Negative energy, superluminosity, and holography

Joseph Polchinski
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030

Leonard Susskind and Nicolaos Toumbas
Department of Physics, Stanford University, Stanford, California 94305-4060

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The holographic connection between large N super Yang-Mills (SYM) theory and gravity in anti–de Sitter (AdS) space requires unfamiliar behavior of the SYM theory in the limit that the curvature of the AdS geometry becomes small. The paradoxical behavior includes superluminal oscillations and negative energy density. These effects typically occur in the SYM description of events which take place far from the boundary of AdS when the signal from the event arrives at the boundary. The paradoxes can be resolved by assuming a very rich collection of hidden degrees of freedom of the SYM theory which store information but give rise to no local energy density. These degrees of freedom, called precursors, are needed to make possible sudden apparently acausal energy momentum flows. Such behavior would be impossible in classical field theory as a consequence of the positivity of the energy density. However we show that these effects are not only allowed in quantum field theory but that we can model them in free quantum field theory. [S0556-2821(99)06616-3]

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I. INTRODUCTION AND REVIEW

That there is some sort of holographic [1,2] correspondence between maximally supersymmetric $SU(N)$ Yang-Mills (SYM) theory and supergravity or string theory on $AdS_5 \times S^5$ [3–5] has been established beyond reasonable doubt. In this paper we will assume the correspondence in the strongest sense, namely, that SYM theory and type IIB string theory are equivalent for all values of N and gauge coupling constant.

Equivalence between theories in different dimensions immediately raises questions about how detailed bulk information in one theory can be completely coded in lower dimensional degrees of freedom. Despite the large amount of evidence that we have for the AdS conformal field theory (CFT) correspondence, there is not yet any direct translation of the configurations of one theory to the other. We will see that the existence of such a dictionary requires behavior for the large N limit of SYM theory which seems very unusual and even unphysical, but we will argue that it is neither.

The parameters of the SYM theory are the gauge coupling constant g and the number of colors N. These are related to the radius of curvature R of the $AdS_5 \times S^5$, the string length scale l_s and the string coupling g_s by

$$g_s = g^2$$
$$R = l_s (N g^2)^{1/4}. \tag{1.1}$$

The 5 and 10 dimensional Newton constants are given by

$$G_5 = G_{10}/R^5$$
$$G_{10} = g_s^2 l_s^8. \tag{1.2}$$

Throughout we will neglect constants of order one.

The features of the correspondence that are most relevant to our discussion are the following:

(1) Certain local gauge invariant SYM operators correspond to bulk supergravity fields evaluated at the boundary of the AdS space. That is, the expectation value of the SYM operator is determined from the boundary value of the field, as explained more fully in Sec. III. For example the energy momentum tensor $T_{\mu\nu}$ of the SYM theory corresponds to the metric perturbation $\gamma_{\mu\nu}$. Similarly $F_{\mu\nu}F^{\mu\nu}$ corresponds to the dilaton field ϕ.

(2) The ultraviolet-infrared connection [6] relates the short wave length ultraviolet modes of SYM theory to the supergravity modes near the boundary of AdS space. To be more precise let us introduce ‘‘cavity coordinates’’ in which the AdS metric ds^2 is written in terms of a dimensionless metric dS^2 and the radius of curvature R:

$$ds^2 = R^2 dS^2$$
$$dS^2 = \left(\frac{1+r^2}{1-r^2}\right)^2 dt^2 - \left(\frac{2}{1-r^2}\right)^2 (dr^2 + r^2 d\Omega^2). \tag{1.3}$$

The coordinates t, r are dimensionless. The center of AdS means the point $r=0$. Near a point of the boundary at $r=1$ the metric has the form

$$ds^2 = R^2 \left[\frac{1}{z^2} (dt^2 - dz^2 - dx^i dx^i) \right] \tag{1.4}$$

where $z = 1 - r$ and x^1, x^2, x^3 replace the coordinates of the 3-sphere. For our purposes the metric (1.4) is to be regarded as a local approximation to the cavity metric. It is true, but irrelevant to our purposes, that the same metric also gives an exact description of a patch of AdS space. In any case we will call these the half-plane coordinates.

The SYM theory will be thought of as living on the dimensionless unit sphere Ω times the dimensionless time t.

