Monopole-antimonopole bound states as a source of ultrahigh-energy cosmic rays

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The electromagnetic decay and final annihilation of magnetic monopole-antimonopole pairs formed in the early universe has been proposed as a possible mechanism to produce the highest energy cosmic rays. We show that for a monopole abundance saturating the Parker limit, the density of magnetic monopolonium formed is many orders of magnitude less than that required to explain the observed cosmic ray flux. We then propose a different scenario in which the monopoles and antimonopoles are connected by strings formed at a low energy phase transition (~ 100 GeV). The bound states decay by gravitational radiation, with lifetimes comparable to the age of the Universe. This mechanism avoids the problems of the standard monopolonium scenario, since the binding of monopoles and antimonopoles is perfectly efficient. [S0556-2821(99)02120-7]

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I. INTRODUCTION

The observation of ultrahigh-energy cosmic rays (UHECR) with energies above 10^{11} GeV [1,2] poses a serious challenge to the particle acceleration mechanisms so far proposed. This fact has motivated the search for non-acceleration models, in which the high energy cosmic rays are produced by the decay of a very heavy particle. Topological defects are attractive candidates for this scenario. Because of their topological stability these objects can retain their energy for very long times and release quanta of their constituents, typically with grand unified theory (GUT) scale masses, which in turn decay to produce the UHECR.

Various topological defect models and mechanisms have been studied by numerous authors [3]. In this paper we investigate two different scenarios involving the annihilation of monopole-antimonopole pairs. We first discuss standard magnetic monopole pair annihilation [4,5], paying particular attention to the kinetics of monopolonium formation. We find that, due to the inefficiency of the pairing process, the density of monopolonium states formed is many orders of magnitude less than the value required to explain the UHECR events.

We then present a different scenario in which very massive monopoles ($m\sim 10^{14}$ GeV) are bound by a light string formed at approximately 100 GeV. These monopoles do not have the usual magnetic charge, or in fact any unconfined flux. Gravitational radiation is the only significant energy-loss mechanism for the bound systems.[1] Their lifetimes can then be comparable to the age of the Universe, and their final annihilation will then contribute to the high energy end of the cosmic ray spectrum.

II. REQUIRED MONOPOLONIUM ABUNDANCE

What density of decaying monopolonium states is required to produce the observed cosmic rays? The monopolonium will behave as a cold dark matter (CDM) component and will cluster in the galactic halo, producing a high energy spectrum of cosmic rays without the Greisen-Zatsepin-Kuzmin (GZK) cutoff [7,8]. Since the observational data does not seem to show any such cutoff, this is an advantage of such topological defect models [9,10].

For a given monopole mass, we can set the lifetime of the monopolonium at least equal to the age of the Universe, and obtain the required density of monopolonium in the halo by normalizing the flux to the observed high energy spectrum [9]. The required number density decreases with the monopole mass, so as a lower limit we can take the required density corresponding to $m_M = 10^{17}$ GeV [9]:

$$N^h_{M\bar{M}}(T_0) > 6\times 10^{-27}\ \mathrm{cm}^{-3}. \tag{1}$$

Since the different components of the CDM cluster in the same way we can use this halo density to get the mean density in the universe, by computing

$$N_{M\bar{M}} = \frac{N^h_{M\bar{M}}\Omega_{CDM}\rho_{\rm cr}}{\rho^h_{CDM}}. \tag{2}$$

For $\Omega_{CDM}h^2 = 0.2$, $\rho^h_{CDM} = 0.3\ \mathrm{GeV\,cm}^{-3}$, and $\rho_{\rm cr} = 10^4 h^2\ \mathrm{eV\ cm}^{-3}$, we get

$$N_{M\bar{M}}(T_0) > 10^{-32}\ \mathrm{cm}^{-3}. \tag{3}$$

We will work with a comoving monopolonium density $\Gamma = N_{M\bar{M}}/s$ where s is the entropy density, currently $s\approx 3\times 10^3\ \mathrm{cm}^{-3}$, so that we require

$$\Gamma > 10^{-35} \tag{4}$$

to explain the observed UHECR.

III. MAGNETIC MONOPOLE STATES

A. Introduction

Monopolonium states are expected to have been formed by radiative capture if there was a non-zero density of free monopoles in the early universe. They will typically be

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[1] Such systems were studied in a different context by Martin and Vilenkin [6].

bound in an orbit with a large quantum number, so we can treat them as classical objects emitting electromagnetic radiation as they spiral down to deeper and deeper orbits, until they annihilate in a final burst of very high energy particles.

