

## Constraining the CKM parameters using $CP$ violation in semileptonic $B$ decays

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We discuss the usefulness of the  $CP$  violating semileptonic asymmetry  $a_{SL}$  not only as a signal of new physics, but also as a tool in constraining the Cabibbo-Kobayashi-Maskawa parameters. We show that this technique could yield useful results in the first years of running at the  $B$  factories. We present the analysis graphically in terms of  $M_{12}$ , the dispersive part of the  $B^0\text{-}\bar{B}^0$  mixing amplitude. This is complementary to the usual unitarity triangle representation and often allows a cleaner interpretation of the data. [S0556-2821(99)01819-6]

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The goal of the  $B$  physics programs soon to begin at the  $e^+e^-$ ,  $ep$ , and  $p\bar{p}$  colliders around the world is to test the standard model's predictions for  $CP$  violation. It is important to have a means of quantifying these tests. One seeks measurements and analyses that would not only offer clean signals of new physics, but would also allow the extraction of fundamental standard model parameters.

A reasonable assumption is that the new physics that affects the  $B^0\text{-}\bar{B}^0$  mixing amplitude does not affect either the  $B$  meson decay amplitudes or Cabibbo-Kobayashi-Maskawa (CKM) unitarity.<sup>1</sup> In that case, one can couple the already measured values of  $|V_{ub}|$  and  $\Delta m_B$  with the measurements of  $a_{\psi K_S}$  and  $a_{\pi\pi}$ , the  $CP$ -violating asymmetries in the decays  $B \rightarrow \psi K_S$  and  $B \rightarrow \pi\pi$ , respectively, to construct the unitarity triangle and also disentangle the new physics contributions to  $B^0\text{-}\bar{B}^0$  mixing from the standard model ones [2]. A drawback of this approach is that the unitarity triangle analysis tends to mix up the experimental errors, which are often quite small, with the theoretical errors that arise in relating these measurements to CKM parameters. An attractive alternative is to focus on the dispersive part of the off-diagonal matrix element,  $M_{12}$ , of the  $B^0\text{-}\bar{B}^0$  mixing matrix [3,4]. In this construction, the data is graphically represented in the complex  $M_{12}$  plane [4]. An advantage of this representation is a separation between the experimental uncertainty in  $\Delta m_B$  from the theoretical uncertainty in its calculation. A shortcoming of both approaches is that discrete ambiguities in relating  $a_{\psi K_S}$  and  $a_{\pi\pi}$  to CKM phases lead to multiple solutions for the standard model and new physics parameters [2,5]. Thus, one needs additional information to try and resolve these.

In this paper we use the graphical representation in the  $M_{12}$  plane to highlight the information that can be obtained from a measurement of  $a_{SL}$ , the  $CP$  violation in semileptonic  $B$  decays. The sensitivity of  $a_{SL}$  to new physics is well known [6,7]. We show, in addition, how one can use constraints on, or the observation of  $a_{SL}$  to restrict allowed regions in the standard model parameter space. Such an analysis requires a precise calculation of  $\Delta\Gamma$ , the  $B^0\text{-}\bar{B}^0$  width difference. This calculation uses the notion of local quark-hadron duality, and moreover depends on certain non-perturbative "bag factors." We propose tests of its consistency, and note that its precision should be significantly improved in the near future by new input from lattice calculations.

Under the assumption that the  $B$  decay amplitudes are not affected, all the new physics effects can be expressed in terms of one complex number: the new contribution to the dispersive part of the  $B^0\text{-}\bar{B}^0$  mixing amplitude,  $M_{12}$ . Explicitly, we write

$$M_{12} = M_{12}^0 + \delta M_{12}, \quad (1)$$

where  $M_{12}^0$  represents the standard model contribution and  $\delta M_{12}$  is a complex number representing the new physics contribution. Also useful is the equivalent representation [2],

$$M_{12} = r^2 e^{i2\theta} M_{12}^0. \quad (2)$$

We will work in the convention where the phase of  $M_{12}^0$  is  $2\beta$ , thus that of  $M_{12}$  is  $2(\beta + \theta) \equiv 2\tilde{\beta}$ . (Note, that these phases are measured relative to that of the  $b \rightarrow c\bar{c}d$  decay amplitude.)

