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# Quintessence, the gravitational constant, and gravity

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Dynamical vacuum energy or quintessence, a slowly varying and spatially inhomogeneous component of the energy density with negative pressure, is currently consistent with observational data. One potential difficulty with the idea of quintessence is that couplings to ordinary matter should be strongly suppressed so as not to lead to observable time variations of the constants of nature. We further explore the possibility of an explicit coupling between the quintessence field and the curvature. Since such a scalar field gives rise to another gravity force of long range ($\gtrsim H_0^{-1}$), the solar system experiments put a constraint on the nonminimal coupling: $|\xi| \lesssim 10^{-2}$. [S0556-2821(99)04318-0]

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## I. INTRODUCTION

Recently a number of observations suggest that the Universe is dominated by an energy component with an effective negative pressure [1]. One possibility for such a component is the cosmological constant. Another possibility is dynamical vacuum energy, or quintessence, a slowly varying and spatially inhomogeneous component with negative pressure [2–7].

We face two problems when we consider such a nonzero vacuum energy. The first is the fine-tuning problem related to the energy scale of the vacuum energy density $\sim 10^{-47}$ GeV. The second is the coincident problem: why the vacuum energy is beginning to dominate presently. While these two problems are separated in quintessence, they are degenerate for the cosmological constant, and one has to introduce the cosmological constant of extremely small energy scale at the very beginning of the universe.

As a solution of the coincidence problem, the notion of a tracker field is introduced in [8]. It is shown that a very wide range of initial conditions approach a common evolutionary track, so that the cosmology is insensitive to the initial conditions similar to inflation. Once one parameter relating to the energy scale of the vacuum energy is fixed, the present-day equation of state $w_Q = p_Q/\rho_Q$ is automatically determined: there is a $\Omega_Q - w_Q$ relation [8].

Direct methods to verify the idea of quintessence are important. Proposed possibilities are the following: the direct reconstruction of the effective potential from the luminosity distance–redshift relation observed for type Ia supernovae [9]; the detection of quintessence from the measurements of a rotation in the plane of polarization of radiation from distant radio sources [10]. The direct interaction of the quintessence field to ordinary matter, however, is found to be strongly suppressed so as not to violate the equivalence principle and the constancy of the constants of nature [10].

The possibility of an explicit coupling between the scalar field and the curvature is not excluded theoretically. It is then natural to consider further the coupling of the quintessence field to the gravity itself. In this paper, we examine the cosmological consequence of the nonminimal coupling of the quintessence field to the gravity. Since such a scalar field gives rise to both the time variation of the gravitational constant and a gravity force of long range, such a coupling should be constrained by experiments.

## II. NONMINIMALLY COUPLED QUINTESSENCE

The action we consider is

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\xi\phi^2 R - \frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi)\right] + S_m, \qquad (2.1)$$

where $\kappa^2 \equiv 8\pi G_{bare}$ is the bare gravitational constant and $S_m$ denotes the action of matter. The effective gravitational "constant" is defined by $\kappa_{eff}^2 \equiv \kappa^2(1-\xi\kappa^2\phi^2)^{-1}$. $\xi$ is the nonminimal coupling between the scalar field and the curvature. In our conventions, $\xi = 1/6$ corresponds to the conformal coupling.

We assume that the universe is described by the flat homogeneous and isotropic universe model with the scale factor $a$. The time coordinate is so normalized that $a=1$ at the present. The field equations are then

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{\kappa^2}{3}[1-\xi\kappa^2\phi^2]^{-1} \times \left(\rho_B + \frac{1}{2}\dot\phi^2 + V(\phi) + 6\xi H\phi\dot\phi\right), \qquad (2.2)$$

$$\dot H = -\frac{\kappa^2}{2}[1-\xi\kappa^2\phi^2]^{-1}[\rho_B + p_B + \dot\phi^2 + 2\xi(H\phi\dot\phi - \dot\phi^2 - \phi\ddot\phi)], \qquad (2.3)$$

$$\ddot\phi + 3H\dot\phi + 6\xi(\dot H + 2H^2)\phi + V' = 0, \qquad (2.4)$$

$$\dot\rho_B + 3H(\rho_B + p_B) = 0, \qquad (2.5)$$

where $\rho_B, p_B$ denotes the background energy density, pressure, respectively, and $V' = dV/d\phi$.

