Is warm inflation possible?

Jun'ichi Yokoyama*
Department of Physics, Stanford University, Stanford, California 94305-4060
and Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Andrei Linde
Department of Physics, Stanford University, Stanford, California 94305-4060
(Received 25 August 1998; published 24 September 1999)

We show that it is extremely difficult and perhaps even impossible to have inflation supported by thermal effects. [S0556-2821(99)00916-9]

PACS number(s): 98.80.Cq, 11.10.Wx

I. INTRODUCTION

The first models of inflationary cosmology [1,2] were based on an assumption that inflation appears as a result of high-temperature phase transitions with supercooling in the early universe [3]. This idea survived less than two years, after which it was replaced by chaotic inflation which does not require initial thermal equilibrium and phase transitions [4]. There were several reasons why this happened; the following is a short list [5].

(1) The assumption that the universe was hot from the very beginning is not necessary in the chaotic inflation scenario. Moreover, the assumption that inflation begins only after a long stage of cooling implies that the universe from the very beginning must be very large and homogeneous. This means that such models do not provide a complete solution to the homogeneity and flatness problems.

(2) The theories where inflation occurs after supercooling require fine tuned effective potentials satisfying "thermal constraints."

(3) The inflaton field typically has a very weak interaction with other fields, so it may be out of thermal equilibrium in the early universe.

(4) Even if the inflaton field is in thermal equilibrium, it takes a lot of time for it to roll to the minimum of the temperature-dependent effective potential. In many cases, inflation occurs while the field rolls down, so when it arrives at the minimum of the effective potential at $\phi=0$, the temperature vanishes.

(5) Even if the temperature does not completely vanish when the field falls to the minimum of the effective potential, it vanishes during supercooling, so the thermal effects become irrelevant for the description of the tunneling and of the subsequent stage of slow rolling of the inflaton field [6].

For all of these five reasons, the idea that inflation is somehow related to high-temperature effects was almost completely abandoned. Recently, however, there was an attempt to revive it in the context of the so-called warm inflation scenario [7].

The main idea of the scenario can be formulated as follows. Suppose the universe is in a state of thermal equilibrium, and the field ϕ slowly rolls down to its minimum. When the universe expands, its temperature tends to decrease as $a(t)^{-1}$, where a is the scale factor. Therefore one expects that the temperature in an inflationary universe falls down exponentially and immediately becomes irrelevant. However, if the scalar field interacts with other particles, it may transfer some of its energy to the thermal bath. This may keep the temperature from falling to zero. This regime may continue in a self-consistent way if the amount of particles produced due to the interaction of the scalar field ϕ with thermal bath is large enough to keep the field from a rapid fall to the minimum of the effective potential.

However, we immediately see a problem here. During each time H^{-1} the universe expands e times, and the energy density of previously existed ultrarelativistic particles becomes e^{-4} times smaller. Therefore most of the particles during warm inflation at any given moment should have been created during the previous time interval $\sim 0.2H^{-1}$. If the energy release is going to keep the field from rapidly falling down, then the energy released each time $0.2H^{-1}$ must be very small. Otherwise the field rapidly falls down, and there is no inflation. But if the energy release is small, then the total number of particles in the warm universe must be very small, and therefore their interaction with the scalar field ϕ may be too small to keep it from rapid falling down.

Despite this problem, the basic idea is rather interesting and it should not be discarded without a serious investigation. It would be very interesting to see, in particular, whether this idea could help to realize inflationary universe scenario in the context of string theory.

The results of investigation of warm inflation indicated, as expected, that this regime is very hard to obtain [8]. However, the methods used in this investigation were rather complex and not very intuitive. Therefore the basic mechanism of warm inflation remained obscure, and there remained a hope that one can obtain a good realization of this scenario by considering slightly more complicated theories of elementary particles.

In this paper we will try to give a simple and intuitive explanation of the mechanism of dissipation of the energy of the inflaton field, which will help us to understand the main problems of the warm inflation scenario. Then we will con-

*Present address: Department of Earth and Space Science, Graduate School of Science, Osaka University, Toyonaka 560-0043, Japan.

firm our expectations using more rigorous methods of nonequilibrium quantum statistics.

II. INTUITIVE ARGUMENT

In order to explain the basic idea of warm inflation, as well as its shortcomings, in this section we will give a simple and intuitive discussion of this scenario in the theory of a massive inflaton field ϕ interacting with a massless field χ:

$$L=\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}(\partial_\mu\chi)^2-\frac{m^2}{2}\phi^2-\frac{g^2}{2}\phi^2\chi^2. \tag{1}$$

If one neglects interaction between these two fields, equation of motion for the scalar field ϕ has a familiar form

$$\ddot{\phi}+3H\dot{\phi}=-m^2\phi. \tag{2}$$

The term $3H\dot{\phi}$ represents energy loss of the field ϕ due to expansion of the universe. This term, which is similar to the viscosity term in an equation of motion of a pendulum in a viscous liquid, slows down the motion of the field ϕ. As a result, the potential energy of the field changes very slowly, which under some circumstances may lead to inflation [5].

One could expect that interaction of the fields ϕ and χ may lead to an additional energy loss of the field ϕ due to creation of χ particles (reheating), which can be represented by adding another friction term to this equation [9,10]:

$$\ddot{\phi}+\Gamma\dot{\phi}+3H\dot{\phi}=-m^2\phi. \tag{3}$$

If $\Gamma \gg H$, the field ϕ will roll down more slowly than in the absence of interaction. Therefore one could expect that inflation may continue for a longer time [9], and may occur even in such theories where otherwise it would be impossible [11].

Further development of the theory of reheating has shown that the situation is more complicated. Addition of the term $\Gamma\dot{\phi}$ effectively describes energy loss due to particle creation only at the stage of oscillations of the scalar field ϕ [12], and only in the case when these oscillations do not lead to parametric resonance [13]. It would be inappropriate to use this equation for the description of energy loss of the field ϕ during inflation, assuming, as usual, that density of all particles during inflation is exponentially small.

However, if the universe is hot, and there are many χ particles in the thermal bath, then in a certain approximation the motion of the scalar field ϕ is indeed described by Eq. (3). This effect is the basic ingredient of the warm inflation scenario. The standard description of this effect is very complicated because it involves summation of series of higher order diagrams in the nonequilibrium quantum statistics [14,15]. We will describe this method later. In this section we would like to give a simple interpretation of this effect which will simultaneously tell us whether this effect can be significant enough to give rise to an inflationary regime.

Let us write equation of motion for the field ϕ taking into account interaction between the scalar fields ϕ and χ in the Hartree approximation:

$$\ddot{\phi}+3H\dot{\phi}+m^2\phi+g^2\phi\langle\chi^2\rangle=0. \tag{4}$$

Here $\langle\chi^2\rangle$ gives the average value of thermal fluctuations of the field χ in the system. If the system is in a state of thermal equilibrium (which is eventually established if the field ϕ is constant) one has [3,5]

$$\langle\chi^2\rangle_{\rm eq}=\frac{1}{(2\pi)^3}\int\frac{n_\chi^{\rm eq}(p)d^3p}{\omega_\chi}, \tag{5}$$

where

$$n_\chi^{\rm eq}=\frac{1}{\exp(\omega_\chi/T)-1},\quad \omega_\chi=\sqrt{p^2+g^2\phi^2}. \tag{6}$$

We will assume that $g\phi \ll T$, because in the opposite limit all thermal effects are exponentially small. In this case

$$\langle\chi^2\rangle_{\rm eq}=\frac{T^2}{12}\left(1-\frac{3g\phi}{\pi T}\right)+\cdots. \tag{7}$$

Now suppose the scalar field ϕ changes by $\Delta\phi$. This changes masses of χ particles $g\phi$, and the equilibrium value of $n_\chi^{\rm eq}$ should change correspondingly. However, this change cannot happen instantaneously; it occurs with a time delay $\Delta t\sim\Gamma_\chi^{-1}$, where Γ_χ is a decay width of χ particles at finite temperature. This means that n_χ deviates from its equilibrium value $n_\chi^{\rm eq}$: when the scalar field reaches its value ϕ, the occupation number remains equal to $n_\chi^{\rm eq}$ at earlier time, when the field was equal to $\phi-\Delta\phi\sim\phi-\dot{\phi}\Delta t$, with $\Delta t\sim\Gamma_\chi^{-1}$ [16]:

$$\Delta n_\chi=n_\chi-n_\chi^{\rm eq}\sim-\frac{dn_\chi^{\rm eq}}{d\phi}\dot{\phi}\Gamma_\chi^{-1}. \tag{8}$$

Therefore $\langle\chi^2\rangle$ in Eq. (4) slightly differs from $\langle\chi^2\rangle_{\rm eq}$:

$$\langle\chi^2\rangle=\frac{1}{(2\pi)^3}\int\frac{n_\chi(p)d^3p}{\omega_\chi}$$
$$=\langle\chi^2\rangle_{\rm eq}-\frac{\dot{\phi}}{(2\pi)^3}\int\frac{d^3p}{\omega_\chi}\frac{dn_\chi^{\rm eq}}{d\phi}\Gamma_\chi^{-1}. \tag{9}$$

Note that Γ_χ^{-1} depends on momentum p, it is small for $p \ll T$. The leading contribution to the last integral is given by particles with typical momentum $p\sim T$. In this case, as we will show in Sec. III D,

$$\Gamma_\chi\simeq\frac{g^4T}{192\pi}. \tag{10}$$

Evaluation of the last integral yields

$$\langle\chi^2\rangle=\langle\chi^2\rangle_{\rm eq}+\frac{C\phi}{g^2T}\dot{\phi}$$
$$=\frac{T^2}{12}\left(1-\frac{3g\phi}{\pi T}\right)+\frac{C\phi}{g^2T}\dot{\phi}+\cdots, \tag{11}$$

where $C = O(10)$.

With an account taken of this correction, Eq. (4) can be represented in the following form [16]:

$$\ddot{\phi} + \frac{C\phi^2}{T}\dot{\phi} + 3H\dot{\phi} + m^2\phi + g^2\phi\langle\chi^2\rangle_{\rm eq} \approx \ddot{\phi} + \frac{C\phi^2}{T}\dot{\phi} + 3H\dot{\phi} + m^2\phi + \frac{g^2T^2}{12}\phi = 0. \tag{12}$$

In Sec. III D we will obtain this equation by a more rigorous, but somewhat more complicated method. The simple and intuitive approach which we have used in this section allows us to easily understand the nature of the viscosity term $(C\phi^2/T)\dot{\phi}$ and evaluate its possible significance. If this additional term is large, it may slow down the rolling of the field ϕ and make inflation longer. It may even lead to inflation in the models where it would be impossible otherwise.

The main idea of the warm inflation scenario is to take this viscosity term into account and to see whether it can be large enough to support inflation in a self-consistent way [7]. In order to do it one should solve Eq. (12) for the scalar field, find out whether it has inflationary solutions, and check whether the corresponding solutions can be self-consistent. According to the warm inflation scenario, the field ϕ should move very slowly (inflation), but still fast enough to transfer sufficiently large amount of energy to thermal fluctuations. If these fluctuations are large enough, they may be responsible for the existence of the large viscosity term $(C\phi^2/T)\dot{\phi}$, which is necessary for the self-consistency of this scenario.

At the first glance, this term could be very large indeed. It is not suppressed by any powers of the small coupling constant g, and in certain cases $(C\phi^2/T)\dot{\phi}$ may be greater than the usual viscosity term $3H\dot{\phi}$. This was the main reason why it was expected that this additional viscosity term could play an important role in inflationary theory.

