QCD-based analysis of the weak kaon decays

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Starting directly from the QCD Lagrangian, effective actions describing weak K-meson decays in different channels have been constructed. To construct effective Lagrangians a bilocal bosonization method, which incorporates the chiral symmetry-breaking mechanism, has been used. The decay rates of K mesons calculated with the derived weak Lagrangians essentially depend on the ''dressing'' mechanism of quarks. This mechanism is generated by the gluonic forces mediated by the two-point gluonic propagators. A reasonable agreement with experiment of calculated decay rates of charged K mesons in leptonic, semileptonic, and the two-pion decay channels has been achieved. The employed parametrization of quark propagators simultaneously gives a correct description of the π-π dynamics. [S0556-2821(99)06517-0]

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I. INTRODUCTION

One of the most interesting problems in mesonic physics is weak decay of K mesons. In this case we are dealing with a great variety of decay channels. The difficulty of the problem lies in the fact that the weak and strong interactions are incorporated in different ways in different decay modes. Roughly speaking, in the leptonic decay channels the strong interactions play a less important role. In the nonleptonic channels ($K\to 2\pi, 3\pi$) the strong interaction effects essentially modify the weak decay processes.

The analysis of the nonleptonic decay modes of K mesons is based mostly on the so-called chiral perturbation theory (ChPT). In Ref. [1] this theory has been applied in calculations of effective Lagrangians describing the dynamics of strong interaction of π mesons in the low-energy region. It was quite natural that this theory made also a proper framework for the analysis of $K\to 2\pi$ and $K\to 3\pi$ decays [2–4]. For a more comprehensive list of references referring to the problem, see [5].

In the derivation of the ChPT Lagrangians the main attention is put on the chiral symmetry of massless QCD. In this approach the mesons are identified with the would-be Goldstone bosons of the $SU(3)_L\times SU(3)_R$ symmetry, and the transition amplitudes (effective actions) are expanded in powers of meson masses and momenta. The coefficients in this expansion are divergent, and so the corresponding renormalization procedure must be applied.

There exists another approach based on the method of the bilocal bosonization [6–8]. In this case the calculated effective Lagrangians are also based on the chiral symmetry of QCD Lagrangian but, contrary to ChPT, the coefficients in the momentum expansion of the corresponding mesonic effective Lagrangians are finite. This is achieved by introducing the intrinsic structure of mesons. Moreover, the corresponding form factors (for a pseudoscalar octet of mesons) are uniquely determined by Bethe-Salpeter-like equations, derived from the Dyson-Schwinger (DS) equations, which define the corresponding vacuum configuration.

As it was shown in [9], the theory based on the bilocal bosonization is in a sense more general than ChPT. The derived mesonic effective action possesses a (chiral) structure identical with the ChPT effective Lagrangians with the finite coefficients entering there. The method presented in [9] has been successfully applied in the description of the π-π dynamics. In this paper we apply the method of the bilocal fields developed in [6–8] to calculate the decay rates of kaons in both leptonic and nonleptonic channels.

II. DETAILS OF CALCULATIONS

Our goal is the calculation of the effective Lagrangians responsible for decay processes of pseudoscalar mesons in different channels.

Following the method proposed in [6] or [7,8], in the QCD generating functional, one introduces supplementary bilocal bosonic fields with the quantum numbers the same as quark-antiquark pairs. This allows us to transform the corresponding generating functional to the Gaussian one with respect to quark variables, and perform a suitable integration. The remaining functional integrations (over the bosonic variables) could be done by the saddle-point method. As a result one obtains the generating functional with an effective action containing only the bosonic fields. This action we shall write in the form

$$S_{\rm eff}(\lambda)=-\,\mathrm{Tr}\ln(1+X_0+X_1)-\frac{1}{2}\mathrm{Tr}(\lambda D^{-1}\lambda), \tag{1a}$$

where

$$X_0=a^{-1}(\lambda_0)\lambda,\quad a(\lambda_0)=i\gamma^\mu\partial_\mu-M-\lambda_0, \tag{1b}$$