The magnitude of  $M_{12}$  is well determined:

$$|M_{12}| = \Delta m_B / 2, \quad (3)$$

<sup>1</sup>For a general analysis of the case where the decay amplitudes are also affected, see [1].

where  $\Delta m_B = 0.470 \pm 0.019 \text{ ps}^{-1} = 3.09 \times 10^{-13} \text{ GeV}$  [8]. We can use this to represent the actual value of  $M_{12} = M_{12}^0 + \delta M_{12}$  as lying somewhere on the unit circle centered at the origin of the complex  $M_{12}$  plane (where all data are rescaled by the experimentally determined central value of  $\Delta m_B/2$ ). The phase of  $M_{12}$ ,  $2\tilde{\beta}$ , will be obtained from the  $CP$  asymmetry in  $B \rightarrow \psi K_S$ :

$$a_{\psi K_S} = \sin 2\tilde{\beta}. \quad (4)$$

We can plot the allowed standard model region in this plane using [9]

$$M_{12}^0 = \frac{G_F^2}{12\pi^2} m_B m_t^2 \eta_B B_{BF}^2 (V_{tb} V_{td}^*)^2 S_0(x_t). \quad (5)$$

Here  $S_0(x_t) \approx 0.784 x_t^{0.76}$  (this is a fit to the exact formula [9]) is a kinematical factor with  $x_t = m_t^2/m_W^2$ . The factor  $\eta_B = 0.55$  is a QCD correction, and typical values for  $\sqrt{B_{BF}}$  are  $200 \pm 40 \text{ MeV}$ . Using  $m_t = 165 \text{ GeV}$ , we find

$$M_{12}^0 = \frac{\Delta m_B}{2} \left| \frac{V_{tb} V_{td}^*}{0.0086} \right|^2 \left( \frac{\sqrt{B_{BF}}}{200 \text{ MeV}} \right)^2 e^{2i\beta}. \quad (6)$$

In the absence of new physics,  $M_{12}^0 = M_{12}$  and one can directly use  $\Delta m_B$  to infer a value for  $|V_{tb} V_{td}^*|$ . Although this is not possible if new physics is present, we can still use the unitarity of the CKM matrix to plot an allowed region for the standard model, and thus constrain  $|V_{tb} V_{td}^*|$ . Using

$$\frac{V_{ub}}{V_{cb}} = a e^{-i\gamma}, \quad 0.06 \leq a \leq 0.10, \quad (7)$$

and considering  $V_{ud} = 0.975$ ,  $V_{cd} = -0.220$ , and  $V_{cb} = 0.0395$  [10] as well determined relative to the other uncertainties in the problem, we obtain

$$\begin{aligned} |V_{tb} V_{td}^*| e^{-i\beta} &= -(V_{cb} V_{cd}^* + V_{ub} V_{ud}^*) \\ &= -0.0395(-0.220 + 0.975 a e^{-i\gamma}). \end{aligned} \quad (8)$$

Using this relation in Eq. (6), we find that as  $a$  covers the stated range and  $\gamma$  varies over 0 to  $2\pi$ ,  $M_{12}^0$  covers a region of the complex  $M_{12}$  plane as shown in Fig. 1.  $M_{12}$ , the full  $B^0-\bar{B}^0$  mixing amplitude can lie anywhere on the solid circle, and  $M_{12}^0$ , the standard model contribution lies somewhere in the region between the two dashed curves. If there were no new physics,  $M_{12}$  would have to lie on the solid circle in one of the two regions where it intersects with the allowed standard model area.

Measuring  $a_{\pi\pi}$ , the  $CP$  asymmetry in  $B \rightarrow \pi\pi$  would give  $\sin 2(\gamma + \tilde{\beta})$  (once the penguin effects are determined). Since, in principle, both  $\tilde{\beta}$  and  $\gamma + \tilde{\beta}$  are known,  $\gamma$  itself is known. For fixed  $\gamma$  the allowed region for  $M_{12}^0$  is a curve extending from the inner to the outer boundary of the  $M_{12}^0$ , as shown in Fig. 2.