However, as we are going to show, this term is actually very small. Indeed, let us remember the procedure of the derivation of the term $(C\phi^2/T)\dot{\phi}$. The leading term in the expression for $\langle\chi^2\rangle_{\rm eq}$ in Eq. (7) is ϕ independent: $\langle\chi^2\rangle_{\rm eq} \approx T^2/12$. The term $(C\phi/g^2T)\dot{\phi}$ appears because the *subleading* term in Eq. (7) depends on ϕ, and because this term *slightly* differs from its thermally equilibrium value $g\phi T/4\pi$. Thus the term $(C\phi^2/T)\dot{\phi}$ appears in Eq. (12) as a correction to a correction, because $(C\phi/g^2T)\dot{\phi} \ll g\phi T/4\pi \ll T^2/12$. In other words, $(C\phi^2/T)\dot{\phi} \ll g^2\phi\langle\chi^2\rangle_{\rm eq}$, so the new viscosity term in Eq. (12) is always much smaller than the usual thermal correction to the equation of motion of the field ϕ. This is an important fact, which will help us to evaluate plausibility of warm inflation.

As an example, let us consider two limiting cases.

Case I. $gT \ll m$. In this case $m^2\phi \gg g^2\phi\langle\chi^2\rangle_{\rm eq} \gg (C\phi^2/T)\dot{\phi}$, so the new viscosity term is completely irrelevant for the description of the evolution of the field ϕ, as is clear from the above intuitive derivation of the equation of motion.

Case II. $gT \gg m$. If $g\phi \gg T$, then all thermal effects disappear. If $g\phi \ll T$, then $m^2\phi^2 \ll g^2\phi^2\langle\chi^2\rangle_{\rm eq} \sim g^2\phi^2T^2 \ll T^4$. In this case equation of state is determined by ultrarelativistic matter, $p \approx \rho/3$, and inflation cannot occur.

In conclusion, new viscosity terms in the equation of motion of the inflaton field appear as a correction to a correction to the usual high temperature terms such as $g^2\phi\langle\chi^2\rangle$. These terms appear if one wants to write equation of motion of the scalar field in terms of the thermal equilibrium quantities such as $\langle\chi^2\rangle_{\rm eq}$, which differs only very slightly from its out-of-equilibrium value $\langle\chi^2\rangle$. If the leading thermal effects cannot result in existence of an inflationary regime of a new type, then the sub-subleading terms cannot do it as well.

In this investigation we did not really evaluate the viscosity term $(C\phi^2/T)\dot{\phi}$; we only used our knowledge that this term appeared as a correction to a correction to the term $(g^2/12)T^2$, so it should be always much smaller than $(g^2/12)T^2$. It is very instructive to study the same issue by solving Eq. (12) directly, without making any *a priori* assumptions about the magnitude of the viscosity term.

Suppose one has a stage of warm inflation in the regime where $T > g\phi$, but $m^2 \gg g^2T^2$ (since otherwise one either does not have thermal corrections, or one has a noninflationary radiation dominated universe). In this regime, and assuming that $C\phi^2/T \gg H$, Eq. (12) acquires the following form:

$$\ddot{\phi} + \frac{C\phi^2}{T}\dot{\phi} + m^2\phi = 0. \tag{13}$$

In the regime of slow rolling one can neglect the first term and obtain a solution

$$\phi^2 = \phi_0^2 - \frac{2m^2Tt}{C}, \tag{14}$$

assuming, for simplicity, that the temperature remains constant. One can easily verify that for $gm < T < Cm/g$ and $\phi^2 > mT/C$ this solution is compatible with the inequalities $T > g\phi$, but $m^2 \gg g^2T^2$, and the term $\ddot{\phi}$ could indeed be neglected in Eq. (13). Since it is not a usual oscillatory regime, one could think that we have an interesting candidate for the warm inflation regime.

However, Eq. (14) tells us that the value of the field ϕ completely changes (vanishes) within the time $\delta t = C\phi_0^2/(2m^2T)$. As one can easily verify, in the regime which we study now ($g\phi \ll T$ and $m^2 \gg g^2T^2$) this time is much smaller than the typical decay time of the χ particles:

$$\Gamma_\chi \delta t \sim \frac{g^4C\phi_0^2}{384\pi m^2} \ll \frac{C}{384\pi} \ll 1. \tag{15}$$

Thus the simple linear approximation $\phi - \Delta\phi \sim \phi - \dot{\phi}\Delta t$, with $\Delta t \sim \Gamma_\chi^{-1}$, which we used for the derivation of Eq. (12) does not work in application to our problem: The typical duration of the linear change of the field ϕ is much shorter than Γ_χ^{-1}. This means that Eq. (12) is incorrect in the only regime where warm inflation could appear in this model. In our previous qualitative analysis of warm inflation in this model we avoided this problem because we did not use the expression $(C\phi^2/T)\dot{\phi}$ for the viscosity term, which we obtained using the incorrect assumption that $\phi - \Delta\phi \sim \phi$

$-\dot{\phi}\Gamma_{\chi}^{-1}$. Instead of that, we used the fact that the viscosity term is always much smaller than $(g^2/12)\,T^2\phi$.

To study motion of the field ϕ in the model (1) one should remember that the typical time when a significant change of the scalar field occurs (such as the oscillation time m^{-1}) is much shorter than Γ_{χ}^{-1}. Therefore the number of χ particles practically does not change during the oscillations. To study the motion of the field ϕ in this case one should evaluate $\langle\chi^2\rangle$ in the regime where n_χ remains constant, see, e.g., Ref. [17]. The behavior of the field ϕ in this regime is completely different from what one could expect on the basis of Eq. (12).

This example shows one of the many obstacles which make it so hard or perhaps even impossible to implement the idea of warm inflation. In order to verify our conclusions, we performed a more detailed investigation of this issue and studied several different models where warm inflation could occur. Before starting its detailed description we remind the reader that the above intuitive derivation of Eq. (12) is based on the following two most fundamental assumptions or conditions: (A) ϕ interacts with particles whose effective mass is much smaller than the temperature; (B) ϕ should not evolve significantly during the relaxation time, Δt, of such particles, namely, $|\dot{\phi}|\Delta t \ll \phi$. (Otherwise the quasistationary inflationary expansion could not be sustained.)

A complete discussion of warm inflation is rather involved; in addition to these two conditions many other constraints have to be satisfied. However, as we will see in order to show that the scenario is not feasible in all models that we will consider it is enough to use only the simplest constraints based on the above two conditions. In the next section we review derivation of the equation of motion of the scalar field in more generic cases in terms of quantum field theory at finite temperature, where these two conditions (A) and (B) are also indispensable. It has been advocated in the literature [8] that this method is quite appropriate for application to warm inflation where the scalar field presumably moves slowly.

As we have demonstrated above for a particular model of two scalar fields with interaction $(g^2/2)\,\phi^2\chi^2$, however, ϕ does evolve significantly during the relaxation time even in its slow roll-over regime, so that the condition (B) is not satisfied. Put it differently, if we attempt to realize warm inflation consistently with the derivation of the equation of motion in the form of Eq. (12), we inevitably find that the e-folding number of inflation is constrained to be much smaller than unity. In fact, a similar result has also been obtained in Ref. [8] for this particular model. However, there was a hope that this result is not generic, and several possible ways out to increase the number of e folds have been proposed [8]. In Sec. IV we demonstrate for several other models that the number of e folds of inflation must be much smaller than unity. As will be seen there, this conclusion remains unchanged even when we take the proposals of Ref. [8] into account.

III. EFFECTIVE EQUATION OF MOTION IN THE THERMAL BATH

As a preparation to quantitative discussion which confirms the above intuitive argument here we review a field theoretic approach to derive an effective equation of motion in the thermal bath. The standard quantum field theory, which is appropriate for evaluating the transition amplitude from an ''in'' state to an ''out'' state for some field operator $\mathcal{O}$, $\langle \mathrm{out}|\mathcal{O}|\mathrm{in}\rangle$, is not suitable to trace time evolution of an expectation value in a nonequilibrium system interacting with a thermal bath. For this purpose we should use the in-in formalism, which was introduced by Schwinger [18] and developed in Refs. [19,20]. Here, extending Gleiser and Ramos [15] and Yamaguchi and Yokoyama [21], who followed Morikawa [14], we derive an effective Langevin-like equation for a coarse-grained field in the case the scalar field is interacting with not only other scalars but also fermions.

A. Nonequilibrium quantum field theory

Let us consider the following Lagrangian density of a singlet scalar field φ interacting with another scalar field χ and a fermion ψ for illustration.

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\varphi)^2-\frac{1}{2}m_\varphi^2\varphi^2-\frac{1}{4!}\lambda\varphi^4 +\frac{1}{2}(\partial_\mu\chi)^2-\frac{1}{2}m_\chi^2\chi^2-\frac{1}{2}g^2\chi^2\varphi^2 +i\bar{\psi}\gamma^\mu\partial_\mu\psi-m_\psi\bar{\psi}\psi-f\varphi\bar{\psi}\psi. \tag{16}$$

In order to follow the time development of φ, only the initial condition is fixed, and so the time contour in a generating functional starting from the infinite past must run to the infinite future without fixing the final condition and come back to the infinite past again. The generating functional is thus given by

$$\begin{aligned} Z[J,\eta,\bar{\eta}]&\equiv \mathrm{Tr}\left[T\left(\exp\left[i\int_c (J\varphi+K\chi+\eta\psi+\bar{\eta}\bar{\psi}]\right)\rho\right]\right.\\ &=\mathrm{Tr}\left\{T_+\left(\exp\left[i\int (J_+\varphi_+ + K_+\chi_+ + \eta_+\psi_+ + \bar{\eta}_+\bar{\psi}_+)\right]\right)\right.\\ &\quad\left.\times T_-\left(\exp\left[i\int (J_-\varphi_- + K_-\chi_- + \eta_-\psi_- + \bar{\eta}_-\bar{\psi}_-)\right]\right)\rho\right\}, \end{aligned} \tag{17}$$

where the suffix c represents the closed time contour of integration and X_+ a field component X on the plus-branch ($-\infty$ to $+\infty$), X_- that on the minus-branch ($+\infty$ to $-\infty$). The symbol T represents the time ordering according to the closed time contour, T_+ the ordinary time ordering, and T_- the antitime ordering. J, K, η, and $\overline{\eta}$ imply the external fields for the scalar and the Dirac fields, respectively. In fact, each external field $J_+(K_+,\eta_+,\overline{\eta}_+)$ and $J_-(K_-,\eta_-,\overline{\eta}_-)$ is identical, but for technical reason we treat them different and set $J_+=J_-(K_+=K_-,\eta_+=\eta_-,\overline{\eta}_+=\overline{\eta}_-)$ only at the end of calculation. ρ is the initial density matrix. Strictly speaking, we should couple the time development of the expectation value of the field with that of the density matrix, which is practically impossible. Accordingly we assume that deviation from the equilibrium is small and use the density matrix corresponding to the finite-temperature state. Then the generating functional is described by the path integral as

$$Z[J,K,\eta,\overline{\eta}]=\exp(iW[J,K,\eta,\overline{\eta}])$$

$$=\int_c\mathcal{D}\varphi\int_c\mathcal{D}\chi\int_c\mathcal{D}\psi\int_c\mathcal{D}\psi^*\times\exp(iS[\varphi,\chi,\psi,\overline{\psi},J,K,\eta,\overline{\eta}]),\tag{18}$$

where the classical action S is given by

$$S[\varphi,\chi,\psi,\overline{\psi},J,K,\eta,\overline{\eta}]=\int_c d^4x\{\mathcal{L}+J(x)\varphi(x)+K(x)\chi(x)+\eta(x)\psi(x)+\overline{\eta}(x)\overline{\psi}(x)\}.\tag{19}$$

As with the Euclidean-time formulation, the scalar field is still periodic and the Dirac field antiperiodic along the imaginary direction, now with $\varphi(t,\boldsymbol{x})=\varphi(t-i\beta,\boldsymbol{x})$, $\chi(t,\boldsymbol{x})=\chi(t-i\beta,\boldsymbol{x})$, and $\psi(t,\boldsymbol{x})=-\psi(t-i\beta,\boldsymbol{x})$.

The effective action for the scalar field is defined by the connected generating functional as

$$\Gamma[\phi]=W[J,K,\eta,\overline{\eta}]-\int_c d^4xJ(x)\phi(x),\tag{20}$$

where $\phi(x)=\delta W[J,K,\eta,\overline{\eta}]/\delta J(x)$.