$$X_1=a^{-1}(\lambda_0)\frac{g}{\sqrt{2}}\gamma^\mu L V^+(T^+W_\mu^+ + T^-W_\mu^-)V, \tag{1c}$$

$$V=\begin{pmatrix}1&0&0\\0&c&s\\0&0&0\end{pmatrix},\quad T^{\pm}=\begin{pmatrix}0&1(0)&0\\0(1)&0&0\\0&0&0\end{pmatrix},$$

$$c=\cos\theta_c,\quad s=\sin\theta_c,\quad L=\frac{1}{2}(1-\gamma_5).$$

In formula (1b) M denotes the mass matrix of u,d, and s quarks, respectively. $a^{-1}(\lambda_0)$ is the diagonal matrix of quark propagators. By D we denote the two-point gluonic propagator. The function λ_0 is determined by a corresponding equa-

TABLE I. Values of constants h,d, and C calculated with the parametrization I [13] and their dependence on the choice of quark masses.

Quark masses (MeV)	h (MeV)	d (MeV)	C (MeV)
$m_u=m_d=10$			
$m_s=150$	82.22	45.99	30.81
$m_u=m_d=m_s=10$	85.56	47.72	32.31

tion resulting from the saddle-point method. $W_{\pm\mu}$ are vector-boson fields. θ_c is the Cabibbo angle which mixes d and s quarks.

For calculations (in details) one needs the matrix elements of λ fields and quark propagators a^{-1}:

$$\hat{a}^{-1}(\lambda_0)=\begin{pmatrix} a_u^{-1}(\lambda_0) & 0 & 0 \\ 0 & a_d^{-1}(\lambda_0) & 0 \\ 0 & 0 & a_s^{-1}(\lambda_0)\end{pmatrix}, \qquad (2)$$

where

$$\langle x|a_u^{-1}(\lambda_0)|y\rangle=\frac{1}{(2\pi)^4}\int d^4q t_u(q)e^{iq(x-y)},$$
$$t_u(q)=[-q^\mu\gamma_\mu A(q^2)+m_u+B(q^2)]^{-1},$$

with a similar expression for the matrix elements of d and s propagators. Functions A and B are determined by solutions of DS equation for the saddle-point configuration λ_0. This equation, written in momentum representation, reduces to a pair of nonlinear integral equations for the form factors $A(q^2)$ and $B(q^2)$. Detailed form of these equations can be found in [9] (we keep the same notation). The solutions for which one is looking strongly depend on the assumed character of the gluonic forces enclosed in the gluonic propagator. The behavior of the form factors A and B was investigated in many papers [10–12,15–17]. In our calculations we explore some of these parametrizations of A and B form factors.

For matrix elements of the mesonic field λ we assume, as in Refs. [7,15], the following separable representation:

$$\langle x|\lambda|y\rangle=\gamma^5\int\frac{d^4p}{(2\pi)^4}e^{ip(x-y)}B(p^2)T^a\theta^a\left(\frac{x+y}{2}\right), \qquad (3)$$

here T^a are the SU(3) flavor matrices. Local fields θ^a represents the octet of π,K, and η pseudoscalar mesons. I must make an important remark. All form factors of π,K, and η mesons in formula (3) are identical and are equal to the form factor B entering expression (2). This is a consequence of Goldstone mechanism of chiral symmetry breaking [6,7].

In our calculations we are concerned with the most realistic simulation of the solution of DS equation. Form factors A and B are given in Ref. [13]. (We shall refer to it as parametrization I). To compare the results of the calculations we also use another form of these form factors taken from Ref. [15] (parametrization II).

Using formulas (2), (3), and the explicit form of form factors A and B one can calculate each term of the series

TABLE II. The dependence of constants h,d, and C on the choice of quark masses with the use of parametrization II [15].