Thus, in principle, the  $CP$ -violating measurements  $a_{\psi K_S}$  and  $a_{\pi\pi}$  allow us to disentangle the standard model con-

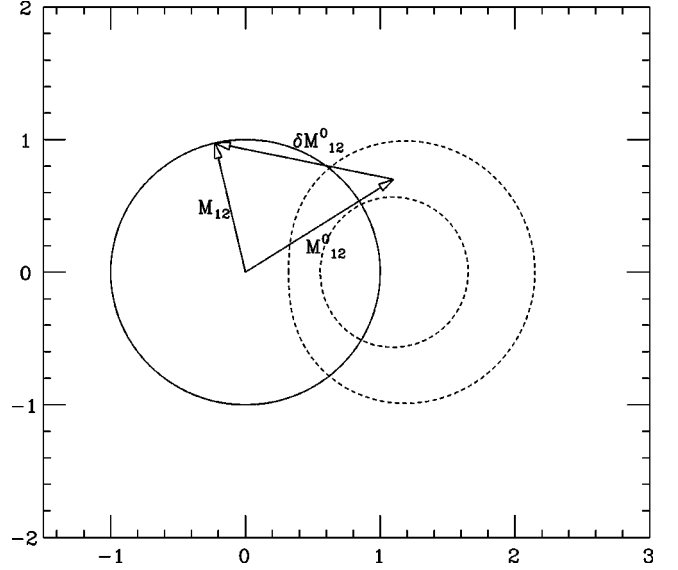


FIG. 1. The complex  $M_{12}$  plane shown in units of  $\Delta m_B/2$ . The measured value of  $M_{12}$  is thus a thin annulus. The standard model contribution,  $M_{12}^0$ , falls within the distorted annulus. The shape is determined by the values of  $V_{td}$  allowed by unitarity, given the measured terms in the CKM matrix. The central value for  $B_{BF}^2 = (200 \text{ MeV})^2$  is used here. The total off-diagonal matrix element is the sum of the standard model contribution and the new physics:  $M_{12} = M_{12}^0 + \delta M_{12}$ .

tribution to  $B^0-\bar{B}^0$  mixing from the new physics contribution. In this construction  $\delta M_{12}$  is the vector extending from the  $\gamma$  curve in the allowed region for  $M_{12}^0$  to the tip of the  $\tilde{\beta}$  vector on the unit circle. This procedure is complementary to that used in [2] to obtain  $r$  and  $\theta$  from these measurements. The advantage in this case is a clean separation of the experimental uncertainties in  $\Delta m_B$  which are small, from the theory errors in the standard model contribution to it. Just as in the unitarity triangle analysis, however, discrete ambiguities in obtaining the phases  $\tilde{\beta}$  and  $\gamma$  lead to multiple allowed regions, thus muddying the situation [2,5]. Without additional inputs, the measurements of  $a_{\psi K_S}$  and  $a_{\pi\pi}$  only allow us to

extract  $2\tilde{\beta}$  up to a twofold ambiguity, and  $\gamma$  up to an eightfold ambiguity. We illustrate this in Fig. 3 based on perfect measurements of the quantities  $a_{\psi K_S} = 0.3$  and  $a_{\pi\pi} = -0.7$ .