We give the finite temperature propagator before the perturbative expansion. For the closed path, the scalar propagator has four components.

$$G_\chi(x-x')=\begin{pmatrix} G_\chi^F(x-x') & G_\chi^+(x-x') \\ G_\chi^-(x-x') & G_\chi^{\overline{F}}(x-x')\end{pmatrix}\tag{21}$$

$$\equiv\begin{pmatrix}\mathrm{Tr}[T_+\chi(x)\chi(x')\rho] & \mathrm{Tr}[\chi(x')\chi(x)\rho] \\ \mathrm{Tr}[\chi(x)\chi(x')\rho] & \mathrm{Tr}[T_-\chi(x)\chi(x')\rho]\end{pmatrix}\tag{22}$$

Similar formulas apply for φ field as well. Also, for a Dirac fermion we find

$$S_\psi(x-x')=\begin{pmatrix} S_\psi^F(x-x') & S_\psi^+(x-x') \\ S_\psi^-(x-x') & S_\psi^{\tilde{F}}(x-x')\end{pmatrix}\tag{23}$$

$$\equiv\begin{pmatrix}\mathrm{Tr}[T_+\psi(x)\overline{\psi}(x')\rho] & \mathrm{Tr}[-\overline{\psi}(x')\psi(x)\rho] \\ \mathrm{Tr}[\psi(x)\overline{\psi}(x')\rho] & \mathrm{Tr}[T_-\psi(x)\overline{\psi}(x')\rho]\end{pmatrix}\tag{24}$$

B. One-loop finite temperature effective action

The perturbative loop expansion for the effective action Γ can be obtained by transforming $\varphi\to\varphi_0+\zeta$ where φ_0 is the field configuration which extremizes the classical action $S[\varphi,J]$ and ζ is small perturbation around φ_0. Up to one loop order and $\mathcal{O}(\lambda^2,g^4,f^2)$ the effective action Γ becomes

$$\begin{aligned}\Gamma[\phi_c,\phi_\Delta]=&\int d^4x\left\{\phi_\Delta(x)[-\Box-M^2]\phi_c(x)-\frac{\lambda}{4!}[4\phi_\Delta(x)\phi_c^3(x)+\phi_c(x)\phi_\Delta^3(x)]\right\}\\&-\int d^4x\int d^4xA_1(x-x')\left[\phi_\Delta(x)\phi_c(x)\phi_c^2(x')+\frac{1}{4}\phi_\Delta(x)\phi_c(x)\phi_\Delta^2(x')\right]\\&-2\int d^4x\int d^4x'A_2(x-x')\phi_\Delta(x)\phi_c(x')\\&+\frac{i}{2}\int d^4x\int d^4x'[B_1(x-x')\phi_\Delta(x)\phi_\Delta(x')\phi_c(x)\phi_c(x')+B_2(x-x')\phi_\Delta(x)\phi_\Delta(x')],\end{aligned}\tag{25}$$

where

$$\phi_c\equiv\frac{1}{2}(\phi_++\phi_-),\tag{26}$$

$$\phi_\Delta\equiv\phi_+-\phi_-,\tag{27}$$

$$M^2=m^2+g^2\int\frac{d^3q}{(2\pi)^3}\frac{1+2n_\chi(\boldsymbol{q})}{2\omega_\chi(\boldsymbol{q})}+\frac{\lambda}{2}\int\frac{d^3q}{(2\pi)^3}\frac{1+2n_\varphi(\boldsymbol{q})}{2\omega_\varphi(\boldsymbol{q})},\tag{28}$$

$$A_1(x-x')=2g^4\,\mathrm{Im}[G^F_\chi(x-x')^2]\,\theta(t-t') + \frac{\lambda^2}{2}\,\mathrm{Im}[G^F_\varphi(x-x')^2]\,\theta(t-t'), \tag{29}$$

$$A_2(x-x')=f^2\,\mathrm{Im}[S^F_{\alpha\beta}(x-x')S^{\beta\alpha}_F(x'-x)]\,\theta(t-t'), \tag{30}$$

$$B_1(x-x')=2g^4\,\mathrm{Re}[G^F_\chi(x-x')^2] + \frac{\lambda^2}{2}\,\mathrm{Re}[G^F_\varphi(x-x')^2], \tag{31}$$

$$B_2(x-x')=-f^2\,\mathrm{Re}[S^F_{\alpha\beta}(x-x')S^{\beta\alpha}_F(x'-x)]. \tag{32}$$

The last term of Eq. (25) gives the imaginary contribution to the effective action Γ. We can attribute these imaginary terms to the functional integrals over real auxiliary fields $\xi_1(x)$ and $\xi_2(x)$ [14] to rewrite Eq. (25) as

$$\exp(i\Gamma[\phi_c,\phi_\Delta])=\int \mathcal{D}\xi_1\int \mathcal{D}\xi_2 P_1[\xi_1]P_2[\xi_2] \times \exp\{iS_{\mathrm{eff}}[\phi_c,\phi_\Delta,\xi_1,\xi_2]\}, \tag{33}$$

where

$$S_{\mathrm{eff}}[\phi_c,\phi_\Delta,\xi_1,\xi_2] \equiv \mathrm{Re}\,\Gamma+\int d^4x[\xi_1(x)\phi_c(x)\phi_\Delta(x)+\xi_2(x)\phi_\Delta(x)]. \tag{34}$$

Here $\xi_1(x)$ and $\xi_2(x)$ are random Gaussian fields with the probability distribution functional

$$P_i[\xi_i]=\mathcal{N}_i\exp\left[-\frac{1}{2}\int d^4x\int d^4x'\,\xi_i(x) \times B_i^{-1}(x-x')\xi_i(x')\right]\quad (i=1,2), \tag{35}$$

where $\mathcal{N}_i$ is a normalization factor. They induce random noise terms in the effective equation of motion of ϕ as a result of the interactions with the thermal bath.

C. Equation of motion

Applying the variational principle to S_{eff}, we obtain the equation of motion for ϕ_c.

$$\left.\frac{\delta S_{\mathrm{eff}}[\phi_c,\phi_\Delta,\xi_1,\xi_2]}{\delta\phi_\Delta}\right|_{\phi_\Delta=0}=0. \tag{36}$$

From Eqs. (34) and (25), it reads

$$(\Box+M^2)\phi_c(x) + \frac{\lambda}{3!}\phi_c^3(x)+\phi_c(x)\int d^3x'\int_{-\infty}^t dt' A_1(x-x')\phi_c^2(x') + 2\int d^3x'\int_{-\infty}^t dt' A_2(x-x')\phi_c(x') = \phi_c(x)\xi_1(x)+\xi_2(x) \tag{37}$$

and

$$\langle\xi_i(x)\xi_i(x')\rangle=B_i(x-x'). \tag{38}$$

Though A_1 and B_1 has two contributions from χ and φ fields, they have the same properties except for the values of coefficients and masses. For the moment, we consider only the contribution from φ field for simplicity and omit the suffix c. The right hand side of Eq. (37) are the noise terms, while the last two terms of the left hand side are combination of dissipation term and one-loop correction to the classical equation of motion which would reduce to a part of the derivative of the effective potential $V'_{\mathrm{eff}}(\phi)$ if we restricted $\phi(x')$ to be a constant in space and time.

The above equation (37) is an extension of equation (3.2) of Gleiser and Ramos [15] in that we have incorporated not only self-interaction but also interactions with a boson χ and a fermion ψ. It is nonlocal in space and time. The spatial nonlocality does not bring any difficulty here since the scalar field is presumably nearly homogeneous during inflation. So we only need to consider contributions with zero external momentum, that is, we can put $\phi_c(\boldsymbol{x}',t')=\phi_c(\boldsymbol{x},t')$ in the integrand.

With this approximation the correlation function of the bosonic noise (31) with $g=0$, for example, becomes

$$\langle\xi_1(x)\xi_1(x')\rangle\Rightarrow\frac{\lambda^2}{2}\int\frac{d^3k}{(2\pi)^3}e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}\int\frac{d^3q}{(2\pi)^3}\mathrm{Re}[G^F_\phi(\boldsymbol{q},t-t')G^F_\phi(\boldsymbol{q}-\boldsymbol{k},t-t')]\bigg|_{\boldsymbol{k}=\boldsymbol{0}}$$

$$=\frac{\lambda^2}{2}\delta^3(\boldsymbol{x}-\boldsymbol{x}')\int\frac{d^3q}{(2\pi)^3}\mathrm{Re}[G^F_\phi(\boldsymbol{q},t-t')]^2. \tag{39}$$

We thus obtain spatially uncorrelated noise. For the scalar field averaged over a volume V, the amplitude of noise reduces in proportion to $V^{-1/2}$, which implies that noise term should be omitted in the equation of motion of spatially averaged homogeneous field. On the other hand, the temporal nonlocality is very important in deriving the dissipation term as seen below.

D. Dissipation term

The equation of motion (37) derived above has contributions representing the dissipative effect in the last two terms of the left hand side. Since these terms are nonlocal in time, in order to extract local terms proportional to $\dot{\phi}$ one should assume that the field changes adiabatically [14,15], or put

$$\phi^n(\boldsymbol{x}',t')\simeq\phi^n(\boldsymbol{x}',t)+n(t'-t)\phi^{n-1}(\boldsymbol{x}',t)\dot{\phi}(\boldsymbol{x}',t) \tag{40}$$

in the integrand of Eq. (37). Then these terms will read

$$\begin{aligned}&\phi\int d^3x'\int_{-\infty}^{t}dt'A_1(x-x')\phi^2(x')+2\int d^3x'\int_{-\infty}^{t}dt'A_2(x-x')\phi(x')\\&=\phi^3(t)\int d^3x'\int_{-\infty}^{t}dt'A_1(x-x')+2\phi(t)\int d^3x'\int_{-\infty}^{t}dt'A_2(x-x')+2\phi^2(t)\dot{\phi}(t)\int d^3x'\int_{-\infty}^{t}dt'(t'-t)A_1(x-x')\\&\quad+2\dot{\phi}(t)\int d^3x'\int_{-\infty}^{t}dt'(t'-t)A_2(x-x').\end{aligned} \tag{41}$$

The last two terms are dissipation terms. They would vanish if we used bare propagators, as a manifestation of the fact that the dissipative effect is intrinsically a nonperturbative phenomenon and cannot be investigated from the perturbation theory [22,23]. In order to obtain a finite result we should use full ‘‘dressed’’ propagators instead [14,15,8].

The viscosity from scalar interactions have been fully investigated in Refs. [15] and [8], so we simply quote their results here. The full propagator of ϕ reads

$$\begin{aligned}G_\phi^F(\boldsymbol{k},t-t')&=\int\frac{d^3k}{(2\pi)^3}e^{i\boldsymbol{k}(x-x')}G_\phi^F(x-x')\\&=\frac{i}{2\omega_\phi}\{[1+n_B(\omega_\phi-i\Gamma_\phi)]e^{-i(\omega_\phi-i\Gamma_\phi)|t-t'|}\\&\quad+n_B(\omega_\phi+i\Gamma_\phi)e^{i(\omega_\phi+i\Gamma_\phi)|t-t'|}\},\end{aligned} \tag{42}$$

where $n_B(\omega_\phi)=(e^{\beta\omega_\phi}-1)^{-1}$ and $\omega_\phi=\sqrt{\boldsymbol{k}^2+m_{\phi T}^2}$ with $m_{\phi T}$ being the finite-temperature effective mass of ϕ. Here Γ_ϕ is the decay width related with the imaginary part of the self-energy Σ as

$$\Gamma_\phi=-\frac{\mathrm{Im}\,\Sigma_\phi}{2\omega_\phi}\simeq\frac{\lambda^2T^2}{1536\pi\omega_\phi}, \tag{43}$$

in the limit $T\gg m_{\phi T}$. We find the contribution of the self-interaction to the viscosity

$$\begin{aligned}&\lambda^2\phi^2(t)\dot{\phi}(t)\int d^3x'\int_{-\infty}^{t}dt'(t'-t)\mathrm{Im}[G_\phi^F(x-x')^2]\\&\simeq\frac{\lambda^2}{8}\phi^2\dot{\phi}\beta\int\frac{d^3k}{(2\pi)^3}\frac{n_B(1+n_B)}{\omega_\phi^2\Gamma_\phi}\\&\simeq\frac{96}{\pi T}\ln\left(\frac{T}{m_{\phi T}}\right)\phi^2\dot{\phi},\end{aligned} \tag{44}$$

in the high temperature limit and in the case $g=f=0$. Note that the above expression was first obtained by Hosoya and Sakagami [16] by a different method which is intuitively more appealing as discussed in Sec. II.

