Understanding radiatively induced Lorentz-*CPT* violation in differential regularization

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We investigate the perturbative ambiguity of the radiatively induced Chern-Simons term in differential regularization. The result obtained by this method contains all those obtained by other regularization schemes and the ambiguity is explicitly characterized by an indefinite ratio of two renormalization scales. It is argued that the ambiguity can only be eliminated by either imposing a physical requirement or resorting to a more fundamental principle. Some calculation techniques in coordinate space are developed in the appendixes. [S0556-2821(99)02218-3]

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In a relativistic quantum field theory, Lorentz and *CPT* violating terms should be strictly prohibited. Otherwise, the status of special relativity, as one of the cornerstones of modern physics, will be challenged. However, recently some investigations have been carried out to consider this possibility [1,2]. Motivated by the proposal put forward about a decade ago of introducing a Chern-Simons term [3], $\mathcal{L}_k = 1/2 k_\mu \epsilon^{\mu\nu\lambda\rho} F_{\nu\lambda} A_\rho$, to violate the Lorentz and *CPT* symmetry of quantum electrodynamics, recently a Lorentz and *CPT* violating extension of the standard model was constructed and some of its quantum aspects were investigated [2]. As pointed out by Jackiw [4], the availability of higher precision instruments nowadays allows a more precise test on some of fundamental principles to be carried out. Such an investigation at least at the theoretical level is not completely unreasonable. The question is how this Lorentz and *CPT* violating term can naturally arise rather than be introduced by hand.

Based on the experience in (2+1)-dimensional QED [5], where a parity-odd Chern-Simons term is induced from the fermionic determinant [6,7], one natural guess is that this Lorentz and *CPT* violation term can come from a Lorentz and *CPT* violation term $\bar{\psi} \not{b} \gamma_5 \psi$ in the fermionic sector. The explicit calculation carried out recently shows that this case can happen [8,9], there has induced a Chern-Simons term with its coefficient k_μ proportional to b_μ. However, since the UV divergence usually emerges in a perturbative quantum correction, one must first choose a regularization scheme to make the theory well defined. It was shown that the coefficient of this radiatively induced Chern-Simons term is regularization dependent [2,8]. In Pauli-Villars regularization, this coefficient is zero, while dimensional regularization combined with the derivative expansion leads to a definite nonzero value [8]. It also seems to us that this induced term cannot be observed by calculating the fermionic determinant in Fock-Schwinger proper time method [10]. In particular, based on the hypothesis that the axial vector current $j_\mu^5 = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)$ should be gauge invariant at arbitrary four-momentum, Coleman and Glashow claimed that the k_μ must be zero [11]. Thus the existence of this radiatively induced Chern-Simons term in perturbative theory is somehow ambiguous. More recently, a new calculation was performed by Jackiw and Kostelecký [9], using the b_μ-exact propagator instead of the free fermionic propagator, in which the Lorentz and *CPT* violating fermionic term was treated nonperturbatively. It was shown that in this nonperturbative approach the Lorentz and *CPT* violating term is generated unambiguously at low energy. Therefore, it is interesting to understand the discrepancies among these various results within the framework of perturbative theory.

One possible way is utilizing an improved regularization scheme. It is known that a regularization is a temporary modification of the original theory. Different regularization schemes have actually provided different methods to calculate a quantum correction. Thus it is possible to incur a regularization dependent result. To avoid this occurrence, one should choose a regularization scheme that modifies the original theory as little as possible and preserves the features of the original theory such as symmetries, etc., as much as possible. In view of this criterion, differential regularization seems to be the most appropriate candidate [12]. This regularization scheme is a relatively new calculation method and it works for a Euclidean field theory in coordinate space. The invention of this regularization is based on the observation that in coordinate space the UV divergence manifest itself in the singularity preventing the amplitude from having a Fourier transform into momentum space. So one can regulate the amplitude by writing its singular term as the derivative of another less singular function, which has a well defined Fourier transform, then performing the Fourier transform and discarding the surface term. In this way one can directly get a well defined amplitude. Up to now this method and its modified version have been applied successfully to almost every aspects of field theories, including chiral anomaly, low-dimensional and supersymmetric field theories [13–17]. One can easily see that this regularization method actually has never introduced an regulator to modify the Lagrangian of the original theory, hence it does not pull the value of a primitively divergent Feynman diagram away from its singularity. In comparison with the usual route of calculating a

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quantum correction, this method has actually skipped over the regularization procedure and straightforwardly yielded the renormalized result. Therefore, the quantum correction obtained in this regularization method should be more universal than any other regularization schemes and hence can provide a better understanding to above ambiguity.

