Conformality and gauge coupling unification

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It has been recently proposed to embed the standard model in a conformal gauge theory to resolve the hierarchy problem, and to avoid assuming either grand unification or low-energy supersymmetry. By model building based on string-field duality, we show how to maintain the successful prediction of an electroweak mixing angle with $\sin^2\theta \simeq 0.231$ in conformal gauge theories with three chiral families. [S0556-2821(99)08220-X]

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Most of the research beyond the standard model [1] is motivated by the hierarchy problem and uses the two assumptions of grand unification and low-energy ($\sim$TeV) supersymmetry. This is, in turn, driven largely by the successful prediction of one number, the $\sin^2\theta$ of the electroweak mixing angle θ. It is proposed to replace the two assumptions of grand unification and low-energy supersymmetry by one assumption, conformality. It therefore is important to show that $\sin^2\theta$ can be derived from conformality alone; that is the principal objective of the model-building in this paper.

Before entering into conformal model-building, let us briefly review the alternative. The experimental data give couplings at the Z pole of [2] $\alpha_3 = 0.118 \pm 0.003$, $\alpha_2 = 0.0338$, $\alpha_1 = \frac{5}{3}\alpha'_Y = 0.0169$ (where the errors on $\alpha_{1,2}$ are less than 1%) and $\sin^2\theta = \alpha'_Y/(\alpha_2 + \alpha'_Y) = 0.231$ with an error less than 0.001. Note that α_2/α_1 is very nearly two; this will be used later. The renormalization group equation (RGE) for the supersymmetric grand unification [3,4] is

$$\frac{1}{\alpha_i(M_G)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi}\ln\left(\frac{M_G}{M_Z}\right). \tag{1}$$

Using the minimal supersymmetric standard model (MSSM) values $b_i = (6\frac{3}{5}, 1, -3)$ and substituting $\alpha_{2,3}$ at $M_Z = 91.187$ GeV gives $M_G = 2.4\times10^{16}$ GeV and $\alpha_{2,3}(M_G)^{-1} = 24.305$. Using Eq. (1) with $i=1$ now *predicts* $\alpha_1(M_Z) = 59.172$ and hence $\sin^2\theta = 0.231$; this is *very* impressive agreement with experiment and is sometimes presented as the accurate meeting of three staight lines on a $\alpha_i^{-1}(\mu)$ vs $\ln\mu$ plot [5,6].

The relationship of the type IIB superstring to conformal gauge theory in $d=4$ gives rise to an interesting class of gauge theories. Choosing the simplest compactification [7] on $AdS_5 \times S_5$ gives rise to an $\mathcal{N}=4$ SU(N) gauge theory which is known to be conformal due to the extended global supersymmetry and non-renormalization theorems. All of the RGE $\beta-$functions for this $\mathcal{N}=4$ case are vanishing in perturbation theory. It is possible to break the $\mathcal{N}=4$ to $\mathcal{N}=2,1,0$ by replacing S_5 by an orbifold S_5/Γ where Γ is a discrete group with $\Gamma \subset SU(2), \subset SU(3), \not\subset SU(3)$ respectively.

In building a conformal gauge theory model [8–10], the steps are (1) choose the discrete group Γ, (2) embed $\Gamma \subset SU(4)$, (3) choose the N of SU(N), and (4) embed the standard model $SU(3)\times SU(2)\times U(1)$ in the resultant gauge group $\otimes SU(N)^p$ (quiver node identification). Here we shall look only at Abelian $\Gamma = Z_p$ and define $\alpha = \exp(2\pi i/p)$. It is expected from the string-field duality that the resultant field theory is conformal in the $N\to\infty$ limit, and will have a fixed manifold, or at least a fixed point, for N finite.

Before focusing on $\mathcal{N}=0$ non-supersymmetric cases, let us first examine an $\mathcal{N}=1$ model first put forward in the work of Kachru and Silverstein [11]. The choice is $\Gamma = Z_3$ and the **4** of SU(4) is **4** = $(1,\alpha,\alpha,\alpha^2)$. Choosing $N=3$ this leads to the three chiral families under $SU(3)^3$ trinification [12]

$$(3,\bar{3},1) + (1,3,\bar{3}) + (\bar{3},1,3). \tag{2}$$

In this model it is interesting that the number of families arises as 4-1=3, the difference between the 4 of SU(4) and $\mathcal{N}=1$, the number of unbroken supersymmetries. However this model has no gauge coupling unification; also, keeping $\mathcal{N}=1$ supersymmetry is against the spirit of the conformality approach. We now present three examples, models A, B and C which accommodate three chiral families, break all supersymmetries ($\mathcal{N}=0$) and possess gauge coupling unification, including the correct value of the electroweak mixing angle.

