeV}^{-1}\times\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1} \tag{12}$$

$$\equiv\alpha_n(E)\left(\frac{M}{\mathrm{TeV}}\right)^{-(n+2)}\mathrm{MeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}. \tag{13}$$

Values for $\alpha_n(E)$ and $f_n(E,T_*=1\ \mathrm{MeV})$ for $n=2,3$ are given in Table I.

The above photon spectrum was derived by calculating the density of KK gravitons produced by annihilation of a single neutrino species. Repeating the same calculation for $\gamma\gamma$ annihilation, we find a spin-summed amplitude squared,

$$\sum|\mathcal{M}|^2=2\frac{s^2}{\bar{M}_P^2}, \tag{14}$$

and, taking into account the symmetry factor of 1/2 due to the initial state photons, a contribution to the spectrum that is larger than that coming from a single neutrino flavor by a factor of 4. When comparing with the observed spectrum, we will take the sum of contributions from photons and three flavors of neutrinos:[5]

$$\left.\frac{dn_\gamma}{dE}\right|_{T_*=1\ \mathrm{MeV}}=7\,\alpha_n(E)\times\left(\frac{M}{\mathrm{TeV}}\right)^{-(n+2)}\mathrm{MeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}. \tag{15}$$

Before comparing our results with the CDG background data, we can already obtain an independent bound on M by requiring that the KK gravitons do not overclose the universe. Contributions from photon and neutrino annihilation give a graviton energy density

$$\rho_G=14S_{n-1}\frac{\bar{M}_P^2}{M^{2+n}}\int_0^\infty dm\,m^n n_0^{(m)}, \tag{16}$$

where $n_0^{(m)}$ is the density defined in Eq. (5). For $n=2$ we obtain

$$\rho_G=14\times10^{-44}\left(\frac{M}{\mathrm{TeV}}\right)^{-4}\mathrm{GeV}^4, \tag{17}$$

which, upon comparison with $\rho_c=8.1h^2\,10^{-47}\ \mathrm{GeV}^4$, leads to $M>6.5/\sqrt{h}$ TeV. Using the relation between the fundamental scale and the radius of compactification,

$$r_n=2\times10^{31/n-16}\left(\frac{1\ \mathrm{TeV}}{M}\right)^{1+2/n}\mathrm{mm}, \tag{18}$$

we obtain

$$r_2<0.015h\ \mathrm{mm}. \tag{19}$$

It may be possible, although certainly challenging, to probe distances of this size in near-future submillimeter gravity experiments. If we take a less conservative bound on T_* and instead use $T_*=2.15$ MeV, the decoupling temperature for the muon and tau neutrinos, we get the more stringent

[5]We neglect an additional contribution from e^+e^- annihilation for the sake of a simplified calculation. Including this contribution enhances the bounds we derive only slightly.

bounds $M > 13.9/\sqrt{h}$ TeV and $r_2 < 3.3h \times 10^{-3}$ mm. Distances this small are likely not to be accessible to those experiments.

IV. COMPARISON WITH DATA

The CDG background has been measured recently in the 800 keV to 30 MeV energy range using the COMPTEL instrument [14]. The authors of [14] find that the photon spectrum is well described by the power-law function $A(E/E_0)^{-a}$, with $a = -2.4 \pm 0.2$, $E_0 = 5$ MeV, and $A = (1.05 \pm 0.2) \times 10^{-4}\,\mathrm{MeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$. They find no evidence for the ''MeV bump'' that was inferred from previous data. Using the COMPTEL results and the calculated contribution to the background from graviton decay in Eq. (15), we can place a lower bound on the gravitational scale M:

$$\left(\frac{M}{\mathrm{TeV}}\right)^{n+2} > 7\,\alpha_n(E)\left(\frac{dn_\gamma/dE|_{\mathrm{measured}}}{\mathrm{MeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}}\right)^{-1}. \quad (20)$$

We find that the most stringent bounds are obtained for $E \simeq 4$ MeV. Using the very conservative upperbound $dn_\gamma/dE|_{\mathrm{measured}} < 10^{-3}\,\mathrm{MeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$ gives, for $n = 2$,

$$M > 110 \text{ TeV}. \quad (21)$$

This corresponds to a bound on the radius of compactification of

$$r_2 < 5.1 \times 10^{-5} \text{ mm}, \quad (22)$$

which is far smaller than the distances at which gravity can be probed in planned experiments.[6] If we instead use $T_* = 2.15$ MeV, we obtain $M > 350$ TeV: M must be about 10^3 or more larger than the electroweak vacuum-expectation value (VEV), reintroducing a mild hierarchy problem and, hence, requiring supersymmetry or some other solution.[7] Applying the same experimental bound to the $n = 3$ case leads to $M > 5.0$ TeV or $M > 13.8$ TeV, for $T_* = 1$ MeV and $T_* = 2.15$ MeV, respectively.