The electromagnetic decay of monopolonium was analyzed by Hill [4] using the dipole radiation formula. The rate of energy loss is[2]

$$\frac{dE}{dt}=\frac{64E^4}{3g_M^2m_M^2}, \tag{5}$$

where g_M is the magnetic charge. From this expression, the lifetime of monopolonium with radius r and binding energy $E=g_M^2/2r$ is [4]

$$\tau_E\sim\frac{m_M^2r^3}{8g_M^4}. \tag{6}$$

For $m_M=10^{16}$ GeV, $g_M=1/(2e)\approx\sqrt{34}$, and an initial radius of $r=10^{-9}$ cm, this gives $\tau_E\sim10^{18}$ sec, comparable to the age of the Universe.

Bhattacharjee and Sigl [5] used a thermodynamic equilibrium approximation to estimate the monopolonium density and argued that the late annihilation of very massive magnetic monopoles could explain the UHECR events observed. Here we recalculate the density of monopolonium states, taking into account the kinematics of formation and the frictional energy loss of monopolonium formed at early times.

B. Friction

Before electron-positron annihilation, monopoles interact with a background of relativistic charged particles. These interactions produce a force which, for a non-relativistic monopole is given by [11]

$$F=\frac{\pi}{18}N_cT^2v\int_{b_{\min}}^{b_{\max}}\frac{db}{b}, \tag{7}$$

where N_c is the number of species of charged particles, v the velocity of the monopole with respect to the background gas of charged particles and b the impact parameter of the incident particles. Since we are interested in the friction that a monopole feels in a bound state orbit of monopolonium, we will not consider the interaction of charged particles with an impact parameter greater than the radius of the monopolonium, so $b_{\max}\approx g_M^2E^{-1}$. Initially, the monopoles are bound with energy $E\sim T$, so $b_{\max}\approx g_M^2T^{-1}$. Equation (7) is derived using the approximation that each charged particle is only slightly deflected. This approximation breaks down for impact parameters that are too small, so we should cut off our integration at [11] $b_{\min}\approx T^{-1}$. Using $N_c=2$ and $g_M^2\approx34$, we get

$$F\approx1.22T^2v \tag{8}$$

so the energy loss rate due to interactions with charged particles in the background is

$$\frac{dE}{dt}\approx1.22T^2v^2. \tag{9}$$

Taking the system to be bound in a circular orbit, we have

$$m_Mv^2\sim E \tag{10}$$

so we can write

$$\frac{dE}{dt}\approx1.22T^2\frac{E}{m_M}. \tag{11}$$

The time scale for this process is

$$\tau_F=\frac{E}{dE/dt}\approx\frac{m_M}{1.22T^2}. \tag{12}$$

If we compare it with the Hubble time,

$$\tau_H=\sqrt{\frac{90}{8\pi^3g_*}}m_{pl}T^{-2}\approx0.184m_{pl}T^{-2}, \tag{13}$$

where m_{pl} is the Planck mass, and g_* is the number of effectively massless degrees of freedom, $g_*=10.75$, we get

$$\frac{\tau_F}{\tau_H}\approx0.15\frac{m_M}{m_{pl}}\ll1. \tag{14}$$

Thus, we see that the damping of the monopolonium energy due to friction is very effective in this regime, and the monopoles spiral down very quickly.

When the distance between monopoles becomes small as compared to T^{-1}, the effect of friction is reduced and Eq. (7) is no longer accurate. However, even for $T=1$ MeV, the radius has been reduced about 2 orders of magnitude to $r\sim2\times10^{-11}$ cm, and the electromagnetic lifetime has been reduced by about 6 orders of magnitude. Thus only monopolonium states formed after electron-positron annihilation can live to decay in the present era.

After electron-positron annihilation the number of charged particles in the thermal background has decreased by a factor $\sim10^{-9}$ so $\tau_F/\tau_H\gg1$ and the monopolonium is little affected by friction.

C. Formation rate

We can obtain an upper limit for the monopolonium density by solving the Boltzmann equation,

$$\frac{dN_{M\bar{M}}}{dt}=\langle\sigma_bv\rangle n_M^2-3HN_{M\bar{M}}, \tag{15}$$

where n_M denotes the free monopole density; $N_{M\bar{M}}$, the monopolonium density; H, the Hubble constant; and $\langle\sigma_bv\rangle$, the average product of the binding cross section times the thermal velocity of the monopoles.

