

Quantum gravity effects near the null black hole singularity

A. Bonanno

*Istituto di Astronomia, Università di Catania, Viale Andrea Doria 6, 95125 Catania, Italy
and INFN Sezione di Catania, Corso Italia 57, 95100 Catania, Italy*

M. Reuter

Institut für Physik, Universität Mainz, Staudingerweg 7, 55099 Mainz, Germany

(Received 3 November 1998; published 24 September 1999)

The structure of the Cauchy horizon singularity of a black hole formed in a generic collapse is studied by means of a renormalization group equation for quantum gravity. It is shown that during the early evolution of the Cauchy horizon the increase of the mass function is damped when quantum fluctuations of the metric are taken into account. [S0556-2821(99)02816-7]

PACS number(s): 04.70.Dy, 04.60.-m, 04.70.Bw, 11.10.Hi

I. INTRODUCTION

Recently much progress has been made in understanding the formation of singularities in realistic black holes. After the seminal work by Poisson and Israel [1], the outcome of several investigations with spherical models (see [2] for a general overview) was that the spacetime develops a null scalar singularity at the Cauchy horizon (CH) whose subsequent evolution eventually stops at the final spacelike singularity at $r=0$ [3].

In particular the Petrov type D component Ψ_2 of the Weyl curvature diverges exponentially with advanced time at this lightlike hypersurface, although the “measured” tidal distortion is bounded. The metric tensor is regular in a suitable local chart adapted to the inner horizon and the metric perturbations are small. This scenario is likely to be essentially the same in more general contexts than spherical symmetry [4], but the structure of the singularity is more complicated than since the square of the Weyl tensor $C_{\mu\nu\tau\lambda}C^{\mu\nu\tau\lambda}$ is dominated by the radiative component $\Psi_0\Psi_4$ of a Petrov type N curvature [5].

It is an interesting question whether quantum effects can modify the classical evolution of the fields in a significant way. As far as the classical evolution is concerned, causality does not permit our ignorance about the correct form of the dynamics in the inner, Planckian curvature regions of the interior to infect the description of the overlaying layers in terms of classical general relativity. The radial coordinate r is in fact timelike in the interior of a spherical hole.

This picture changes in quantum field theory because in loop calculations even states localized outside the light cone have an impact on the value of the renormalized quantities. One can then imagine that the metric fluctuations near the inner horizon modify the infrared region where the Weyl curvature is still growing but it has not yet reached Planckian levels. In particular it is interesting to see if the presence of some “self-regulator” mechanism could prevent the local curvature from diverging at the CH. An indication has been given in [6] where it has been noticed that the classical divergence of the mass function in an evaporating black hole (BH) can be damped out by the contribution of the blue-shifted influx of the Hawking radiation at late advanced

times. More complete investigations in four dimensions [7] have been performed in the semiclassical approximation by considering a massless minimally coupled scalar field, but they were inconclusive about the “sign” of the quantum correction, i.e. about whether it would lead to a stronger or to a weaker divergence.

The objective of the present investigation is to show that quantum fluctuations of the gravitational field indeed weaken the strength of the singularity at the inner horizon. This result is obtained by studying the running of the Newton constant at large momenta by means of the non-perturbative renormalization-group equation [8–10] which governs the scale dependence of the effective average action Γ_k for gravity [11]. Γ_k is a Wilson-type effective action with a built-in infrared (IR) cutoff at the mass scale k . The functional Γ_k is obtained by integrating out the quantum fluctuations with momenta between a fixed ultraviolet (UV) cutoff k_{UV} and the variable IR cutoff k . In this framework, a renormalizable theory with the classical action S is quantized by solving the flow equation subject to the initial condition $\Gamma_{k_{UV}}=S$ and letting then $k_{UV}\rightarrow\infty, k\rightarrow 0$ (after suitable renormalization).

What makes the effective average action an ideal tool for studying quantum gravity is the fact that this method can also be used in order to renormalization-group-evolve (coarse grain) the actions of non-renormalizable effective field theories. In this case one assumes that there is some fundamental theory which has been “partially quantized;” i.e., its quantum fluctuations with momenta from infinity down to a fixed scale k_{UV} have been integrated out already. This leads to an effective action S_{eff} which, when evaluated in tree approximation, correctly describes all phenomena with typical momenta of the order k_{UV} . If we are interested in processes at smaller momenta $k < k_{UV}$, we can construct a new effective action, appropriate for the lower scale by setting $\Gamma_{k_{UV}}=S_{\text{eff}}$ and solving the flow equation for Γ_k with this initial condition. It is clear that for effective theories the limit $k_{UV}\rightarrow\infty$ should not be performed; hence the non-renormalizability of a theory does not pose any problems in this context.