Model A. Choose $\Gamma = Z_7$, embed the 4 of SU(4) as $(\alpha^2, \alpha^2, \alpha^{-3}, \alpha^{-1})$, and choose $N=3$ to aim at a trinification $SU(3)_C \times SU(3)_W \times SU(3)_H$.

The seven nodes of the quiver diagram will be identified as *C-H-W-H-H-H-W*.

The behavior of the 4 of SU(4) implies that the bifundamentals of chiral fermions are in the representations

$$\sum_{j=1}^{7} [2(N_j, \bar{N}_{j+2}) + (N_j, \bar{N}_{j-3}) + (N_j, \bar{N}_{j-1})]. \tag{3}$$

Embedding the C, W and H SU(3) gauge groups as indicated by the quiver mode identifications then gives the seven quartets of irreducible representations

$$[3(3,\overline{3},1)+(3,1,\overline{3})]_1 +[3(1,1,1+8)+(\overline{3},1,3)]_2 +[3(1,3,\overline{3})+(1,1+8,1)]_3 +[(2(1,1,1+8)+(1,\overline{3},3)+(\overline{3},1,3)]_4 +[2(1,1,1+8)+2(1,\overline{3},3)]_5 +[2(\overline{3},1,3)+(1,1,1+8)+(1,\overline{3},3)]_6 +[4(1,3,\overline{3})]_7. \quad (4)$$

Combining terms gives, aside from (real) adjoints and overall singlets

$$3(3,\overline{3},1)+4(\overline{3},1,3)+(3,1,\overline{3})+7(1,3,\overline{3})+4(1,\overline{3},3). \quad (5)$$

Canceling the real parts (which acquire Dirac masses at the conformal symmetry breaking scale) leaves under trinification $SU(3)_C\times SU(3)_W\times SU(3)_H$

$$3[(3,\overline{3},1)+(1,3,\overline{3})+(\overline{3},1,3)] \quad (6)$$

which are the desired three chiral families.

Given the embedding of Γ in SU(4) it follows that the 6 of SU(4) transforms as $(\alpha^4,\alpha,\alpha,\alpha^{-1},\alpha^{-1},\alpha^{-4})$. The complex scalars therefore transform as

$$\sum_{j=1}^{7}[(N_j,\overline{N}_{j\pm 4})+2(N_j,\overline{N}_{j\pm 1})]. \quad (7)$$

These bifundamentals can by their vacuum expectation values (VEVs) break the symmetry $SU(3)^7=SU(3)_C \times SU(3)^2_W\times SU(3)^4_H$ down to the appropriate diagonal subgroup $SU(3)_C\times SU(3)_W\times SU(3)_H$.

Now to the final aspect of model A which is its motivation, the gauge coupling unification. The embedding in $SU(3)^7$ of $SU(3)_C\times SU(3)^2_W\times SU(3)^4_H$ means that the couplings $\alpha_1,\alpha_2,\alpha_3$ are in the ratio $\alpha_1/\alpha_2/\alpha_3=1/2/4$. Using the phenomenological data given at the beginning, this implies that $\sin^2\theta=0.231$. On the other hand, the QCD coupling is $\alpha_3=0.0676$ which is too low unless the conformal scale is at least 10 TeV. We prefer a scale ~ 1 TeV for conformal breaking where α_3 is nearer to 0.10. This motivates our models B and C below which have larger α_3 but are otherwise more complicated.

Model B. Choose $\Gamma=Z_{10}$ and embed $Z_{10}\subset SU(4)$ such that $4=(\alpha^4,\alpha^4,\alpha^{-3},\alpha^{-5})$. The chiral fermions are therefore

$$\sum_{j=1}^{10}[2(N_j,\overline{N}_{j+4})+(N_j,\overline{N}_{j-3})+(N_j,\overline{N}_{j-5})]. \quad (8)$$

To attain trinification we identify the quiver nodes as *C-H-H-H-W-W-H-W-H-H* and then the chiral fermions are in the ten quartets of irreducible representations

$$[4(3,\overline{3},1)]_1 +[2(1,\overline{3},3)+(1,1,1+8)]_2 +[2(1,1,1+8)+(1,\overline{3},3)]_3 +[2(1,\overline{3},3)+(\overline{3},1,3)+(1,1,1+8)]_4 +[4(1,3,\overline{3})]_5+[3(1,3,\overline{3})+(\overline{3},3,1)]_6 +[2(\overline{3},1,3)+(1,1,1+8)]_7 +[3(1,3,\overline{3})+(1,1+8,1)]_8 +[3(1,1,1+8)+(1,\overline{3},3)]_9 +[3(1,1,1+8)+(1,\overline{3},3)]_{10}. \quad (9)$$

Removing the (real) octets and singlets leaves

$$4(3,\overline{3},1)+(\overline{3},3,1)+3(\overline{3},1,3)+10(1,3,\overline{3})+7(1,\overline{3},3) \quad (10)$$

so that the chiral (complex) part is again

$$3[(3,\overline{3},1)+(1,3,\overline{3})+(\overline{3},1,3)] \quad (11)$$

which are three chiral families.