As shown in the figure, the true value of the  $B^0-\bar{B}^0$  mixing amplitude,  $M_{12}$  could be either of the points labeled  $a$  or  $b$ . The standard model contribution to it,  $M_{12}^0$  could lie on any one of the curves labeled  $\gamma_1$  through  $\gamma_8$ . If there is no new physics, one can use information from  $K-\bar{K}$  mixing as well as the fact that  $\alpha$ ,  $\beta$ , and  $\gamma$  are the angles of a triangle to reduce these ambiguities to a simple two-fold ambiguity in  $\gamma$ . This is not possible in the presence of new physics, and one needs additional information in order to extract the standard model parameters from the  $CP$ -violating measurements  $a_{\psi K_S}$  and  $a_{\pi\pi}$ . Note, that the  $a_{\psi K_S}$  value chosen here already tells us that there is new physics present in the  $B^0-\bar{B}^0$  mixing amplitude. This can be seen from the fact that neither of the points  $a$  and  $b$  on the  $M_{12}$  circle lies within the allowed

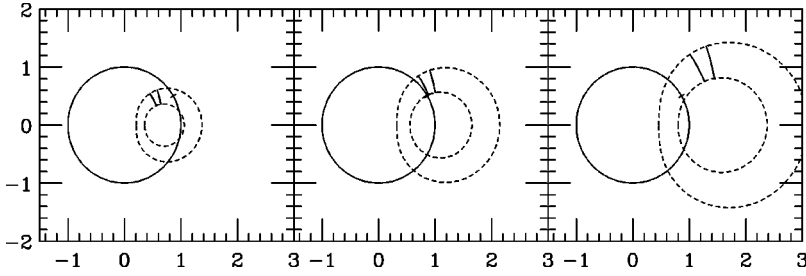


FIG. 2. The complex  $M_{12}$ , in units of  $\Delta m_B/2$ . The value of  $\tilde{\beta}=0.262$  is indicated by the tick mark on the unit circle. The allowed range of  $\gamma$  derived from  $\sin 2(\gamma+\tilde{\beta})=0.43\pm 0.20$  is a slice of the annular region. The three figures correspond, left to right, to the values  $\sqrt{B_B f_B}=160,200,240$  MeV.

standard model region. Although there exist techniques that allow a direct extraction of the angle  $\gamma$ , these are either experimentally difficult [11], or suffer from theoretical uncertainties and sensitivity to new physics [12]. We will now discuss how a measurement of, or constraints on  $a_{SL}$  restricts the allowed standard model parameter space and helps resolve discrete ambiguities.

In machines running at the  $Y(4s)$ ,  $a_{SL}$  is measured by the asymmetry in dilepton events with same sign leptons coming from both  $B$  decays:

$$a_{SL} \equiv \frac{N(l^+l^+) - N(l^-l^-)}{N(l^+l^+) + N(l^-l^-)}, \quad (9)$$

where  $N(l^+l^+)$  [ $N(l^-l^-)$ ] defines the number of times a  $B^0-\bar{B}^0$  pair decays into a pair of positively [negatively] charged leptons. The source of  $a_{SL}$  is  $CP$  violation in the  $B^0-\bar{B}^0$  mixing matrix and it arises due to a phase between the absorptive and dispersive parts of the  $B^0-\bar{B}^0$  mixing amplitude

$$a_{SL} = \text{Im} \left( \frac{\Gamma_{12}}{M_{12}} \right) = \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin \phi_{12}, \quad (10)$$

where  $\phi_{12}$  is the phase between  $\Gamma_{12}$  and  $M_{12}$ . In the standard model,  $a_{SL}$  is unobservably small,  $\sim 10^{-3}$  because  $|\Gamma_{12}/M_{12}| \sim 10^{-2}$  and because the Glashow-Iliopoulos-