Quite generally, the effective action Γ or the average action Γ_k encapsulates *all* physical effects of a given theory.

Once we have identified its leading terms for a given range of momenta, no other quantum corrections beyond those which are already contained in the running coupling constants parametrising the approximate form of Γ_k need to be taken into account. In the case at hand, a truncated derivative expansion in powers of the curvature tensor and its covariant derivatives are a sensible approximation as long as $k \ll m_p$ since the terms omitted are suppressed by higher powers of k/m_p . (m_p denotes the Planck mass.) Within this approximation, the most relevant term in the action is the Einstein-Hilbert term since it has the smallest canonical dimension. As a consequence, the most important effects of quantum gravity are encoded in the running of the associated coupling, i.e. Newton's constant. Thus, when we increase k from the classical (i.e., IR) domain to larger values in order to explore gravity at smaller distances, the first sign of a non-classical behavior is a changing value of Newton's constant. In the present paper we investigate the regime of momenta where, on the one hand, the first quantum gravitational effects appear already while on the other hand higher order invariants (R^2 terms, etc.) are not yet important.

In the following we shall consider Einstein gravity as an effective field theory and we identify the standard Einstein-Hilbert action with the average action $\Gamma_{k_{\text{obs}}}$. Here k_{obs} is some typical "observational scale" at which the classical tests of general relativity have confirmed the Einstein-Hilbert action. We assume that also for $k > k_{\text{obs}}$, i.e. at higher energies, Γ_k is well approximated by an action of the Einstein-Hilbert form as long as k is not too close to the Planck scale. The two parameters in this action, Newton's constant and the cosmological constant, will depend on k , however, and the flow equation will tell us how the running Newton constant $G(k)$ and the running cosmological constant $\Lambda(k)$ depend on the cutoff. Their experimentally observed values are $G(k_{\text{obs}}) = G_{\text{obs}}$ and $\Lambda(k_{\text{obs}}) = \Lambda_{\text{obs}} \approx 0$. The Newton constant defines the Planck mass according to $m_p = 1/\sqrt{32\pi G_{\text{obs}}}$.

We fix a scale $k_{\text{UV}} \gg k_{\text{obs}}$ in such a way that it is still sufficiently below the Planck scale so that Γ_k is not yet very different from the Einstein-Hilbert action, but is already large enough for quantum gravitational effects to play a role. We start the renormalization group evolution at the scale k_{UV} with bare parameters $G(k_{\text{UV}}) = \bar{G}$ and $\Lambda(k_{\text{UV}}) = \bar{\Lambda}$ which should be thought of as functions of G_{obs} and Λ_{obs} . We shall use our result for the function $G(k)$, $k \in [k_{\text{obs}}, k_{\text{UV}}]$, in order to study the impact of the scale dependent Newton constant on the mass-inflation scenario.

In a sense, we shall "renormalization-group-improve" the classical metric describing the late-time behavior of the spacetime near the CH. Our method is similar to the following renormalization-group based derivation of the Uehling correction to the Coulomb potential in massless QED [12]. One starts from the classical potential energy $V_{\text{cl}}(r) = e^2/4\pi r$ and replaces e^2 by the running gauge coupling in the one-loop approximation:

$$e^2(k) = e^2(k_0)[1 - b \ln(k/k_0)]^{-1}, \quad b \equiv e^2(k_0)/6\pi^2. \quad (1)$$

Hereby one may identify the renormalization point k with the

inverse of the distance r because in the massless theory this is the only relevant scale. The result of this substitution reads

$$V(r) = -e^2(r_0^{-1})[1 + b \ln(r_0/r) + O(e^4)]/4\pi r \quad (2)$$

where the IR reference scale $r_0 \equiv 1/\mu_0$ has to be kept finite in the massless theory. Note that Eq. (1) is the correct (one-loop, massless) Uehling potential which is usually derived by standard perturbative methods [12]. Obviously the position dependent renormalization group improvement $e^2 \rightarrow e^2(k)$, $k \propto 1/r$ encapsulates the most important effects which the quantum fluctuations have on the electric field produced by a point charge. We argue that analogous substitution $G \rightarrow G(k)$ with an appropriate $k = k(x^\mu)$ yields the leading modification of the spacetime metric.