The 6 of SU(4) transforms under $\Gamma=Z_{10}$ as $6=(\alpha^8,\alpha,\alpha,\alpha^{-1},\alpha^{-1},\alpha^{-8})$ and so the complex scalars are

$$\sum_{j=1}^{10}[(n_j,\overline{N}_{j\pm 8})+2(N_j,\overline{N}_{j\pm 1})]. \quad (12)$$

With the given quiver node identification VEVs for these scalars can break $SU(3)^{10}=SU(3)_C\times SU(3)^3_W\times SU(3)^6_H$ to the diagonal subgroup $SU(3)_C\times SU(3)_W\times SU(3)_H$.

The couplings $\alpha_1,\alpha_2,\alpha_3$ are in the ratio $\alpha_1/\alpha_2/\alpha_3=1/2/6$ corresponding to $\sin^2\theta=0.231$ and $\alpha_3=0.101$. This is within the range of a TeV conformal breaking scale. Nevertheless, it is numerically irresistible to notice that the Z-pole values satisfy $\alpha_1/\alpha_2/\alpha_3=1/2/7$ which leads naturally to Model C.

Model C. Choose $\Gamma=Z_{23}$ and embed in SU(4) by $4=(\alpha^6,\alpha^6,\alpha^{-5},\alpha^{-7})$. Given this embedding the quiver nodes can be chosen as *C-C-X-X-X-H-H-W-H-X-X-X-X-X-X-X-W-H-H-W-X-X-X* where the thirteen *X*'s denote any distribution of of four *W*'s and nine *H*'s that allows breaking by the complex scalars cited below. The quiver is arranged such that according to the rule of $(3_C-\overline{3}_W)$ minus $(3_W-\overline{3}_C)$ there are three chiral families. (The model in [10] did not follow this rule and has two families.) Note that because of anomaly cancellation and the occurrence of only bifundamentals the remainder of trinification is automatic and need not be checked in every case.

The chiral families are as in models A and B.

The 6 of SU(4) transforms as $(\alpha^{12},\alpha,\alpha,\alpha^{-1},\alpha^{-1},\alpha^{-12})$. This implies complex scalars whose VEVs can break $SU(3)^{23}=SU(3)^{2}_{C}\times SU(3)^{7}_{W}\times SU(3)^{14}_{W}$ to $SU(3)_C \times SU(3)_W \times SU(3)_H$ with a suitable distribution of W and H nodes on the quiver.

With this choice of diagonal subgroups the couplings are in the ratio $\alpha_1/\alpha_2/\alpha_3=1/2/7$ corresponding to $\sin^2\theta = 0.231$ and $\alpha_3=0.118$ which coincide with the Z-pole values.

Discussion. We have given three examples of building conformal models from Abelian Γ with acceptable values of the couplings at the conformal scale, assuming that the SU(3) gauge couplings are all equal at the conformal scale. Model A is the simplest but its α_3 is too small unless the conformal scale is taken up to at least 10 TeV. Models B and C can accommodate a lower conformal scale but are more complicated.

There are two features of conformal models which bear repetition:

(1) Bifundamentals prohibit representations like (8,2) or (3,3) in the standard model consistent with Nature.

(2) Charge quantization is incorporated since the Abelian $U(1)_Y$ group has a positive-definite $\beta-$function and cannot be conformal until it is embedded in a non-Abelian group.

There are three questions which merit further investigation:

(1) The first question bears on whether there is a fixed manifold (line, plane, ...) with respect to the renormalization group or only a fixed point which is, in any case, sufficient to apply our conformality constraints. In perturbation theory, do the $\beta-$functions vanish?

(2) Are the additional particles necessary to render the standard model conformal consistent with the stringent constraints imposed by the precision electroweak data?

(3) Coefficients of dimension-4 operators are prescribed by group theory and all dimensionless properties such as quark and lepton mass ratios and mixing angles are calculable. Do these work and, if not, can one refine the model-building to obtain a best fit?

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