 

We consider a potential of inverse power as an example of the tracker field for $\xi=0$ [2,8]:

$$V(\phi)=M^4(\phi/M)^{-\alpha}. \tag{2.6}$$

For $\xi=0$, there exists the following scaling solution during the background dominated epoch

$$H/H_0=a^{-3(1+w_B)/2}, \tag{2.7}$$

$$\phi/\phi_0=a^{3(1+w_B)/(\alpha+2)}, \tag{2.8}$$

$$\phi_0=\left(\frac{2\alpha(\alpha+2)^2M^{\alpha+4}}{9H_0^2(1+w_B)[4+(1-w_B)\alpha]}\right)^{1/(\alpha+2)}. \tag{2.9}$$

The equation of state $w_Q$ is

$$w_Q=\frac{\alpha w_B-2}{\alpha+2}. \tag{2.10}$$

Since we consider a potential whose present mass scale is extremely small ($\lesssim H_0\sim 10^{-33}$ eV), the force mediated by the scalar field is of long range, and hence the usual solar system limit on $\xi$, likewise the Brans-Dicke parameter, does apply. The correspondence to the Brans-Dicke field $\Phi_{BD}$ and the coupling function $\omega(\Phi_{BD})$ of scalar-tensor theories of gravity [11] is given by

$$\Phi_{BD}=8\pi(1-\xi\kappa^2\phi^2)/\kappa^2, \tag{2.11}$$

$$\omega(\Phi_{BD})=\frac{1-\xi\kappa^2\phi^2}{4\xi^2\kappa^2\phi^2}=\frac{\kappa^2\Phi_{BD}}{4\xi(8\pi-\kappa^2\Phi_{BD})}. \tag{2.12}$$

### A. Perturbative analysis

To consider the effect of $\xi$ qualitatively, we consider the case of $|\xi|\kappa^2\phi^2\ll 1$. Then during the background dominated epoch, Eq. (2.2) and Eq. (2.4) are approximated to

$$H^2=\frac{\kappa^2}{3}\rho_B, \tag{2.13}$$

$$\ddot\phi+3H\dot\phi+\xi\kappa^2(1-3w_B)\rho_B\phi+V'=0, \tag{2.14}$$

where we have used Eq. (2.3) to derive Eq. (2.14). It is recently established that the scaling solutions Eqs. (2.7), (2.8) with the same power-index persist even if $\xi\neq 0$ and that the stability of them does not depend on $\xi$ [12].

To the lowest order in $\xi$, the corresponding Brans-Dicke parameter is given by

$$\omega_0=\frac{1-\xi\kappa^2\phi_0^2}{4\xi^2\kappa^2\phi_0^2}\simeq\frac{3}{4\alpha(\alpha+2)}\frac{1}{\xi^2}, \tag{2.15}$$

where we have used the relation that holds for the potential of inverse power [8] to estimate the present-day value of the scalar field:

$$V''=\alpha(\alpha+1)\frac{V}{\phi^2}=\frac{9}{2}(1-w_Q^2)\frac{\alpha+1}{\alpha}H^2. \tag{2.16}$$

Up to $\mathcal{O}(\xi)$, the time variation of the gravitational constant is given by

$$\left.\frac{\dot G}{G}\right|_0=\left.\frac{2\xi\kappa^2\phi\dot\phi}{1-\xi\kappa^2\phi^2}\right|_0\simeq 2\xi\alpha H_0. \tag{2.17}$$

Hence, for $\xi>0$ the gravitational ''constant'' increases with time, while it decreases for $\xi<0$. Equation (2.17) also shows that $|\dot G/G|$ is larger for larger $\alpha$ since the potential then becomes steeper and the scalar field rolls down the potential more rapidly.

### B. Constraining $\xi$

We perform the numerical calculation to examine in detail the time variation of $G$ and the deviation from general relativity induced by the nonminimal coupling of the quintessence field to the curvature. The initial condition is set at $a=10^{-14}$. We vary the fraction of the energy density of the quintessence field relative to radiation from $10^{-9}$ to $10^{-30}$. We also choose various initial $\phi$ and $\dot\phi$. We confirmed the tracking behavior: convergence to a common evolutionary track [8,12]. Below we show typical results for the potential Eq. (2.6) with $\alpha=4$. We choose the following parameters: $\Omega_M\equiv\kappa_{eff}^2\rho_M/3H^2|_0=0.3$ and $H_0=100h$ km/sec/Mpc with $h=0.6$.