Not only these favourable features, differential regularization has another great advantage over other regularization schemes. When implementing a differential regularization on a quantum amplitude, one can introduce a renormalization scale for each singular term. These individual renormalization scales are not independent and the relations among them can be fixed by the symmetries of theory. In other words, the maintenance of the symmetries in the theory such as gauge symmetry etc., can be achieved by choosing the indefinite renormalization scales at the final stage of the calculation. As will be shown later, this special feature of differential regularization is not only the reason why the regularization ambiguity can be explicitly parameterized by the ratio of two renormalization scales, but also provide a guide for us to search for a natural setting to eliminate this ambiguity.

In view of this, in this paper we shall investigate this radiatively induced ambiguity in terms of differential regularization. The model we shall start from is quantum electrodynamics with the inclusion of a Lorentz-and CPT-violating axial vector term in the fermionic sector [8,9,18],

$$\mathcal{L}_{\text{fermion}}=\bar{\psi}(\partial\!\!\!/-A\!\!\!/-b\!\!\!/\gamma_5)\psi, \tag{1}$$

where b_μ is a constant four-vector with a fixed orientation in space-time. The term $\bar{\psi}b\!\!\!/\gamma_5$ is gauge invariant, but it explicitly violates Lorentz and CPT symmetries, since b_μ picks up a preferred direction in space-time. We will see that this Lorentz and CPT violation in the fermionic sector is the origin of the induced Chern-Simons term.

In Ref. [9], it was found that the radiatively induced Chern-Simons term can arise in the low-energy, or equivalently in the large fermionic mass limit. In principle, we can also utilize the b_μ-exact propagator in coordinate space to calculate the vacuum polarization tensor. However, the existence of the b term make it impossible to write out this b_μ-exact propagator in coordinate space, and hence one cannot proceed parallel to Ref. [5] in coordinate space. Thus we have to adopt a free fermionic propagator. The Feynman diagram that will be calculated is the vacuum polarization tensor but with an insertion of a zero-momentum composite operator $\int d^4z\bar{\psi}b\!\!\!/\gamma_5\psi$ in the internal fermionic line (Fig. 1), since only this kind of diagram can give the lowest order contribution in b and hence possibly leads to the induced Chern-Simons term. Equivalently, this kind of Feynman diagram can also be thought as the triangle diagram composed of two vector currents and one axial vector currents but with zero momentum transfer between the vector currents. In fact, the explicit calculation in Ref. [9] is very similar to that for the chiral anomaly, only the zero-momentum transfer between two vector gauge field vertices was achieved naturally

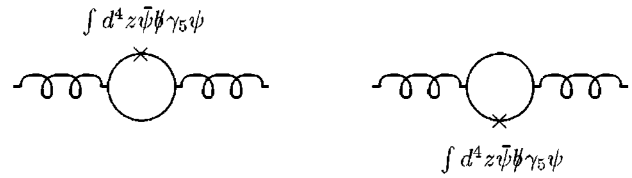

FIG. 1. Vacuum polarization contributed by fermionic loop with an insertion of zero-momentum composite operator $\int d^4z\bar{\psi}b\!\!\!/\gamma_5\psi$ in either of the internal fermionic lines, $\times$ denoting the zero-momentum composite operator $\int d^4z\bar{\psi}b\!\!\!/\gamma_5\psi$.

due to the utilization of the b_μ-exact propagator. Here we can also get a natural zero-momentum transfer by considering the above Feynman diagrams.

We need the free fermionic propagator

$$S(x)=\frac{1}{4\pi^2}\partial\!\!\!/\frac{1}{x^2} \tag{2}$$

for the massless case, and

$$(x)=(\partial\!\!\!/-m)\Delta(x)=\frac{m}{4\pi^2}(\partial\!\!\!/-m)\left[\frac{K_1(mx)}{x}\right] \tag{3}$$

for the massive case, here and later on we denote $x \equiv |x|$, $K_1(x)$ is the first-order modified Bessel function of the second kind. The short-distance expansion of the massive scalar propagator $\Delta(x)$ is

$$\begin{aligned}\Delta(x)&=\frac{1}{4\pi^2}\frac{m}{x}K_1(mx)\\&=\frac{1}{4\pi^2}\left[\frac{1}{x^2}+\frac{1}{2}m^2\ln(mx)\right.\\&\qquad\left.+\frac{m^2}{4}[1-2\psi(2)]+\text{regular terms}\right].\end{aligned} \tag{4}$$

We first have look at the massless case. The vacuum polarization with an insertion of the zero-momentum composite operator $\int d^4z\bar{\psi}b\!\!\!/\gamma_5\psi$ on either of the fermionic lines is read as