V. COSMOLOGICAL UNCERTAINTIES

Are there ways to evade our bounds on M? The authors of [3] have pointed out that there may be additional branes, in addition to our own, on which gravitons can decay. Depending on the decay rate on these branes, their existence can greatly reduce the number of gravitons that decay on our brane. If $1/\Gamma'$, the decay lifetime onto the other brane(s), is significantly longer than the age of the universe t_0, then the number of decays on our brane will not be substantially reduced. If $1/\Gamma' \ll t_0$, on the other hand, the number of decays on our brane—and thus the contribution to the photon background—is reduced by a factor $\sim 1/(\Gamma' t_0)$. Moreover, in this case nearly all of the gravitons decay at large redshift, so that for $T_* \sim 1$ MeV the redshifted photon energies fall below the MeV range.

We know of two scenarios that give the large Γ' required to evade the CDG bound. In the first, Γ' is large because the extra brane(s) have higher dimension than ours [3]. If one of these so called ''fat-branes'' has thickness W in a single extra dimension, the probability that a graviton will decay on it is enhanced over its probability of decaying on our brane by a factor $\sim WT_*$. For $WT_* \sim 5 \times 10^6$, we find that the graviton contribution to the CDG is consistent with the COMPTEL result for M as low as ~ 1 TeV. Taking $T_* = 1$ MeV, this corresponds to a thickness $W > 1\ \mu$m. Note that introducing a higher dimensional brane does *not* enable us to evade the bound obtained by considering overclosure of the universe, Eq. (19). Because the fat-brane is higher dimensional, the decay products have a momentum component that is perpendicular to our brane and which, therefore, does not redshift (recall that the extra spatial dimensions are frozen). Thus the energy density of these decay products will go as R^{-3} rather than R^{-4}, regardless of whether or not the particles are relativistic, and we cannot eliminate the graviton contribution to Ω.

In the second scenario, Γ' is large because there exist a very large number of 4D branes in addition to our own. More precisely, we need at least $\sim 5 \times 10^6$ additional branes to have a graviton contribution to the CDG background that is consistent with the COMPTEL result when $M \sim 1$ TeV. An important distinction between this scenario and the one involving higher dimensional branes is that now, provided the foreign branes are parallel to our own, relativistic decay products on them *do* redshift, and the bound in Eq. (19) can be evaded.

VI. CONCLUSIONS

We have examined two cosmological constraints on the theories with large extra dimensions proposed in [1,2,3]. To place limits on M, we apply a conservative lower bound on the normalcy temperature, $T_* > 1$ MeV, as required by BBN. We find that, ignoring the possible existence of additional branes, the radius of compactification of the extra dimensions for $n = 2$ is bound by the cosmic diffuse gamma ray background to be $r_2 < 5.1 \times 10^{-5}$ mm, well beyond the reach of planned submillimeter gravity experiments. From the constraint that gravitons do not overclose the universe, we derive a milder bound, $r_2 < 0.015h$ mm, albeit one that is less dependent on our assumptions regarding foreign branes. If one instead insists on a normalcy temperature above the decoupling temperature for the muon and tau neutrinos, $T_* > 2.15$ MeV, these bounds become $r_2 < 5.2 \times 10^{-6}$ mm and $r_2 < 3.3h \times 10^{-3}$ mm, respectively.

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[6] The authors of [8] estimate $M > 100$ TeV, by requiring $T_* \gtrsim 3$ MeV.

[7] The string scale may be lower than M, in which case the hierarchy is alleviated slightly. At least in the string scenario described in [2], where standard model particles are localized on a three-brane, the factor one might gain in this way is ~ 10 rather than $\sim 10^3$ [3]. If the standard model particles are instead localized on a brane of higher dimension, one can achieve further suppression of the string scale relative to M [15].

A recent calculation has given the bound $M > 50$ TeV for $n = 2$, from the requirement that supernovas do not cool too rapidly by graviton emission [5]. This astrophysical constraint complements the cosmological ones we have studied: it is subject to larger technical calculational uncertainties, while our analysis is subject to uncertainties in the global cosmological picture. In either case, a bound on M can only be translated into a limit on r_n if it is assumed that the extra dimensions have the same size. No matter how large n is taken to be, it is always possible that one extra dimension has a size in the millimeter to micrometer range, while the others are much smaller [16]. However, in a framework involving vastly different radii, we are unable to argue why gravity would be expected to diverge from r^{-2} behavior specifically at those distance scales accessible to planned experiments.

ACKNOWLEDGMENTS

We are grateful to Nima Arkani-Hamed for useful discussions. D. S. also thanks Michael Graesser for helpful conversations. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and in part by the National Science Foundation under Grant No. PHY-95-14797.

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