All quantities such as energy, distance and time in the SYM theory are regarded as dimensionless. To relate them to corresponding bulk quantities the conversion factor is R. Thus for example, a time interval δt corresponds to a proper interval $R\,\delta t$ in the bulk theory. Similarly an energy $E_{\rm SYM}$ in the SYM theory is related to the bulk energy by $E_{\rm SYM} = E_{\rm bulk}R$.

The UV-IR connection states that supergravity degrees of freedom at $1-r=\delta$ correspond to SYM degrees of freedom with wavelength $\sim\delta$. The UV-IR connection is at the heart of the holographic requirement that the number of degrees of freedom should be of order the area of the boundary measured in Planck units. It also suggests that physical systems near the center of the AdS space should be described by modes of the SYM theory of the longest wavelength, that is the homogeneous modes on Ω.

(3) The existence of a flat space limit [7,8]. This limit involves $N\to\infty$ but is not the 't Hooft limit in which g^2N and the energy are kept fixed. The flat space limit is given by

$$g^2 \to \text{fixed}$$
$$N\to\infty. \tag{1.5}$$

In addition, all energy scales are kept fixed in string units. This means that the dimensionless SYM energy scales like R, or using Eq. (1.1)

$$E_{\rm SYM}\to N^{1/4}. \tag{1.6}$$

As argued in [7,8] an S-matrix can be defined in this limit in terms of SYM correlation functions (see Refs. [9] for related developments). We will outline the construction but the reader is referred to the references for details.

A bulk massless particle of energy k is described by a SYM excitation of energy ω given by

$$\omega = Rk = (Ng^2)^{1/4} l_s k \quad . \tag{1.7}$$

In order to obtain definite kinematics in the flat space limit, the scattering must occur in a known position due to the position-dependence of the metric. Therefore, we require the particles to collide within a space-time region called the ''lab.'' The lab is centered at $t=r=0$ and has a large but fixed size L in string units. At the end we may take L/l_s as big as we like.[1] In terms of dimensionless coordinates the lab dimensions are

$$\delta t\sim\delta r\sim L/R. \tag{1.8}$$

Since L is fixed in string units

$$\delta t\sim\delta r\sim (Ng^2)^{-1/4}. \tag{1.9}$$

[1]For generic wave packets the analysis in Refs. [7,8] shows that they grow as $N^{1/8}$ due to geometric optics effects, but for simplicity we imagine special packets chosen to intersect in a volume of order N^0. The size and duration of any collision process is determined by the external energies and so is of order N^0.

Creation and annihilation operators for emitting and absorbing particles at the AdS boundary can be defined. In order that the particles pass through the lab they must be emitted at time $t_{\rm in}\approx -\pi/2$. The collision process lasts for a fixed time in string units which means a dimensionless time of order $N^{-1/4}$. Thus in the dimensionless time of the SYM theory the duration of the collision becomes negligible as $N\to\infty$. The outgoing particles will arrive at the boundary at time $t_{\rm out}\approx\pi/2$. The graviton creation operators corresponding to this situation are given by

$$A_{\rm in} = \int dt\, d^3x\, T_{\mu\nu}(x,t) e^{i\omega t} G(x-x_0, t+\pi/2) \tag{1.10}$$

where the integration is over the boundary and x_0 is the point from which the graviton is emitted. Annihilation operators $A_{\rm out}$ at $t=\pi/2$ are defined in a similar manner. In order to make sure that the particles pass through the origin the functions G in Eq. (1.8) must not be too sharply peaked at x_0. We refer the reader to [7,8] for a discussion of this point.

Let us consider a state involving a packet of gravitons emitted at $t=-\pi/2$ from the boundary at point x_0, in a uniform state on the S^5. The packet is very well concentrated in the dimensionless coordinates t,r,Ω. In order to translate this into the SYM theory we need to compute the gravitational field of such a source. Fortunately this has been done in [10]. The result is an AdS generalization of the Aichelburg Sexl metric and like that metric, it is described by a shock wave that propagates in the bulk with the particle. The thickness of the shock in dimensionless coordinates tends to zero with the spread of the packet. The intersection of the shock wave with the boundary forms a 2-sphere or shell which expands away from the point x_0 with the speed of light. Both in front of the shell and behind it $\langle T_{\mu\nu}\rangle$ vanishes. The shell expands to its maximum size at $t=0$ and then contracts to the antipodal point at $t=+\pi/2$. Thus the expectation value of the SYM energy momentum tensor has its support on such a moving shell and is zero everywhere else.