II. QUANTUM GRAVITY EFFECTS BEHIND THE INNER POTENTIAL BARRIER

Considering a spherically symmetric, charged BH the metric can be conveniently expressed using the coordinates x^a ($a, b = 0, 1$) on the radial two-spaces $(\theta, \phi) = \text{const}$ and the function $r(x^a)$ that measures the area of those two-spheres whose line element is $r^2 d\Omega^2$. The metric element is then $ds^2 = g_{ab} dx^a dx^b + r^2 d\Omega^2$. By defining the scalar fields $f(x^a), m(x^a)$ and $-2\kappa(x^a) = \partial f / \partial r$ through $f = 1 - 2G_{\text{obs}}m/r + G_{\text{obs}}e^2/r^2$ the Einstein equations reduce to the two-dimensionally covariant equations

$$\begin{aligned} r_{;ab} + \kappa g_{ab} &= -4\pi G_{\text{obs}} r (T_{ab} - g_{ab} T) \\ R - 2\partial_r \kappa &= 8\pi G_{\text{obs}} (T - 2P) \end{aligned} \quad (3)$$

where the static electro-magnetic field is generated by a charge of strength e and T_{ab} is the stress-energy tensor of the matter field whose two-dimensional trace is T and tangential pressure is P . From the conservation laws one finds the following two-dimensional wave equation for the mass function

$$\begin{aligned} \square m &= -16\pi^2 r^3 G_{\text{obs}} T_{ab} T^{ab} + 8\pi G_{\text{obs}} f (P - T) \\ &+ 4\pi r^2 G_{\text{obs}} \kappa T - 4\pi r^2 G_{\text{obs},a} T^{,a}. \end{aligned} \quad (4)$$

This latter equation is the key to understanding the phenomenon of the mass inflation. The late time behavior of the external gravitational field produced during the collapse of a star is that of a (Kerr-Newman, in general) black hole of external mass m_0 perturbed by a tail of gravitational waves whose flux decays as $\sim v^{-p}$ with $p = 4(l+1)$ for a multipole of order l . As a consequence of the boundary conditions set at the event horizon, the $T_{ab} T^{ab}$ interaction term between the influx and out-flux of gravitational waves scattered from the inner potential barrier triggers a divergent source term for the local mass function $m(u, v)$. The outflow can be modelled as a radial stream of light-like material particles because of the infinite blue-shift near the Cauchy Horizon. It is possible to show [1] that near the CH

$$m(v, r) \sim v^{-p} e^{\kappa_0 v} \quad (v \rightarrow \infty) \quad (5)$$

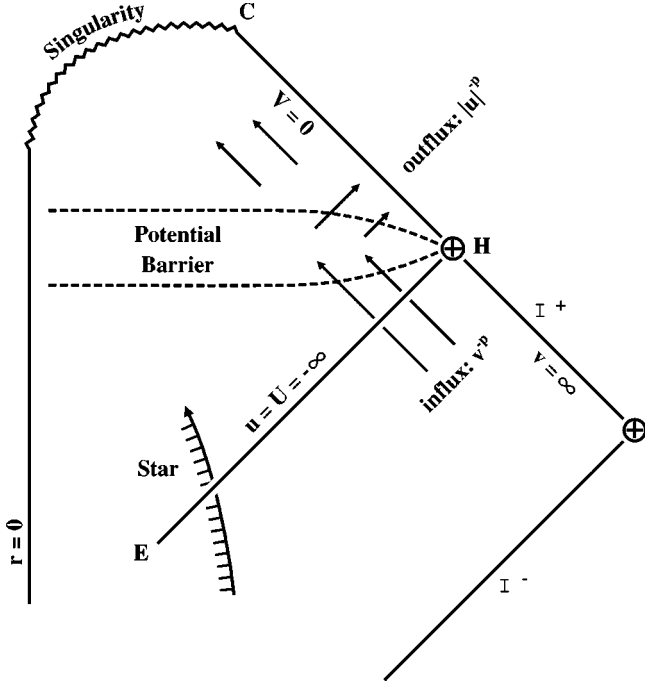


FIG. 1. Penrose conformal diagram of a collapsing star. Note that the point H is not part of the manifold, but a singular point of this mapping.

where v is the standard advanced time Eddington-Finkelstein coordinate. (κ_0 denotes the surface gravity of the Reissner-Nordström static black hole that characterises the external field configuration.)