The contribution from the interaction with χ can be obtained similarly. Assuming χ has no self-interaction its width is given by

$$\Gamma_\chi\simeq\frac{g^4T}{192\pi} \tag{45}$$

in the high-temperature limit and the relevant part of the viscosity terms is given by

$$\begin{aligned}&4g^4\phi^2(t)\dot{\phi}(t)\int d^3x'\int_{-\infty}^{t}dt'(t'-t)\mathrm{Im}[G_\chi^F(x-x')^2]\\&\simeq\frac{48}{\pi T}\ln\left(\frac{T}{m_{\chi T}}\right)\phi^2\dot{\phi}.\end{aligned}$$

In both cases the viscosity terms due to scalar interactions are of the form $\phi^2\dot{\phi}/T$ [8].

On the other hand, the dissipation due to Yukawa interaction is calculated in the Appendix. In the high temperature limit $T\gg m_{\psi T},\ m_{\phi T}$ we find

$$\begin{aligned}&2f^2\dot{\phi}\int d^3x'\int_{-\infty}^{t}dt'(t'-t)\mathrm{Im}[S_{\alpha\beta}^F(x-x')S_F^{\beta\alpha}(x'-x)]\\&\simeq\frac{288}{\pi^3}\zeta(3)T\dot{\phi}\simeq 11T\dot{\phi}.\end{aligned} \tag{46}$$

In both cases the viscosity coefficients would be exponentially suppressed and would not play any important role if the high-temperature conditions were not satisfied.

IV. FEASIBILITY OF WARM INFLATION

In this section we study if the warm inflation driven by the viscosity term is possible in the case the viscosity term in the effective equation of motion is most effective, namely, in the high-temperature limit when the viscosity coefficient is given either by Eq. (44) or (47) depending on the interaction. We consider these cases separately for both new [2] and chaotic [4] inflation scenarios.

Before analyzing specific models we formulate generic conditions to satisfy. We are interested in the new possibility that the slow-rollover inflation is realized due to the thermal viscosity (44) or (47), so the effective equation of motion should read

$$C_v\dot{\phi}=-V'[\phi], \tag{47}$$

that is, we require $C_v \gg 3H$. The inflaton's energy released through this viscosity term presumably goes to radiation, whose energy density, ρ_r, satisfies

$$\frac{d\rho_r}{dt}=-4H\rho_r+C_v\dot{\phi}^2. \tag{48}$$

Since C_v strongly depends on the radiation temperature, ρ_r should not change too rapidly in time in order to sustain quasi-stationary inflation. So the creation term in Eq. (48) should balance the redshift term. As a result we find

$$\rho_r=\frac{\pi^2 g_*}{30}T^4\simeq\frac{C_v\dot{\phi}^2}{4H}, \tag{49}$$

which gives the radiation temperature as a function of ϕ. In contrast to Ref. [8], where the temperature has been fixed to its initial value, we perform a consistent analysis by using the value of T calculated from Eq. (49), which would help to increase the viscosity from bosonic interactions [8]. Here g_* is the total effective number of relativistic degrees of freedom. We normalize it by 150 and denote $g_*/150\equiv g_{*N}$ below. In order that the universe is inflating the potential energy density should dominate over ρ_r and the kinetic energy density.

Finally a number of conditions must be satisfied to justify the derivation of the effective equation of motion. In order that the radiation created from the inflaton thermalizes sufficiently rapidly, we need

$$\Gamma_\phi\simeq\frac{\lambda^2 T}{1536\pi}\gg H,\quad \Gamma_\chi\simeq\frac{g^4 T}{192\pi}\gg H,\quad \Gamma_\psi\simeq\frac{\pi}{64}\frac{f^2 m_{\psi T}^2}{T}\gg H, \tag{50}$$

depending on the nature of interaction. In addition, the adiabatic conditions

$$\Gamma_\phi,\Gamma_\chi,\Gamma_\psi\gg\left|\frac{\dot{\phi}}{\phi}\right|, \tag{51}$$

must be fulfilled so that the viscosity term is proportional to $\dot{\phi}$. The inequalities (50) and (51) are essential to realize quasistationary state of inflationary expansion. Finally, but most importantly, the high-temperature conditions $T\gg m_{\phi T}$, $m_{\chi T}$, $m_{\psi T}$ must also be satisfied, otherwise the viscosity term would be exponentially small and our entire discussion would break down.

Since the failure of case II in Sec. II is evident, we here consider the case I, where finite-temperature correction to the effective potential is sub-dominant but still the viscosity term appears sizable at first glance (until we convince ourselves it is not, through the intuitive argument in Sec. II.) Thus we can make the following list of the inequalities to be satisfied to realize the desired scenario.

(i) $C_v\gg 3H$,

(ii) $V[\phi]\gg\frac{1}{2}\dot{\phi}^2$,

(iii) $V[\phi]\gg\rho_r$,

(iv) $\Gamma_\phi\gg H$, $\Gamma_\chi\gg H$, or $\Gamma_\psi\gg H$,

(v) $\Gamma_\phi\gg|\dot{\phi}/\phi|$, $\Gamma_\chi\gg|\dot{\phi}/\phi|$, or $\Gamma_\psi\gg|\dot{\phi}/\phi|$,

(vi) $T\gg m_{\phi T}$,

(vii) $T\gg m_{\chi T}$ or $T\gg m_{\psi T}$,

(viii) Finite-temperature correction to the effective potential being subdominant.

Inequalities (v) are nothing but the condition (B) in Sec. II, and (vi) and (vii) stand for the condition (A). As will be seen below, we can essentially rule out all the models we consider only in terms of these two conditions.

In addition to the constraints listed above, there is a constraint following from the investigation of density perturbations produced during warm inflation. The standard expression for density perturbations produced during inflation in the cold-matter dominated universe is [5]

$$\frac{\delta\rho}{\rho}=\frac{6}{5}\frac{H\delta\phi}{\dot{\phi}}. \tag{52}$$

If one uses the standard estimate for inflationary perturbations $\delta\phi\sim H/2\pi$, one gets

$$\frac{\delta\rho}{\rho}=\frac{6}{5}\frac{H^2}{2\pi\dot{\phi}}. \tag{53}$$

The first of these two equations holds for the warm inflation as well. However, the second one should be modified because the amplitude of fluctuations during warm inflation is greater than $H/2\pi$ for two different reasons.

First of all, the wavelength of perturbations of a field with mass $m^2\ll H^2$ which freeze during inflation usually is $O(H^{-1})$ because of the friction term $3H\dot{\phi}$. This allows one to make the standard estimate $\delta\phi\sim H/2\pi$ by calculating the amplitude of vacuum fluctuations with the wavelength $O(H^{-1})$. However, during warm inflation the amplitude of perturbations of scalar fields with momenta $k\ll T$ is enhanced by the factor $\sim\sqrt{T/k}$ because of the additional contribution of thermal fluctuations. This leads to an estimate $\delta\phi\sim\sqrt{3HT/4\pi}$ [24].

But this is not the only effect which should be taken into account. Indeed, during warm inflation the friction term is $(C_v+3H)\dot{\phi}$, and it is assumed that $C_v \gg 3H$. As a result, it may happen that the fluctuations of the scalar field freeze not when their wavelength approaches H^{-1}, but much earlier, when their amplitude can be much greater. This may lead to an additional increase of the magnitude of perturbations of scalar fields produced during inflation.

We will not perform here a detailed evaluation of density perturbations in warm inflation because we did not find any model where this scenario can be realized. Indeed, we will be able to rule out warm inflation in all models which we will consider even without using the theory of density perturbations. However, one should keep in mind the necessity to study constraints based on the theory of density perturbations, because usually these constraints lead to the strongest restrictions on the structure of inflationary models.

A. Chaotic inflation with viscosity from bosonic interaction

Here we consider chaotic-type inflation with a potential $V[\phi]=(\lambda/4!)\phi^4$ and the viscosity arising from self-interaction or other bosonic interactions proportional to ϕ^2, such as $\frac{1}{2}g^2\phi^2\chi^2$. In the usual chaotic inflation slow-roll of the inflaton is realized due to the Hubble friction and it is effective only when $\phi \gtrsim 0.3M_{Pl}$, but if thermal viscosity discussed above is effective, we might have inflation with much smaller ϕ.

In this case the viscosity coefficient and the Hubble parameter are given, respectively, by

$$C_v=\frac{96c_\phi}{\pi T}\phi^2, \quad H=\frac{\sqrt{\pi\lambda}}{3M_{Pl}}\phi^2. \tag{54}$$

Here $c_\phi=\ln(T/m_{\phi T})$ if ϕ has self-interaction only. But c_ϕ can be increased if it interacts with additional scalar fields χ_i's through the interaction

$$\mathcal{L}_{\rm int}=-\sum_j \frac{1}{2}g_j^2\phi^2\chi_j^2. \tag{55}$$

Then C_v is given by

$$C_v=\frac{96}{\pi T}\left[\frac{\lambda^2}{\lambda^2+\Sigma_j g_j^4}\ln\left(\frac{T}{m_{\phi T}}\right)+\sum_j \frac{1}{2}\ln\left(\frac{T}{m_{\chi_j T}}\right)\right]\phi^2, \tag{56}$$

where $m_{\chi_j T} \ll T$ is the finite-temperature effective mass of χ_j field [8]. For simplicity, we identify $c_\phi(\gg 1)$ with the number of χ_j fields, which corresponds to the case $\ln(T/m_{\chi_j T})=2$. We also assume all the coupling constants g_j take the same value $g_j \equiv g$.

From Eqs. (47) and (49) the temperature is given by

$$T=1.6\times 10^{-2}g_{*N}^{-1/3}\,c_\phi^{-1/3}\lambda^{1/2}M_{\rm Pl}^{1/3}\phi^{2/3}. \tag{57}$$

We can then convert the inequalities (i)–(viii) to the constraints on the range of ϕ and on other model parameters as follows.

(i) $\phi \ll 3.4\times 10^4 g_{*N}^2 c_\phi^2 \lambda^{-3/2}M_{\rm Pl}$,
(ii) $\phi \gg 2.9\times 10^{-11} g_{*N}^{-1} c_\phi^{-4}\lambda^3 M_{\rm Pl}$,
(iii) $\phi \gg 1.3\times 10^{-3} g_{*N}^{-1/4} c_\phi^{-1}\lambda^{3/4}M_{\rm Pl}$,
(iva) $\phi \ll 1.2\times 10^{-4} g_{*N}^{-1/4} c_\phi^{-1/4}\lambda^{3/2}M_{\rm Pl}$,
(ivb) $\phi \ll 5.5\times 10^{-4} g_{*N}^{-1/4} c_\phi^{-1/4} g^3 M_{\rm Pl}$,
(va) $\lambda \gg 26 c_\phi^{-1}$,
(vb) $\lambda g^{-4} \ll 0.30 c_\phi$,
(vi) $\phi \ll 1.2\times 10^{-5} g_{*N}^{-1} c_\phi^{-1} M_{\rm Pl}$,
(vii) $\phi \ll 4.3\times 10^{-6} g_{*N}^{-1} c_\phi^{-1}\lambda^{3/2} g^{-3} M_{\rm Pl}$,
(viii) $\lambda \gtrsim 3.8\times 10^{-2} c_\phi g^4$.