The description given above is somewhat surprising in view of the UV-IR connection. We might have expected that in the SYM description the energy would be transferred from the short wave length modes of the field theory to long wave lengths as the graviton moves toward $r=0$. In this event the sharp features of the shell should have dissipated. However the energy stays concentrated in a thin shell whose thickness tends to zero with N. This in itself is somewhat puzzling.

II. HISTORY OF A COLLISION

Paradoxes become apparent when we consider the collision of two packets. The packets are emitted from points x_1, x_2 at time $t=-\pi/2$. The first thing to notice is that from the viewpoint of the boundary SYM theory the behavior of the system after $t=0$ becomes infinitely sensitive to the details of the emission process. As an example suppose x_1 and x_2 are separated by an angle of 90 degrees on the 3-sphere Ω. If the two particles are emitted at the same time they will reach $r=0$ simultaneously and collide. But suppose the emission processes are separated by time $\epsilon\ll 1$. In this case the arrivals will be separated by a time *in string units* of order $\epsilon(Ng^2)^{1/4}$. This means that if $\epsilon > (Ng^2)^{-1/4}$ the particles will miss each other and pass essentially unscattered. On the other hand if $\epsilon < (Ng^2)^{-1/4}$ a collision will take place leading to a very different final state. In other words shifting the parameters of the emission process by tiny amounts will lead to large differences in the outcome.

Consider a process in which two packets of fixed energy in string units are emitted from diametrically opposing points in such a way that they pass through the lab. In the SYM theory the process starts out as a pair of expanding thin shells of energy. At time $t=0$ the shells meet. Now from what was said above, one might expect the evolution just after $t=0$ to be supersensitive to to the initial conditions. However this is not true. In fact the two shells just pass through one another without any apparent interaction. The reason is that as $R \to \infty$, points near the boundary are so far (again in string units) from the sources that the gravitational field equations linearize. The energy-momentum continues to be concentrated on the thin shells which are now contracting toward the points antipodal from where they originated. Not only is $\langle T_{\mu\nu} \rangle$ zero everywhere off the shells but so are all other SYM fields that correspond to classical supergravity fields.

We now come to a critical question. The vanishing of $\langle T_{\mu\nu} \rangle$ in classical field theory would indicate that the local state of the system is vacuum-like. In other words all fields or functionals of fields supported off the shells should have their vacuum values. As we shall see, this can not be true in the quantum theory. We will find that after the shells pass through each other the region between them must be excited away from the local vacuum configuration despite the fact that the expectation value of the energy density (as well as the value of every SYM field which corresponds to classical supergravity) vanishes.

A simple illustrative example is the case where the two packets are prepared so as to collide head-on at $t=r=0$. Assume the particles have a fixed energy which is much larger than the 10-dimensional Planck mass. In this case they will form a 10-dimensional Schwarzschild black hole. Not all the energy will go into the black hole but much of it will continue to propagate as gravitational bremsstrahlung. According to the assumption of a flat space limit [7,8] the percentage of energy trapped in the black hole can be calculated in the flat space limit [11]. The black hole quickly becomes spherically symmetric and then decays by Hawking evaporation. The entire history of the black hole lasts a fixed time in string units and therefore a dimensionless time which tends to zero like $N^{-1/4}$.

How does the creation and evaporation of the black hole affect the metric and other supergravity fields at the AdS boundary? The answer is that it does not, at least at first. In fact the supergravity fields do not respond until light has had a chance to propagate from $r=0$ to the boundary. The evaporating black hole sends out a spherically symmetric signal which arrives at the boundary at $t=\pi/2$. The arrival of the signal is very sudden, occupying a time $\delta t \sim (Ng^2)^{-1/4}$. At this time the entire boundary suddenly ''lights up'' with a spherically symmetric distribution of energy which in total equals the mass of the black hole. In other words a fraction of the energy originally stored in the collapsing shells very quickly flows and is redistributed into a homogeneous component. We will call this phenomenon ''light-up.''

