Finding the CP-violating Higgs bosons at e^+e^- colliders

Bohdan Grzadkowski*
Institute of Theoretical Physics, Warsaw University, Warsaw, Poland

John F. Gunion†
Davis Institute for High Energy Physics, University of California at Davis, California 95616

Jan Kalinowski‡
Institute of Theoretical Physics, Warsaw University, Warsaw, Poland

(Received 11 February 1999; published 10 September 1999)

We discuss a general two-Higgs-doublet model with CP violation in the Higgs sector. In general, the three neutral Higgs fields of the model all mix and the resulting physical Higgs bosons have no definite CP properties. We derive a new sum rule relating Yukawa and Higgs-Z couplings which implies that a neutral Higgs boson cannot escape detection at an e^+e^- collider if it is kinematically accessible in $Z+$Higgs boson, $b\bar{b}+$Higgs boson and $t\bar{t}+$Higgs boson production, irrespective of the mixing angles and the masses of the other neutral Higgs bosons. We also discuss modifications of the sum rules and their phenomenological consequences in the case when the two-doublet Higgs sector is extended by adding one or more singlets. A brief discussion of the implications of the sum rules for Higgs boson discovery at the Fermilab Tevatron and CERN LHC is given. [S0556-2821(99)05917-2]

PACS number(s): 14.80.Cp, 11.30.Er, 11.30.Qc, 12.60.Fr

I. INTRODUCTION

Despite the spectacular successes of high-energy physics (e.g. precision tests of the standard model), the origins of mass and of CP violation still remain mysteries from both the experimental and the theoretical points of view. Models of mass generation by electroweak symmetry breaking driven by elementary scalar dynamics predict the existence of one or more physical Higgs bosons. The minimal model is a one-doublet Higgs sector as employed in the standard model (SM), which gives rise to fermion masses and to a single physical CP-even Higgs scalar boson, $h_{\rm SM}$. But, a Higgs boson has yet to be observed. Regarding CP, there is only one solid experimental signal of CP violation, namely $K_L^0 \to \pi^+\pi^-$ decay [1]. The classical method for incorporating CP violation into the SM is to make the Yukawa couplings of the Higgs boson to quarks explicitly complex, as built into the Kobayashi-Maskawa mixing matrix [2] proposed more than two decades ago. However, CP violation could equally well be partially or wholly due to other mechanisms. The possibility that CP violation derives largely from the Higgs sector itself is particularly appealing. Even the simple two-Higgs-doublet model (2HDM) extension of the one-doublet SM Higgs sector provides a much richer framework for describing CP violation; in the 2HDM, spontaneous and/or explicit CP violation is possible in the scalar sector [3].

The CP-conserving (CPC) version of the 2HDM has received considerable attention, especially in the context of the minimal supersymmetric model (MSSM) [4]. It predicts[1] the existence of two neutral CP-even Higgs bosons (h^0 and H^0, with $m_{h^0} \leq m_{H^0}$), one neutral CP-odd Higgs boson (A^0) and a charged Higgs pair ($H^\pm$). However, in a general 2HDM with CP-violation (CPV) in the scalar sector, the three electrically neutral Higgs fields mix and the physical mass eigenstates, h_i ($i=1,2,3$), have undefined CP properties.

The absence of any $e^+e^- \to Zh_{\rm SM}$ signal in CERN e^+e^- collider LEP1 data (where the Z is virtual) and LEP2 data (where the Z is real) translates into a lower limit on $m_{h_{\rm SM}}$ which has been increasing as higher energy data becomes available. The latest analysis of four LEP experiments at $\sqrt{s}$ up to 189 GeV implies $m_{h_{\rm SM}}$ greater than 87.5 GeV (ALEPH), 94.1 GeV (DELPHI), 95.5 GeV (L3), 94.0 GeV (OPAL) [6]. The negative results of Higgs boson searches at LEP can be formulated as restrictions on the parameter space of the 2HDM and more general Higgs sector models. As has been shown in Refs. [7,8], the sum rules for the Higgs-boson–Z-boson couplings derived in the CP-conserving 2HDM can be generalized to the CP-violating case to yield a sum rule [see Eq. (14)] that requires at least one of the ZZh_i, ZZh_j and Zh_ih_j (*any* $i\neq j$, $i,j=1,2,3$) couplings to be substantial in size. Very roughly, this implies that if there are two light Higgs bosons with $m_{h_i}+m_{h_j}$, $m_{h_i}+m_Z$ and $m_{h_j}+m_Z$ all sufficiently below $\sqrt{s}$, then at least one will be observable. A recent analysis of LEP data shows that the 95% confidence level exclusion region in the (m_{h_i},m_{h_j}) plane that results from the general sum rule is quite significant [9].

*Email address: bohdang@fuw.edu.pl
†Email address: jfgucd@pc90.ucdavis.edu
‡Email address: kalino@fuw.edu.pl

[1]However, with soft-supersymmetry CP-violating phases, the h^0, H^0 and A^0 will mix beyond the Born approximation [5].

It is also appropriate to consider the implications of the precision LEP and Tevatron electroweak data for the general 2HDM. In the context of the SM, $m_{h_{\rm SM}} \leqslant 260$ GeV is required for $\Delta\chi^2 \leqslant (1.64)^2$ (corresponding to 95% C.L. for a one-sided distribution) [10]. In the 2HDM, any neutral Higgs boson with significant ZZh coupling (g_{WWh}/g_{ZZh} is the same as in the SM) contributes to $\Delta\rho$ an amount given by $g^2_{ZZh}/g^2_{ZZh_{\rm SM}}$ times the contribution of a SM Higgs boson of the same mass. In the absence of additional contributions to $\Delta\rho$, the SM limit roughly converts to the requirement that at least one of the neutral h_i must have mass below 260 GeV *and* have substantial ZZh_i coupling. However, if the Higgs bosons of the 2HDM are not all degenerate, there can be additional positive contributions to $\Delta\rho$ which compensate an enhanced negative contribution to $\Delta\rho$ (by virtue of larger m_h) from the diagrams involving the ZZh and WWh couplings. Very roughly [4], substantial extra contributions arise when there is a (neutral) h_i with $|m_{h_i} - m_{H^\pm}|$ and $g_{h_i H^\pm W^\mp}$ both large or when there is a neutral h_i-h_j pair with $|m_{h_i} - m_{h_j}|$ and $g_{h_i h_j Z}$ both large. In the MSSM, one is protected against such situations by the natural ''decoupling'' limit of the model. In the general 2HDM, significant extra positive $\Delta\rho$ contributions are possible in a general scan over model parameters. Thus, constraints from the precision data are complicated and will not be directly implemented here.

In this paper, we consider the 2HDM in the context of higher energy e^+e^- linear colliders ($\sqrt{s} \sim 350 - 1600$ GeV). The general question we wish to address is whether we are guaranteed to see any neutral Higgs boson that is light. Two scenarios give cause for concern.