The last condition is from the requirement that radiative correction due to χ does not change λ. The e-folding number of warm inflation, if any, is calculated as

$$N \equiv \int_{\phi_i}^{\phi_f} H\frac{d\phi}{\dot{\phi}}=5.1\times 10^3 g_{*N}^{1/3} c_\phi^{4/3}\lambda^{-1}M_{\rm Pl}^{-4/3}(\phi_i^{4/3}-\phi_f^{4/3}), \tag{58}$$

where ϕ_i and ϕ_f are upper and lower bounds of ϕ that satisfy all the above inequalities.

Using (vii) and (vb) in Eq. (58) we find

$$N \ll 3.6\times 10^{-4} g_{*N}^{-1}\lambda g^{-4} \ll 1.1\times 10^{-4} g_{*N}^{-1} c_\phi. \tag{59}$$

Since χ's also contribute to g_*, $g_{*N}^{-1}c_\phi$ cannot be larger than 150 and it is apparent that N cannot even exceed unity. Hence we cannot realize warm inflation in this model. This conclusion is independent of the simplification $\ln(T/m_{\chi T})=2$, for the above constraint on N is primarily due to the condition (vii) or $T \gg m_{\chi T}$, and the fact that this condition is hardly satisfied implies we have overestimated the duration of warm inflation.

Next for completeness we consider the potential $V[\phi]=\frac{1}{2}m^2\phi^2+(\lambda/4!)\phi^4$ with $m^2 \gg \lambda\phi^2/12$. In this case the temperature is given by

$$T=4.3\times 10^{-2} g_{*N}^{-1/3} c_\phi^{-1/3}\, m M_{\rm Pl}^{1/3}\phi^{-1/3}, \tag{60}$$

and the inequalities read as follows.

(i) $\phi \gg 2.8\times 10^{-2} g_{*N}^{-1/4} c_\phi^{-1} m^{3/4} M_{\rm Pl}^{1/4}$,
(ii) $\phi \gg 6.0\times 10^{-2} g_{*N}^{-1/7} c_\phi^{-4/7} m^{6/7} M_{\rm Pl}^{1/7}$,
(iii) $\phi \gg 9.1\times 10^{-2} g_{*N}^{-1/10} c_\phi^{-2/5} m^{3/5} M_{\rm Pl}^{2/5}$,
(iva) $\phi \ll 9.5\times 10^{-5} g_{*N}^{-1/4} c_\phi^{-1/4}\lambda^{3/2} M_{\rm Pl}$,
(ivb) $\phi \ll 4.5\times 10^{-4} g_{*N}^{-1/4} c_\phi^{-1/4} g^3 M_{\rm Pl}$,
(va) $\phi \gg 13 c_\phi^{-1/2}\lambda^{-1} m$,
(vb) $\phi \gg 4.4 c_\phi^{-1/2} g^{-2} m$,
(vi) $\phi \ll 8.0\times 10^{-5} g_{*N}^{-1} c_\phi^{-1} M_{\rm Pl}$,
(vii) $\phi \ll 9.4\times 10^{-2} g_{*N}^{-1/4} c_\phi^{-1/4} g^{-3/4} m^{3/4} M_{\rm Pl}^{1/4}$,
(viii) $\phi < \sqrt{12}\lambda^{-1/2} m$.

The number of e folds of inflation is formally given by

$$N=\int_{\phi_i}^{\phi_f} H\frac{d\phi}{\dot{\phi}}=4.4\times10^2 m^{-2}M_{\rm Pl}^{-4/3}\,(\phi_i^{10/3}-\phi_f^{10/3}). \tag{61}$$

From (vb) and (vii) we find $m\ll 2.1\times10^{-7}g_{*N}^{-1}c_\phi g^5 M_{\rm Pl}$. Then inequality (vii) reads $\phi\ll 9.2\times10^{-7}g_{*N}^{-1}c_\phi^{1/2}g^3M_{\rm Pl}$. Using these inequalities in Eq. (61) we find

$$N\ll 7.3\times10^{-5}g_{*N}^{-4/3}\,c_\phi^{-1/3}. \tag{62}$$

Thus no matter how many scalar fields are interacting with the inflaton we cannot find warm inflation solution. Note that the above constraint on the e-folding number primarily comes from the condition (vii) or $T\gg m_{\chi T}$. This means that if we had used the correct value of $\ln(T/m_{\chi T})$ instead of putting it to be 2, the viscosity term would have been smaller and warm inflation would have been even more unlikely. So we can justify our simplification. Therefore the conclusion that the number of e folds of warm inflation with chaotic-type potentials is constrained to be much smaller than unity [8] remains unchanged even when we use the consistent value of the cosmic temperature obtained from Eq. (49), rather than fixing it to its initial value [8].

Note that these considerations look different from the arguments used in the Sec. II, but they lead to the same final conclusion. Now let us see how one can reach the same conclusions directly, using the arguments of Sec. II. We will assume for simplicity that the field ϕ interacts with one field χ, and $\lambda\ll g^2$. In this theory Eq. (12) looks as follows:

$$\ddot{\phi}+C_v\dot{\phi}+3H\dot{\phi}+m^2\phi+\lambda\phi^3+g^2\phi\langle\chi^2\rangle_{\rm eq}=0. \tag{63}$$

Let us consider two limiting cases.

Case I. $g^2T^2\ll\max\{m^2,\lambda\phi^2\}$. In this case $m^2\phi+\lambda\phi^3\gg g^2\phi\langle\chi^2\rangle_{\rm eq}\gg C_v\dot{\phi}$, so the new viscosity term is completely irrelevant for the description of the evolution of the field ϕ, as is clear from the intuitive derivation of the equation of motion in Sec. II.

Case II. $g^2T^2\gg\max\{m^2,\lambda\phi^2\}$. If $g\phi\gg T$, then all thermal effects disappear. If $g\phi\ll T$, then $m^2\phi^2/2+\lambda\phi^4/4\ll g^2\phi^2\langle\chi^2\rangle_{\rm eq}\sim g^2\phi^2T^2\ll T^4$. In this case equation of state is determined by ultrarelativistic matter, $p\approx\rho/3$, and inflation cannot occur.

B. Chaotic inflation with viscosity from Yukawa interaction

We next study the case the dominant contribution of viscosity arises from Yukawa interaction, so that we find $C_v\simeq 11c_\psi T$ where $c_\psi=1$ in the case ϕ has a Yukawa coupling to one species of Dirac fermion, but it can be larger if ϕ interacts with more fermions. We identify c_ψ with the number of fermion species interacting with ϕ with the universal coupling strength f.

First we consider the case inflation is driven by a quartic potential $V[\phi]=(\lambda/4!)\,\phi^4$. The temperature is given by

$$T=0.12g_{*N}^{-1/5}\,c_\psi^{-1/5}\lambda^{3/10}\phi^{4/5}M_{\rm Pl}^{1/5}. \tag{64}$$

The following inequalities must be simultaneously satisfied.

(i) $\phi\ll 0.78g_{*N}^{-1/6}c_\psi^{2/3}\lambda^{-1/6}M_{\rm Pl}$,
(ii) $\phi\ll 62g_{*N}^{-1/5}c_\psi^4\lambda^{-1}M_{\rm Pl}$,
(iii) $\phi\gg 0.15g_{*N}^{1/4}c_\psi^{-1}\lambda^{1/4}M_{\rm Pl}$,
(iv) $\phi\ll 0.63g_{*N}^{1/4}c_\psi^{1/4}f^5\lambda^{-1}M_{\rm Pl}$,
(v) $f^4\gg 0.31c_\psi^{-1}\lambda$,
(vi) $\phi\ll 1.4\times10^{-4}g_{*N}^{-1}c_\psi^{-1}\lambda^{-1}M_{\rm Pl}$,
(vii) $\phi\ll 2.5\times10^{-5}g_{*N}^{-1}c_\psi^{-1}f^{-5}\lambda^{3/2}M_{\rm Pl}$,
(viii) $\phi\gg 1.6\times10^{-6}g_{*N}^{-1}c_\psi^{3/2}f^5\lambda^{-1}M_{\rm Pl}$.

The last inequality comes from the condition that the finite-temperature correction to the effective potential is small, that is, $c_\psi f^2T^2/6\ll\lambda\phi^2/2$.

The number of e folds of inflation is expressed by

$$N\equiv\int_{\phi_i}^{\phi_f}H\frac{d\phi}{\dot{\phi}}=5.8g_{*N}^{-1/5}\,c_\psi^{4/5}\lambda^{-1/5}M_{\rm Pl}^{-4/5}(\phi_i^{4/5}-\phi_f^{4/5}). \tag{65}$$

Using (v) in (vii) we find $\phi\ll 1.1\times10^{-4}g_{*N}^{-1}c_\psi^{1/4}\lambda^{1/4}M_{\rm Pl}$. Inserting this limit to ϕ_i in Eq. (65) we obtain an upper bound on N as $N\ll 3.9\times10^{-3}g_{*N}^{-1}c_\psi$. Since $g_{*N}^{-1}c_\psi$ cannot be larger than 300/7 we can conclude warm inflation is impossible here. In fact, we can explicitly show that there is no open parameter region that satisfy all the inequalities even in the case $g_{*N}^{-1}c_\psi$ is maximal.

Next we analyze inflation driven by the mass term $V[\phi]=\frac{1}{2}m^2\phi^2$, in which the temperature is given by

$$T=0.19g_{*N}^{-1/5}c_\psi^{-1/5}\,m^{3/5}M_{\rm Pl}^{1/5}\phi^{1/5}. \tag{66}$$

The following inequalities must be satisfied for successful warm inflation.

(i) $\phi\ll 0.26g_{*N}^{-1/4}c_\psi m^{-1/2}M_{\rm Pl}^{3/2}$,
(ii) $\phi\gg 0.025g_{*N}c_\psi^{-4}m^2M_{\rm Pl}^{-1}$,
(iii) $\phi\gg 0.17g_{*N}^{1/6}c_\psi^{-2/3}m^{1/3}M_{\rm Pl}^{2/3}$,
(iv) $\phi\gg 13g_{*N}^{-1/4}c_\psi^{-1/4}f^{-5}m^2M_{\rm Pl}^{-1}$,
(v) $\phi\gg 1.4c_\psi^{-1/2}f^{-2}m$,
(vi) $\phi\gg 4.2\times10^3g_{*N}c_\psi m^2M_{\rm Pl}^{-1}$,
(vii) $\phi\ll 0.13g_{*N}^{-1/4}c_\psi^{-1/4}f^{-5/4}m^{3/4}M_{\rm Pl}^{1/4}$,
(viii) $\phi<3.6\times10^5g_{*N}c_\psi^{-3/2}f^{-5}m^2M_{\rm Pl}^{-1}$.

The last inequality is from the requirement $m^2\gg(c_\psi/6)\,f^2T^2$.

It is not impossible to find the values of model parameters that satisfy all the above inequalities. This does not imply, however, that warm inflation is feasible in this model because the e-folding number of inflation,

$$N\equiv\int_{\phi_i}^{\phi_f}H\frac{d\phi}{\dot{\phi}}=3.6g_{*N}^{-1/5}\,c_\psi^{4/5}m^{-2/5}M_{\rm Pl}^{-4/5}(\phi_i^{6/5}-\phi_f^{6/5}), \tag{67}$$

turns out to be smaller than unity as seen below, where ϕ_i and ϕ_f are upper and lower bounds satisfying all the inequalities (i)–(viii) as before.

Inserting the upper bound (vii) to ϕ_i in Eq. (67), we find

$$N\ll 0.31g_{*N}^{1/2}c_\psi^{1/2}f^{-3/2}m^{1/2}M_{\rm Pl}^{-1/2}. \tag{68}$$

From the consistency between (v) and (vii) we find $m\ll 7.4\times10^{-5}g_{*N}^{-1}c_\psi f^3M_{\rm Pl}$, which, together with Eq. (68), implies $N \ll 2.7\times10^{-3} g_{*N}^{-1} c_\psi$. Since $g_{*N}^{-1} c_\psi$ cannot exceed 300/7 we can conclude warm inflation is impossible in this model, too.