This behavior seems extremely bizarre. The instantaneous rearrangement of energy appears to violate causality. However this may not be so. To better understand it we will describe an analogous example involving a sudden flow of electric charge. Consider an example in which initially there is a concentration of charge in some region R_1. At time $t=0$ the charge is found to disappear from R_1 and reappear at R_2 which is outside the forward light cone of R_1. To see how this can happen, imagine a wire connecting R_1 and R_2. The wire is full of electrons and positive ions so that it is electrically neutral. Now we prearrange observers at each point of the wire so that at $t=0$ they move each electron slightly toward R_2. The result is a sudden appearance of charge at the ends with no charge density ever occurring anywhere else. If there was already a charge at R_1 it would be cancelled by the new charge at that point. The net result would be a sudden redistribution of charge.

Two ingredients are necessary for such behavior. The first is that the current vector j^μ be spacelike. Since the charge density on the wire is always zero it is clear that the current is purely in the spacelike direction. This also means that in some frame the charge density was negative. This of course is not a difficulty since charge density can be either positive or negative.

The other ingredient is *prearrangement*. The physical conditions along the wire must include agents with synchronized clocks that are instructed in advance to act simultaneously.

The sudden flow of energy requires the same two ingredients. In order that the energy is rearranged so suddenly the flux of energy $\langle T^{0i} \rangle$ must be much larger than the energy density itself. This means that the energy density can be made negative by a Lorentz boost. However, unlike electric charge, energy is not allowed to be negative in SYM theory. Since in classical SYM theory the energy density is positive this kind of flow of energy is absolutely forbidden in the classical theory. However quantum theory allows local negative energy densities as long as they are (over)compensated by nearby positive energy density [12]. In the next section we will analyze this in more detail and show that the bounds as in Ref. [12] are consistent with the behavior required of the SYM theory.

A second puzzle concerns locality. Let us suppose that before light-up the region between the separating shells is physically indistinguishable from the vacuum of the SYM theory. By this we mean that all expectation values of functionals of fields in this region are identical to their vacuum values. Then the light-up is impossible. To see this we consider a point (x,t) on the boundary just after light-up. The point is not near the points where the shells are localized. The SYM Heisenberg equations of motion can be used to express the energy density at this point in terms of local fields at a time just before light-up. Furthermore, causality requires that the only fields that can be involved are in the region between the shells where we have assumed vacuum conditions. It follows that the energy density at (x,t) must be the same as for the vacuum, that is, zero.

The resolution of these paradoxes, assuming the correspondence is really as strong as we believe, must be that the region between the shells at time $0<t<\pi/2$ must not be vacuum-like even though the expectation value of the energy-momentum tensor vanishes. Thus we are forced to postulate that in a region in which $\langle T_{\mu\nu}\rangle=\langle F^{\mu\nu}F_{\mu\nu}\rangle=\cdots=0$, the vacuum is excited to a non-vacuum-like local state which provides the precursor for the later event that we called light-up. The precursor fields must play the role of the prearranged agents which simultaneously move charges in the electric example. Furthermore there must be a very rich manifold of such precursor configurations. To see this suppose we change the initial emission parameters by a small amount of order $N^{-1/4}$. As we have seen this can lead to a very large change in the results of the collision. For example such a change can cause an increase of the impact parameter so that a peripheral grazing collision results. In this case the particles may get deflected through a small angle. Again, the news of the collision does not arrive at the boundary until $t=\pi/2$. As before the energy must suddenly rearrange but this time the result is not a spherically symmetric component but a new pair of localized small shells at shifted positions. Also as before, the information must be locally stored in a configuration with vanishing expectation value for the energy momentum tensor. Evidently all the local physical processes that can take place in the lab are coded in precursor configurations.

III. GRAVITATIONAL WAVE

The principal

In this section we will consider a simplified example in which a gravitational wave propagates radially outward from $r\sim 0$. We begin with the wave in the linearized theory in which the field equations are treated to lowest order in the deviations from AdS space. Assume that the wave is in one of the lowest spherical harmonics on the 3–sphere. In half-plane coordinates the plane fronted wave has the form

$$\gamma_{\mu\nu}(z,x,t)=\xi_{\mu\nu}R^2z^2f(z,t) \tag{3.1}$$

where $\gamma_{\mu\nu}(z,x,t)$ is defined by

$$ds^2=R^2\left[\frac{1}{z^2}(dt^2-dz^2-dx^idx^i)\right]+\gamma_{\mu\nu}(z,x,t)dx^\mu dx^\nu \tag{3.2}$$

and $\xi_{\mu\nu}$ is a transverse traceless polarization tensor with non-vanishing components in the x directions. The polarization tensor is assumed normalized to unity.