First, the precision electroweak suggestion of a light h_i with significant ZZh_i coupling could prove correct, in which case the h_i will be seen in $e^+e^- \to Z^* \to Zh_i$ Higgs-strahlung production. However, it could happen that there are actually two light Higgs bosons. We denote the second by h_j. There are then two possibilities allowed by the above-mentioned sum rule [Eq. (14)]. (a) If the h_i observed in Zh_i does not have full strength ZZh_i coupling then either the Zh_ih_j or ZZh_j coupling (or both) must be substantial and h_j will be observable in the h_ih_j or Zh_j final state (or both) provided $m_{h_i} + m_{h_j} < \sqrt{s} - \Delta$ and $m_{h_j} + m_Z < \sqrt{s} - \Delta'$, where Δ and Δ' generically represent the subtractions from the absolute kinematic limits due to backgrounds, efficiencies and finite luminosity. (b) If the h_i has full strength ZZh_i coupling, then the sum rule guarantees that the Zh_ih_j and Zh_j couplings vanish and, therefore, the h_j will not be discovered via Higgs-strahlung (Zh_j) or pair (h_ih_j) production. (Note that the above conclusions hold regardless of the mixing structure of the neutral Higgs boson sector.) It is case (b) that causes concern.

A second, and even worse scenario, is the following. It could happen that there is only one light h_i but model parameters conspire so that it has a ZZh_i coupling that is too weak for its detection in Higgs-strahlung production while at the same time precision electroweak constraints are satisfied.

The primary result of the present paper is the derivation of new sum rules that relate the Yukawa and Higgs-Z couplings of the 2HDM [see Eq. (17)] in such a way as to guarantee that any h_i that is sufficiently light ($m_{h_i} + 2m_t < \sqrt{s} - \Delta$) will be observable regardless of the mixing structure of the neutral Higgs boson sector and independent of the masses of the other Higgs bosons. Very roughly, this new sum rule implies that if the Higgs-strahlung cross section for h_i is small because of small ZZh_i coupling, then the cross section for either $b\bar{b}h_i$ or $t\bar{t}h_i$ (dominated by Higgs radiation from the final state fermions) will be large enough to be detected in the clean environment of e^+e^- collisions.

We shall also discuss the extension of these sum rules to the two-doublet plus one-singlet CP-violating model. We find that there is no guarantee that a single light Higgs boson will be observable. However, the extended sum rules do imply that if there are three light (as defined above) Higgs bosons, then at least one will be observable in e^+e^- collisions via production in association with $b\bar{b}$ or $t\bar{t}$.

Before proceeding, it should be emphasized that our results make no assumption as to the nature of the model at energies above the Higgs boson masses. As shown in Ref. [11], demanding perturbativity for all couplings up to a scale of order the Planck mass places strong constraints on the spectrum of those Higgs bosons that have substantial ZZ coupling. These constraints are such that the next generation of e^+e^- collider would be able to see Zh production for at least one Higgs boson or collection of Higgs bosons. Our focus here is on results that apply purely as a result of the structure of the low-energy Higgs sector model.

The paper is organized as follows. In Sec. II, we outline how CP violation arises in the 2HDM and give the general forms of the ZZ-Higgs, Z-Higgs-Higgs, and Higgs Yukawa couplings in terms of the matrix specifying the mixing of the neutral Higgs bosons. In Sec. III, we present the crucial sum rules for these couplings. In Sec. IV, we specify the existing experimental constraints that we require be satisfied as we scan over Higgs masses and mixing parameters. Numerical results for Zh_1h_2, $b\bar{b}h_1$ and $t\bar{t}h_1$ cross sections resulting from the scan over 2HDM parameter space are presented and discussed in Sec. V. In Sec. VI, we extend the sum rules to the case of the two-doublet plus one-singlet Higgs sector and outline implications. In Sec. VII, we present an outline of the impact of the sum rules for Higgs boson discovery at the Fermilab Tevatron and CERN Large Hadron Collider (LHC). Concluding remarks are given in Sec. VIII. The Appendix presents the detailed cross section formula for the $e^+e^- \to f\bar{f}h_i$ process allowing for Higgs boson mixing and CP violation.

II. THE TWO-HIGGS-DOUBLET MODEL WITH CP VIOLATION

The 2HDM of electroweak interactions contains two SU(2) Higgs doublets denoted by $\Phi_1 = (\phi_1^+, \phi_1^0)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$ and is defined by Yukawa couplings and the Higgs potential. The most general renormalizable scalar potential for the model has the following form:

$$V(\Phi_1,\Phi_2)=V_{symm}(\Phi_1,\Phi_2)+V_{soft}(\Phi_1,\Phi_2)+V_{hard}(\Phi_1,\Phi_2) \quad (1)$$

$$V_{symm}(\Phi_1,\Phi_2)=-\mu_1^2\Phi_1^\dagger\Phi_1-\mu_2^2\Phi_2^\dagger\Phi_2+\lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4|\Phi_1^\dagger\Phi_2|^2+\frac{1}{2}[\lambda_5(\Phi_1^\dagger\Phi_2)^2+\mathrm{H.c.}]$$

$$V_{soft}(\Phi_1,\Phi_2)=-\mu_{12}^2\Phi_1^\dagger\Phi_2+\mathrm{H.c.}$$

$$V_{hard}(\Phi_1,\Phi_2)=\frac{1}{2}\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)+\frac{1}{2}\lambda_7(\Phi_2^\dagger\Phi_2)\times(\Phi_1^\dagger\Phi_2)+\mathrm{H.c.}$$

If both of the two Higgs boson doublets couple to up- or to down-type quarks (or to both types), flavor changing neutral currents (FCNC) are generated at tree level. To avoid FCNC, it is customary to impose a discrete Z_2 symmetry under which

$$\Phi_2\to-\Phi_2,\quad u_{iR}\to-u_{iR} \quad (2)$$

and the other fields are unchanged. Then, Φ_2 couples only to up-type quarks and Φ_1 couples only to down-type quarks and leptons. The resulting invariant fermion-Higgs Yukawa interactions can be written in the form

$$\mathcal{L}_Y=-(\overline{u_i},\overline{d_i})_L\Gamma_u^{ij}\tilde{\Phi}_2u_{jR}-(\overline{u_i},\overline{d_i})_L\Gamma_d^{ij}\Phi_1d_{jR}-(\overline{\nu_i},\overline{e_i})_L\Gamma_e^{ij}\Phi_1e_{jR}+\mathrm{H.c.}, \quad (3)$$

where i,j are generation indices and $\tilde{\Phi}_2$ is defined as $i\sigma_2\Phi_2^*$. Only the first term $V_{symm}(\Phi_1,\Phi_2)$ in Eq. (2) is symmetric under Z_2. However, if the Z_2 symmetry is broken only softly (that is by operators of dimension 3 and less) then renormalizability is preserved [12] and FCNC effects remain small. The unique soft-breaking term is that appearing in $V_{soft}(\Phi_1,\Phi_2)$. The dimension 4 terms contained in $V_{hard}(\Phi_1,\Phi_2)$ break the Z_2 symmetry in a hard way and therefore cannot be accepted.[2]