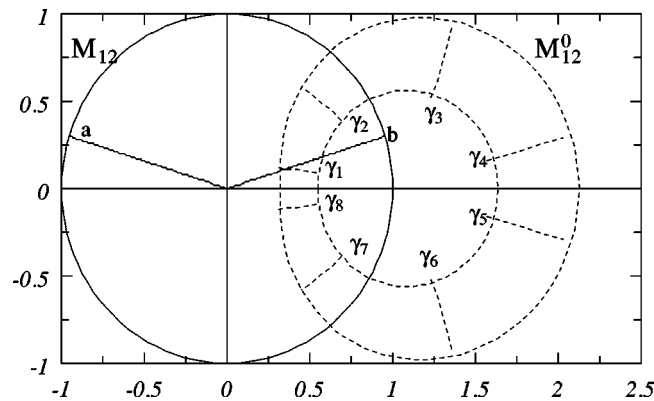


FIG. 3. The complex  $M_{12}$  plane in units of  $\Delta m_B/2$ . We show the twofold discrete ambiguity in the value of  $2\tilde{\beta}$  (the points  $a$  and  $b$ ) and the eightfold ambiguity in  $\gamma$  (the curves labeled  $\gamma_1 \dots \gamma_8$ ) resulting from the measurements  $a_{\psi K_S}=0.3$  and  $a_{\pi\pi}=-0.7$ . We have used  $\sqrt{B_B f_B}=200$  MeV in obtaining the standard model region.

Mal'ani mechanism results in  $\sin \phi_{12} \sim m_c^2/m_b^2 \sim 10^{-1}$ . Thus, new physics can enhance  $a_{SL}$  by increasing  $|\Gamma_{12}/M_{12}|$  and/or  $\sin \phi_{12}$ .

In order for new physics to significantly affect  $\Gamma_{12}$ , one would need either large new decay amplitudes into known states that are common to both  $B^0$  and  $\bar{B}^0$ , or to introduce additional, exotic common final states. Such a scenario could enhance both the factors mentioned above, and could lead to  $a_{SL} \sim 0.1$  [13]. This would be detected in the very early stages of data taking at the asymmetric  $B$  factories, with only about  $10^6$   $B^0-\bar{B}^0$  pairs. Here we concentrate on the more likely possibility where the new heavy particles contribute to  $M_{12}$  but not  $\Gamma_{12}$ . This could lead to enhancements of  $\sin \phi_{12}$ , thus allowing  $a_{SL} \sim 0.01$  [6,7], which would be observable in about one year of running at the  $B$  factories.

Within the standard model, at leading order we have [14–16]

$$\begin{aligned} \Gamma_{12}^0 = & -\frac{G_F^2 m_b^2 m_B}{24\pi} \left[ \frac{5}{3} \frac{m_B^2}{(m_b+m_d)^2} (K_2 - K_1) f_B^2 B_S (V_{tb} V_{td}^*)^2 \right. \\ & + \frac{8}{3} \left( K_1 + \frac{K_2}{2} \right) \\ & \times f_B^2 B_B (V_{tb} V_{td}^*)^2 + 8(K_1 + K_2) \\ & \left. \times f_B^2 B_B \frac{m_c^2}{m_b^2} V_{cb} V_{cd}^* V_{tb} V_{td}^* \right]. \quad (11) \end{aligned}$$

Here  $K_1 = -0.39$  and  $K_2 = 1.25$  [16] are combinations of Wilson coefficients.  $B_S$  and  $B_B$  are the bag factors corresponding to the matrix elements of the operators  $Q_S \equiv (\bar{b}d)_{S-P}(\bar{b}d)_{S-P}$  and  $Q \equiv (\bar{b}d)_{V-A}(\bar{b}d)_{V-A}$ . Combining Eqs. (11) and (5), and using  $m_b = 4.5$  GeV, we have<sup>2</sup>

$$\frac{\Gamma_{12}^0}{M_{12}^0} = -5.0 \times 10^{-3} \left( 1.4 \frac{B_S}{B_B} + 0.24 + 2.5 \frac{m_c^2}{m_b^2} \frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*} \right). \quad (12)$$

In the vacuum saturation approximation one has  $B_S/B_B=1$  at some typical hadronic scale, and this expectation is confirmed by a leading order lattice calculation [17]. Although

<sup>2</sup>Note that obtaining the numerical result requires using  $\eta_B = 0.88$  in Eq. (5) due to the different definition of  $B_B$  in Eq. (11). See Ref. [16] for details.