There exist a lot of experimental limits on the time variation of $G$ [13]. Radar ranging data to the Viking landers on Mars gives $|\dot G/G|=(2\pm 4)\times 10^{-12}$ yr$^{-1}$ [14]. Lunar laser ranging experiments yield $|\dot G/G|=(0\pm 11)\times 10^{-12}$ yr$^{-1}$ [15] and recently updated as $|\dot G/G|=(1\pm 8)\times 10^{-12}$ yr$^{-1}$ [16]. More recently, a tighter bound is found by analyzing the measurements of the masses of young and old neutron stars in binary pulsars: $|\dot G/G|=(0.6\pm 2.0)\times 10^{-12}$ yr$^{-1}$ [17], although the uncertainties in the age estimation may weaken the constraint. Considering these experimental results, we will adopt the limit: $|\dot G/G|=(0\pm 8)\times 10^{-12}$ yr$^{-1}$, and the limit by Thorsett is treated separately.

In Fig. 1, we show the numerical results of $\dot G/G$. The shaded region is already excluded by the current experimental limits. To examine the model dependencies of the results, we also show $\dot G/G$ for the potential of the form $M^4[\exp(1/\kappa\phi)-1]$ [8] by a dotted curve. We find that negative $\xi$ is severely constrained, while positive $\xi$ is loosely constrained and the limit is dependent on the potential.

These results are intuitively understood via a conformally transformed picture [18]. If we perform the conformal transformation so that the scalar field is minimally coupled,

$$g_{ab}=\widetilde{g_{ab}}|1-\kappa^2\xi\phi^2|^{-1}. \tag{2.18}$$

Then the action Eq. (2.1) becomes

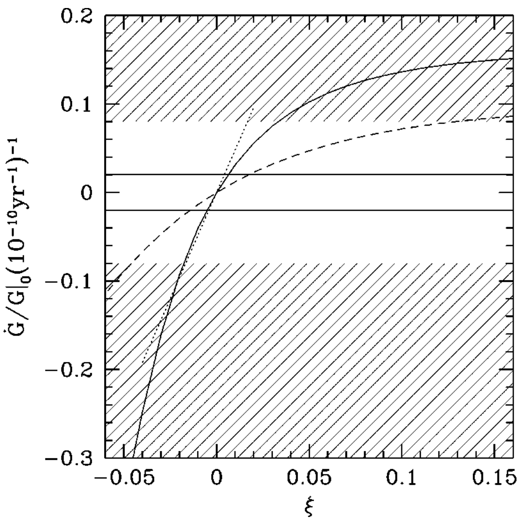


FIG. 1. The present-day $\dot{G}/G$ as a function of $\xi$ for the potential of inverse power with $\alpha=4$ (solid curve) and for the exponential potential (dashed curve). An approximated relation for $|\xi|\kappa^2\phi^2 \ll 1$ [Eq. (2.17)] is plotted as a dotted line. The shaded region is already excluded by the current experimental limits: $|\dot{G}/G|=(0\pm 8)\times 10^{-12}\ \mathrm{yr}^{-1}$. The limit by Thorsett is also shown: $|\dot{G}/G|=(0\pm 2)\times 10^{-12}\ \mathrm{yr}^{-1}$ [17].

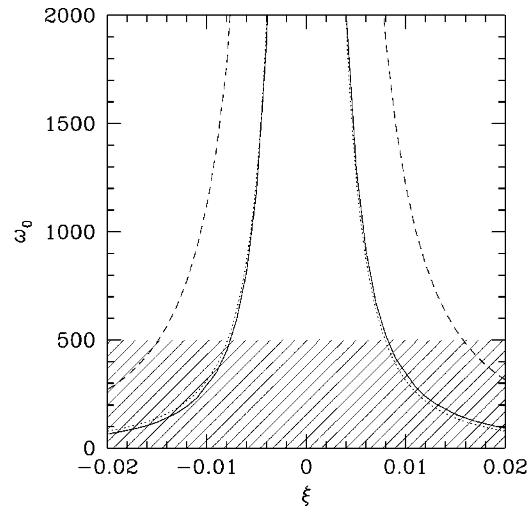


FIG. 2. The present-day Brans-Dicke parameter. The limit by the solar system experiments is $\omega_0>500$ [19]. The solid curve is for the potential of inverse power with $\alpha=4$; the dashed curve is for the exponential potential. An approximated relation for $|\xi|\kappa^2\phi^2\ll 1$ [Eq. (2.15)] is plotted as a dotted curve.