Quark masses (MeV)	h (MeV)	d (MeV)	C (MeV)
$m_u=m_d=10$			
$m_s=150$	68.54	38.24	27.7
$m_u=m_d=m_s=10$	71.87	39.55	29.2

generated by expansion of the logarithmic factor in formula (1a). In this way one can obtain effective Lagrangians describing the decay rates of K mesons in all channels. We list below the following sequence of effective actions responsible for different decay modes:

$$L(K^\pm\to\ell^\pm+\nu)$$
$$=2iG_F\sin\theta_c f_k\left(\frac{f_{\ell k}}{f_k}\right)^2\int dz(\partial^\mu K(x))j_{\ell\mu}(z)+\text{H.c.}, \qquad (4)$$

$$j_{\ell\mu}(z)=\overline{u_\ell}(z)\gamma_\mu L u_\nu(z),\quad \theta_K(z)=\frac{\sqrt{2}}{f_k}K(z);$$

$$L(K^\pm\to\pi^0+\ell^\pm+\nu)$$
$$=iG_F\sin\theta_c H_1\left\{\int dz j_{\ell\mu}(z)[\pi^0(z)(\partial^\mu K(z))\right.$$
$$\left.-(\partial^\mu\pi^0(z))K(z)]\right\}+\text{H.c.}, \qquad (5)$$

$$H_1=\frac{2}{3}\frac{1}{f_k f_\pi}h^2;$$

$$L(K^\pm\to\pi^\pm+\pi^0)$$
$$=G_F\sin\theta_c\cos\theta_c H_2\int dz[\pi^0(z)(\partial_\mu\partial^\mu\pi(z))K^*(z)$$
$$-\pi^0(z)(\partial_\mu\pi(z))(\partial^\mu K^*(z))]+\text{H.c.}, \qquad (6)$$

$$H_2=\frac{N_c}{3}\frac{f_{\ell\pi}^2 d^2}{f_k f_\pi^2};$$

$$L(K^\pm\to 3\pi)$$
$$=G_F\sin\theta_c\cos\theta_c\left\{-\frac{1}{2}H_3\int dz\pi^0[(\partial_\mu K^*)\pi^0(\partial^\mu\pi)\right.$$
$$-(\partial_\mu K^*)\pi(\partial^\mu\pi^0)](z)+H_3'\int dz[K^*\pi^0(\partial_\mu\pi)$$
$$\left.\times(\partial^\mu\pi^0)+3K^*(\partial_\mu\pi)\pi(\partial^\mu\pi)\pi^*](z)\right\}+\text{H.c.}, \qquad (7)$$

$$H_3=\frac{N_c}{\sqrt{2}}\frac{f_{\ell\pi}^2 C^2}{f_k f_\pi^3},\quad H_3'=\frac{N_c^2}{3\sqrt{2}}\frac{d^4}{f_k f_\pi^3}.$$

TABLE III. Decay widths of kaons. Indices I and II mean that in calculations the parametrization I or II was used.

The decay widths	I (keV$\times 10^{-10}$)	II (keV$\times 10^{-10}$)	Experiment (keV$\times 10^{-10}$)
$\Gamma(K^{\pm}\to\mu^{\pm}+\nu)$	0.225	0.094	0.3380 ± 0.00101
$\Gamma(K^{\pm}_{e3})$	0.0166	0.00827	0.0256 ± 0.00032
$\Gamma(K^{\pm}_{\mu 3})$	0.0127	0.00632	0.0169 ± 0.00043
$\Gamma(K^{\pm}\to\pi^{\pm}+\pi^{0})$	0.0636	0.0144	0.1126 ± 0.00085

Constants $f_{\ell\pi}, f_{\ell k}, h, d$, and C entering formulas (4)–(7) were calculated independently using parametrizations I [13] and II [15].

Normalization constants f_π and f_k enter the free Lagrangians of corresponding mesonic fields. Constants $f_{\ell\pi}$ and $f_{\ell k}$ determine the effective Lagrangians for pion and kaon decays in leptonic channels ($\pi^{\pm}K^{\pm}\to\ell^{\pm}+\nu$). In PCAC (partial conservation of axial vector current) theory we are dealing with only one kind of normalization constant f_π and f_k, different for π and K mesons. Their experimental values $f_\pi = 93.5$ MeV and $f_k = 113$ MeV are fixed by the corresponding decay widths of π and K meson (in the purely leptonic channels). These constants must be obviously identical with the normalization factors entering the free Lagrangians of π and K mesonic fields.