The 2HDM Higgs sector can exhibit either explicit or spontaneous *CP* violation. *CP* violation is explicit if there is no choice of phases such that all the potential parameters are real. *CP* violation is said to be spontaneous if the potential minimum is such that one of the two vacuum expectation values is complex, even though all the potential parameters can be chosen to be real. If only V_{symm} is present then neither explicit nor spontaneous *CP* violation can be present in the Higgs sector [13]. In fact, when FCNC are suppressed by imposing *exact* Z_2 symmetry, one must introduce a third Higgs doublet in order to allow for *CP* violation in the Higgs sector. However, both explicit and spontaneous *CP* violation in the 2HDM become possible even if the Z_2 symmetry is only broken softly. The *CP* violation will be explicit in $V_{symm}+V_{soft}$ if $\mathrm{Im}(\mu_{12}^{*\,4}\lambda_5)\neq0$. When $\mathrm{Im}(\mu_{12}^{*\,4}\lambda_5)=0$, spontaneous *CP* violation can arise as follows. Without loss of generality, the phase of Φ_1 can be chosen such that its vacuum expectation value is real and positive, $\langle\Phi_1\rangle=v_1/\sqrt{2}$ (with $v_1>0$), and the phase of Φ_2 such that the λ_5 coupling is real and positive. Then, the second Higgs doublet will have a complex vacuum expectation value, $\langle\Phi_2\rangle=v_2e^{i\theta}/\sqrt{2}$ ($v_2>0$ by convention),[3] provided

$$\left|\frac{\mu_{12}^2}{\lambda_5v_1v_2}\right|<1, \quad (4)$$

since, then, the minimum of the potential occurs for [14]

$$\cos\theta=\frac{\mu_{12}^2}{\lambda_5v_1v_2}. \quad (5)$$

Therefore, the 2HDM with Higgs potential given by $V_{soft}+V_{symm}$ is a very attractive and simple model in which to explore the implications of *CP* violation in the Higgs sector.

After SU(2)×U(1) gauge symmetry breaking, one combination of neutral Higgs fields, $\sqrt{2}(c_\beta\,\mathrm{Im}\,\phi_1^0+s_\beta\,\mathrm{Im}\,\phi_2^0)$, becomes a would-be Goldstone boson which is absorbed in giving mass to the Z gauge boson. (Here, we use the notation $s_\beta\equiv\sin\beta$, $c_\beta\equiv\cos\beta$, where $\tan\beta=v_2/v_1$.) The same mixing angle, β, also diagonalizes the mass matrix in the charged Higgs sector. If either explicit or spontaneous *CP* violation is present, the remaining three neutral degrees of freedom,

$$(\varphi_1,\varphi_2,\varphi_3)\equiv\sqrt{2}(\mathrm{Re}\,\phi_1^0,\mathrm{Re}\,\phi_2^0,s_\beta\,\mathrm{Im}\,\phi_1^0-c_\beta\,\mathrm{Im}\,\phi_2^0) \quad (6)$$

are not mass eigenstates. The physical neutral Higgs bosons h_i ($i=1,2,3$) are obtained by an orthogonal transformation, $h=R\varphi$, where the rotation matrix is given in terms of three Euler angles $(\alpha_1,\alpha_2,\alpha_3)$ by

$$R=\begin{pmatrix} c_1 & -s_1c_2 & s_1s_2 \\ s_1c_3 & c_1c_2c_3-s_2s_3 & -c_1s_2c_3-c_2s_3 \\ s_1s_3 & c_1c_2s_3+s_2c_3 & -c_1s_2s_3+c_2c_3 \end{pmatrix}, \quad (7)$$

where $s_i\equiv\sin\alpha_i$ and $c_i\equiv\cos\alpha_i$. Without loss of generality, we assume $m_{h_1}\leqslant m_{h_2}\leqslant m_{h_3}$.

As a result of the mixing between real and imaginary parts of neutral Higgs fields, the Yukawa interactions of the h_i mass eigenstates are not invariant under *CP*. They are given by

$$\mathcal{L}=h_i\overline{f}(S_i^f+iP_i^f\gamma_5)f \quad (8)$$

where the scalar (S_i^f) and pseudoscalar (P_i^f) couplings are functions of the mixing angles. For up-type quarks we have

[2]If V_{hard} is present, there is no argument for dropping the FCNC Yukawa terms which are also of dimension 4.

[3]In this normalization $v\equiv\sqrt{v_1^2+v_2^2}=2m_W/g=246$ GeV.

$$S_i^u = -\frac{m_u}{v s_\beta} R_{i2}, \quad P_i^u = -\frac{m_u}{v s_\beta} c_\beta R_{i3}, \tag{9}$$

and for down-type quarks one finds

$$S_i^d = -\frac{m_d}{v c_\beta} R_{i1}, \quad P_i^d = -\frac{m_d}{v c_\beta} s_\beta R_{i3}, \tag{10}$$

and similarly for charged leptons. For large $\tan\beta$, the couplings to down-type fermions are typically enhanced over the couplings to up-type fermions.

In the following analysis we will also need the couplings of neutral Higgs and Z bosons; they are given by

$$g_{ZZh_i} \equiv \frac{g m_Z}{c_W} C_i = \frac{g m_Z}{c_W} (s_\beta R_{i2} + c_\beta R_{i1}) \tag{11}$$

$$g_{Zh_ih_j} \equiv \frac{g}{2c_W} C_{ij} = \frac{g}{2c_W} (w_i R_{j3} - w_j R_{i3}) \tag{12}$$

$$g_{ZZh_ih_j} \equiv \frac{g^2}{2c_W^2} X_{ij} = \frac{g^2}{2c_W^2} \sum_{k=1}^{3} R_{ik} R_{jk} \tag{13}$$

where $w_i = s_\beta R_{i1} - c_\beta R_{i2}$, $c_W = \cos\theta_W$, g is the SU(2) gauge coupling constant and m_Z denotes the Z-boson mass. In the case of the 2HDM, $X_{ij} = \delta_{ij}$ by virtue of the orthogonality of R and its 3×3 dimensionality; in particular, the ZZh_ih_j coupling is not suppressed by mixing angles.