C. New inflation

Next we consider new inflation driven by the potential

$$V[\phi]=\frac{\lambda}{4}\left(\phi^2-\frac{m^2}{\lambda}\right)^2. \tag{69}$$

Inflation is possible only for

$$\phi \ll \lambda^{-1/2} m. \tag{70}$$

First we analyze the case viscosity is dominated by bosonic interaction. The temperature is constant in this case:

$$T=4.8\times10^{-2} g_{*N}^{-1/3} c_\phi^{-1/3} \lambda^{1/6} m^{2/3} M_{\rm Pl}^{1/3}. \tag{71}$$

We find the following inequalities.

(i) $\phi \gg 8.3\times10^{-2} g_{*N}^{-1/6} c_\phi^{-2/3} \lambda^{-1/6} m^{4/3} M_{\rm Pl}^{-1/3}$,
(ii) $\phi \gg 2.2\times10^{-3} g_{*N}^{-1/3} c_\phi^{2/3} \lambda^{2/3} m^{2/3} M_{\rm Pl}^{1/3}$,
(iii) $m \gg 6.0\times10^{-3} g_{*N}^{-1/4} c_\phi^{-1} \lambda^{1/2} M_{\rm Pl}$,
(iva) $m \ll 1.3\times10^{-4} g_{*N}^{-1/4} c_\phi^{-1/4} \lambda^2 M_{\rm Pl}$,
(ivb) $m \ll 6.4\times10^{-4} c_\phi^{-1/4} g_{*N}^{-1/4} \lambda^{1/2} g^3 M_{\rm Pl}$,
(va) $\phi \gg 13 c_\phi^{-1/2} \lambda^{-1} m$,
(vb) $\phi \gg 4.4 c_\phi^{-1/2} g^{-2} m$,
(vi) $m \ll 1.1\times10^{-4} g_{*N}^{-1} c_\phi^{-1} \lambda^{1/2} M_{\rm Pl}$,
(vii) $\phi \ll 4.8\times10^{-2} g_{*N}^{-1/3} c_\phi^{-1/3} \lambda^{1/6} g^{-1} m^{2/3} M_{\rm Pl}^{1/3}$,
(viiia) $m \gg 1.4\times10^{-5} g_{*N}^{-1} c_\phi^{-1} \lambda^2 M_{\rm Pl}$,
(viiib) $m \gg 2.7\times10^{-6} g_{*N}^{-1} c_\phi^{1/2} g^3 M_{\rm Pl}$.

Here the conditions (viii) are required to ensure the symmetry remains broken and warrant the use of the zero-temperature potential, that is, (viiia) comes from the condition $m^2 \gg \lambda T^2/4$ and (viiib) from $m^2 \gg c_\phi g^2 T^2/12$.

The number of e folds of inflation is expressed as

$$N=4.6\times10^2 g_{*N}^{1/3} c_\phi^{4/3} \lambda^{-1/6} m^{-2/3} M_{\rm Pl}^{-4/3} (\phi_f^2-\phi_i^2), \tag{72}$$

where ϕ_i and ϕ_f are now the lower and upper bounds of ϕ which satisfy all the above inequalities, respectively. Inserting (vii) to Eq. (72) we find $N \ll 1.1 g_{*N}^{-1/3} c_\phi^{2/3} \lambda^{1/6} g^{-2} m^{2/3} M_{\rm Pl}^{-2/3}$. The consistency between (vb) and (vii) sets an upper bound on m as $m \ll 1.3\times10^{-6} g_{*N}^{-1} c_\phi^{-1} \lambda^{1/2} g^3 M_{\rm Pl}$. These two inequalities implies $N \ll 1.3\times10^{-4} g_{*N}^{-1} \lambda^{1/2}$; thus warm inflation is not feasible.

So far we have used only inequalities (v) and (vii) as promised, apart from the generic condition (70) for new inflation. If we use other inequalities in addition, we can completely close the allowed region of the parameter space as follows. The consistency between Eq. (70) and (ii) sets a lower bound on m as

$$m \gg 1.1\times10^{-8} g_{*N}^{-1} c_\phi^2 \lambda^{7/2} M_{\rm Pl}. \tag{73}$$

From (vi) and Eq. (73) we find $\lambda \ll 22 c_\phi^{-1}$. On the other hand, from Eq. (70) and (v) we find $\lambda \gg 1.7\times10^2 c_\phi^{-1}$. Thus there is no allowed region for λ to realize warm inflation consistently.

Next we move on to the case viscosity is dominated by Yukawa interaction, when the temperature is given by

$$T=0.20 g_{*N}^{-1/5} c_\psi^{-1/5} \lambda^{1/10} m^{2/5} \phi^{2/5} M_{\rm Pl}^{1/5}. \tag{74}$$

The following inequalities must be satisfied.

(i) $\phi \gg 5.5 g_{*N}^{1/2} c_\psi^{-2} \lambda^{-3/2} m^4 M_{\rm Pl}^{-3}$,
(ii) $\phi \ll 2.1 g_{*N}^{-1/3} c_\psi^{4/3} \lambda^{-2/3} m^{2/3} M_{\rm Pl}^{1/3}$,
(iii) $\phi \ll 2.1 g_{*N}^{-1/8} c_\psi^{1/2} \lambda^{-7/8} m^{3/2} M_{\rm Pl}^{-1/2}$,
(iv) $\phi \gg 3.0 g_{*N}^{-1/8} c_\psi^{-1/8} \lambda^{-1/4} f^{-5/2} m^{3/2} M_{\rm Pl}^{-1/2}$,
(v) $\phi \gg 1.4 c_\psi^{-1/2} f^{-2} m$,
(vi) $\phi \gg 56 g_{*N}^{1/2} c_\psi^{1/2} \lambda^{-1/4} m^{3/2} M_{\rm Pl}^{-1/2}$,
(vii) $\phi \ll 6.8\times10^{-2} g_{*N}^{-1/3} c_\psi^{-1/3} f^{-5/3} \lambda^{1/6} m^{2/3} M_{\rm Pl}^{1/3}$,
(viiia) $\phi \ll 3.2\times10^2 g_{*N}^{1/2} c_\psi^{1/2} \lambda^{-3/2} m^{3/2} M_{\rm Pl}^{-1/2}$,
(viiib) $\phi \ll 5.3\times10^2 g_{*N}^{1/2} c_\psi^{-3/4} f^{-5/2} \lambda^{-1/4} m^{3/2} M_{\rm Pl}^{-1/2}$.

(viiia) comes from the condition $m^2 \gg \lambda T^2/4$ and (viiib) from $m^2 \gg c_\psi f^2 T^2/6$.

In this case, contrary to the case the viscosity arises from bosonic interactions, there exists some allowed region in the parameter space, but the number of e folds of inflation,

$$N \equiv \int_{\phi_i}^{\phi_f} \frac{H}{\dot\phi} d\phi = 8.0 g_{*N}^{-1/5} c_\psi^{4/5} \lambda^{-2/5} m^{2/5} M_{\rm Pl}^{-4/5} (\phi_f^{2/5}-\phi_i^{2/5}), \tag{75}$$

turns out to be much smaller than unity as seen below. Here ϕ_i and ϕ_f are lower and upper bounds on ϕ obtained from the inequalities (i) through (viiib) and Eq. (70) as before.

From (vii) we find

$$N \ll 2.7 g_{*N}^{-1/3} c_\psi^{2/3} \lambda^{-1/3} f^{-2/3} m^{2/3} M_{\rm Pl}^{-2/3}. \tag{76}$$

Consistency between (v) and (vii) imposes an upper bound on m as $m \ll 1.2\times10^{-4} g_{*N}^{-1} c_\psi^{1/2} \lambda^{1/2} f M_{\rm Pl}$. Inserting it into Eq. (76) we find $N \ll 6.6\times10^{-3} g_{*N}^{-1} c_\psi$. Thus N is much smaller than unity no matter how many fermions are interacting with the inflaton.

D. Shifted field model

So far we have studied the possibilities of chaotic inflation and new inflation models driven by thermal viscosity term. As a result we have shown that none of the above models can accommodate inflationary expansion which lasts more than one e fold. In fact, we have been able to rule out them simply from the high-temperature condition (vii) and the adiabatic condition (v). To cure this problem another model has been suggested in Ref. [8] in which the scalar field χ has the interaction term

$$\mathcal{L}_{\rm int}=-\sum_j \frac{1}{2} g^2 (\phi-M)^2 \chi_j^2. \tag{77}$$

We are not aware of any particle physics motivation for this kind of interaction. Nevertheless it may be worthwhile to study this model because it helps to reduce the effective mass of χ when ϕ is large and close to M. Thus one could hope that one may relax a constraint from the high-temperature condition (vii) and find a sensible solution for warm inflation [8].

The situation, however, is not that simple, as one immediately recognizes once he writes down the effective equation of motion in this model. Indeed instead of Eq. (12), with $C=96c_\phi/\pi$, we find

$$\ddot{\phi}+\frac{96c_\phi}{\pi T}(\phi-M)^2\dot{\phi}+3H\dot{\phi}+m^2\phi+g^2(\phi-M)\sum_j \langle\chi_j^2\rangle_{\rm eq}=0, \tag{78}$$

Thus the viscosity term vanishes at $\phi=M$, which makes the dissipation inefficient in the region $\phi\approx M$.

Nevertheless one may still want to consider the possibility of warm inflation in the vicinity of $\phi=M$, where the high temperature condition, $m_\chi=g|\phi-M|\ll T$, is satisfied but the slight deviation from $\phi=M$ makes the viscosity nonvanishing. Below we consider this possibility in chaotic inflation driven by the mass term for illustration and work out inequalities to be satisfied as we did above.

In this model the effective potential of ϕ is given by

$$V_{\rm eff}[\phi]=\frac{1}{2}m^2\phi^2+\frac{c_\phi g^2}{24}T^2(\phi-M)^2+\cdots \tag{79}$$

in the high-temperature limit. We must treat the cases with $c_\phi g^2T^2\ll m^2$ and with $c_\phi g^2T^2\gg m^2$ separately.

1. The case $c_\phi g^2T^2\ll m^2$

First we consider the case $c_\phi g^2T^2\ll m^2$. Since we are interested in the regime $\phi>|\phi-M|$, the second term in Eq. (79) is entirely negligible in the inflaton's dynamics. The effective equation of motion in the slow-roll regime is given by

$$C_v\dot{\phi}=\frac{96c_\phi}{\pi T}(\phi-M)^2\dot{\phi}=-m^2\phi, \tag{80}$$

and the radiation temperature is calculated from Eq. (49) as

$$T=4.3\times10^{-2}c_\phi^{-1/3}g_{*N}^{-1/3}m\phi^{1/3}(\phi-M)^{-2/3}M_{\rm Pl}^{1/3}. \tag{81}$$

Apparently it is divergent at $\phi=M$. However, since $\phi=M$ is out of the slow-roll regime and will be automatically excluded using the inequalities below, this does not cause any problem.

Now we list the required inequalities relevant to χ.

(i) $|\phi-M|\gg0.17c_\phi^{-1/8}g_{*N}^{-1/8}m^{3/4}\phi^{1/2}M_{\rm Pl}^{-1/4}$,

(ii) $|\phi-M|\gg8.5\times10^{-2}c_\phi^{-1/2}g_{*N}^{-1/8}m^{3/4}\phi^{1/8}M_{\rm Pl}^{1/8}$,

(iii) $|\phi-M|\gg5.0\times10^{-2}g_{*N}^{-1/8}m^{3/4}\phi^{-1/4}M_{\rm Pl}^{1/2}$,

(iv) $|\phi-M|\ll2.1\times10^{-7}c_\phi^{-1/2}g_{*N}^{-1/2}g^6\phi^{-1}M_{\rm Pl}^2$,

(v) $|\phi-M|\gg2.5g^{-2}c_\phi^{-1/2}m$,

(vii) $|\phi-M|\ll0.15c_\phi^{-1/5}g_{*N}^{-1/5}m^{3/5}\phi^{1/5}M_{\rm Pl}^{1/5}$.