In our approach all the normalization factors for pions and kaons have to be calculated independently. In fact, the normalization constants entering the free Lagrangians (f_π, f_k) and the purely leptonic decay actions ($f_{\ell\pi}, f_{\ell k}$) are not identical.

With the parametrization I [13], preserving as much as possible the characteristic features of QCD, it was possible to reproduce quite satisfactorily the pion-pion dynamics in the low-energy region. The calculations with the use of this parametrization (I) give $f_\pi \approx 91$ MeV [13]. Moreover, we have, with an accuracy of the order of 1%, $f_k \approx f_\pi$ (for the quark masses $m_u = m_d = 10$ MeV and $m_s = 150$ MeV).

For the weak decay constants one obtains $f_{\ell\pi} \approx 102.1$ MeV, $f_{\ell k} \approx 101.13$ MeV (for $m_u = m_d = 10$ MeV, $m_s = 150$ MeV). The discrepancy (≈ 10 MeV) between the numerical values of the f_π and $f_{\ell\pi}$ is rather big. For comparison, using the parametrization II [15], one obtains the values $f_\pi = 84.54$ MeV and $f_{\ell\pi} \approx 82.11$ MeV, which are very close.

Numerical values of the constants h, d, and C in formulas (5)–(7), calculated with the use of parametrization I [13] and II [15], are given in Tables I and II.

Using the Lagrangians (4)–(7), the decay rates of charged K mesons in leptonic, semileptonic, and two-pion channels were calculated. The results are presented in Table III. In the calculations for normalization constants f_π and f_k, we use their experimental values 93.5 and 113 MeV, respectively. The Cabibbo angle θ_c is given by $\sin\theta_c \approx 0.224$.

As can be seen, the calculated decay widths essentially depend on the choice of the parametrization of the quark propagators. Parametrization I gives results closer to the experimental values. Nevertheless, even in this case, the calculated decay widths of K mesons in the leptonic and semileptonic channels differ from the experimental values by 30%. For the two-pion decay channels of charged K mesons, the discrepancy is bigger (of the order of 40%).

To find the reason for this discrepancy, we would like to call our attention to two circumstances. The first and probably the most important is that in our approach the weak decay processes are governed by the simplest Weinberg-Salam standard model of electroweak interactions. In this case, the effect of the renormalization of weak-coupling constant g due to the gluonic exchange has not been included.

The second circumstance is that the gluonic effects are included only through the dressing mechanism of quarks. The problem of renormalization of weak vertices due to gluonic exchange is very important and represents a separate interest. In this respect, one of the most important contributions was given in Ref. [18], where the effective Hamiltonian describing the $\Delta S = 1$ strangeness changing weak processes was calculated. To understand the renormalization mechanism, one has to include also other exchange processes, in which the gluon-gluon interactions are taken into account, see [19]. A solution of this problem can allow one to understand the reason for the difference between normalization constants f_π and f_k for π and K mesons.

To include effects of renormalization of weak vertices on the decay rates of K mesons in different channels, we shall carry out some crude phenomenological estimations.

The relation $G_F = (1/4\sqrt{2})(g^2/M_W^2)$, used in the derivation of the Lagrangians (4)–(7), could be modified as

$$G_F \to G'_F = \frac{1}{4\sqrt{2}}\frac{g^2}{M_W^2}(1+t_1) = G_F(1+t_1) \qquad (8a)$$

for leptonic and semileptonic decays of K mesons,

$$G_F \to G''_F = \frac{1}{4\sqrt{2}}\frac{g^2}{M_W^2}(1+t_2) = G_F(1+t_2) \qquad (8b)$$

for leptonic decays of π mesons, and

$$G_F \to G_F(1+t_1)(1+t_2) \qquad (8c)$$

for nonleptonic decay modes of K mesons.