The CP-conserving limit can be obtained as a special case: $\alpha_2 = \alpha_3 = 0$. Then, if we take $\alpha_1 = \pi/2 - \alpha$, α is the conventional mixing angle that diagonalizes the mass-squared matrix for $\sqrt{2}\mathrm{Re}\,\phi_1^0$ and $\sqrt{2}\mathrm{Re}\,\phi_2^0$. The resulting mass eigenstates are $h_1 = -h^0$, $h_2 = H^0$ and $\sqrt{2}(s_\beta \mathrm{Im}\,\phi_1^0 - c_\beta \mathrm{Im}\,\phi_2^0) = -A^0$, where h^0, H^0 (A^0) are the CP-even (CP-odd) Higgs bosons defined earlier for the CPC 2HDM.

III. SUM RULES FOR THE HIGGS BOSON COUPLINGS

As discussed earlier, we wish to determine whether or not the additional freedom in Higgs boson couplings in the general CP-violating 2HDM (by tuning the mixing angles one can suppress certain couplings) is sufficient to jeapordize our ability to find light neutral Higgs bosons. We will show that the unitarity of R_{ij} implies a number of interesting sum rules for the Higgs couplings which prevent the hiding of any neutral Higgs boson that is sufficiently light to be kinematically accessible (a) in Higgs-strahlung *and* Higgs pair production, or (b) Higgs-strahlung *and* $b\bar{b}$+Higgs boson *and* $t\bar{t}$+Higgs boson.

(a) Let us first recall the sum rule for Higgs-Z couplings that requires at least one of the ZZh_i, ZZh_j and Zh_ih_j (*any* $i\neq j$, $i,j=1$, 2,3) couplings to be substantial in size [8], namely

$$C_i^2 + C_j^2 + C_{ij}^2 = 1 \tag{14}$$

where $i\neq j$ are any two of the three possible indices.[4] The power of Eq. (14) with $i,j=1,2$ for LEP physics derives from two facts: it involves only two of the neutral Higgs bosons; and the experimental upper limit on any one C_i^2 derived from $e^+e^-\to Zh_i$ data is very strong —$C_i^2 \lesssim 0.1$ for $m_{h_i} \lesssim 70$ GeV. Thus, if h_1 and h_2 are both below about 70 GeV in mass, then Eq. (14) requires that $C_{12}^2 \sim 1$, whereas for such masses the limits on $e^+e^-\to h_1h_2$ from LEP2 data require $C_{12}^2 \ll 1$. As a result, there cannot be two light Higgs bosons even in the general CP-violating case; the excluded region in the (m_{h_1}, m_{h_2}) plane that results from a recent analysis by the DELPHI Collaboration can be found in Ref. [9].

At a higher energy e^+e^- collider, Eq. (14) will have many possible applications. If no Higgs boson is discovered in Higgs-strahlung or Higgs pair production, Eq. (14) will imply that at least one of $m_{h_i}+m_{h_j}$, $m_{h_i}+m_Z$ and $m_{h_j}+m_Z$ must be $>\sqrt{s}-\Delta$ for any choice of i and j. However, as noted earlier, this does not preclude the possibility that there is a light h_i with $m_{h_i}+m_Z<\sqrt{s}-\Delta$ but with small ZZh_i coupling. More likely, the precision electroweak suggestion will turn out to be correct and at the e^+e^- collider we will find at least one Higgs boson in $e^+e^-\to Zh_i$ production (note that h_i need not be the lightest neutral Higgs boson) and measure its C_i with good accuracy. If the observed h_i has $C_i\sim 1$, then Eq. (14) implies that any other h_j must have small ZZh_j and Zh_ih_j couplings and will not be observable in Higgs-strahlung or Higgs pair production (in association with the observed h_i). If the measured C_i is substantially smaller than 1, then Eq. (14) implies that either $e^+e^-\to h_ih_j$ or $e^+e^-\to Zh_j$ would have a substantial rate for any sufficiently light h_j ($j\neq i$). If a second h_j has not been detected, we would then conclude that $m_{h_j} > \min[\sqrt{s}-m_{h_i}-\Delta, \sqrt{s}-m_Z-\Delta']$ for the other two $j\neq i$ neutral Higgs bosons.

(b) If even one of the three processes, Zh_1, Zh_2 (Higgs-strahlung) and h_1h_2 (pair production), is beyond the collider's kinematical reach, the sum rule in Eq. (14) is not sufficient to guarantee h_1 or h_2 discovery. For example, suppose that h_1h_2 production is not kinematically allowed. Equation (14) can be satisfied by taking $C_{12}\sim 1$ and $C_{1,2}\sim 0$. For these choices, Zh_1 and Zh_2 production would be suppressed and unobservable (even if kinematically allowed) because of small C_1 and C_2, respectively. However, we find that the Yukawa and ZZ couplings of any one Higgs boson also obey sum rules which require that at least one of these couplings has to be sizable; i.e., if $C_i\sim 0$ at least one h_i Yukawa coupling must be large. Thus, if an h_i is sufficiently light, its detection will be possible, irrespective of the neutral Higgs sector mixing.

To derive the relevant sum rules, it is convenient to introduce rescaled couplings

[4]Another interesting sum rule reads $C_{ij}^2 = C_k^2$ for (i,j,k) being any permutation of (1,2,3).

$$\hat{S}_i^f \equiv \frac{S_i^f v}{m_f}, \quad \hat{P}_i^f \equiv \frac{P_i^f v}{m_f}, \tag{15}$$

where $f=t,b$.[5] Using Eqs. (9) and (10), one finds

$$(\hat{S}_i^t)^2+(\hat{P}_i^t)^2=\left(\frac{\cos\beta}{\sin\beta}\right)^2[R_{i3}^2+R_{i2}^2/\cos^2\beta]; \tag{16}$$

$$(\hat{S}_i^b)^2+(\hat{P}_i^b)^2=\left(\frac{\sin\beta}{\cos\beta}\right)^2[R_{i3}^2+R_{i1}^2/\sin^2\beta].$$

Using the unitarity of R_{ij}, these can be written as

$$(\hat{S}_i^t)^2+(\hat{P}_i^t)^2=\left(\frac{\cos\beta}{\sin\beta}\right)^2\left[1+\frac{C_i}{\cos^2\beta}(2\hat{S}_i^b\cos^2\beta+C_i)\right]; \tag{17}$$

$$(\hat{S}_i^b)^2+(\hat{P}_i^b)^2=\left(\frac{\sin\beta}{\cos\beta}\right)^2\left[1+\frac{C_i}{\sin^2\beta}(2\hat{S}_i^t\sin^2\beta+C_i)\right].$$

From Eq. (17), we see that if a light Higgs boson h_i has suppressed coupling to ZZ, $C_i\to 0$, then $(\hat{S}_i)^2+(\hat{P}_i)^2$ for the top and bottom quark rescaled couplings behaves as $\cot^2\beta$ and $\tan^2\beta$, respectively. If $C_i=\pm 1$, i.e., full strength ZZh_i coupling, one finds that $(\hat{S}_i)^2+(\hat{P}_i)^2\to 1$, for both the top and the bottom quark couplings, in the limit of either very large or very small $\tan\beta$. More generally, combining the two sum rules, as written in Eq. (16), and using unitarity again, we find

$$\sin^2\beta[(\hat{S}_i^t)^2+(\hat{P}_i^t)^2]+\cos^2\beta[(\hat{S}_i^b)^2+(\hat{P}_i^b)^2]=1 \tag{18}$$

independently of C_i. Equation (18) implies that the Yukawa couplings to top and bottom quarks cannot be simultaneously suppressed. As the earlier examples show, the relative weighting is a sensitive function of both $\tan\beta$ and C_i. In some sense, the most pessimistic case for measuring the Yukawa couplings is $|C_i|=1$ in that it forbids significant enhancement for either the top or the bottom Yukawa couplings — both are SM-like in the limit of large or small $\tan\beta$. Still, Eq. (18) guarantees that, with sufficient integrated luminosity, determination of at least one of the two Yukawa couplings will be possible for any h_i kinematically accessible in $t\bar{t}h_i$ (as well as $b\bar{b}h_i$) production.