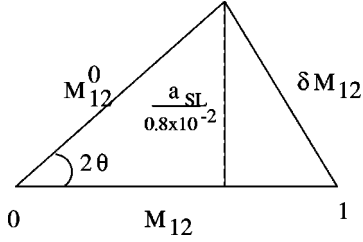


FIG. 4. The relationship between  $M_{12}$ ,  $M_{12}^0$ , and  $a_{SL}$ . The perpendicular distance between  $M_{12}$  and the tip of the  $M_{12}^0$  vector is given by  $a_{SL}/0.8 \times 10^{-2}$ , where  $0.8 \times 10^{-2}$  is the calculated central value of  $\Gamma_{12}^0/M_{12}^0$ .

corrections to the vacuum saturation value are unknown, a more precise lattice calculation of this ratio should be available soon. This would result in a more reliable central value with well defined errors [which are expected to be  $\lesssim \mathcal{O}(25\%)$ ] [17]. Note, however, that the uncertainty due to the ratio of bag factors is restricted to  $\text{Re}(\Gamma_{12}/M_{12}) \approx \Delta\Gamma/\Delta m$ , and that  $\text{Im}(\Gamma_{12}/M_{12})$  which arises from the third term in the parenthesis does not suffer from this uncertainty. Thus,  $a_{SL}$  is precisely calculated in the standard model. From the measured value of  $|V_{ub}/V_{cb}|$  and CKM unitarity we know that  $|\sin\beta| < 0.45$ . Then, using  $m_c^2/m_b^2 = 0.085$  and  $\text{Im}(V_{cb}V_{cd}^*/V_{tb}V_{td}^*) \sim \sin\beta$  leads to the limit  $a_{SL}^{SM} < 10^{-3}$  which is unobservably small. To simplify matters, we will ignore this small phase in the standard model value of  $\Gamma_{12}/M_{12}$ . One can then write

$$\frac{\Gamma_{12}}{M_{12}} = \frac{\Gamma_{12}}{M_{12}^0} \frac{M_{12}^0}{M_{12}} = -(0.8 \pm 0.2) \times 10^{-2} \frac{e^{-i2\theta}}{r^2}, \quad (13)$$

where we have used Eq. (2) and  $B_S/B = 1 \pm 0.25$  in Eq. (12). Thus, Eqs. (10) and (13) lead to

$$\begin{aligned} a_{SL} &= (0.8 \pm 0.2) \times 10^{-2} \text{Im} \left( \frac{M_{12}^0}{M_{12}} \right) \\ &= (0.8 \pm 0.2) \times 10^{-2} \frac{\sin 2\theta}{r^2}. \end{aligned} \quad (14)$$

Combining Eqs. (2) and (14) one sees that  $M_{12}^0$  is given by a vector at an angle  $2\theta$  from  $M_{12}$  and whose tip is a perpendicular distance  $a_{SL}/0.8 \times 10^{-2}$  from it. In Fig. 4 we demonstrate this relation between  $M_{12}$ ,  $M_{12}^0$ , and  $a_{SL}$ .

In Figs. 5, 6, and 7 we use three hypothetical scenarios to highlight the effects of combining  $a_{\psi K_S}$  and  $a_{\pi\pi}$  with  $a_{SL}$  in constraining the allowed standard model parameter space. As before, we use  $\sqrt{B_B}f_B = 200$  MeV and  $0.06 \leq a \leq 0.10$  to construct the allowed standard model region, and assume that  $a_{\psi K_S} = 0.3$ , and  $a_{\pi\pi} = -0.7$  have been measured. In all three figures, the points labeled  $a$  and  $b$  correspond to the