$$\Pi_{\mu\nu}(x,y)=-b_\lambda\int d^4z\{\mathrm{Tr}[\gamma_5\gamma_\lambda S(z-x)\gamma_\mu S(x-y)\gamma_\nu S(y-z)]+\mathrm{Tr}[\gamma_5\gamma_\lambda S(z-y)\gamma_\nu S(y-x)\gamma_\mu S(x-z)]\}$$
$$=-\frac{1}{(4\pi^2)^3}b_\lambda\Bigg[\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c)\int d^4z\frac{\partial}{\partial z_a}\frac{1}{(z-x)^2}\frac{\partial}{\partial x_b}\frac{1}{(x-y)^2}\frac{\partial}{\partial y_c}\frac{1}{(y-z)^2}$$
$$+\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)\int d^4z\frac{\partial}{\partial z_a}\frac{1}{(z-y)^2}\frac{\partial}{\partial y_b}\frac{1}{(y-x)^2}\frac{\partial}{\partial x_c}\frac{1}{(x-z)^2}\Bigg]$$
$$=\frac{4}{(4\pi^2)^3}b_\lambda\Bigg[\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c)\int d^4z\frac{(z-x)_a}{(z-x)^4}\frac{(z-y)_c}{(z-y)^4}\frac{\partial}{\partial x_b}\frac{1}{(x-y)^2}$$
$$+\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)\int d^4z\frac{(z-y)_a}{(z-y)^4}\frac{(z-x)_c}{(z-x)^4}\frac{\partial}{\partial y_b}\frac{1}{(x-y)^2}\Bigg]. \tag{5}$$

Using the convolution integral given by Eq. (A6),

$$\int d^4z\frac{(z-x)_\mu(z-y)_\nu}{(z-x)^4(z-y)^4}=\frac{\pi^2}{2(x-y)^2}\left[\delta_{\mu\nu}-2\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^2}\right], \tag{6}$$

we obtain

$$\Pi_{\mu\nu}(x,y)=\Pi_{\mu\nu}(x-y)=\frac{1}{32\pi^4}b_\lambda\times\mathrm{Tr}[\gamma_5\gamma_\lambda(\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c-\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]\times\left[\delta_{ac}-2\frac{(x-y)_a(x-y)_c}{(x-y)^2}\right]\times\frac{1}{(x-y)^2}\frac{\partial}{\partial x_b}\frac{1}{(x-y)^2}. \tag{7}$$

For convenience, denoting $x-y$ as x and employing the differential operation

$$\frac{x_ax_bx_c}{x^8}=-\frac{1}{48}\frac{\partial^3}{\partial x_a\partial x_b\partial x_c}\frac{1}{x^2}-\frac{1}{24}\times\left(\delta_{ab}\frac{\partial}{\partial x_c}+\delta_{bc}\frac{\partial}{\partial x_a}+\delta_{ca}\frac{\partial}{\partial x_b}\right)\frac{1}{x^4}, \tag{8}$$

we can write the above vacuum polarization tensor in the following form:

$$\Pi_{\mu\nu}(x)=\frac{1}{64\pi^4}b_\lambda\,\mathrm{Tr}[\gamma_5\gamma_\lambda(\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c-\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]\times\left[\frac{\partial}{\partial x_b}\frac{1}{x^4}\delta_{ac}-\frac{1}{6}\frac{\partial^3}{\partial x_a\partial x_b\partial x_c}\frac{1}{x^2}-\frac{1}{3}\times\left(\delta_{ab}\frac{\partial}{\partial x_c}+\delta_{bc}\frac{\partial}{\partial x_a}+\delta_{ca}\frac{\partial}{\partial x_b}\right)\frac{1}{x^4}\right]. \tag{9}$$

Obviously, $1/x^4$ is too singular to have a Fourier transform into momentum space, so we must replace it by its differential regulated version

$$\left(\frac{1}{x^4}\right)_R=-\frac{1}{4}\Box\frac{\ln(x^2M^2)}{x^2}, \tag{10}$$

where $\Box\equiv\partial^2$ denotes the four-dimensional Laplacian operator. Thus the differential regulated version of the vacuum polarization tensor with an insertion of the zero-momentum composite operator $\int d^4z\bar{\psi}\not{b}\gamma_5\psi$ is

$$\Pi_{\mu\nu}(x)=\frac{1}{64\pi^4}b_\lambda\,\mathrm{Tr}[\gamma_5\gamma_\lambda(\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c-\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]\times\left[-\frac{1}{4}\delta_{ca}\frac{\partial}{\partial x_b}\Box\frac{\ln(x^2M_1^2)}{x^2}-\frac{1}{6}\frac{\partial^3}{\partial x_a\partial x_b\partial x_c}\frac{1}{x^2}+\frac{1}{12}\left(\delta_{ab}\frac{\partial}{\partial x_c}+\delta_{bc}\frac{\partial}{\partial x_a}+\delta_{ca}\frac{\partial}{\partial x_b}\right)\Box\frac{\ln(x^2M_2^2)}{x^2}\right]. \tag{11}$$

Note that we chosen two different renormalization scales for $1/x^4$ in the first and the third term of Eq. (9) since two singular terms can differ from a finite quantity. After contracting with the external γ-matrix trace, we get

$$\Pi_{\mu\nu}(x)=\frac{1}{16\pi^4}b_\lambda\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a}\left(\frac{1}{3}+\ln\frac{M_1^2}{M_2^2}\right)\Box\frac{1}{x^2}$$
$$=-\frac{1}{4\pi^2}\left(\frac{1}{3}+2\ln\frac{M_1}{M_2}\right)b_\lambda\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a}\delta^{(4)}(x). \tag{12}$$