It must be observed that in Eq. (4) the strength of the gravitational interactions between out-flux and in-flux is proportional to the Newton constant. Hence small changes in G due to renormalization effects are then exponentially amplified by the mass function like in a magnifying lens. In particular if gravity is asymptotically free, the classical divergence of the mass function can be weakened by the decreasing of the Newton constant at small distances.

In order to discuss this phenomenon in the mass-inflation scenario we consider the model analyzed in [3] for the scalar field collapse although our result should not depend on this particular framework. We are interested in the asymptotic portion of the spacetime at late retarded times (the ‘‘corner’’ region near the point H in Fig. 1) before the strong focusing region where $r \rightarrow 0$. The null Kruskal coordinates U, V are thus introduced, being

$$\kappa_0 U = -\exp(\kappa_0 u), \quad \kappa_0 V = -\exp(\kappa_0 v) \quad (6)$$

where (u, v) are the retarded and advanced time coordinates. In a neighborhood of $(U = -\infty, V = 0)$ an approximate analytical solution of the Einstein equations and the wave equation for a massless minimally coupled scalar field Φ can be found [3]. The explicit asymptotic expression for the metric is

$$ds^2 = -2 \frac{r_0}{r} dU dV + r^2 d\Omega^2 \quad (7)$$

$$r^2 = r_0^2 - 2G_{\text{obs}}[A(U) + B(V)] \quad (8)$$

where r_0 is the location of the CH in the static BH spacetime configuration. The dimensionless functions $A(U)$ and $B(V)$ are regular at the CH, $A(-\infty) = B(0) = 0$, but \dot{B} diverges like $1/V[-\ln(-\kappa_0 V)]^{(p+2)}$ as $V \rightarrow 0^-$ while A is positive definite and \dot{A} is bounded. Even though the metric coefficients and the scalar field Φ are both regular at the CH, the mass function is divergent for $V \rightarrow 0^-$, being

$$m(U, V) \approx \frac{G_{\text{obs}}}{r_0} \dot{A} \dot{B}. \quad (9)$$

We consider the evolution of the above geometry in the mass-inflating regime starting from a value of the coordinate $V = V_{\text{IR}}$ for which the mass function is already exponentially growing $m(U, V)/m_0 \gg 1$ (we assume $r_0^{-1} \sim m_0 \gg 1$ in Planck units) but the curvature has not yet reached Planckian values.

At this point we need an explicit expression for the running Newton constant. We use the result obtained in [11] where, for pure gravity, the evolution of Γ_k has been obtained in the ‘‘Einstein-Hilbert approximation’’ where only the $\sqrt{-g}$ and $\sqrt{-g}R$ operators are considered in the renormalization group flow. This amounts to truncating the space of all the actions to those of the form

$$\Gamma_k[g, \bar{g}] = (16\pi G(k))^{-1} \int d^4x \times \sqrt{g} \{-R(g) + 2\Lambda(k)\} + S_{\text{gf}}[g, \bar{g}] \quad (10)$$

where S_{gf} is the classical background gauge fixing term. For this truncation the flow equation reads

$$k \partial_k \Gamma_k[g, \bar{g}] = \frac{1}{2} \text{Tr} \{ (\Gamma_k^{(2)}[g, \bar{g}] + \mathcal{R}_k^{\text{grav}}[\bar{g}])^{-1} k \partial_k \mathcal{R}_k^{\text{grav}}[\bar{g}] \} - \text{Tr} \{ (-\mathcal{M}[g, \bar{g}] + \mathcal{R}_k^{\text{gh}}[\bar{g}])^{-1} k \partial_k \mathcal{R}_k^{\text{gh}}[\bar{g}] \} \quad (11)$$