The above makes it apparent that the complete Higgs hunting strategy at e^+e^- colliders, and at hadron colliders as well, should include not only the Higgs-strahlung process and Higgs pair production but also the Yukawa processes[6] with Higgs radiation off top and bottom[7] quarks. Details of this strategy at a future e^+e^- collider are discussed in Sec. V.

For definiteness, in what follows we will consider the high luminosity option that has been examined in the context of the DESY TeV Energy Superconducting Linear Accelerator (TESLA) design, for which one expects $L=500\ \mathrm{fb}^{-1}\mathrm{y}^{-1}$ at $\sqrt{s}=500$ GeV [17]. We will consider Higgs discovery possibilities after combining $Z+$Higgs boson, Higgs pair, $b\bar{b}+$Higgs boson and $t\bar{t}+$Higgs boson production. Throughout this paper, we will employ the criterion of requiring 50 events (before cuts and various detection efficiencies, such as those associated with b tagging) for observation of any one of the above production processes. Clearly, this is an over simplification given that these four production processes will have different overall efficiencies and different background levels. However, we believe that 50 raw events is a conservative requirement. Past experimental studies for e^+e^- colliders have usually found techniques for uncovering a signal starting with a substantially smaller raw number of events. In particular, as regards the specific processes considered here, it seems likely that a $t\bar{t}+$Higgs event would be very hard to miss and would have very small background. We hope that experimentalists will refine the results we obtain in what follows based on the 50 event criterion.

IV. EXPERIMENTAL CONSTRAINTS

In the numerical analyses that follow we will include constraints on the model parameters that result from the current experimental limits. Thus, in this section, we briefly discuss the experimental data that will be taken into account and how they constrain the general CPV 2HDM model. For given Higgs boson masses, we must consider all nonredundant values of the mixing angles α_i. Existing data already exclude certain configurations of masses and angles, see e.g. [8,9]. We will follow the method used in Ref. [8], with updated experimental input. The constraints that we impose on the mixing angles are as follows:

The C_i^2 are restricted by non-observation of Higgs-strahlung events at LEP1 and LEP2. We take the limits presented in Fig. 16 of Ref. [18] for the case when no b-tagging has been used. By doing this, we avoid potential problems concerning the dependence of the Higgs-$b\bar{b}$ boson and Higgs-$\tau^+\tau^-$ boson branching ratios on the mixing angles.[8]

The contribution to the total Z-width from $Z\to Z^*h_i \to f\bar{f}h_i$ (summed over $i=1,2,3$) and $Z\to h_ih_j$ (summed over $i,j=1,2,3{:}i>j$) is required to be below 7.1 MeV; see Ref. [19].

[5]For obvious reasons we consider the third generation of quarks. Similar expressions hold for for lighter generations.

[6]The importance of the Yukawa processes in the context of a CP conserving 2HDM for large $\tan\beta$ has been stressed in the past many times [15,16].

[7]Looking for radiation off the tau leptons in the case of large $\tan\beta$ may also help.

[8]We thank F. Richard for discussions on this point.

For any given values of (m_1,m_2) and the α_i, we calculate the number of expected events in the processes $e^+e^-\to h_1h_2\to b\overline{b}b\overline{b}+b\overline{b}\tau^+\tau^-$ at the LEP2 energies $\sqrt{s}=133$, 161, 170, 172, 183 GeV using the corresponding integrated luminosities $L=5.2$, 10.0, 1.0, 9.4, 54 pb^{-1}, assuming efficiency $\epsilon=.52$ in the individual $b\overline{b}b\overline{b}$ and $b\overline{b}\tau^+\tau^-$ channels. Our calculations take into account the mixing-angle dependence of the Higgs-boson branching ratios to $b\overline{b}$ or $\tau^+\tau^-$.[9] If the probability of observing zero events (after summing the rates for all energies) is below 5%, the set of masses and mixing angles is assumed to be excluded.

V. HIGGS BOSON PRODUCTION IN e^+e^- COLLIDERS

As we argued above, in e^+e^- collisions production of light neutral Higgs boson(s) can proceed via three important mechanisms: (a) bremsstrahlung off the Z boson, $e^+e^-\to Zh_1$, (b) Higgs pair production, $e^+e^-\to h_1h_2$, and (c) the Yukawa processes with Higgs radiation off a heavy fermion line in the final state, $e^+e^-\to f\overline{f}h_1$. The Yukawa processes are particularly important if (a) is dynamically suppressed by the mixing and (b) is kinematically forbidden.

In order to treat the three processes on the same footing, we will discuss the production of h_1 in association with heavy fermions:

$$e^+e^-\to f\overline{f}h_1. \tag{19}$$

Feynman diagrams for processes (a) and (b) contribute to this final state when $Z\to f\overline{f}$ and $h_2\to f\overline{f}$, respectively. If $|C_1|$ is not too near 1, Eqs. (9),(10) imply that radiation diagrams (c) are enhanced when the Higgs boson is radiated off top quarks for small $\tan\beta$ and off bottom quarks or τ leptons for large values of $\tan\beta$.