In the above calculation, we have used

$$\mathrm{Tr}[\gamma_5\gamma_\lambda(\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c-\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]\delta_{ca}\frac{\partial}{\partial x_b}=-16\epsilon_{\lambda\mu\nu b}\frac{\partial}{\partial x_b},$$

$$\mathrm{Tr}[\gamma_5\gamma_\lambda(\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c-\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]\delta_{bc}\frac{\partial}{\partial x_a}=-16\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a},$$

$$\mathrm{Tr}[\gamma_5\gamma_\lambda(\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c-\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]\delta_{ab}\frac{\partial}{\partial x_c}=-16\epsilon_{\lambda\mu\nu c}\frac{\partial}{\partial x_c},$$

$$\mathrm{Tr}[\gamma_5\gamma_\lambda(\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c-\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]\frac{\partial^3}{\partial x_a\partial x_b\partial x_c}=-8\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a}\Box. \tag{13}$$

One may wonder why we only adopt two mass scales in Eq. (11) for those four short-distance singular terms of Eq. (9). Of course, we can introduce four distinct mass scales, then there appears

$$\Pi_{\mu\nu}(x)=\frac{1}{64\pi^4}b_\lambda\,\mathrm{Tr}[\gamma_5\gamma_\lambda(\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c-\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]$$
$$\times\left[-\frac{1}{4}\delta_{ac}\frac{\partial}{\partial x_b}\Box\frac{\ln(x^2M_1^2)}{x^2}-\frac{1}{6}\frac{\partial^3}{\partial x_a\partial x_b\partial x_c}\frac{1}{x^2}\right.$$
$$+\frac{1}{12}\left(\delta_{ab}\frac{\partial}{\partial x_c}\Box\frac{\ln(x^2M_3^2)}{x^2}+\delta_{bc}\frac{\partial}{\partial x_a}\Box\frac{\ln(x^2M_4^2)}{x^2}\right.$$
$$\left.\left.+\delta_{ca}\frac{\partial}{\partial x_b}\Box\frac{\ln(x^2M_5^2)}{x^2}\right)\right]$$
$$=-\frac{1}{4\pi^2}\left[\frac{1}{3}+2\ln\frac{M_1}{(M_3M_4M_5)^{1/3}}\right]$$
$$\times b_\lambda\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a}\delta^{(4)}(x). \tag{14}$$

With the definition $M_2\equiv(M_3M_4M_5)^{1/3}$, we still obtain the same result as Eq. (12). It can be easily checked that any other differential operations on $1/x^4$ will lead to the same conclusion: the ambiguity is only relevant to two independent mass scales and uniquely parametrized by their ratio $\ln M_1/M_2$. The physical renormalization conditions or symmetries will fix this ambiguity.

The vacuum polarization tensor (12) shows that the following Lorentz and CPT violated action are indeed induced:

$$S_{\mathrm{ind}}=\frac{1}{8\pi^2}\left(\frac{1}{3}+2\ln\frac{M_1}{M_2}\right)\int d^4x\,\epsilon_{\mu\nu\lambda\rho}b_\mu A_\nu F_{\lambda\rho}. \tag{15}$$

It is remarkable that this radiatively induced Lagrangian has an ambiguity parametrized by an indefinite coefficient $\ln M_1/M_2$. It is just the case recently pointed out by Jackiw that the radiative correction is finite but undetermined [18].

The more interesting case is when the fermion is massive, where Jackiw and Kostelecký [9] successfully escaped from the ''no-go'' theorem proposed by Coleman and Glashow [11] and found the existence of the radiatively induced Chern-Simons term in a nonperturbative way, so this case has a direct physical relevance. The corresponding vacuum polarization tensor is