where $\bar{g}_{\mu\nu}$, $\Gamma_k^{(2)}$ and \mathcal{M} denote the background metric, the Hessian of Γ_k with respect to the ‘‘ordinary’’ metric argument $g_{\mu\nu}$, and the Faddeev-Popov ghost operator, respectively. The operators $\mathcal{R}_k^{\text{grav}}$ and $\mathcal{R}_k^{\text{gh}}$ are the IR cutoffs in the graviton and the ghost sector, respectively. They are defined in terms of an arbitrary smooth function $\mathcal{R}_k(p^2)$ (interpolating between zero for $p^2 \rightarrow \infty$ and a constant $\propto k^2$ at $p^2 = 0$) by replacing p^2 with the graviton and ghost kinetic operator, respectively. Inside loops, they suppress the contribution from modes with covariant momenta $p < k$.

Upon projecting the renormalization group flow on the two dimensional space spanned by the operators $\sqrt{-g}$ and $\sqrt{-g}R$ the functional flow equation becomes two ordinary differential equations for $G(k)$ and $\Lambda(k)$. The equation for the scale derivative of the running dimensionless Newton constant $g(k) = k^2 G(k)$ is found to be

$$k \partial_k g(k) = [2 + \eta(k)] g(k) \quad (12)$$

where $\eta(k) \equiv gB_1/(1 - gB_2)$ is an anomalous dimension involving two known functions [11] of the cosmological constant, B_1 and B_2 , which depend on the choice for $\mathcal{R}_k(p^2)$. Contrary to the running of the dimensionless gauge coupling $e(k)$ in QED, the beta function describing the running of the Newton constant is not universal. It is scheme dependent even in the lowest order of the loop expansion. In our framework this is reflected by the \mathcal{R}_k dependence of η . To lowest order of an expansion in powers of k/m_p one may ignore the impact of the running cosmological constant on $\eta(k)$ and set $\Lambda(k) \approx 0$. Thus, returning to physical units and retaining only the leading term of the k/m_p expansion the solution to Eq. (12) reads

$$G(k) = G_{\text{obs}}[1 - \omega G_{\text{obs}}k^2 + O(k^4/m_p^4)]. \quad (13)$$

For pure gravity one obtains $\omega = \omega_G \equiv -B_1(\Lambda=0)/2 > 0$ which assumes the numerical value $\omega_G = 4(1 - \pi/144)/\pi$ for a standard exponential cutoff [11]. While ω depends on the shape of \mathcal{R}_k , it can be shown that ω is positive for any choice of this function. Consequently pure gravity is ‘‘anti-screening:’’ Newton’s constant decreases as k increases; i.e., it is large in the IR and becomes smaller in the UV.

Equation (13) is believed to be reliable as long as k_{UV} is still below the Planck mass. If $k < k_{\text{UV}} \ll m_p$, the effect, in the renormalized system, of higher curvature invariants such as R^2 , $R_{\mu\nu}R^{\mu\nu}$ or R^3 which were omitted from the ansatz (10) is indeed small. In fact, those invariants have been classified accordingly to their anomalous scaling dimension which characterize the linearised renormalization group flow near the Gaussian fixed point [10]. The result is that the flow in the UV region is determined only by the ‘‘relevant’’ operators \sqrt{g} and $\sqrt{g}R$, and that any other invariant with a higher canonical dimension is suppressed by additional powers of k/m_p [10]. We shall then assume that

$$k < k_{\text{UV}} = m_p/a \quad (14)$$

with a a fixed number well above unity. This defines the domain of validity of our approximation.

It is straightforward to include matter fields. In our model it might appear natural to keep the electro-magnetic field classical but quantize the full scalar field. The only effect on the running of G is to shift the parameter ω . Using the same cutoff as above, one finds [13] $\omega = \omega_{GS} = 4/\pi - 3\pi/72$ which, again, is positive and leads to the same qualitative features as pure gravity.

The running of G has dramatic consequences for the mass-inflation scenario. The leading quantum correction of the metric is obtained by replacing G_{obs} in Eq. (9) for the mass function by the running Newton constant $G(k)$ with an appropriately chosen scale k . Since $G(k) < G_{\text{obs}}$, for any value of $k > k_{\text{obs}}$, we conclude that the *quantum corrections tend to damp the increase of the mass function*. This qualitative conclusion is independent of the precise definition of the cutoff k . It is a rather robust result therefore.