The number of e-folds of inflation is given by

$$N\equiv\int_{|\phi-M|_i}^{|\phi-M|_f}H\frac{d|\phi-M|}{\dot{\phi}}\simeq4\times10^2c_\phi^{1/3}g_{*N}^{1/3}m^{-2}M_{\rm Pl}^{-4/3}\phi^{-1/3}\times(|\phi-M|_i^{11/3}-|\phi-M|_f^{11/3}), \tag{82}$$

where the integration has been done near $|\phi-M|$ assuming $\phi\sim M$. Inserting (vii) to $|\phi-M|_i$ into Eq. (82) one finds

$$N\ll0.4c_\phi^{-2/5}g_{*N}^{-2/5}m^{1/5}\phi^{2/5}M_{\rm Pl}^{-3/5}. \tag{83}$$

From the consistency between (v) and (vii) it follows that

$$m\ll8.8\times10^{-4}c_\phi^{3/4}g_{*N}^{-1/2}g^5\phi^{1/2}M_{\rm Pl}^{1/2}. \tag{84}$$

Using it in Eq. (83) one finds that

$$N\ll9\times10^{-2}c_\phi^{-1/4}g_{*N}^{-1/2}g\phi^{1/2}M_{\rm Pl}^{-1/2}. \tag{85}$$

This means that unless $\phi\sim M\gg M_{\rm Pl}$ one cannot obtain sufficiently long period of warm inflation. But then it is not interesting because for $\phi\gg M_{\rm Pl}$ we can realize chaotic inflation without the help of thermal viscosity. This conclusion follows from (v) and (vii).

But if we consider other inequalities the situation becomes even worse. From the consistency between (i) and (iv) we find

$$m\phi^2\ll1.3\times10^{-8}c_\phi^{-1/2}g_{*N}^{-1/2}g^8M_{\rm Pl}^3. \tag{86}$$

Inserting it into Eq. (83) we obtain

$$N\ll0.01c_\phi^{-1/2}g_{*N}^{-1/2}g^{8/5}. \tag{87}$$

Thus warm inflation is ruled out for perturbatively meaningful values of the coupling constant.

2. The case $c_\phi g^2T^2\gg m^2$

Next we consider the opposite limit, $c_\phi g^2T^2\gg m^2$. If $g|\phi-M|\gg T$, then all thermal effects disappear. If $g|\phi-M|\ll T$, one may still neglect contribution of the second term in Eq. (79) to the total energy density of the universe, because otherwise the universe would be dominated by radiation and there will be no inflation. This does not mean, however, that this term does not affect the motion of ϕ. On the contrary, the potential force is dominated by its derivative. As a result the minimum of the potential is shifted from the origin to the vicinity of $\phi=M$:

$$\phi(t)=\frac{M}{1+12m^2/c_\phi g^2T^2}\cong M-\frac{12m^2M}{c_\phi g^2T^2}. \tag{88}$$

One may encounter two possible regimes by comparing the time scale of the friction, $\tau_f\equiv C_v^{-1}\propto T(\phi-M)^{-2}$, with that of ϕ's oscillation, $\tau_0\equiv\sqrt{12}/(c_\phi^{1/2}gT)$. If the deviation from $\phi=M$ is sufficiently large to warrant $\tau_f<\tau_0$, the field is in the slow roll-over regime toward the minimum (88) and warm inflation might be possible.[1] If $\tau_f>\tau_0$, on the other

[1]Since we are interested in the feasibility of viscosity-driven warm inflation we assume $C_v>3H$ and hence the only relevant time scale of friction is $\tau_f=C_v^{-1}$.

hand, ϕ sits in the minimum (88) which is time-dependent through the temperature. Let us consider these two regimes in turn.

First, if ϕ is rolling toward the minimum, we find the following effective equation of motion

$$\frac{96c_\phi}{\pi T}(\phi-M)^2\dot{\phi}=-m^2\phi-\frac{c_\phi}{12}g^2T^2(\phi-M). \quad (89)$$

The second term in the right-hand-side (RHS) dominates the potential force except in the close vicinity of $\phi=M$, and we neglect the first term in RHS. In the regime where the first term dominates over the second term we would come to the same conclusion as in Sec. IV D 1.

From Eq. (49) one can find the temperature

$$T=1.8\times10^6 g_{*N}c_\phi^{-1}m\phi M_{\rm Pl}^{-1}. \quad (90)$$

We write down the required inequalities relevant to χ.

(i) $|\phi-M|\gg6.0\times10^2 g_{*N}^{1/2}c_\phi^{-1}m\phi M_{\rm Pl}^{-1}$,
(ii) $|\phi-M|\gg1.6\times10^{16}g_{*N}^3c_\phi^{-3}g^2m^2\phi^2M_{\rm Pl}^{-3}$,
(iii) $\phi\ll3.1\times10^{-14}g_{*N}^{-5/2}c_\phi^2m^{-1}M_{\rm Pl}^2$,
(iv) $g^4\gg3.3\times10^{-4}c_\phi g_{*N}^{-1}$,
(v) $|\phi-M|\gg5.3\times10^{12}g_{*N}^2c_\phi^{-2}m^2\phi M_{\rm Pl}^{-2}$,
(vii) $|\phi-M|\ll1.8\times10^6 g_{*N}c_\phi^{-1}g^{-1}m\phi M_{\rm Pl}^{-1}$.

The number of e folds of inflation is calculated as

$$N\equiv\int_{|\phi-M|_i}^{|\phi-M|_f}H\frac{d|\phi-M|}{\dot{\phi}}=6.4\times10^{-17}g_{*N}^{-3}c_\phi^3m^{-2}\phi^{-2}\times M_{\rm Pl}^2(|\phi-M|_i^2-|\phi-M|_f^2), \quad (91)$$

where the integration has been done over $|\phi-M|$ assuming $\phi\sim M$. Inserting (vii) into Eq. (91) we find $N\ll 2.1\times10^{-4}g_{*N}^{-1}c_\phi$. Hence this regime does not lead to inflation.

Next we suppose that ϕ has fallen to the minimum (88). Since the temperature is presumably gradually decreasing during the warm inflation, ϕ changes with time according to

$$\dot{\phi}\cong\frac{24m^2M}{c_\phi g^2T^3}\dot{T}. \quad (92)$$

We would like to see whether this time-variation of ϕ may create a sufficient amount of radiation by the energy release through the viscosity term to support quasistationary stage of warm inflation.

If such a stage exists at all, we find from Eqs. (49) and (92)

$$\dot{T}=-1.3\times10^{-2}g_{*N}^{1/2}c_\phi^{3/2}g^4m^{-7/2}M^{-3/2}M_{\rm Pl}^{-1/2}T^{15/2}. \quad (93)$$

In this case it is convenient to express the required conditions in terms of the inequalities on the temperature.

(i) $T\ll3.7c_\phi^{-1/5}g^{-4/5}m^{3/5}M^{1/5}M_{\rm Pl}^{1/5}$,
(ii) $T\ll1.3g_{*N}^{1/9}c_\phi^{1/9}g^{4/9}m^{5/9}M^{1/3}M_{\rm Pl}^{1/9}$,
(iii) $T\ll0.32g_{*N}^{-1/2}m^{1/2}M^{1/2}$,
(iv) $T\gg1.2\times10^3g^{-4}mMM_{\rm Pl}^{-1}$,
(v) $T\ll0.22g_{*N}^{-1/7}c_\phi^{-1/7}g^{4/7}m^{3/7}M^{3/7}M_{\rm Pl}^{1/7}$,
(vii) $T\gg2.3c_\phi^{-1/3}g^{-1/3}m^{2/3}M^{1/3}$.

In addition, consistency of the assumption that ϕ stays in the time-dependent minimum (88) requires $\tau_f>\tau_0$, namely,

$$T>5.0c_\phi^{-1/4}g^{-5/6}m^{2/3}M^{1/3}. \quad (94)$$

The number of e folds in this regime is expressed by an integral over the temperature as

$$N\equiv\int_{T_i}^{T_f}H\frac{dT}{\dot{T}}=25g_{*N}^{-1/2}c_\phi^{-3/2}g^{-4}m^{9/2}M^{5/2}M_{\rm Pl}^{-1/2}\times(T_f^{-13/2}-T_i^{-13/2}), \quad (95)$$

where T_i and T_f are the upper and the lower bounds on the temperature satisfying all the above inequalities, and we have set $\phi\sim M$. Using Eqs. (94) and (95) we obtain $N\ll7.2\times10^{-4}g_{*N}^{-1/2}c_\phi^{1/8}g^{17/12}m^{1/6}M^{1/3}M_{\rm Pl}^{-1/2}$. Now the consistency between (i) and (iv) imposes an upper bound on mM^2 as $mM^2\ll5.3\times10^{-7}c_\phi^{-1/2}g^8M_{\rm Pl}^3$. From these two inequalities we find $N\ll6.5\times10^{-5}g_{*N}^{-1/2}c_\phi^{1/24}g^{33/12}$, which is much smaller than unity. Thus we do not see any possibility to implement the idea of warm inflation even in this model.

V. CONCLUSION

In the present paper we have examined feasibility of the warm inflation scenario [7] from various view points. First we discussed how the viscosity term arises in the equation of motion of a scalar field in a thermal bath following Hosoya and Sakagami [16]. Indeed such a term as $C_v\dot{\phi}$ appears because it takes finite time for number density of particles interacting with ϕ to relax to its thermally equilibrium value while ϕ is in motion.

The viscosity term thus obtained could be very large at first glance, because it is not suppressed by any small coupling constants. As is evident in its derivation, however, this term appears as a result of a small correction to a subleading thermal correction term in the equation of motion. Since we know that even the leading thermal correction term does not lead to inflation, such a subleading viscosity term is not expected to play an important role and to yield an inflationary regime of a new type.

If one neglects this fundamental feature and solves the overdamped equation of motion including the term $C_v\dot{\phi}$, one may find solutions indicating a possible emergence of warm inflation. However, we have found that such solutions violate the adiabatic condition that the scalar field should not change significantly in the relaxation time of the particles interacting with it. This condition is necessary for the derivation of the viscosity term $C_v\dot{\phi}$. Thus, the equation of motion incorporating the viscosity term $C_v\dot{\phi}$ fails in the regime where it could describe warm inflation.

If, on the other hand, we attempt to realize warm inflation in a manner fully consistent with the field theoretic derivation of the equation of motion, we inevitably find that the number of e folds of inflation is constrained to be much smaller than unity, mainly due to the difficulty to satisfy the high-temperature condition and the adiabatic condition simultaneously. Even the shifted field model, which has been proposed to relax the high-temperature condition [8], turned out to be no exception. Thus, our results as a whole show that it is extremely difficult to realize the idea of warm inflation in realistic models of elementary particles.

Note added. After we submitted this paper for publication, we learned that the authors of Ref. [25] proposed the first possible implementation of the warm inflation scenario. They did not study the issue of density perturbations in their model and investigated only the possibility to achieve 60 e folds of inflation. They assumed that the field ϕ interacts with scalar fields χ_{jk} as follows: $\Sigma_j\Sigma_k(g_{jk}^2/2)(\phi - M_j)^2\chi_{jk}^2$. As we have shown in Sec. IV D, the number of e folds of inflation in this type of model with one shift parameter M is much smaller than unity. In order to have 60 e folds of inflation, the authors of Ref. [25] were forced to introduce 10^4 different fields χ_{jk} with 10^3 specifically adjusted parameters M_j. We believe that this can serve as a good illustration to our conclusion.

ACKNOWLEDGMENTS

The authors are thankful to A. Berera for the discussion of warm inflation. J.Y. is grateful to the Institute of Theoretical Physics at Stanford University for the hospitality during the time when this work was done, and to the Monbusho for financial support. The work by A.L. was supported in part by NSF Grant No. PHY-9870115.