Since all fermion and Higgs boson masses in the final state must be kept nonzero, the formulas for the cross section are quite involved. For the CPC case, they can be read off from Ref. [16]. In the CPV case, they are more complicated due to mixing of all neutral Higgs bosons. Therefore, for completeness, we will present the formula for the cross section. Let Q_f denote the electric charge, N_c the number of colors, a_f and v_f the axial and vector Z charges of the fermion f normalized as

$$a_f=\frac{2I_L^f}{4s_Wc_W},\quad v_f=\frac{2I_L^f-4Q_fs_W^2}{4s_Wc_W}, \tag{20}$$

with $I_L^f=\pm 1/2$ being the weak isospin of the left-handed fermions. The total cross section for the process (19) can be written as follows:

$$\begin{aligned}\sigma=\int dx_1dx_2N_c\frac{\sigma_0}{4\pi}\Bigg\{&\left[q_e^2q_f^2+2\frac{q_eq_fv_ev_f(1-z)}{(1-z)^2+z\gamma_z}+\frac{(v_e^2+a_e^2)(v_f^2+a_f^2)}{(1-z)^2+z\gamma_z}\right](G_1+F_1)\\&+\frac{a_e^2+v_e^2}{(1-z)^2+z\gamma_z}\left[a_f^2(G_2+F_2+G_3+G_4+G_5+G_6)+v_f^2(G_4+G_6)+\frac{1}{16s_W^2c_W^2}(G_7+F_3)+\frac{a_f}{4s_Wc_W}(F_4+G_8)\right]\\&+\frac{q_fq_ev_ev_f(1-z)}{(1-z)^2+z\gamma_z}G_6\Bigg\},\end{aligned} \tag{21}$$

where $\sigma_0=4\pi\alpha^2/3s$ is the standard normalization cross section. Here, $\sqrt{s}$ is the total c.m. energy, $x_{1,2}=2E_{f,\overline{f}}/\sqrt{s}$ are the reduced energies of fermions in the final state and $z=m_Z^2/s$, $\gamma_z=\Gamma_Z^2/s$ are the reduced mass and width of the Z boson, respectively. The functions G_i and F_i are given in the Appendix: $G_{1,2}$ and $F_{1,2}$ arise from squaring graphs where h_1 is radiated from the fermion; $G_{3,4}$ arise from squaring $Z\to Zh_1$ graphs; $G_{5,6}$ arise from interference between fermion-radiation and Zh_1 graphs; the remaining G's and F's involve Higgs pair production graphs and their interference with fermion-radiation and Zh_1 graphs.

If the coupling of the h_1 to the Z boson is not dynamically suppressed, i.e. C_1 is substantial, then the Higgs-strahlung process, $e^+e^-\to Zh_1$, will be sufficient to find it. In the opposite case, our focus in this paper, one has to consider the other processes (b) and/or (c), for which the sum rules (14) and (17) will imply that the neutral Higgs boson(s), if kinematically accessible, will be produced at a comfortably high rate at a high luminosity future linear e^+e^- collider. Below we will consider two situations: (i) two light Higgs bosons, and (ii) one light Higgs boson.

(i) $m_{h_1}+m_{h_2}, m_{h_1}+m_Z, m_{h_2}+m_Z<\sqrt{s}$:

If the Higgs-strahlung processes are suppressed by mixing angles, $C_1,C_2\ll 1$, then from Eq. (14) it follows that Higgs pair production is at full strength, $C_{12}\sim 1$. In particular, we will retain only those configurations of angles and masses for which, at a given value of $\sqrt{s}$, the total numbers of $e^+e^-\to Zh_1$ and (separately) Zh_2 events are both less than 50 for an integrated luminosity of 500 fb^{-1}. In Fig. 1 we show contour plots for the minimum value of the pair production cross section, $\sigma(e^+e^-\to h_1h_2)$, as a function of Higgs boson

[9] In the previous analysis [8] the SM branching ratios for the Higgs boson decays were used. We find, however, that our final results for cross sections are nearly insensitive to this modification.

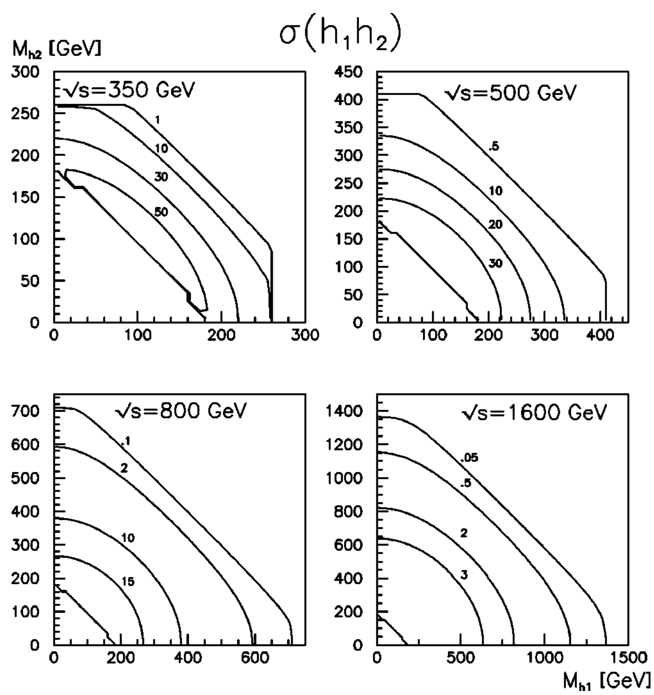

FIG. 1. Contour lines for min$[\sigma(e^+e^-\to h_1h_2)]$ as functions of Higgs boson masses for the indicated $\sqrt{s}$ values. The number next to each contour is the minimum cross section in units of fb. In scanning over mixing angles α_i, we respect the experimental constraints listed in Sec. IV, and we require that at the given $\sqrt{s}$ the number of $e^+e^-\to Zh_1$ or Zh_2 events is less than 50 for total luminosity $L=500\ \mathrm{fb}^{-1}$. The contour lines are plotted for $\tan\beta=0.5$; the plots are virtually unchanged for larger values of $\tan\beta$. The contour lines overlap in the inner corner of each plot as a result of excluding mass choices inconsistent with experimental constraints from LEP2 data.

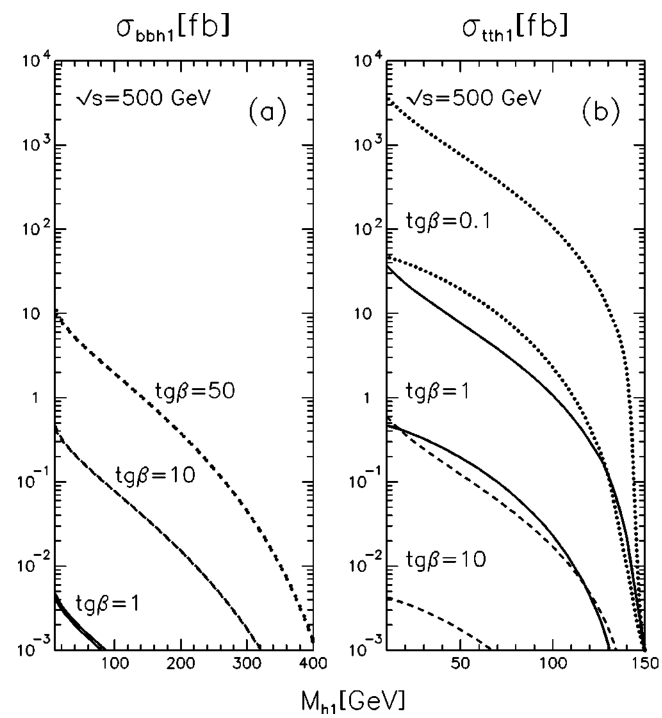

FIG. 2. The minimal and maximal values (after requiring fewer than 50 Zh_1 events for $L=500\ \mathrm{fb}^{-1}$) of the cross sections for $e^+e^-\to b\bar{b}h_1$ (a) and $e^+e^-\to t\bar{t}h_1$ (b) are plotted for $\sqrt{s}=500$ GeV. For a given value of $\tan\beta$, the same type of line (dots for $\tan\beta=0.1$ and $t\bar{t}h_1$, solid for $\tan\beta=1$, dashes for $\tan\beta=10$, dots for $\tan\beta=50$ and $b\bar{b}h_1$) is used for the minimal and maximal values of the cross sections. In the case of $b\bar{b}h_1$, the minimal and maximal values of the cross sections are almost the same. Masses of the remaining Higgs bosons are assumed to be 1000 GeV.