$$\Pi_{\mu\nu}(x-y)=-\left(\frac{m}{4\pi^2}\right)^3 b_\lambda\int d^4z\Bigg\{\mathrm{Tr}\Bigg[\gamma_5\gamma_\lambda\left(\gamma_a\frac{\partial}{\partial z_a}-m\right)\frac{K_1[m(z-x)]}{z-x}\gamma_\mu\left(\gamma_b\frac{\partial}{\partial x_b}-m\right)\frac{K_1[m(x-y)]}{x-y}\gamma_\nu$$
$$\times\left(\gamma_c\frac{\partial}{\partial y_c}-m\right)\frac{K_1[m(y-z)]}{y-z}\Bigg]+\mathrm{Tr}(\mu\leftrightarrow\nu,x\leftrightarrow y)\Bigg\}$$
$$=-\left(\frac{m}{4\pi^2}\right)^3 b_\lambda\int d^4z\Bigg\{\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c)\frac{\partial}{\partial z_a}\frac{K_1[m(z-x)]}{z-x}\frac{\partial}{\partial x_b}\frac{K_1[m(x-y)]}{x-y}\frac{\partial}{\partial y_c}\frac{K_1[m(y-z)]}{y-z}$$
$$+\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)\frac{\partial}{\partial z_a}\frac{K_1[m(z-y)]}{z-y}\frac{\partial}{\partial y_b}\frac{K_1[m(y-x)]}{y-x}\frac{\partial}{\partial x_c}\frac{K_1[m(x-z)]}{x-z}$$
$$+8m^2\epsilon_{\lambda\mu\nu a}\Bigg[\left(\frac{\partial}{\partial z_a}\frac{K_1[m(z-x)]}{z-x}\right)\frac{K_1[m(x-y)]}{x-y}\frac{K_1[m(y-z)]}{y-z}$$
$$+\frac{K_1[m(z-y)]}{z-y}\left(\frac{\partial}{\partial y_a}\frac{K_1[m(y-x)]}{y-x}\right)\frac{K_1[m(x-z)]}{x-z}+\frac{K_1[m(z-x)]}{z-x}\frac{K_1[m(x-y)]}{x-y}\left(\frac{\partial}{\partial y_a}\frac{K_1[m(y-z)]}{y-z}\right)\Bigg]\Bigg\}. \tag{16}$$

Using the convolution integral (A14) for the massive case and denoting $x-y$ as x, we can write the vacuum polarization tensor (16) as follows:

$$\Pi_{\mu\nu}(x)=-\left(\frac{m}{4\pi^2}\right)^3 b_\lambda\Bigg\{\frac{2\pi^2}{m^2}[\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c)-\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\nu\gamma_b\gamma_\mu\gamma_c)]\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial^2}{\partial x_a\partial x_c}K_0(mx)$$
$$-\frac{2\pi^2}{m^2}\epsilon_{\lambda\mu\nu a}\left[16m^2\frac{K_1(mx)}{x}\frac{\partial}{\partial x_a}K_0(mx)+8m^2K_0(mx)\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\right]\Bigg\}. \tag{17}$$

One natural way to perform the operation on Eq. (17) is to expand the term $\partial/\partial x_b[K_1(mx)/x]\partial^2/(\partial x_a\partial x_c)K_0(mx)$, write its singular terms in a derivative form and then contract it with γ-matrix trace. However, we have no way to realize this due to the difficulty in solving a differential equation with the modified Bessel function. Neither can we do it for the terms $K_1(mx)/x\,\partial/\partial x_a K_0(mx)$ and $K_0(mx)\partial/\partial x_a K_0(mx)$. Therefore, in contrast to the massless case, we shall first carry out the trace calculation. Making use of the techniques collected in Eqs. (B1)–(B4), we can work out above vacuum polarization tensor as follows:

$$\Pi_{\mu\nu}(x)=-\frac{m}{32\pi^4}b_\lambda\Bigg\{16\epsilon_{\lambda ab\nu}\frac{\partial}{\partial x_a}\left[\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial}{\partial x_\mu}\frac{K_0(mx)}{x}\right]-16\epsilon_{\lambda ab\mu}\frac{\partial}{\partial x_a}\left[\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial}{\partial x_\nu}\frac{K_0(mx)}{x}\right]$$
$$-16\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial^2}{\partial x_a\partial x_b}K_0(mx)+8\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\Box K_0(mx)-8m^2\epsilon_{\lambda\mu\nu a}\Bigg[2\frac{K_1(mx)}{x}\frac{\partial}{\partial x_a}K_0(mx)$$
$$+K_0(mx)\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\Bigg]\Bigg\}$$
$$=\frac{m^2}{2\pi^4}b_\lambda\epsilon_{\lambda\mu\nu a}\left\{-2\left[\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2-\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2\right]+m\frac{\partial}{\partial x_a}\left[\frac{K_1(mx)K_0(mx)}{x}\right]\right\}. \tag{18}$$

Obviously, due to the asymptotic expansion (4) the function $[K_1(mx)/x]^2$ is singular as $x\sim 0$ and has no Fourier transform into the momentum space. It should be emphasized that in deriving Eqs. (B2)–(B4) and (18) the substraction operation among the singular terms such as $[K_1(mx)/x]^2$ should not be naively carried out. It is analogous to the fact that in momentum space two divergent terms with the same form but opposite sign cannot be canceled, until after a regularization scheme is implemented so that they become well defined and the substraction operation can work safely. Otherwise, a finite term will probably be lost since in general the difference of two infinite quantities is not zero. In fact, the operation keeping the singular terms untouched before performing the regularization is a crucial point in the differential regularization method.