APPENDIX

In this appendix we derive the viscosity coefficient arising from Yukawa interaction. First we calculate the fermion self-energy Σ_ψ, which is expressed as

$$\Sigma_\psi(\boldsymbol{p},\tau_2-\tau_1)=f^2\int\frac{d^3k}{(2\pi)^3}S(\tau_2-\tau_1,\boldsymbol{k})G(\tau_2-\tau_1,\boldsymbol{p}-\boldsymbol{k}), \tag{A1}$$

in terms of temperature Green function of ψ, $S(\tau,\boldsymbol{k})$ and that of ϕ, $G(\tau,\boldsymbol{q})$. We use the following spectral representation of Green functions:

$$S(\tau,\boldsymbol{k})=\int\frac{d\omega}{2\pi}\sigma(\omega,\boldsymbol{k})e^{-\omega\tau}\times\{[1-n_F(\omega)]\theta(\tau)-n_F(\omega)\theta(-\tau)\}, \tag{A2}$$

$$\sigma(\omega,\boldsymbol{k})=i\left[\frac{\omega\gamma_0-\boldsymbol{k\gamma}+m_\psi}{(\omega+i\epsilon)^2-\boldsymbol{k}^2-m_\psi^2}-\frac{\omega\gamma_0-\boldsymbol{k\gamma}+m_\psi}{(\omega-i\epsilon)^2-\boldsymbol{k}^2-m_\psi^2}\right], \tag{A3}$$

$$G(\tau,\boldsymbol{q})=\int\frac{d\omega}{2\pi}\rho(\omega,\boldsymbol{q})e^{-\omega\tau}\times\{-[1+n_B(\omega)]\theta(\tau)-n_B(\omega)\theta(-\tau)\}, \tag{A4}$$

$$\rho(\omega,\boldsymbol{q})=i\left[\frac{1}{(\omega+i\epsilon)^2-\boldsymbol{q}^2-m_\phi^2}-\frac{1}{(\omega-i\epsilon)^2-\boldsymbol{q}^2-m_\phi^2}\right]. \tag{A5}$$

Inserting them to Eq. (A1) and applying the Fourier transform we find the self-energy in Matsubara representation

$$\begin{aligned}\Sigma_\psi(i\omega_l,\boldsymbol{p})&=\int_0^\beta d\tau e^{i\omega_l\tau}\Sigma_\psi(\tau,\boldsymbol{p})\\&=-f^2\int\frac{d^3k}{(2\pi)^3}\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\\&\quad\times\sigma(\omega_1,\boldsymbol{k})\rho(\omega_2,\boldsymbol{p}-\boldsymbol{k})\frac{1+n_B(\omega_2)-n_F(\omega_1)}{i\omega_l-(\omega_1+\omega_2)},\end{aligned} \tag{A6}$$

with $\omega_l=(2l+1)\pi T$. The above expression can be analytically continued to the retarded self-energy $\Sigma_\psi(p_0+i\epsilon,\boldsymbol{p})$, and the imaginary part of the self-energy is given by the discontinuity

$$\mathrm{Im}\,\Sigma_\psi(p)=\frac{1}{2i}[\Sigma_\psi(p_0+i\epsilon,\boldsymbol{p})-\Sigma_\psi(p_0-i\epsilon,\boldsymbol{p})]. \tag{A7}$$

Explicitly, we find

$$\begin{aligned}\mathrm{Im}\,\Sigma_\psi(p)&=\pi f^2\int\frac{d^3k}{(2\pi)^3}\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\sigma(\omega_1,\boldsymbol{k})\rho(\omega_2,\boldsymbol{p}-\boldsymbol{k})\\&\quad\times[1+n_B(\omega_2)-n_F(\omega_1)]\delta(p_0-\omega_1-\omega_2)\\&=\pi f^2\int\frac{d^3k}{(2\pi)^3}\int d\omega_1 d\omega_2(\omega_1\gamma_0-\boldsymbol{k\gamma}+m_\psi)\\&\quad\times[1+n_B(\omega_2)-n_F(\omega_1)]\mathrm{sgn}\,\omega_1\\&\quad\times\delta(\omega_1^2-\boldsymbol{k}^2-m_\psi^2)\mathrm{sgn}\,\omega_2\,\delta[\omega_2^2-(\boldsymbol{p}-\boldsymbol{k})^2-m_\phi^2].\end{aligned} \tag{A8}$$

Putting $p=(p_0,0,0,p_3)$, $|\boldsymbol{k}|\equiv k_m$, and $|\boldsymbol{p}|\equiv p_m(=p_3)$, and in the limit m_ψ and m_ϕ are negligible compared with p_m, we find

$$\mathrm{Im}\,\Sigma_\psi(p)\simeq\frac{\pi}{64}\frac{f^2T^2}{p_m}(\gamma_0-\gamma_3)=\frac{\pi}{64}\frac{f^2T^2}{p_0^2}\not{p}\equiv\hat{\Gamma}\not{p}, \tag{A9}$$

for $p_m=p_0\lesssim T$.

Then the dressed retarded Green function reads

$$S^R(p_0,\boldsymbol{p})=\frac{1}{(1-i\hat{\Gamma})((p_0+i\epsilon)\gamma^0-\boldsymbol{p\gamma})-m_{\psi T}}, \tag{A10}$$

in momentum representation, where $m_{\psi T} \equiv m_\psi + \mathrm{Re}\,\Sigma_\psi$ is the finite-temperature effective mass. The dressed spectral function is therefore given by

$$\sigma(p) = i(S^R(p) - S^{R\dagger}(p)) = i\left[\frac{(1-i\hat{\Gamma})\not{p} + m_{\psi T}}{(1-i\hat{\Gamma})^2[(p_0+i\epsilon)^2 - \boldsymbol{p}^2] - m_{\psi T}^2} - \frac{(1+i\hat{\Gamma})\not{p} + m_{\psi T}}{(1+i\hat{\Gamma})^2[(p_0-i\epsilon)^2 - \boldsymbol{p}^2] - m_{\psi T}^2}\right]$$
$$= i\left[\frac{(1-i\hat{\Gamma})\not{p} + m_{\psi T}}{2\omega_p(1-i\hat{\Gamma})^2}\left(\frac{1}{p_0-\omega_p} - \frac{1}{p_0+\omega_p}\right) - \frac{(1+i\hat{\Gamma})\not{p} + m_{\psi T}}{2\omega_p^*(1+i\hat{\Gamma})^2}\left(\frac{1}{p_0-\omega_p^*} - \frac{1}{p_0+\omega_p^*}\right)\right], \tag{A11}$$

with $\omega_p \equiv p_m + i m_{\psi T}^2 \hat{\Gamma}/p_m$. The final expression applies in the limit $|\boldsymbol{p}| \equiv p_m \gg m_{\psi T}$.

In terms of this spectral function the real-time finite-temperature Green function reads in momentum representation

$$S^F(t,\boldsymbol{p}) = -i\int \frac{dp_0}{2\pi} e^{-ip_0 t}\{[1-n_F(p_0)]\theta(t) - n_F(p_0)\theta(-t)\}\sigma(p)$$
$$= i\left[\frac{(1-i\hat{\Gamma})(-\omega_p - \boldsymbol{p\gamma}) + m_{\psi T}}{2\omega_p(1-i\hat{\Gamma})^2} n_F(\omega_p) e^{i\omega_p t} + \frac{(1+i\hat{\Gamma})(\omega_p^* - \boldsymbol{p\gamma}) + m_{\psi T}}{2\omega_p(1+i\hat{\Gamma})^2}[1-n_F(\omega_p^*)]e^{-i\omega_p^* t}\right]\theta(t)$$
$$- i\left[\frac{(1-i\hat{\Gamma})(\omega_p - \boldsymbol{p\gamma}) + m_{\psi T}}{2\omega_p(1-i\hat{\Gamma})^2} n_F(\omega_p) e^{-i\omega_p t} + \frac{(1+i\hat{\Gamma})(-\omega_p^* - \boldsymbol{p\gamma}) + m_{\psi T}}{2\omega_p^*(1+i\hat{\Gamma})^2}[1-n_F(\omega_p^*)]e^{i\omega_p^* t}\right]\theta(-t). \tag{A12}$$

Therefore the reading term to the viscosity due to Yukawa coupling is given by

$$2f^2\dot{\phi}\int_{-\infty}^{t} dt'(t'-t)\int \frac{d^3p}{(2\pi)^2}\mathrm{Im}\,S^F_{\mu\nu}(t-t',\boldsymbol{p})S^F_{\nu\mu}(t'-t,\boldsymbol{p}) \simeq 2f^2\dot{\phi}\int \frac{d^3p}{(2\pi)^2}\int_{-\infty}^{t} dt'(t'-t)e^{-2\Gamma(t'-t)}\frac{4m_{\psi T}^2}{p_m^2}\beta\Gamma\frac{e^{\beta p_m}}{(e^{\beta p_m}+1)^2}$$
$$= \frac{64}{\pi^3}\dot{\phi}\int_0^\infty dp_m p_m^3 \frac{e^{\beta p_m}}{(e^{\beta p_m}+1)^2 T^3} = \frac{288}{\pi^3}\zeta(3)T\dot{\phi} \simeq 11.2T\dot{\phi}, \tag{A13}$$

where

$$\Gamma_\psi \equiv \frac{m_{\psi T}^2}{p_m}\hat{\Gamma} = \frac{\pi f^2 T^2 m_{\psi T}^2}{64 p_m^3}. \tag{A14}$$

[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981); K. Sato, Mon. Not. R. Astron. Soc. **195**, 467 (1981).

[2] A. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[3] D. A. Kirzhnits, JETP Lett. **15**, 529 (1972); D. A. Kirzhnits and A. D. Linde, Phys. Lett. **42B**, 471 (1972); Sov. Phys. JETP **40**, 628 (1974); Ann. Phys. (N.Y.) **101**, 195 (1976); S. Weinberg, Phys. Rev. D **9**, 3320 (1974); L. Dolan and R. Jackiw, *ibid.* **9**, 3357 (1974).

[4] A. D. Linde, Phys. Lett. **129B**, 177 (1983).

[5] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).

[6] A. D. Linde, Phys. Lett. **116B**, 335 (1982); **116B**, 340 (1982).

[7] A. Berera, Phys. Rev. Lett. **75**, 3218 (1995); Phys. Rev. D **54**, 2519 (1996); **55**, 3346 (1997); H. P. de Oliveira and R. O. Ramos, *ibid.* **57**, 741 (1998).

[8] A. Berera, M. Gleiser, and R. O. Ramos, Phys. Rev. D **58**, 123508 (1998).

[9] A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982).

[10] M. Morikawa and M. Sasaki, Prog. Theor. Phys. **72**, 782 (1984).

[11] J. Yokoyama and K. Maeda, Phys. Lett. B **207**, 31 (1988).

[12] A. D. Dolgov and A. D. Linde, Phys. Lett. **116B**, 329 (1982); L. F. Abbott, E. Farhi, and M. B. Wise, *ibid.* **117B**, 29 (1982).

[13] L. Kofman, A. Linde, and A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994).

[14] M. Morikawa, Phys. Rev. D **33**, 3607 (1986).

[15] M. Gleiser and R. O. Ramos, Phys. Rev. D **50**, 2441 (1994).

[16] A. Hosoya and M. Sakagami, Phys. Rev. D **29**, 2228 (1984).

[17] L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. D **56**, 3258 (1997).

[18] J. Schwinger, J. Math. Phys. **2**, 407 (1961).
[19] P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. **4**, 1 (1963); **4**, 12 (1963).
[20] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964).
[21] M. Yamaguchi and J. Yokoyama, Phys. Rev. D **56**, 4544 (1997).
[22] D. Boyanovsky, H. J. de Vega, R. Holman, D.-S. Lee, and A. Singh, Phys. Rev. D **51**, 4419 (1995).
[23] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974); S. Jeon, *ibid.* **47**, 4586 (1993); N. P. Landsman and Ch. G. van Weert, Phys. Rep. **145**, 141 (1987).
[24] A. Berera and L. Z. Fang, Phys. Rev. Lett. **74**, 1912 (1995); A. Berera, hep-ph/9807523.
[25] A. Berera, M. Gleiser, and R. O. Ramos, Phys. Rev. Lett. **83**, 264 (1999).