masses at $\sqrt{s}=350$, 500, 800 and 1600 GeV. With integrated luminosity of 500 fb^{-1}, a large number of events (large enough to allow for selection cuts and experimental efficiencies) is predicted for all the above energies over a broad range of Higgs boson masses. If 50 events before cuts and efficiencies prove adequate, one can probe reasonably close to the kinematic boundary defined by requiring that $m_{h_1}+m_Z$, $m_{h_2}+m_Z$ and $m_{h_1}+m_{h_2}$ all be less than $\sqrt{s}$.

(ii) $m_{h_1}+m_Z<\sqrt{s}$, $m_{h_1}+m_{h_2}$, $m_{h_2}+m_Z>\sqrt{s}$:

In this case, if C_1 is small the sum rules (17) imply that Yukawa couplings may still allow detection of the h_1. We illustrate this in Fig. 2 by plotting the minimum and maximum values of $\sigma(e^+e^-\to f\bar{f}h_1)$ for $f=t,b$ as a function of the Higgs boson mass, where the scan over the mixing angles α_1, α_2 and α_3 at a given $\tan\beta$ is constrained by present experimental constraints and by the requirement that fewer than 50 Zh_1 events are predicted for $\sqrt{s}=500$ GeV and $L=500\ \mathrm{fb}^{-1}$. [The results are essentially independent of m_{h_2} (and m_{h_3}) for $m_{h_1}+m_{h_2}>\sqrt{s}$.] Overall, Fig. 2 shows that if m_{h_1} is not large there will be sufficient events in either the $b\bar{b}h_1$ or the $t\bar{t}h_1$ channel (and perhaps both) to allow h_1 discovery. The smallest reach in m_{h_1} arises if $1\lesssim\tan\beta\lesssim 10$ and the α_i's are such that the $t\bar{t}h_1$ cross section is minimal. For example, let us assume $L=500\ \mathrm{fb}^{-1}$ at $\sqrt{s}=500$ GeV and take 50 events (before cuts and efficiencies) as the criteria (i.e. we require $\sigma>0.1$ fb). If $\tan\beta=1$, $\sigma(b\bar{b}h_1)\ll 0.1$ fb for all m_{h_1} while $\sigma_{\min}(t\bar{t}h_1)$ falls below 0.1 fb for $m_{h_1}>70$ GeV. At $\tan\beta=10$, $\sigma_{\min}(t\bar{t}h_1)\ll 0.1$ fb and $\sigma_{\min}(b\bar{b}h_1)\simeq\sigma_{\max}(b\bar{b}h_1)$ falls below 0.1 fb for $m_{h_1}>80$ GeV. A $\sqrt{s}=1$ TeV machine would considerably extend this mass reach.

For a given $\tan\beta$ value, especially interesting features of the $C_1\sim 0$ cross sections of Fig. 2 are the following. (a) The minimal and maximal $b\bar{b}h_1$ cross sections are almost equal. Further, for any given values of $\tan\beta$ and m_{h_1}, by explicit computation one finds $\sigma(b\bar{b}A^0)\simeq\sigma(b\bar{b}h_1)$, where $\sigma(b\bar{b}A^0)$ is the cross section computed in the CP-conserving two-doublet model. (b) Similarly, one finds $\sigma(t\bar{t}A^0)\simeq\sigma_{\min}(t\bar{t}h_1)$. These features can be understood as follows.

That the $C_1\sim 0$ cross sections should be related to the A^0 cross sections is not altogether surprising given that in the limit of $C_1\to 0$ the h_1 behaves like the A^0 in that it decouples from ZZ. However, to understand why (for $C_1\sim 0$) the minimal and maximal h_1 cross sections and the A^0

cross section are all numerically essentially the same in the $b\bar{b}$ final state, despite the fact that the h_1 possesses non-zero S and P Yukawa couplings (and therefore is not a genuine pseudoscalar) requires more discussion. First, we note that, for $C_1 \to 0$, Eq. (17) implies

$$(\hat{S}_1^t)^2 + (\hat{P}_1^t)^2 \to (\hat{P}_{A^0}^t)^2, \quad (\hat{S}_1^b)^2 + (\hat{P}_1^b)^2 \to (\hat{P}_{A^0}^b)^2, \tag{22}$$

where $P_{A^0}^{t,b}$ are the t and b couplings of the A^0 in the CP-conserving version of the 2HDM. Second, we note that in Eq. (21) only $G_{1,2} = (S_i^f)^2 g_{1,2}$ and $F_{1,2} = (P_i^f)^2 f_{1,2}$ [where $g_{1,2}$ and $f_{1,2}$ are functions of kinematic variables only, defined by Eqs. (A1) and (A2) in the Appendix] remain nonzero as $C_1 \to 0$ (see Appendix), implying in rough notation.

$$\frac{d\sigma(e^+e^- \to f\bar{f}h_1)}{d\phi} \sim (S_1^f)^2 (Ag_1 + Bg_2) + (P_1^f)^2 (Af_1 + Bf_2), \tag{23}$$

where A, B, $f_{1,2}$ and $g_{1,2}$ are all positive and ϕ denotes a point in phase space. Thirdly, it is easily verified that $g_1 - f_1$ and $g_2 - f_2$ are both of order m_f^2/s and thus these differences are very small in the case of the $b\bar{b}$ final state. As a result, inserting the $C_1 \to 0$ limit of Eq. (22) into Eq. (21) [or Eq. (23)] implies that the minimal and maximal values of $\sigma(e^+e^- \to b\bar{b}h_1)$ are essentially the same and that both are very nearly equal to $\sigma(e^+e^- \to b\bar{b}A^0)$.

Next, we would like to understand why the minimum $t\bar{t}h_1$ cross section is obtained by taking $S_1^t \sim 0$, equivalent to [see Eq. (22)] $(P_1^t)^2 \sim (P_{A^0}^t)^2$. Referring to Eq. (23), we see that this will be the case if $\int (Ag_1 + Bg_2) d\phi > \int (Af_1 + Bf_2) d\phi$, as is easily verified.