Unfortunately, as above, due to the difficulty in solving a differential equation with the modified Bessel function, we still cannot write the singular function $[K_1(mx)/x]^2$ as the derivative of another less singular function. However, we can consider the asymptotic expansion (4). One can easily see that in Eq. (18) the singularity at short distance is only carried by the leading term $1/x^4$, the other terms are finite and hence they are exactly canceled. Therefore, making use of Eq. (10) again, we obtained the regulated form for the vacuum polarization tensor in the massive case:

$$\begin{aligned}\Pi_{\mu\nu}(x)&=\frac{m^2}{2\pi^4}b_\lambda\epsilon_{\lambda\mu\nu a}\left\{-\frac{1}{2m^2}\frac{\partial}{\partial x_a}\left[\Box\frac{\ln x^2M_1^2}{x^2}-\Box\frac{\ln x^2M_2^2}{x^2}\right]+m\frac{\partial}{\partial x_a}\left[\frac{K_1(mx)K_0(mx)}{x}\right]\right\}\\&=\frac{1}{2\pi^4}b_\lambda\epsilon_{\lambda\mu\nu a}\left\{-\ln\frac{M_1}{M_2}\frac{\partial}{\partial x_a}\Box\frac{1}{x^2}+m^3\frac{\partial}{\partial x_a}\left[\frac{K_1(mx)K_0(mx)}{x}\right]\right\}\\&=\frac{1}{2\pi^4}b_\lambda\epsilon_{\lambda\mu\nu a}\left\{4\pi^2\ln\frac{M_1}{M_2}\frac{\partial}{\partial x_a}\delta^{(4)}(x)+m^3\frac{\partial}{\partial x_a}\left[\frac{K_1(mx)K_0(mx)}{x}\right]\right\}.\end{aligned}\tag{19}$$

The above vacuum polarization tensor can be expressed in momentum space by performing its Fourier transform. According to the standard differential regularization procedure [12], we have

$$\begin{aligned}\Pi_{\mu\nu}(p)&=\int d^4xe^{-ip\cdot x}\Pi_{\mu\nu}(x)\\&=\frac{2}{\pi^2}b_\lambda\epsilon_{\lambda\mu\nu a}ip_a\left[\ln\frac{M_1}{M_2}\right.\\&\quad\left.+\frac{m^3}{4\pi^2}\int d^4xe^{-ip\cdot x}\frac{K_1(mx)K_0(mx)}{x}\right]\\&=\frac{2}{\pi^2}b_\lambda\epsilon_{\lambda\mu\nu a}ip_a\left[\ln\frac{M_1}{M_2}+\frac{m}{2p}\frac{\operatorname{arcsin\,h}[p/(2m)]}{\sqrt{1+p^2/(4m^2)}}\right]\\&=\frac{2}{\pi^2}b_\lambda\epsilon_{\lambda\mu\nu a}ip_a\left[\ln\frac{M_1}{M_2}+\frac{m}{4p\sqrt{1+p^2/(4m^2)}}\right.\\&\quad\left.\times\ln\frac{\sqrt{1+p^2/(4m^2)}+p/(2m)}{\sqrt{1+p^2/(4m^2)}-p/(2m)}\right].\end{aligned}\tag{20}$$

As Ref. [5], the radiatively induced Chern-Simons term can be defined at low energy $p^2=0$ (or equivalently at large-m limit)

$$\Pi_{\lambda\mu\nu}(p)|_{p^2=0}=\frac{2}{\pi^2}\epsilon_{\lambda\mu\nu a}ip_a\left(\ln\frac{M_1}{M_2}+\frac{1}{4}\right).\tag{21}$$

Equation (21) shows that the coefficient of the induced Chern-Simons term has a finite ambiguity, which was explicitly parametrized by the ratio of two renormalization scales, M_1/M_2. Especially, Eq. (21) has contained all the results obtained in other regularization schemes. For $M_1=e^{-1/4}M_2$, we get the conclusion in Pauli-Villars regularization:

$$\Pi_{\lambda\mu\nu}(p)|_{p^2=0}=0,\tag{22}$$

while if we choose $M_1=e^{-1/16}M_2$, then the result in dimensional regularization [8] and the nonperturbative approach [9] is reproduced,

$$\Pi_{\lambda\mu\nu}(p)|_{p^2=0}=\frac{3}{8\pi^2}\epsilon_{\lambda\mu\nu a}ip_a\,.\tag{23}$$

It is remarkable that a natural choice $M_1=M_2$ does not correspond to a subtraction in the dispersive representation given in Ref. [9].