With the comoving monopole density $\gamma=n_M/s$, we can rewrite the equation above as

[2]Here and throughout we use units where $\hbar=c=k_B=1$.

$$\frac{d\Gamma}{dt}=\langle\sigma_b v\rangle\gamma n_M=\langle\sigma_b v\rangle\gamma^2 s. \tag{16}$$

Using the approximation for the classical radiative capture cross section of monopoles with thermal velocities given by [12],

$$\langle\sigma_b v\rangle\approx\frac{\pi^{7/5}}{2}\frac{g_M^4}{m_M^2}\left(\frac{m_M}{T}\right)^{9/10}, \tag{17}$$

and with

$$s=\frac{2\pi^2}{45}g_{*S}T^3, \tag{18}$$

where g_{*S} is the number of degrees of freedom contributing to the entropy, we get

$$\frac{d\Gamma}{dt}=\frac{\pi^{17/5}}{45}\frac{g_M^4\gamma^2}{m_M^2}\left(\frac{m_M}{T}\right)^{9/10}g_{*S}T^3. \tag{19}$$

Since we are interested in the evolution of the monopolonium density after electron-positron annihilation, we will take a constant value $g_{*S}\approx 3.91$ to get

$$\frac{d\Gamma}{dt}\approx 4.25\frac{g_M^4\gamma^2}{m_M^2}\left(\frac{m_M}{T}\right)^{9/10}T^3. \tag{20}$$

As we will see, only a tiny fraction of the monopoles will ever be bound, so we can consider the comoving number of monopoles γ to be constant. To integrate Eq. (20), we will make a change of variable

$$t=\sqrt{\frac{90}{32\pi^3 g_*}}m_{pl}T^{-2}\approx 0.164 m_{pl}T^{-2}, \tag{21}$$

appropriate to times after electron-positron annihilation, to get

$$\frac{d\Gamma}{dT}\approx -1.34\frac{g_M^4 m_{pl}\gamma^2}{m_M^2}\left(\frac{m_M}{T}\right)^{9/10}, \tag{22}$$

and thus

$$\Gamma_f\approx 13.4 g_M^4\left(\frac{m_{pl}}{m_M}\right)\left(\frac{T_i}{m_M}\right)^{1/10}\gamma^2. \tag{23}$$

We now take $T_i\sim 1$ MeV and $g_M^2=34$, and note that to produce the observed UHECR, we must have $m_M>10^{11}$ GeV, so that for a fixed monopole comoving density γ, we have the bound

$$\Gamma_f<10^{11}\gamma^2. \tag{24}$$

D. Monopole density bound

The formation of magnetic monopoles via the Kibble mechanism [13] is inevitable in all GUT models of the early universe, and annihilation mechanisms are not efficient in a rapidly expanding background [14,15], so that the typical initial density of monopoles produced at a GUT phase transition will very soon dominate the energy density of the Universe. The most attractive solution for this problem is the inflationary scenario [16]. In standard inflation, the exponential expansion of the Universe reduces the monopole density to a completely negligible value. However, it is possible for new monopoles to be formed at the end of inflation [17–21]. The exact relic abundance of monopoles created in this period is very model dependent, but its value is constrained by the Parker limit [22]: To prevent the acceleration of monopoles from eliminating the galactic magnetic field, the monopole flux into the galaxy must be limited by

$$F<10^{-16}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}. \tag{25}$$

Assuming a monopole velocity with respect to the galaxy of $\sim 10^{-3}c$, we can translate this bound into a limit on the monopole density,

$$n_M<10^{-23}\ \mathrm{cm}^{-3}, \tag{26}$$

and thus $\gamma<10^{-26}$. Then, from Eq. (24) we have

$$\Gamma_f<10^{-41}. \tag{27}$$

Since this conflicts with Eq. (4) by 10 orders of magnitude, we conclude that primordial bound states of magnetic monopoles cannot explain the UHECR.

We note that we have used several approximations which overstate the possible value of Γ_f: First, we have considered the total classical radiative capture cross section. This takes into account not only the monopolonium formed with the right energy to decay at present, but all the possible binding energies, clearly overestimating the value of Γ_f. Second, it has been argued that the classical cross section given in Eq. (17) overestimates its real value due to photon discreteness effects [12]. Finally, some of the monopolonium will have decayed before the present time, reducing the value of Γ. All of these effects make the conflict above more serious.

IV. MONOPOLES CONNECTED BY STRINGS

We present now a different scenario for the formation and annihilation of monopole-antimonopole bound states. The main problem in explaining the UHECR by the conventional magnetic monopolonium system is the inefficiency of the binding mechanism. This can be solved if we assume that all the monopoles get connected by strings in a later phase transition. Since the U(1) symmetry of the monopoles would be broken by the second phase transition, this U(1) must be a field other than the usual electromagnetism.[3] We furthermore assume that these monopoles will not have any other unconfined charge, so that they will feel almost no frictional force moving in a background of particles.