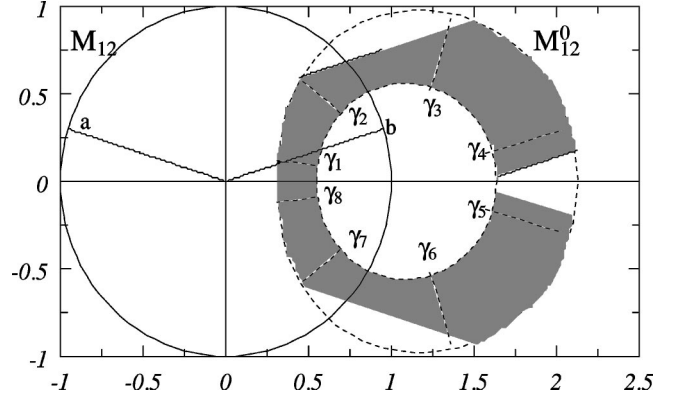


FIG. 5. The complex  $M_{12}$  plane in units of  $\Delta m_B/2$ . The points  $a$  and  $b$  and the curves  $\gamma_1 \dots \gamma_8$  result from the measurements  $a_{\psi K_S} = 0.3$  and  $a_{\pi\pi} = -0.7$ . The shaded region corresponds to the allowed standard model parameter space coming from a measurement of  $|a_{SL}| < 5 \times 10^{-3}$ . We have used  $\sqrt{B_B}f_B = 200$  MeV in obtaining the standard model region.

twofold ambiguity in obtaining the phase of  $M_{12}$  and  $\gamma_1 \dots \gamma_8$  represent the eightfold ambiguity in obtaining  $M_{12}^0$ .

We first discuss what the constraint  $|a_{SL}| < 5 \times 10^{-3}$  would teach us. This should be achievable in one year running at the asymmetric  $B$  factories [18]. As can be seen from Fig. 4,  $M_{12}^0$  must lie in a band of width  $a_{SL}/(0.8 \times 10^{-2})$  above or below  $M_{12}$ . We illustrate this in Fig. 5 where we can see that some of the allowed parameter space is ruled out by this constraint.

Next, in Fig. 6, we illustrate what a measurement of  $a_{SL} < 0$  would teach us. From Eq. (14) we see that  $a_{SL} < 0$  implies  $-\pi < 2\theta < 0$ , thus  $M_{12}^0$  must be either above the  $M_{12}$  vector labeled  $b$ , or below the one labeled  $a$ . This corresponds to the shaded region in the figure, where we see four of the allowed  $\gamma$  curves for  $M_{12}^0$  have been ruled out. The fact that one can obtain this significant restriction on the

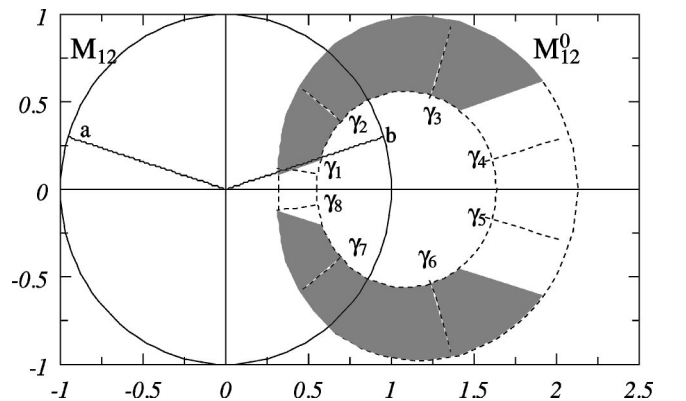


FIG. 6. The complex  $M_{12}$  plane in units of  $\Delta m_B/2$ . The points  $a$  and  $b$  and the curves  $\gamma_1 \dots \gamma_8$  result from the measurements  $a_{\psi K_S} = 0.3$  and  $a_{\pi\pi} = -0.7$ . The shaded region corresponds to the allowed standard model parameter space coming from a measurement of  $a_{SL} < 0$ . We have used  $\sqrt{B_B}f_B = 200$  MeV in obtaining the standard model region.