The above conclusion is not strange to us and the profound reason lies in the excellent features possessed by differential regularization. As it is shown above, the basic operation in differential regularization is replacing a singular term by the derivative of another less singular function. This operation has provided a possibility to add arbitrary local terms to the higher order amplitude since we have to solve a differential equation for noncoincident points [19]. When performing such a operation, we are introducing a new arbitrary local term into the quantum effective action. According to renormalization theory, the introduction of an arbitrary local term into the amplitude of a Green function is equivalent to the addition of a finite counterterm to the Lagrangian. Therefore, from this viewpoint, differential regularization can lead to a more general quantum effective action than any other regularization schemes. In particular, differential regularization keeps all the ambiguities to the final stage of the calculation, and these ambiguities can only be fixed by imposing some additional physical requirements or resorting to some more fundamental principle. This special feature presented by differential regularization has formed a sharp contrast to other regularization schemes such as dimensional, Pauli-Villars, and cutoff regularization, etc. These regularization methods, together with the given renormalization prescription, can fix the those arbitrary terms automatically at the beginning. In different regularization schemes, these local terms are different. This is the reason why different regularization schemes can induce different Chern-Simons terms. It should be emphasized that no regularization can claim that it gives the right value for this induced term. In differential regularization, this ambiguity is explicitly parametrized by the ratio of two indefinite renormalization scales and the results obtained in other regularization schemes can be reproduced by an appropriate choices on this arbitrary ratio. Therefore, one can say that differential regularization has yielded a more universal result than any other regularization method, since it does not impose any preferred choice on the Green function at the beginning.

In summary, we have investigated the Lorentz and CPT violating Chern-Simons term induced by radiative corrections in differential regularization. The ambiguous results obtained in other regularization schemes are universally obtained and especially, the ambiguity is quantitively parametrized by the ratio of two renormalization scales. This ambiguity should be fixed by renormalization conditions or certain fundamental physical symmetries rather than an arbitrary choice on the mass scales. For example, if one requires the Lagrangian density rather than the action to be gauge invariant, one must choose $M_1 = e^{-1/4} M_2$, and hence the generated Chern-Simons term vanishes. Another choice is, to require the action and $\int d^4 x j^5_\mu$ (i.e., the axial vector current $j^5_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$ at zero momentum) to be gauge invariant, as done by Jackiw and Kostelecký [9]. In this case, one must choose $M_1 = e^{-1/16} M_2$ and consequently, the Chern-Simons term is generated unambiguously. It should be emphasized that the natural prescription on the renormalization scales, $M_1 = M_2$, can be taken only when it corresponds to certain physical renormalization condition.

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APPENDIX A: DERIVATION OF CONVOLUTION INTEGRAL

One important technique in our calculation is the application of the convolution integrals. Here we give a detail derivation.

In the massless case, there exists that

$$\int d^4 z \frac{(z-x)_\mu (z-y)_\nu}{(z-x)^4 (z-y)^4} = \frac{1}{4} \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial y_\nu} \int d^4 z \frac{1}{(z-x)^2 (z-y)^2}. \tag{A1}$$

We need to write the integration $\int 1/[(z-x)^2 (z-y)^2]$ as an explicit function of $x-y$. So we first assume

$$f(x-y) = \int d^4 z \frac{1}{(z-x)^2 (z-y)^2}. \tag{A2}$$

Acting the four-dimensional Laplacian operator $\Box_x$ on the both sides of Eq. (A2) and using the formula

$$\Box_x \frac{1}{x^2} = -4\pi^2 \delta^{(4)}(x), \tag{A3}$$

we obtain

$$\Box_x f(x-y) = -4\pi^2 \frac{1}{(x-y)^2}. \tag{A4}$$

With the aid of the (four-dimensional) spherical symmetric form of the Laplacian operator $\Box_x = 4/x^2 d/dx^2 [(x^2)^2 d/dx^2]$, the solution to the differential equation (A4) can be easily found,

$$f(x-y) = -\pi^2 \ln[\Lambda^2 (x-y)^2], \tag{A5}$$

Λ being the cutoff. Thus we obtain the convolution integral formula

$$\int d^4 z \frac{(z-x)_\mu (z-y)_\nu}{(z-x)^4 (z-y)^4} = \frac{\pi^2}{2}\left[\delta_{\mu\nu} - 2\frac{(x-y)_\mu (x-y)_\nu}{(x-y)^4}\right]. \tag{A6}$$

Now we turn to the massive case, where the situation is quite complicated. From Eq. (16), what we need to determine is

$$g(x-y) = \int d^4 z \frac{K_1[m(z-x)]}{z-x} \frac{K_1[m(z-y)]}{z-y}. \tag{A7}$$

According to the property

$$(\Box - m^2)\Delta(x) = (\Box - m^2)\left[\frac{1}{4\pi^2} \frac{m}{x} K_1(mx)\right] = \delta^{(4)}(x), \tag{A8}$$