In formulas (8a) and (8b) one introduces two different parameters t_1 and t_2 which correspond to vertices with (quark u)-(quark s)-(vector boson) and (quark u)-(quark d)-(vector boson), respectively. In general, these vertices could be renormalized (by gluon exchange) in different ways. The dimensionless parameters t_1 and t_2 measure the deviation of the weak-coupling constant g from its undressed value.

The nonlinear dependence of the weak-coupling constant in formula (8c) on parameters (t_1, t_2) is due to the fact that the corresponding transition-matrix element is generated by the product of two quark currents. The linear dependence on t_1, t_2 in formulas (8a) and (8b) for the leptonic decay processes is obviously due to the fact that the corresponding matrix elements are linear in quark currents. (The leptonic current coupled to the vector boson is not renormalizable, due to the gluon exchange.) Taking for the parameters in question values

$$t_1=\left(\frac{f_k}{f_{\ell k}}\right)^2-1\simeq 0.25,\quad t_2=\left(\frac{f_\pi}{f_{\ell\pi}}\right)^2-1\simeq -0.15,\qquad (9)$$

one reproduces experimental values of purely leptonic decay widths of $K^\pm$ and $\pi_\pm$ mesons. The semileptonic decay rates of charged K mesons, calculated with t_1 given by Eq. (9), are very close to the experimental data.

$$\begin{aligned}\Gamma_I(K^\pm\to\pi^0+e^\pm+\nu)&=0.0259\times 10^{-10}\ \text{keV}\\&\approx 0.0166(1+t_1)^2\times 10^{-10}\ \text{keV},\qquad (10)\\ \Gamma_I(K^\pm\to\pi^0+\mu^\pm+\nu)&=0.0198\times 10^{-10}\ \text{keV}\\&\approx 0.0127(1+t_1)^2\times 10^{-10}\ \text{keV}.\end{aligned}$$

For the two-pion decay width of charged kaons one obtains

$$\Gamma_I(K^\pm\to\pi^\pm+\pi^0)=0.0636(1+t_1)^2(1+t_2)^2\approx 0.0712\times 10^{-10}\ \text{keV}.$$

This value differs from the experimental one (0.113×10^{-10} keV) by 30%. Agreement with experiment is obtained with $t_1\approx t_2\approx 0.155$.

Using the effective action (7) and parametrization I for the three-pion decay widths of charged K mesons, one obtains

$$\Gamma(K^\pm\to\pi^\pm+\pi^\mp+\pi^\pm)=10^{-3}\times 0.0119\times 10^{-10}\ \text{keV},$$

and

$$\Gamma(K^\pm\to\pi^\pm+2\pi^0)=10^{-3}\times 0.002\,48\times 10^{-10}\ \text{keV}.$$

The corresponding experimental values are [14] $(0.0297\pm 0.000\,27)\times 10^{-10}$ keV and $(0.009\,21\pm 0.00021)\times 10^{-10}$ keV, respectively.

The nonleptonic decay modes of neutral K mesons are described by Lagrangians similar to that given by formulas (6), (7). Calculated decay widths (using these Lagrangians) are three orders of magnitude smaller than the corresponding experimental values in both two- and three-pion channels. To explain the discrepancy with experimental data in nonleptonic decay modes of K mesons, the effect of strong interaction must be included. Within the framework of ChPT, this problem was analyzed in Refs. [3,4,20].

III. CONCLUSIONS

Starting from the QCD effective Lagrangian it is possible to calculate effective actions describing in a unique way weak decay processes of K mesons in all channels. Calculated decay widths essentially depend on the character of the ''dressing'' mechanism of quark propagators. It is possible to reproduce reasonable values of the decay widths in leptonic, semileptonic, and also in the two-pion decay channels of charged kaons. A quantitative agreement (within the accuracy of 30%) with experimental data can be achieved with parametrization I. Calculated two-pion decay widths of neutral K_s mesons and the three-pion decay rates (of kaons) are three orders of magnitude smaller than the experimental ones.

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