Constraint on the magnetic moment of the top quark

R. Martínez, J.-Alexis Rodríguez, and M. Vargas
Departamento de Física, Universidad Nacional, Bogotá, Colombia

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We derive a bound on the magnetic dipole moment of the top quark in the context of the effective Lagrangian approach by using the ratios $R_b = \Gamma_b/\Gamma_h$, $R_l = \Gamma_h/\Gamma_l$, and the Z width. We take into account the vertex and oblique corrections. We obtain the allowed region for the magnetic dipole moment of top quark as $-0.79 \leqslant \delta\kappa \leqslant 1.3$. [S0556-2821(99)00817-6]

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The most recent analyses of precision measurements at the CERN Large Electron Positron (LEP) collider led to the conclusion that the predictions of the standard model (SM) of electroweak interactions, based on the gauge group $SU(2)_L \otimes U(1)_Y$, are in excellent agreement with experimental results. Recently the discovery of the top quark has been announced by the Collider Detector at Fermilab (CDF) and D0 Collaborations [1]. The direct measurement of the top quark mass m_t is in agreement with the indirect estimates derived by confronting the SM m_t dependent higher order corrections with the LEP and other experimental results. The measurement of the top quark mass reduced the number of free parameters of the SM. A precise knowledge of the value of the top mass will improve the sensitivity of searches of new physics through small indirect effects.

The precise measurements of the $g-2$ value of the electron provides a test of its pointlike character. Similarly, measurements of the electric and chromomagnetic moments of the quarks can be important to study physics beyond the SM. In particular, the chromomagnetic moment of the top quark can affect its production in the $p\bar{p}$ and e^+e^- reactions [2].

The SM predicts how the top quark should behave under these interactions, so any deviation from this behavior would provide us with a probe of new physics beyond the SM. If new physics is found in this sector, it probably originates from a nonstandard symmetry breaking mechanism. This is because the top mass is of the order of the electroweak (EW) breaking scale, and hence it is conceivable that the top-quark properties are sensitive to unsuppressed EW breaking effects [3].

The aim of the present work is to extract indirect information on the magnetic dipole moment of the top quark from LEP data, specifically we use the ratios R_b and R_l defined by

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadron})},$$

$$R_l = \frac{\Gamma(Z \to \text{hadron})}{\Gamma(Z \to l\bar{l})}, \tag{1}$$

and the Z width, in the context of an effective Lagrangian approach. The oblique and QCD corrections to the b quark and hadronic Z decay widths cancel off in the ratio R_b. This property makes R_b very sensitive to direct corrections to the $Zb\bar{b}$ vertex, especially those involving the heavy top quark [4], while Γ_Z and R_l are more sensitive to oblique corrections.

The effective Lagrangian approach is a convenient model independent parametrization of the low-energy effects of the new physics that may show up at high energies [5]. Effective Lagrangians, employed to study processes at a typical energy scale E can be written as a power series in $1/\Lambda$, where the scale Λ is associated with the heavy particles masses of the underlying theory [6]. The coefficients of the different terms in the effective Lagrangian arise from integrating out the heavy degrees of freedom that are characteristic of a particular model for new physics.

In order to define an effective Lagrangian it is necessary to specify the symmetry and the particle content of the low-energy theory. In our case, we require the effective Lagrangian to be CP conserving, invariant under SM symmetry $SU(2)_L \otimes U(1)_Y$, and to have as fundamental fields the same ones appearing in the SM spectrum. Therefore we consider a Lagrangian in the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \alpha_n \mathcal{O}^n, \tag{2}$$

where the operators $\mathcal{O}^n$ are of dimension greater than four. In the present work, we consider the following dimension six and CP-conserving operators:

$$O_{uW}^{ab} = \bar{Q}_L^a \sigma^{\mu\nu} W_{\mu\nu}^i \tau^i \tilde{\phi} U_R^b,$$

$$O_{uB}^{ab} = \bar{Q}_L^a \sigma^{\mu\nu} Y B_{\mu\nu} \tilde{\phi} U_R^b, \tag{3}$$

where Q_L^a is the quark isodoublet, U_R^b is the up quark isosinglet, a, b are the family indices, $B_{\mu\nu}$ and $W_{\mu\nu}$ are the $U(1)_Y$ and $SU(2)_L$ field strengths, respectively, and $\tilde{\phi} = i\tau_2\phi^*$. We use the notation introduced by Buchmüller and Wyler [7]. In the case of the operators O_{uB}^{ab} and O_{uW}^{ab}, some degree of family mixing is made explicit (corresponding to $a \neq b$) without breaking SM gauge invariance. After spontaneous symmetry breaking, these fermionic operators also generate effective vertices proportional to the anomalous magnetic moments of quarks. The above operators for the third family give rise to the anomalous $t\bar{t}\gamma$ vertex and the unknown coefficients ϵ_{uB}^{ab} and ϵ_{uW}^{ab} are related, respectively, with the anomalous magnetic moment of the top quark through