we act the operator $(\Box_x - m^2)$ on both sides of Eq. (A7) and obtain

$$(\Box_x - m^2) g(x-y) = 4\pi^2 \frac{K_1[m(x-y)]}{m(x-y)}. \tag{A9}$$

Repeating the above operation on Eq. (A9), we get

$$(\Box_x - m^2)^2 g(x) = \frac{16\pi^4}{m^2} \delta^{(4)}(x). \tag{A10}$$

Upon considering the Fourier transform of $g(x)$,

$$g(x) = \int \frac{d^4 p}{(2\pi)^4} g(p) e^{ip\cdot x}, \tag{A11}$$

Eq. (A11) directly yields

$$g(p) = \frac{16\pi^4}{m^2} \frac{1}{(p^2 + m^2)^2}. \tag{A12}$$

$g(x)$ can be obtained by performing the Fourier transform

$$g(x)=\frac{16\pi^4}{m^2}\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+m^2)^2}e^{ip\cdot x}$$
$$=\frac{1}{m^2}\int_0^\infty dp\frac{p^3}{(p^2+m^2)^2}\int_0^\pi d\theta\sin^2\theta e^{ipx\cos\theta}$$
$$\times\int_0^\pi\sin\varphi d\varphi\int_0^{2\pi}d\phi$$
$$=\frac{4\pi^2}{m^2x}\int_0^\infty dp\frac{p^2}{(p^2+m^2)^2}J_1(px)=\frac{2\pi^2}{m^2}K_0(mx). \tag{A13}$$

Thus we have finally worked out the important convolution integral formula

$$\int d^4z\frac{K_1[m(z-x)]}{z-x}\frac{K_1[m(z-y)]}{z-y}=\frac{2\pi^2}{m^2}K_0[m(x-y)], \tag{A14}$$

$K_0(x)$ being the zeroth-order modified Bessel function of the second kind.

APPENDIX B: DIFFERENTIAL OPERATIONS

Some differential calculation techniques used in deriving the massive vacuum polarization tensor (19) is collected in this appendix:

$$\mathrm{Tr}(\gamma_5\gamma_\lambda\gamma_a\gamma_\mu\gamma_b\gamma_\nu\gamma_c)\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial^2}{\partial x_a\partial x_c}K_0(mx)$$
$$=8\epsilon_{\lambda a\mu b}\frac{\partial}{\partial x_a}\left[\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial}{\partial x_\nu}\frac{K_0(mx)}{x}\right]+8\epsilon_{\lambda ab\nu}\frac{\partial}{\partial x_a}\left[\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial}{\partial x_\mu}\frac{K_0(mx)}{x}\right]$$
$$+8\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial^2}{\partial x_a\partial x_b}K_0(mx)+4\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\Box K_0(mx), \tag{B1}$$

$$\epsilon_{\lambda ab\nu}\frac{\partial}{\partial x_a}\left[\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial}{\partial x_\mu}K_0(mx)\right]=\epsilon_{\lambda ab\nu}\frac{\partial}{\partial x_a}\left[-mx_\mu\frac{K_1(mx)}{x}\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\right]=-\frac{1}{2}m\epsilon_{\lambda ab\nu}\frac{\partial}{\partial x_a}\left[x_\mu\frac{\partial}{\partial x_b}\left(\frac{K_1(mx)}{x}\right)^2\right]$$
$$=\frac{1}{2}m\epsilon_{\lambda\mu\nu b}\frac{\partial}{\partial x_b}\left(\frac{K_1(mx)}{x}\right)^2, \tag{B2}$$

$$\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\frac{\partial^2}{\partial x_a\partial x_b}K_0(mx)=-m\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_b}\frac{K_1(mx)}{x}\left\{\delta_{ab}\frac{K_1(mx)}{x}+x_ax_b\frac{1}{x}\frac{d}{dx}\left[\frac{K_1(mx)}{x}\right]\right\}$$
$$=-m\epsilon_{\lambda\mu\nu a}\left[\frac{1}{2}\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2+\left(\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\right)x\frac{d}{dx}\frac{K_1(mx)}{x}\right]$$
$$=-m\epsilon_{\lambda\mu\nu a}\left\{\frac{1}{2}\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2+\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\left[-\frac{K_1(mx)}{x}+\frac{d}{dx}K_1(mx)\right]\right\}$$
$$=-m\epsilon_{\lambda\mu\nu a}\left[\frac{1}{2}\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2-\frac{1}{2}\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2+\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\frac{d}{dx}K_1(mx)\right]$$
$$=-m\epsilon_{\lambda\mu\nu a}\left\{\frac{1}{2}\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2-\frac{1}{2}\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2\right.$$
$$\left.-\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\left[mK_0(mx)+\frac{K_1(mx)}{x}\right]\right\}$$
$$=\epsilon_{\lambda\mu\nu a}\left[-\frac{1}{2}m\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2+m\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2+m^2K_0(mx)\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\right], \tag{B3}$$

$$\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\Box K_0(mx)=-m\epsilon_{\lambda\mu\nu a}\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\left[4\frac{K_1(mx)}{x}+x\frac{d}{dx}\frac{K_1(mx)}{x}\right]$$
$$=\epsilon_{\lambda\mu\nu a}\left[-2m\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2+m\frac{\partial}{\partial x_a}\left(\frac{K_1(mx)}{x}\right)^2+m^2K_0(mx)\frac{\partial}{\partial x_a}\frac{K_1(mx)}{x}\right]. \quad \text{(B4)}$$

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