According to the AdS-CFT correspondence, the wave makes a contribution to the SYM energy momentum tensor given by [13–15]

$$\langle T_{ij}\rangle\sim-\frac{R}{G_5}z^{-2}\gamma_{ij}\bigg|_{z=0}=-\xi_{ij}\frac{R^3}{G_5}f(0,t). \tag{3.3}$$

We assume that at some initial time t_0 in the past, the function $f(z,t)$ describes a wave propagating toward $z=0$ and that it vanishes for $z<t_0$. Thus at the initial time the contribution to $\langle T\rangle$ from the wave vanishes. Furthermore causality of the bulk theory insures that $f(0,t)$ will remain exactly zero until $t=0$. At that time $f(0,t)$ and the SYM stress tensor begin to oscillate. After a time the wave will be reflected and the value at $z=0$ will return to zero exponentially. Thus far we have considered the gravitational wave in the linearized approximation. By analogy with terminology introduced in [12] the contribution to $\langle T\rangle$ in this approximation is called the *principal*.

The interest

The nonlinear corrections to the gravitational field equations give rise to a correction to the metric which in turn corrects the SYM energy density. Again by analogy with [12] this term is called the *interest*. It is smaller than the principal by a factor G_5. To compute the interest let us return to cavity coordinates. Consider any spherical[2] distribution of energy which is non-vanishing only for $r<r_0$. Then, as in flat space, the gravitational field at $r>r_0$ is completely determined to be that of a neutral non-rotating black hole with the same total energy. The metric of a black hole in AdS is given in Schwarzschild-like coordinates by

$$ds^2=R^2\left[\left(1+b^2-\frac{2MG_5}{R^2b^2}\right)dt^2-\left(1+b^2-\frac{2MG_5}{R^2b^2}\right)^{-1}\times db^2-b^2d\Omega^2\right] \tag{3.4}$$

where b is the radial coordinate. The coordinates b and r are related by

$$1+b^2=\frac{(1+r^2)^2}{(1-r^2)^2}. \tag{3.5}$$

Near the boundary this becomes

$$b=\frac{1}{1-r}. \tag{3.6}$$

Thus near the boundary the time-time component of the metric has the form

$$g_{00}=g_{00}^{AdS}-2MG_5(1-r)^2. \tag{3.7}$$

This gives a contribution to γ_{00} given by

$$\gamma_{00}=-2MG_5(1-r)^2=-2MG_5z^2. \tag{3.8}$$

[2]Near the boundary, the gravitational wave looks like a plane sheet with uniform energy density. The correction to the metric at a given point should be the same with that of a large spherical distribution of the same energy density.

Using the time-time component of Eq. (3.3) we find the interest

$$\langle T_{00}^{int}\rangle \sim MR. \tag{3.9}$$

Thus to compute the interest we need to compute the energy M stored in the gravitational wave. Our goal will be to compute the energy in terms of data on the boundary. This will facilitate comparison with the boundary CFT. For our purposes, it is convenient to set

$$\gamma_{\mu\nu}=\frac{h_{\mu\nu}}{z^2}=\xi_{\mu\nu}R^2\frac{\Phi(z,t)}{z^2}. \tag{3.10}$$

We work in the gauge $h_{z\mu}=0$. In the case when $\xi_{\mu\nu}$ is traceless with no timelike components, the constraints from the $z\mu$ components of the Einstein equations are automatically satisfied. The linearized equations for h_{ij} reduce to the following differential equation for $\Phi(z,t)$:

$$\partial_z^2\Phi-\frac{3}{z}\partial_z\Phi-\partial_t^2\Phi=0. \tag{3.11}$$

The corresponding (1+1) dimensional Lagrangian density is obtained to be

$$\mathcal{L}=\frac{R^2}{2G_5z^3}[(\partial_t\Phi)^2-(\partial_z\Phi)^2]. \tag{3.12}$$

This is just the Lagrangian density of a minimally coupled massless scalar field in AdS. Using the equation of motion, we can write the energy stored in the gravitational wave as

$$M=\frac{R^2}{2G_5}\int\frac{dz}{z^3}[(\partial_t\Phi)^2-\Phi\partial_t^2\Phi]. \tag{3.13}$$