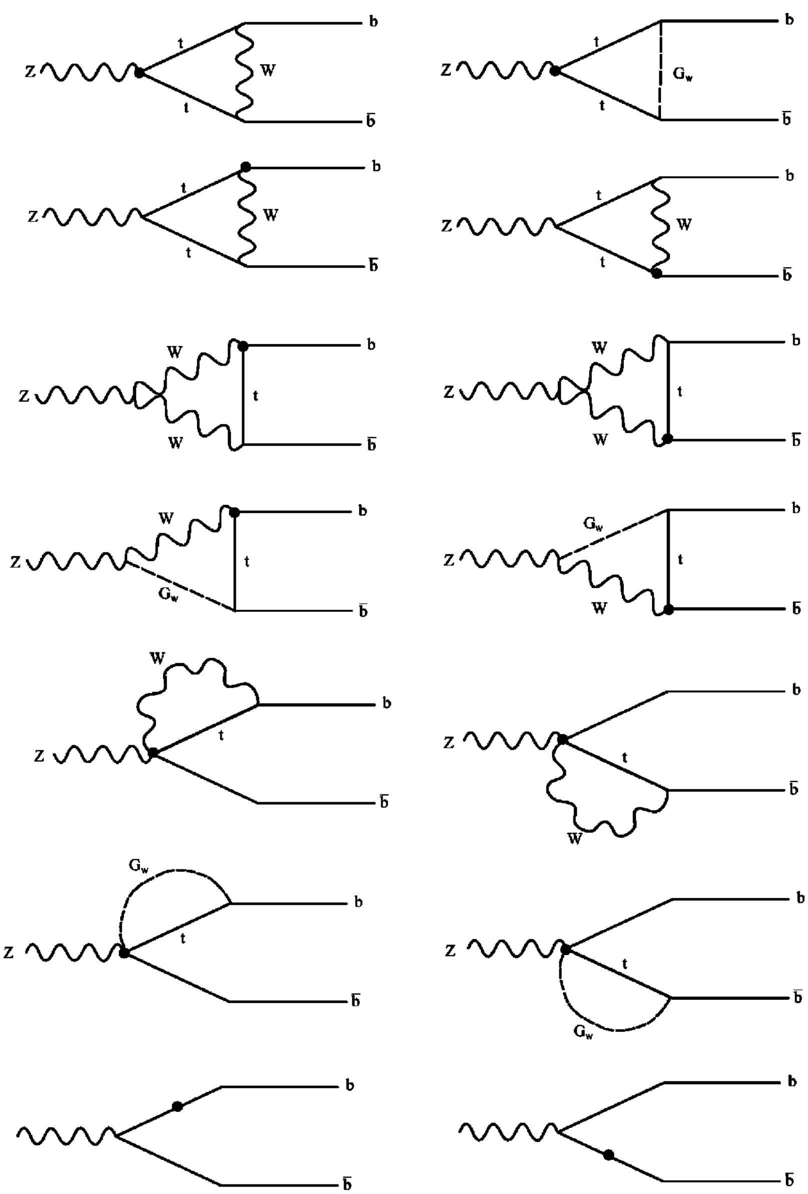

FIG. 1. Feynman diagrams contributing to the $Z\to b\bar{b}$ decay. The heavy dots denote an effective vertex.

$$\delta\kappa_t=-\sqrt{2}\,\frac{m_t}{m_W}\,\frac{g}{eQ_t}(s_W\epsilon_{uW}^{33}+c_W\epsilon_{uB}^{33}), \tag{4}$$

where s_W denotes the sine of the weak mixing angle.