III. IMPROVED MODEL

We are now going to implement this mechanism in an iterative calculation, where the zeroth-order solution for the metric is substituted into the running of G in order to calculate the first-order correction of the metric. For the sake of simplicity let us now consider the simpler case of the cross-flow model discussed in [3].

The first question to be answered is what is the analogue of the identification $k \propto 1/r$ which we used in QED. We are looking for a x^μ -dependent cutoff $k = k(x^\mu)$ which respects general coordinate invariance and which measure the typical mass scale set by the curvature of spacetime. Since in the case at hand the metric is spherically symmetric, the natural candidate for the cutoff is the ‘‘Coulombian’’ component of the Weyl curvature [7]: $k^2 \propto |\Psi_2| = G_{\text{obs}}m(U, V)/r_0^3$. More precisely we use the classical metric (zeroth-order approximation) to define the position dependent IR cutoff by

$$k^2(V) = \max_U \{b^2 |\Psi_2|\} = \max_U \{b^2 G_{\text{obs}}m(U, V)/r_0^3\} \quad (15)$$

with b another fixed number much larger than unity and the maximum is performed over the region near $U \rightarrow -\infty$. Here we are invoking a kind of adiabatic approximation where the use of a position dependent cutoff is justified because the mass function $m(U, V)$ is almost constant on the length scales at which the eigenmodes integrated out are varying. (A similar approximation has already been used in [7] in a semi-classical calculation.) From $k(V)$ one obtains a running Newton constant as a function of the V coordinate

$$G(V) = G_{\text{obs}}[1 - A\dot{B}(V)] \quad (16)$$

where

$$A = \omega b^2 \max_U \{G_{\text{obs}}^3 \dot{A}(U)/r_0^4\}. \quad (17)$$

It is now possible to evolve the classical geometry in Eq. (7) by considering the running Newton’s constant in the Einstein equations. Within our approximation, the improvement amounts to replacing

$$G_{\text{obs}}T_{ab} = G_{\text{obs}}T_{ab}^{\text{in}}(V) + G_{\text{obs}}T_{ab}^{\text{out}}(U) \quad (18)$$

with

$$T_{ab}^{\text{imp}} = G(V)T_{ab}^{\text{in}}(V) + G_{\text{obs}}T_{ab}^{\text{out}}(U). \quad (19)$$

This modified energy-momentum tensor is then covariantly conserved since it satisfies $(T_{ab}^{\text{imp}}r^2)^{;b} = 0$.

From the Bianchi identities one obtains the following wave equation for the mass function:

$$\square(Gm) = -16\pi^2 r^3 G^2 T_{ab}T^{ab} + e^2 \frac{G_{,a}{}^{;a}}{2r}. \quad (20)$$

The general solution is uniquely determined once the value of the fields along the characteristic $U = U_{\text{IR}}$ and $V = V_{\text{IR}}$ are

given. Asymptotically the improved metric is still of the form (7) but Eq. (8) is now replaced by

$$r^2 = r_0^2 - 2G(V)B(V) - 2G_{\text{obs}}A(U). \quad (21)$$

By noticing that G approaches its bare value very rapidly one finds that the leading term (as $V \rightarrow 0^-$) on the right hand side of Eq. (20) is now given by the classically divergent $T_{ab}T^{ab}$ contribution. Thus, after the inner potential barrier, one finds the solution

$$m(U, V) \simeq \frac{G_{\text{obs}}}{r_0} (1 - \mathcal{A}\dot{B})\dot{A}\dot{B} \quad (22)$$

which replaces the classical expression (9). Inserting the function B one has more explicitly for the leading term

$$m(U, V) \simeq \frac{G_{\text{obs}}}{r_0} \left(1 - \frac{\mathcal{A}}{-V[-\ln(-\kappa_0 V)]^{(p+2)}} \right) \times \frac{\dot{A}(U)}{-V[-\ln(-\kappa_0 V)]^{(p+2)}}. \quad (23)$$

This is our main result. It confirms our earlier conclusion about the damped increase of the mass function within an improved approximation which takes the back reaction of the metric into account.