We now look for solutions that fall like z^4 near the boundary so that $\gamma_{\mu\nu}$ falls like z^2. If we set

$$\Phi=z^2\chi(z)e^{-i\omega t}, \tag{3.14}$$

then χ satisfies a standard Bessel differential equation

$$z^2\partial_z^2\chi+z\partial_z\chi-(4-z^2\omega^2)\chi=0 \tag{3.15}$$

with solution $J_2(\omega z)$. Thus, the most general solution can be written as follows:

$$\Phi(z,t)=\frac{z^2}{2}\int d\omega\,\phi(\omega)J_2(\omega z)e^{-i\omega t}+\text{c.c.} \tag{3.16}$$

Near the boundary, $J_2(\omega z)\sim(\omega z)^2$, and so

$$f(\omega)=\omega^2\phi(\omega) \tag{3.17}$$

is the Fourier transform of the boundary data.

It is a simple exercise to compute the energy, and, therefore, the interest in terms of the boundary data. Using Eqs. (3.13), (3.16) and the orthogonality relations for Bessel functions gives

$$\langle T_{00}^{int}\rangle \sim \frac{R^3}{G_5}\int d\omega\frac{|f(\omega)|^2}{\omega^3}. \tag{3.18}$$

There are two things to note about the interest. First is that it is completely featureless having neither space nor time dependence. The second is that in a certain sense it can be made arbitrarily small. To see this let us compare the interest with the principal obtained in Eq. (3.3)

$$\langle T_{ij}\rangle \sim -\xi_{ij}\frac{R^3}{G_5}f(0,t)=-\xi_{ij}\frac{R^3}{2G_5}\int d\omega f(\omega)e^{-i\omega t}+\text{c.c.} \tag{3.19}$$

Defining

$$P=\langle T_{ij}\rangle$$

$$I=T_{00}^{int} \tag{3.20}$$

we note that the ratio I/P^2 is given by

$$\frac{I}{P^2}\sim G_5/R^3. \tag{3.21}$$

Now using Eqs. (1.1) and (1.2) we can rewrite this in terms of SYM quantities

$$\frac{I}{P^2}\sim 1/N^2. \tag{3.22}$$

The point is that if we hold fixed the principal then the interest goes to zero like $1/N^2$. Therefore in the large N limit the interest becomes negligible.

To summarize, the AdS-CFT correspondence requires the following behavior for the SYM energy momentum tensor. For $t<0$ the energy density and pressure are constant with respect to spatial and temporal position. For large N they are vanishingly small $\sim N^{-2}$. At $t=0$ the stress tensor begins to oscillate simultaneously over all space with magnitude of order unity. After a time the wave is reflected and the oscillations cease.

IV. SQUEEZED STATES IN FIELD THEORY

The superluminal behavior and non-positivity of the energy-momentum tensor are incompatible with classical field theory. However as we will see in this section, they are not only compatible with quantum field theory but can even be found in the theory of free fields. For notational simplicity we will use the theory of N^2 free scalar fields ϕ_{nm}. Thus, we will be able to obtain a field theoretic model for the gravitational wave we have just described.

Conformal invariance requires that we use the improved form of the energy-momentum tensor, which in four dimensions is given by

$$T_{\mu\nu}=\mathrm{Tr}\bigg(\frac{2}{3}\partial_\mu\phi\partial_\nu\phi-\frac{1}{6}\eta_{\mu\nu}(\partial_\sigma\phi)^2-\frac{1}{3}\phi\partial_\mu\partial_\nu\phi+\frac{1}{12}\eta_{\mu\nu}\phi\partial^2\phi\bigg). \tag{4.1}$$

In this form, the energy-momentum tensor is traceless and satisfies the usual conservation law. In what follows we normal-order the energy-momentum tensor so that all creation operators appear to the left of all annihilation operators. This is equivalent to setting the vacuum energy to zero.

Consider the squeezed state[3]

$$|\psi\rangle=\exp\bigg[\frac{1}{2}\int d^3\vec{k}d^3\vec{k}'F(\vec{k},\vec{k}')a^\dagger_{mn}(\vec{k})a^\dagger_{nm}(\vec{k}')\bigg]|0\rangle \tag{4.2}$$

for the particular choice

$$F(\vec{k},\vec{k}')=F(\vec{k})\,\delta^3(\vec{k}+\vec{k}') \tag{4.3}$$

such that $F(\vec{k})=F(-\vec{k})$. We consider the case when F is small; then,

$$\langle\psi|T_{\mu\nu}|\psi\rangle=T^{(1)}_{\mu\nu}+T^{(2)}_{\mu\nu} \tag{4.4}$$

with the first piece being linear in F and the second piece being quadratic in F.