The expression for R_b is given by

$$R_b=R_b^{\mathrm{SM}}[1+(1-R_b^{\mathrm{SM}})\,\delta_b], \tag{5}$$

where R_b^{SM} is the value predicted by the SM and δ_b is the factor which contains the new physics contribution, and it is defined as follows:

$$\delta_b=\frac{2(g_V^{\mathrm{SM}}g_V^{\mathrm{NP}}+g_A^{\mathrm{SM}}g_A^{\mathrm{NP}})+(g_V^{\mathrm{NP}})^2+(g_A^{\mathrm{NP}})^2}{(g_V^{\mathrm{SM}})^2+(g_A^{\mathrm{SM}})^2} \tag{6}$$

and g_V^{SM} and g_A^{SM} are the vector and axial vector couplings of the $Zb\bar{b}$ vertex normalized as

$$g_V^{\mathrm{SM}}=-\frac{1}{2}\left(1-\frac{4}{3}(1+\Delta\kappa)s_W^2+\varepsilon_b\right)\left(1+\frac{\Delta\rho}{2}\right),$$

$$g_A^{\mathrm{SM}}=-\frac{1}{2}\left(1+\frac{\Delta\rho}{2}\right)(1+\varepsilon_b).$$

where we are including the radiative corrections from the SM, in the parameters $\Delta\kappa$, $\Delta\rho$, and ε_b, because in the expression for δ_b we take the second order of $g_{V(A)}^{\mathrm{NP}}$. The contributions from new physics, Eq. (3), to R_l and Γ_Z are of two classes. One from vertex correction to $Zb\bar{b}$ in the Γ_{hadr} and the other from the oblique corrections through $\Delta\kappa$ in the $\sin^2\theta_W$ and $\Delta\rho$. These can be written as

$$R_l=R_l^{\mathrm{SM}}(1-0.1851\;\;\Delta\kappa+0.2157\;\;\delta_b),$$

$$R_b=R_b^{\mathrm{SM}}(1-0.03\;\;\Delta\kappa+0.7843\;\;\delta_b), \tag{7}$$

$$\Gamma_Z=\Gamma_Z^{\rm SM}(1-0.2351\ \ \Delta\kappa+0.1506\ \ \delta_b),$$

where $\Delta\rho$ is equal to zero for the operators that we are considering.

The contribution of the above effective operators to the $Zb\bar{b}$ vertex is given by the Feynman diagrams shown in Fig. 1, where a heavy dot denotes an effective vertex. After evaluating the Feynman diagrams, with insertions of the effective operators O_{uB}^{ab} and O_{uW}^{ab} we obtain

$$\begin{aligned} g_V^{\rm NP}=&4\sqrt{2}\epsilon_{uW}^{33}G_F m_W^3 m_t\Bigg\{3c_W(\tilde{C}_{12}-\tilde{C}_{11})-\frac{m_t^2}{\sqrt{2}m_W^2}\\ &\times(C_{12}-C_{11}+C_0)+\frac{(1+a)}{8c_W}(C_{11}+C_{12}+C_0)\\ &+\frac{1}{\sqrt{2}}(C_{12}-C_{11}-C_0)-\frac{3a}{4c_W m_Z^2}(B_1-B_0)\Bigg\},\\ g_A^{\rm NP}=&4\sqrt{2}\epsilon_{uW}^{33}G_f m_W^3 m_t\Bigg\{-\frac{a}{2c_W}(C_0+C_{12}-C_{11})\\ &-\frac{1}{\sqrt{2}}(C_{12}-C_0-C_{11})+\frac{m_t^2}{\sqrt{2}m_W^2}(C_{12}-C_{11}+C_0)\\ &-\frac{2m_t s_W^2}{m_W}(\tilde{C}_0+\tilde{C}_{12}-\tilde{C}_{11})-\frac{3}{4c_W m_Z 2}(B_1-B_0)\Bigg\} \end{aligned} \tag{8}$$

for the operator O_{uW} and

$$\begin{aligned} g_V^{\rm NP}=g_A^{\rm NP}=&\frac{4\sqrt{2}}{3}\epsilon_{uB}^{33}G_F m_W^3 m_t\frac{s_W}{c_W}\Bigg[\frac{m_t^2}{\sqrt{2}m_W^2}(C_{12}-C_{11}+C_0)\\ &-\frac{1}{\sqrt{2}}(-C_{11}+C_{12}-C_0)\Bigg], \end{aligned} \tag{9}$$

for the operator O_{uB}. In the above equations $a=1-\frac{8}{3}s_W^2$ while $C_{ij}=C_{ij}(m_W,m_t,m_t)$, $\tilde{C}_{ij}=\tilde{C}_{ij}(m_t,m_W,m_W)$, and $B_i=B_i(0,m_t,m_W)$ are the Passarino-Veltman scalar integral functions [8]. The combination B_0-B_1 has a pole in $d=4$ dimensions that is identified with the logarithmic dependence on the cutoff. Using the prescription given in Ref. [9], the pole can be replaced by $\ln\Lambda^2/m_Z^2$.

The operators (3) contribute to the fermion processes at one loop level, giving oblique corrections to the gauge boson self-energies. The contribution is essentially coming from the $\Sigma_{\gamma Z}(m_Z^2)$ self-energy. Therefore these operators only contribute to the $\Delta\kappa$ parameter [10]. For $\Delta\kappa$ we have obtained the same results of the Eqs. (50) and (51) of Ref. [11].