A priori it might have appeared equally possible to perform the substitution $G_{\text{obs}} \rightarrow G(k)$ directly in the Einstein equations. The identification $k^2 = G(k)m/r^3$ leads to a non-linear equation for k that up to $O(k^2/m_p^2)$ is equivalent to $k^2 = G_{\text{obs}}m/r_0^3$ which was used before. However, it is important to observe that in this case a decreasing G leads to the additional effect of lowering the value of the surface gravity at the inner horizon which, too, damps the increase of the mass function.

It should also be stressed that the above results were obtained by integrating out only the field modes with momenta between $b\sqrt{|\Psi_2|}$ and m_p/a . While lowering the IR cutoff even further is difficult from the technical point of view (the

adiabatic approximation is not available any longer) the monotonicity of the function in Eq. (13) suggests that taking additional modes into account will lead to an even stronger damping of the classical increase of the mass function. On the other hand, by adding further matter fields the antiscreening nature of the gravitational interaction could be destroyed in principle. (In Ref. [14] a condition on the number of the various species of fields implying $\omega > 0$ can be read off.) We nevertheless believe that the qualitative features of our discussion will hold for arbitrary matter systems with $\omega > 0$. In particular for the matter system consisting of a massless minimally coupled scalar field considered in this investigation the quantum-corrected geometry is less singular than its classical counterpart.

IV. CONCLUSIONS

We have discussed a possible physical mechanism which has the effect of damping the classical increase of the mass function behind the potential barrier inside a realistic black hole. We believe that this mechanism is operative already for black holes with masses $M \gg m_p$ and before the Cauchy horizon singularity is reached, i.e. in a regime of sub-Planckian curvatures where it can be calculated reliably. On the basis of the present investigation we cannot make any claim about the fate of the singularity at the CH. However, one can speculate that if the decrease of G continues and that $G \rightarrow 0$ at the CH, the geometry of the spacetime near the late-time portion of the CH is regular with a sub-Planckian Weyl curvature. In order to settle this issue completely a more complete calculation is needed and we hope to address this problem in the future.

ACKNOWLEDGMENTS

It is a pleasure to thank W. Israel for stimulating discussions. One of us (M.R.) would like to thank the Department of Physics of Catania University for the cordial hospitality extended to him while this work was in progress. He is also grateful to INFN, Sezione di Catania, for the financial support which made this visit possible.

-
- [1] E. Poisson and W. Israel, Phys. Rev. D **41**, 1796 (1990).
 - [2] *Internal Structure of Black Holes and Spacetime Singularities*, edited by L. M. Burko and A. Ori, the Annals of the Israel Physical Society, Vol. XIII (Institute of Physics, Bristol, 1997).
 - [3] A. Bonanno, S. Droz, W. Israel, and S.M. Morsink, Proc. R. Soc. London **A450**, 553 (1995).
 - [4] A. Ori, Phys. Rev. Lett. **68**, 2117 (1992); A. Bonanno, Phys. Rev. D **53**, 7373 (1996).
 - [5] P.R. Brady, S. Droz, and S.M. Morsink, Phys. Rev. D **58**, 084034 (1998).
 - [6] R. Balbinot, P.R. Brady, W. Israel and E. Poisson, Phys. Lett. A **161**, 223 (1991).
 - [7] W.G. Anderson, P.R. Brady, W. Israel, and S.M. Morsink, Phys. Rev. Lett. **70**, 1041 (1993); R. Balbinot and E. Poisson, *ibid.* **70**, 13 (1993).
 - [8] For an introduction see M. Reuter, in ‘‘Proceedings of the 5th Hellenic School and Workshop on Elementary Particle Physics,’’ Corfu, Greece, 1995; and hep-th/9602012.
 - [9] M. Reuter and C. Wetterich, Nucl. Phys. **B417**, 181 (1994); **B427**, 291 (1994).
 - [10] A. Bonanno, Phys. Rev. D **52**, 969 (1995); A. Bonanno and D. Zappala, *ibid.* **55**, 6135 (1997).
 - [11] M. Reuter, Phys. Rev. D **57**, 971 (1998).
 - [12] W. Dittrich and M. Reuter, *Effective Lagrangians in Quantum Electrodynamics* (Springer, Berlin, 1985).
 - [13] L.N. Granda, Europhys. Lett. **42**, 487 (1998).
 - [14] D. Dou and R. Percacci, Class. Quantum Grav. **15**, 3449 (1998).