The effect linear in F is obtained from contracting two annihilation operators in $T_{\mu\nu}$ with two creation operators in $|\psi\rangle$, or vice versa with $\langle\psi|$. The gauge indices in each pair are contracted among themselves, and so $T^{(1)}$ is of order N^2. All timelike components, $T^{(1)}_{0\mu}$, are zero as one can easily verify. The nonzero diagonal components are given by

$$T^{(1)}_{ii}=\frac{N^2}{2}\int\frac{d^3\vec{p}}{\omega_{\vec{p}}}\bigg(p_i^2-\frac{1}{3}\omega_{\vec{p}}^{\,2}\bigg)F(\vec{p})e^{-2i\omega_{\vec{p}}t}+\text{c.c.}, \tag{4.5}$$

and the off-diagonal components

$$T^{(1)}_{ij}=\frac{N^2}{2}\int\frac{d^3\vec{p}}{\omega_{\vec{p}}}p_ip_jF(\vec{p})e^{-2i\omega_{\vec{p}}t}+\text{c.c.} \tag{4.6}$$

As expected $T^{(1)}$ is homogeneous, traceless and oscillatory in time. In all, five independent components are non-zero as in the case of the gravitational wave studied above. Therefore, we identify this piece with the principal.

The only non-vanishing piece which is quadratic in F has the form $\langle F|T|F\rangle$. It receives contributions from terms in $T_{\mu\nu}$ with one creation and one annihilation operator only. It is also of order N^2 and homogeneous. In particular,

$$T^{(2)}_{00}=N^2(2\pi)^3\int d^3\vec{p}\,\omega_{\vec{p}}|F(\vec{p})|^2 \tag{4.7}$$

is positive and time-independent. We identify this piece with the interest.

We are now ready to compare with the results found in the previous section. First, we note that the ratio

$$\frac{T_2}{(T_1)^2}\sim\frac{1}{N^2} \tag{4.8}$$

has the same scaling with N as before. Moreover, we can obtain a consistent relation between the Fourier transform of the boundary data, $f(\omega)$, and the angle average of $F(\vec{p})$. If we compare $T^{(1)}$ with the principal, we find

$$f\sim F\omega^3. \tag{4.9}$$

Now, if we substitute f/ω^3 for F in Eq. (4.7), we obtain the same connection between interest and principal as given in Eq. (3.18).

We proceed now to study the time dependence of the principal. We wish to show that it can be oscillatory for a certain period of time and then vanishingly small. Take

$$F(\vec{p})=\alpha\frac{p_3^2}{p^3}(p^2\lambda)^{n/2}e^{-\lambda p^2} \tag{4.10}$$

as an example. Here, α has units of mass and λ has units of inverse mass squared. We consider the case for which the number n is even. We focus on a particular non-zero component. For example,

$$T^{(1)}_{33}=\frac{4\alpha\lambda^{-3/2}\pi N^2}{45}\int_{-\infty}^{+\infty}dx\,x^{(2+n)}e^{-x^2}e^{2ix(t/\sqrt{\lambda})}. \tag{4.11}$$

This in turn is equal to

$$T^{(1)}_{33}=\frac{8\alpha\lambda^{-3/2}\pi N^2}{45}\bigg(\frac{\sqrt{\lambda}}{2i}\bigg)^{(2+n)}\partial_t^{(2+n)}\int_{-\infty}^{+\infty}dx\,e^{-x^2}e^{2ixt/\sqrt{\lambda}}$$
$$=\frac{8\alpha\lambda^{-3/2}\pi N^2}{45}\bigg(\frac{\sqrt{\lambda}}{2i}\bigg)^{(2+n)}\partial_t^{(2+n)}(\sqrt{\pi}e^{-t^2/\lambda}). \tag{4.12}$$

The final result is a polynomial in t times a Gaussian. This means that the principal is oscillatory for some period of time near $t=0$ (depending on λ) and, then, it vanishes exponentially as $t\to\infty$.