Now we have various posibilities to explore the space of the parameters ε_{uW}^{33}, ε_{uB}^{33}, and $\delta\kappa_t$, which are related by Eq. (4). Further we have that the parameters ε_{uW}^{33}, ε_{uB}^{33} are involved with the measured quantities $Q_i=(\Gamma_Z,R_b,R_l)$ through Eq. (7) and they form a surface in the respective space (ε_{uW}^{33}, ε_{uB}^{33},Q_i). Thus the experimental planes for the quantities Q_i cut the surface defined in that space and it is the allowed region in the plane $\varepsilon_{uW}^{33}-\varepsilon_{uB}^{33}$. If we do not neglect the term of the order $(g^{\rm NP})^2$ in Eq. (7) we get the following expressions:

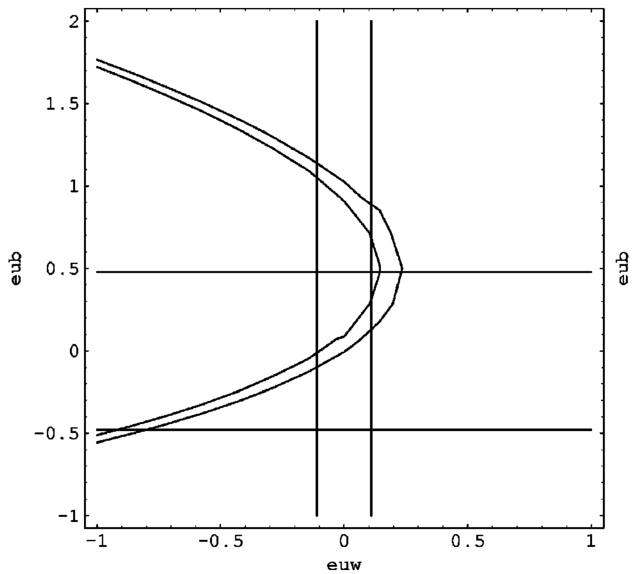

FIG. 2. The projection on the plane $\varepsilon_{uW}^{33}-\varepsilon_{UB}^{33}$ of the first expression of Eq. (10), i.e., the cut with the experimental values of Γ_Z. The horizontal and vertical lines are the constraints from Eq. (11).

$$\begin{aligned} &-0.054\varepsilon_{uB}+0.054\varepsilon_{uB}^2+0.054\varepsilon_{uW}+0.0015\varepsilon_{uW}^2\\ &\quad+0.01\varepsilon_{uB}\varepsilon_{uW}=1-(\Gamma_Z^{\rm exp}/\Gamma_Z^{\rm SM}),\\ &-0.023\varepsilon_{uB}+0.024\varepsilon_{uB}^2+0.016\varepsilon_{uW}+0.0007\varepsilon_{uW}^2\\ &\quad+0.0046\varepsilon_{uB}\varepsilon_{uW}=1-(R_b^{\rm exp}/R_b^{\rm SM}),\\ &-0.626\varepsilon_{uB}+0.635\varepsilon_{uB}^2+0.537\varepsilon_{uW}\\ &\quad+0.018\varepsilon_{uW}^2+0.1218\varepsilon_{uB}\varepsilon_{uW}=1-(R_l^{\rm exp}/R_l^{\rm SM}), \end{aligned} \tag{10}$$

where we have omitted the superindex 33 and each expression define two planes, with the upper and lower experimental limits, respectively. The SM values for the parameters that we have used are $\Gamma_Z=2.4972$ GeV, $R_l=20.747$, $R_b=0.2157$, $\Gamma_{\rm hadr}=1743.4$ MeV, and $\Gamma_l=84.03$ MeV with the input parameters $m_t=175$ GeV, $\alpha_s(m_Z)=0.118$, $m_Z=91.1861$ GeV, $m_H=100$ GeV, and $\Lambda=1$ TeV. The experimental values are $\Gamma_Z=2.4946\pm0.0027$ GeV, $R_l=20.778\pm0.029$, $R_b=0.2178\pm0.0011$.