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{2\kappa^2}-\frac{1}{2}F^2(\phi)(\tilde{\nabla}\phi)^2-\tilde{V}(\phi)\right]+S_m, \tag{2.19}$$

where

$$F^2(\phi)=\frac{1-\xi(1-6\xi)\kappa^2\phi^2}{(1-\xi\kappa^2\phi^2)^2}, \tag{2.20}$$

$$\tilde{V}(\phi)=\frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}. \tag{2.21}$$

Hence, after redefining the scalar field so that the kinetic term is canonical,

$$\Phi=\int d\phi F(\phi), \tag{2.22}$$

the action is reduced to that of the scalar field minimally coupled to the Einstein gravity. We can follow the dynamics qualitatively by simply looking at the effective potential $\tilde{V}(\Phi)$. Note that $1/(1-\xi\kappa^2\phi^2)^2$ is a decreasing function of $\phi$ for $\xi<0$, while an increasing function for $\xi>0$. For $\xi<0$ the effective potential $\tilde{V}(\phi)$ decreases more rapidly than $V(\phi)$ [in particular, $\tilde{V}(\Phi)$ decreases exponentially for large $\kappa\phi$], and consequently the scalar field rolls down the potential more rapidly. On the other hand, for $\xi>0$, $\tilde{V}(\Phi)$ diverges at $\kappa^2\phi^2=1/\xi$, so the slope of the effective potential becomes gentler and the scalar field rolls down the potential more slowly, and hence $|\dot{G}/G|$ becomes smaller than that for $\xi<0$.

We may summarize that the current experimental limits on the time variation of $G$ constrain the nonminimal coupling as

$$-10^{-2}\lesssim\xi\lesssim 10^{-2}\sim 10^{-1}, \tag{2.23}$$

while if the tighter limit by Thorsett is adopted, then we have

$$-10^{-2}\lesssim\xi\lesssim 10^{-2}. \tag{2.24}$$

However, the limit is sensitive to the shape of the potential.

The most important experimental limits we must consider are the solar system experiments, such as the Shapiro time delay and the deflection of light [19] because the nonminimally coupled scalar field can mediate the long range gravity force in addition to that mediated by a metric tensor. The recent experiments set a constraint on the parametrized-post-Newtonian (PPN) parameter $\gamma_{\mathrm{PPN}}$ as [20]

$$|\gamma_{\mathrm{PPN}}-1|<2\times 10^{-3}, \tag{2.25}$$

which constrains the Brans-Dicke parameter through the relation $\gamma_{\mathrm{PPN}}=(\omega+1)/(\omega+2)|_0$ [19]

$$\omega_0>500. \tag{2.26}$$

In Fig. 2, we show the present-day Brans-Dicke parameter defined by Eq. (2.12) as a function of $\xi$. We also plot a curve derived under the assumption of $|\xi|\kappa^2\phi^2\ll 1$, Eq. (2.15). We find a good agreement. Thus, using Eq. (2.15) and Eq. (2.26), the nonminimal coupling $\xi$ is found to be constrained as

$$|\xi|<3.9\times 10^{-2}\frac{1}{\sqrt{\alpha(\alpha+2)}}\leqslant 2.2\times 10^{-2}, \tag{2.27}$$

as long as $\alpha\geqslant 1$. The limit is less sensitive to the potential than that derived from $|\dot{G}/G|$ because $\omega$ does not explicitly depend on $\dot{\phi}$ unlike $|\dot{G}/G|$. We note that the limit is found to be insensitive to $\Omega_M$ as long as $\Omega_M \lesssim 0.7$. There is another PPN parameter $\beta_{\rm PPN}$ which is written in terms of $\omega$ as $\beta_{\rm PPN}-1=(d\omega/d\Phi_{BD})(2\omega+4)^{-1}(2\omega+3)^{-2}|_0$ [19]. The most recent results of the lunar laser ranging [21], combined with Eq. (2.25), yield

$$|\beta_{\rm PPN}-1|<6\times 10^{-4}. \tag{2.28}$$

We find that $|\beta_{\rm PPN}-1|\sim\mathcal{O}(\xi^3)$ and consequently the experimental limit on $\beta_{\rm PPN}$ is always satisfied if the condition Eq. (2.25) is satisfied.

## III. SUMMARY

We have explored the possibility of an explicit coupling between the quintessence field and the curvature. Because the force mediated by the scalar field is of long range ($\gtrsim H_0^{-1}$), such a coupling is constrained by the solar system experiments. Through both analytical estimate and numerical integration of the equations, we have found that the limit is given by $|\xi|\lesssim 10^{-2}$. The current limit on the nonminimal coupling, $|\xi|\lesssim 10^{-2}$, is not so strong when compared with other couplings to ordinary matter. For example, a coupling with the electromagnetic field is suppressed at the level of $\lesssim 10^{-6}$; the coupling with QCD is at most $\lesssim 10^{-4}$ [10]. We have also found that the induced time variation of $G$ is sensitive to the shape of the potential. The future improvements in the limit of $\dot{G}/G$ may further constrain negative $\xi$ or might lead to a detection of $\dot{G}/G<0$ depending on the potential of the quintessence field.

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