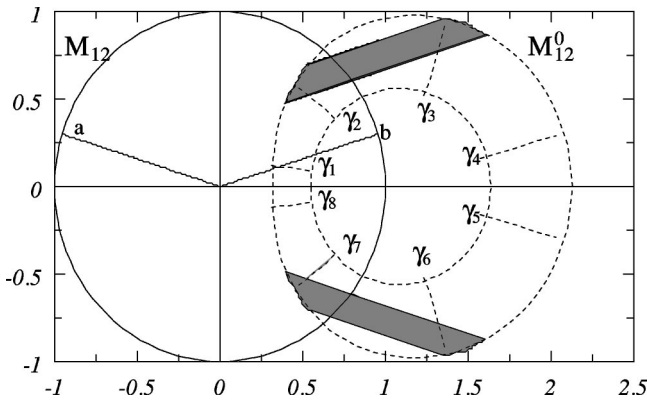


FIG. 7. The complex  $M_{12}$  plane in units of  $\Delta m_B/2$ . The points  $a$  and  $b$  and the curves  $\gamma_1 \dots \gamma_8$  result from the measurements  $a_{\psi K_S} = 0.3$  and  $a_{\pi\pi} = -0.7$ . The shaded region corresponds to the allowed standard model parameter space coming from a measurement of  $a_{SL} = (-5 \pm 1) \times 10^{-3}$ . We have used  $\sqrt{B_B} f_B = 200$  MeV in obtaining the standard model region.

standard model allowed region just from the sign of  $a_{SL}$  has the major advantage that one does not need a very precise measurement of  $a_{SL}$ , just one that is  $3\sigma$  from zero. This would be useful if  $a_{SL}$  turns out to be large.

Finally, in Fig. 7 we show the constraints for the same value of  $a_{\psi K_S}$  and  $a_{\pi\pi}$ , but now with a measurement of  $a_{SL} = (-5 \pm 1) \times 10^{-3}$ . In this case the standard model point must lie in one of the two shaded bands parallel to the  $M_{12}$  vectors  $a$  and  $b$ , respectively. The width of the bands includes both the assumed experimental error in the measurement of  $a_{SL}$ , as well as the theoretical uncertainty in the coefficient of  $a_{SL}$  [cf. Eq. (14)]. Notice that for particular values of  $\gamma$  we now know both  $\sin 2\theta$  and  $r^2$ , hence one has not only resolved the standard model parameters, but also the new physics ones.

A crucial ingredient in the discussion so far is the reliability of the standard model calculation of  $\Gamma_{12}^0$ , which is essentially a long distance quantity. The calculation consists of an inclusive sum over final states that are common to the  $B$  and the  $\bar{B}$ . Thus, it is reliable to the extent that one can use the

notion of local quark-hadron duality in doing such a calculation. Although this is expected to be correct [19], there have been objections to this calculation [20,21], and it is important to be able to test its accuracy. One such test is by measuring  $CP$  violation in semi-inclusive hadronic  $B$  decays as proposed in [22]. The  $B$  decays into these semi-inclusive channels are precisely those that are summed to give  $\Delta\Gamma$  and  $a_{SL}$ . An agreement between the measurements and theoretical expectation would support the use of local quark-hadron duality in this calculation. Another test is available within the  $B_s$  system, where there exist two complementary calculations of the quantity  $\Delta\Gamma = \text{Re}\Gamma_{12}$ . One done by actually summing over common final states [23] and one using quark-hadron duality [16]. The fact that both give similar answers and with the same sign could be an indication of the reliability of the quark level calculation. More importantly,  $\Delta\Gamma_s$  may actually be large enough to be measurable. A measurement of  $\Delta\Gamma_s$  which agrees with the quark level prediction would be a further indication of the correctness of the calculation.

To conclude, we have discussed the information one can obtain from a measurement of  $a_{SL}$ . The standard model prediction  $a_{SL} \lesssim 10^{-3}$  is robust within the assumption of quark-hadron duality. Thus, a measurement in contradiction with this limit would indicate the presence of physics beyond the standard model. Within this theoretical framework, we have shown how a measurement of  $a_{SL}$  could help constrain the CKM parameters, and remove some of the discrete ambiguities in their phases. This method depends on the ratio of bag factors  $B_S/B_B$  which is poorly known at present, but for which there should be improved lattice calculations available soon. Finally, we have presented the analysis in terms of  $M_{12}$ , which affords us something quite beyond what is available with the unitarity triangle. This graphical representation [4] correctly represents where the real uncertainties lie. They are not in  $\Delta m_B$ , which is known quite well, but in our estimation of the standard model prediction of  $M_{12}$ .

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