It is interesting to consider gravitational waves which propagate along a direction which is not perpendicular to the boundary. Suppose the wave vector has components in the (x^1,z) plane. In this case the wave fronts do not simultaneously arrive at the boundary. In fact the boundary data itself becomes a wave propagating in the x^1 direction. Furthermore the wave has both group and phase velocity greater than 1. It is superluminal. To see how this happens in the CFT we can compute the principal with the delta function in

[3]The terminology ‘‘squeezed state’’ originated in the quantum optics literature. The squeezing refers to the shape of the oscillator phase space probability distributions.

Eq. (4.3) replaced by $\delta(\vec{k}+\vec{k}'-\vec{q})$ where $\vec{q}$ lies along the x^1 direction. The resulting principal is easily computed and forms a superluminal wave. Actually it is not necessary to do any further calculation to see the bulk-boundary agreement in this case. The gravitational wave propagating in the z,x^1 direction can be obtained by an AdS-Lorentz transformation from the original wave propagating along z. This maps into a conformal transformation of the boundary. Since the field theory we are using is conformally invariant the agreement for the transformed wave follows from the agreement in the original case.

In addition to being superluminal, the wave in the x^1 direction also has nonvanishing oscillating time-time component so that the energy density oscillates from positive to negative [12].

V. PRECURSORS

The profile of the gravitational wave obviously carries information. We would like to understand what precursor degrees of freedom carry that information, particularly during the time $t<0$ before the wave arrives at the boundary. The interest $T^{int}_{\mu\nu}$ is completely featureless in both space and time and cannot be relevant here. Furthermore all bulk fields vanish within a neighborhood of the boundary. This means that the local SYM fields that can be identified with boundary values of bulk fields also vanish.

Evidently the precursors are nonlocal. In the free field example a convenient example is given by

$$\Phi(x,x')=\left(\frac{2}{3}\partial_\mu\partial'_\nu-\frac{1}{6}\eta_{\mu\nu}\partial_\sigma\partial'^{\sigma}-\frac{1}{3}\partial'_\mu\partial'_\nu\right)\times\langle\phi(\vec{x},t)_{mn}\phi(\vec{x}',t')_{nm}\rangle|_{t=t'} \quad (5.1)$$

where the expectation value is taken in the squeezed state. This quantity obeys a wave equation with respect to t and $(\vec{x}-\vec{x}')/2$. Furthermore at $\vec{x}-\vec{x}'=0$ it is given by $\langle T_{\mu\nu}\rangle$. At early times when $\langle T_{\mu\nu}\rangle$ vanishes $\Phi(x,x')$ is nonzero for $|\vec{x}-\vec{x}'|\approx 2|t|$. In other words the precursor becomes increasingly nonlocal the further the wave is from the boundary. This is of course a manifestation of the IR-UV connection [6]. Furthermore at any time $\Phi(x,x')$ has the same information as the function F that characterizes the squeezed state.

VI. CONCLUSION

The main purpose of the free field model is to demonstrate that some of the odd superluminal and negative behavior of T predicted by the AdS-CFT correspondence is consistent with the principles of quantum field theory. It is somewhat surprising that the free field model works so well and captures the detailed relation between principal and interest. We do not really know why this is so but it seems to be part of a pattern. For example, free field theory describes the thermodynamics of AdS black holes correctly apart from a well known factor of 3/4 in the effective number of degrees of freedom. More exactly, free field theory agrees with AdS/CFT predictions for the two and three point functions of chiral primaries. We suspect that if the model scalar field theory is replaced by free SYM theory the numerical relation between interest and principal may be exact as a consequence of the non-renormalization theorems of the two and three point functions.

Perhaps the most interesting result of this paper is the identification of nonlocal precursor fields such as

$$\phi_{mn}(x)\phi_{nm}(x'). \quad (6.1)$$

These fields would have to be modified in the interacting SYM since they are not gauge invariant as they stand. A candidate would be

$$\phi_{mn}(x)\phi_{rs}(x')W_{mr}W'_{sn} \quad (6.2)$$

where W and W' are Wilson lines along two paths connecting the points x,x'. In fact the entire object (6.2) can be thought of as a single Wilson loop. This suggests that the nonlocal precursors which code local bulk information are expectation values of Wilson loops of size dictated by the UV-IR connection.

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