In Fig. 2 we plot these curves in the plane $\varepsilon_{uW}^{33}-\varepsilon_{uB}^{33}$ corresponding to the cut with the experimental values of Γ_Z. Figures 3 and 4 are the same as Fig. 2 for R_b and R_l, respectively. In this kind of scenario new physics is explored assuming that its effects are smaller than the SM effects, consequently one expect that $|g_{V,A}^{\rm NP}/g_{V,A}^{\rm SM}|\ll 1$ and then we get from $g_{V(A)}^{\rm SM}$ and the expressions (10), the inequalities

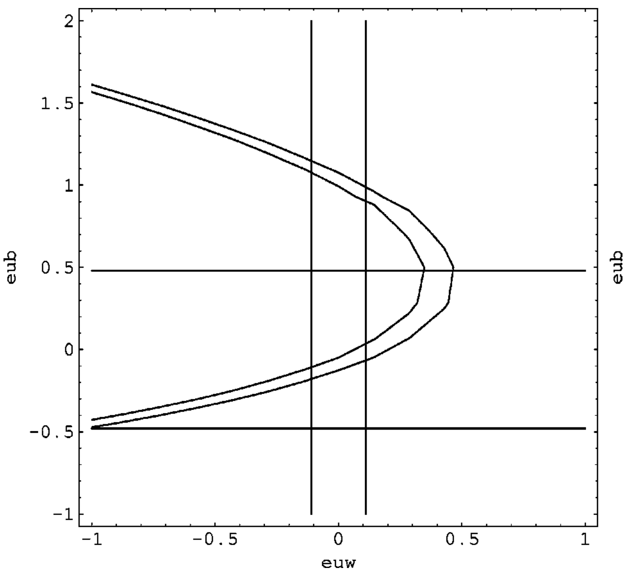

FIG. 3. The same as Fig. 2 for the ratio R_b.

$$|\varepsilon_{uW}^{33}| \leqslant 0.11,$$
$$|\varepsilon_{uB}^{33}| \leqslant 0.48. \tag{11}$$

which are plotted in Figs. 2, 3, and 4 as vertical and horizontal lines, respectively. The allowed region in the plane $\varepsilon_{uW}^{33} - \varepsilon_{uB}^{33}$ in Figs. 2, 3, and 4 is the intersected area between the curves and the limits of Eq. (11).

We try to get other constraints to the parameter ε_{uW}^{33} using other measured processes. For instance, we calculate the contribution to the top quark decay $t \to bW$ but with the available measurements by CDF and D0 of $B(t \to bW)$, it is not possible to get a better constraint for ε_{uW}^{33}, obtaining the value $|\varepsilon_{uW}^{33}| \leqslant 0.7$. We also inspect the system $B^0 - \overline{B}^0$ where we get a contribution identical to zero for the operators under consideration.

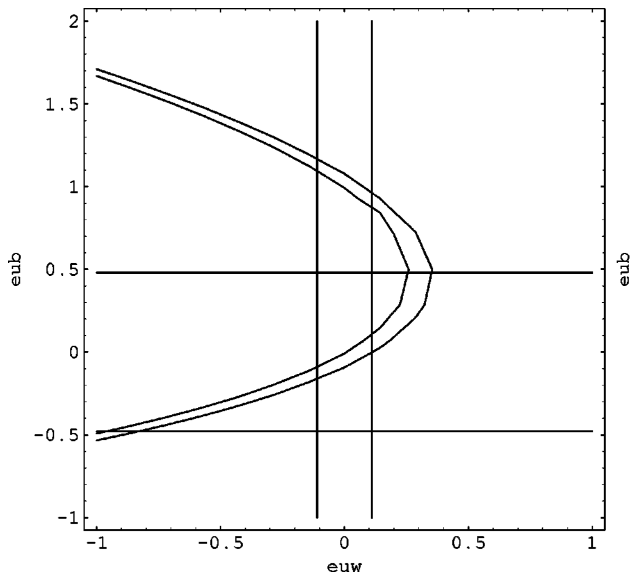

FIG. 4. Same as Fig. 2 for R_l.

On the other hand, the allowed regions given in Figs. 2, 3, and 4 give maximal and minimal bounds for the parameters ε_{uW}^{33} and ε_{uB}^{33}. By using Eq. (4) and the bounds showed in the figures, we obtain for $\delta\kappa_t$ the following values:

$$-3 \leqslant \delta\kappa_t \leqslant 1.3,$$
$$-0.79 \leqslant \delta\kappa_t \leqslant 1.4, \tag{12}$$
$$-1.4 \leqslant \delta\kappa_t \leqslant 1.8,$$

which correspond to Γ_Z, R_b, and R_l, respectively. Therefore for these observables Γ_Z, R_b, and R_l, we get the allowed region $-0.79 \leqslant \delta\kappa \leqslant 1.3$. These bounds agrees also with the one obtained of the same effective operators in Refs. [12,13], by using the CLEO result on $B(b \to s\gamma)$.

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