[3]This is different from the Langacker-Pi scenario [23], where electromagnetism is broken and then restored at a lower temperature, and monopoles do feel large frictional forces.

We take the comoving density of bound monopole systems Γ to be constant. With a monopole mass of 10^{14} GeV the calculation of Sec. II gives $\Gamma \sim 10^{-33}$, and with all monopoles bound, $\gamma = 2\Gamma$. The proper density at the time of string formation is then

$$n_M(T_s) = \gamma s = \frac{2\pi^2}{45} g_{*S} T_s^3 \gamma \sim 10^{-32} T_s^3. \tag{28}$$

We can then compute the mean separation between monopoles at the time the string is formed,

$$L_i \sim [n_M(T_s)]^{-1/3}. \tag{29}$$

If we take $T_s \sim 100$ GeV, we obtain

$$L_i \sim 10^{-6} \text{ cm}, \tag{30}$$

which is much smaller than the horizon distance, $d_H \sim 3$ cm at $T \sim 100$ GeV. We will assume that there are no light ($m \sim T_s$ or less) particles that are charged under the string flux. This means that there will be no charged particles that interact with the monopoles and cause the system to lose energy, so that gravitational radiation will be the only energy loss mechanism. When the strings are formed they may have excitations on scales smaller than the distance between monopoles, but these will be quickly smoothed out by gravitational radiation, leaving a straight string. The energy stored in the string is then μL_i, where $\mu \sim T_s^2$ is the energy per unit length of the string. This is smaller than the monopole mass by the ratio

$$\frac{\mu L_i}{m_M} \sim 10^{-2} \tag{31}$$

so the monopoles will move non-relativistically.

In order to estimate the radiation rate we can assume that the monopoles are moving in straight lines. In fact, at the time of string formation the monopoles will have thermal velocities, so that in general the system will be formed with some non-zero angular momentum. However, in general this will be small compared to the linear motion due to the string tension, so we will ignore it, except to note that the monopoles will pass by without collision. The half oscillation of one monopole is parametrized by

$$x(t) = (2aL)^{1/2} t - \frac{1}{2} a t^2 \tag{32}$$

with $a = \mu / m_M$ and $0 < t < (8L/a)^{1/2}$. Using the quadrupole approximation,[4] the rate of energy loss of the system is

$$\frac{dE}{dt} = \frac{288}{45} G \mu^2 \left(\frac{\mu L}{m_M} \right). \tag{33}$$

[4]The fully relativistic situation was considered in [6].

Since μL is the energy in the string, we can integrate this equation to obtain

$$L = L_i e^{-t/\tau_g} \tag{34}$$

with

$$\tau_g = \frac{45}{288} \frac{m_M}{G\mu^2} = \frac{45}{288} \frac{m_{pl}^2 m_M}{T_s^4}. \tag{35}$$

The lifetime of the state will thus be $\tau_g \ln(L_i/r_M)$, where $r_M \sim m_M^{-1}$ is the radius of the monopole core. For $T \sim 100$ GeV and $m_M \sim 10^{14}$ GeV, Eq. (35) gives $\tau_g \sim 10^{17}$ sec, comparable to the age of the Universe.

This suggests that the bound system formed by a monopole-antimonopole pair connected by a string can slowly decay gravitationally, and release the energy stored in the monopoles in a final annihilation when the two monopole cores become close enough.

V. CONCLUSIONS

We showed that is not possible to construct a consistent model for the origin of the UHECR based on the electromagnetic decay and final annihilation of magnetic monopole-antimonopole bound states formed in the early universe. We have obtained an upper limit for the monopolonium density today, taking into account its enhancement in the galactic halo and the maximum average free monopole density consistent with the Parker limit. Because of the small radiative capture cross section for the monopoles and the rapid expansion of the Universe, the maximum density of monopolonium is many orders of magnitude below the concentration required to explain the highest energy cosmic ray events.

We then proposed a different scenario in which the monopoles are connected by strings that form at a relatively low energy. This mechanism solves the problem of the inefficiency of the binding process, since every monopole will be attached to an antimonopole at the other end of the string. Because of the confinement of the monopole flux inside of the string, the main source of energy loss for these bound systems will be gravitational radiation. If we assume a monopole mass of 10^{14} GeV and a string energy scale of the order of 100 GeV, the lifetime of the bound states would be comparable with the age of the Universe, making them a possible candidate for the origin of the ultrahigh-energy cosmic rays.

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