We note that the minimum cross section values would be altered if the scan over the α_i is not restricted by requiring small C_1. In particular, if one observes Zh_1 events and finds $|C_1| \sim 1$, then, as outlined earlier, both $(\hat{S}_1^t)^2 + (\hat{P}_1^t)^2$ and $(\hat{S}_1^b)^2 + (\hat{P}_1^b)^2$ will be of order unity, approaching 1 exactly if $\tan\beta$ is either very large or very small. This implies minimum cross sections values similar to the $\tan\beta = 1$ $f\bar{f}A^0$ cross sections. Thus, at a $\sqrt{s} = 500$ GeV machine with integrated luminosity of order $L = 500$ fb^{-1}, it would almost certainly not be possible to use $b\bar{b}h_1$ production to measure the h_1's $b\bar{b}$ coupling and, if m_{h_1} is significantly above 70 GeV, it would also be difficult to measure its $t\bar{t}$ Yukawa coupling. Of course, increasing the $\sqrt{s}$ will extend the range of m_{h_1} for which the $t\bar{t}h_1$ process will have a useful rate.

As a final aside, we emphasize that even if $L = 500$ fb^{-1} cannot be achieved in a single year of operation at $\sqrt{s} \sim 500$ GeV, one can envision accumulating such an integrated luminosity over a period of several years. Of course, for $\sqrt{s} \sim 1$ TeV and above, our results may be conservative given that the e^+e^- collider will very probably be designed to have a yearly integrated luminosity that scales with energy like s.

VI. THE TWO-DOUBLET+ONE-SINGLET (2D1S) HIGGS SECTOR MODEL

We do not go into the details of the most general Higgs potential for the 2D1S model, but simply state the well-known fact that explicit or spontaneous CP violation is entirely possible for a 2D1S Higgs sector. The primary change relative to the formalism given for the two-doublet model is that the R matrix is extended to a 5×5 matrix. The formulas for the couplings of a given physical eigenstate h_i to ZZ and to the quarks remain unchanged relative to the two-doublet case, being entirely determined by R_{i1}, R_{i2} and R_{i3} in the basis where

$$(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5) \equiv \sqrt{2} (\mathrm{Re}\,\phi_1^0, \mathrm{Re}\,\phi_2^0, s_\beta \,\mathrm{Im}\,\phi_1^0 - c_\beta \,\mathrm{Im}\,\phi_2^0, \mathrm{Re}\,N, \mathrm{Im}\,N), \tag{24}$$

with N being the singlet Higgs field. In general, the only constraints on the parameters of the model are that R must, as before, be an orthogonal matrix and the masses-squared of the physical Higgs eigenstates must be non-negative. Physically, this means that we can have two light Higgs bosons that reside entirely within the singlet sector and therefore do not couple to either quarks or gauge bosons. As a result, one can only guarantee discovery of a neutral Higgs boson if at least three of the five physical states are light. Further, we shall show that this guarantee is possible only by employing the Yukawa radiation processes. No statement will be possible for just one or two light Higgs bosons.

We begin by focusing on the generalization of the Yukawa sum rules to the 2D1S case. Starting from Eq. (16) (which still applies), one finds

$$\sin^2\beta[(\hat{S}_i^t)^2 + (\hat{P}_i^t)^2] + \cos^2\beta[(\hat{S}_i^b)^2 + (\hat{P}_i^b)^2] = R_{i1}^2 + R_{i2}^2 + R_{i3}^2 \equiv R_i^2, \tag{25}$$

where R_i^2 is a measure of the extent to which h_i resides in the two-doublet portion of the Higgs sector. We will refer to R_i^2 as the two-doublet content of h_i. In the 2HDM model $R_i^2 = 1$ ($i = 1,2,3$) was automatic by virtue of the orthogonality of R and its 3×3 dimensionality. However, in the present case $R_i^2 = R_{i1}^2 + R_{i2}^2 + R_{i3}^2 = 1 - R_{i4}^2 - R_{i5}^2$ could be zero if the h_i Higgs boson resides entirely in the singlet sector ($R_{i4}^2 + R_{i5}^2 = 1$). We only know that after summing over all the physical Higgs bosons we must get the full two-doublet content: $\Sigma_{i=1}^5 R_i^2 = 3$. Results analogous to the $C_i = 0$ limits of Eqs. (17) can also be obtained. For $C_i = 0$,

$$(\hat{S}_i^t)^2 + (\hat{P}_i^t)^2 = \left(\frac{\cos\beta}{\sin\beta}\right)^2 R_i^2, \quad (\hat{S}_i^b)^2 + (\hat{P}_i^b)^2 = \left(\frac{\sin\beta}{\cos\beta}\right)^2 R_i^2. \tag{26}$$

Note that both could be zero for a pure singlet h_i. Summing over two Higgs bosons does not help, since both Higgs boson could reside entirely in the singlet sector. However, if we sum over three Higgs bosons (we use $i = 1,2,3$ in what follows), one finds

$$\sum_{i=1,2,3} R_i^2 = 1 + (R_{44}^2 + R_{45}^2 + R_{54}^2 + R_{55}^2) \geqslant 1. \tag{27}$$

In the worst case, $R_{44}^2 = R_{45}^2 = R_{54}^2 = R_{55}^2 = 0$, i.e. the singlet Higgs field N is entirely contained in the three light Higgs bosons. The two most important implications of these results are the following.

(1) Equation (25) implies that our ability to observe a Yukawa radiation process and measure either the $b\bar{b}$ or the $t\bar{t}$ Yukawa coupling of a Higgs boson h_i is determined by its two-doublet content, R_i^2. For substantial two-doublet content, and $m_{h_i} + 2m_t < \sqrt{s} - \Delta$, we are guaranteed that at least one of these two Yukawa couplings will be measurable.

(2) If there are three light Higgs bosons (light being defined by $m_{h_i} + 2m_t < \sqrt{s} - \Delta$), and two have small Yukawa couplings, then Eq. (27) implies that at least one of the Yukawa couplings of the third will be large enough to detect the Higgs boson in association with $b\bar{b}$ or $t\bar{t}$.

Of course, the Yukawa couplings (squared) could be apportioned more or less equally among the three light Higgs bosons, in which case observation of a Yukawa radiation process of any one of the three would require substantially more luminosity than if the two-doublet content resides primarily in just one of the three.

The generalization to more singlets is clear. Each singlet field introduces two more physical neutral Higgs bosons. At least $1 + 2N_{\text{singlet}}$ of the neutral Higgs bosons must be light in order to guarantee that $\Sigma_{i=1}^{1+2N_{\text{singlet}}} R_i^2 \geqslant 1$, implying definite opportunity for observing at least one in $t\bar{t}h_i$ or $b\bar{b}h_i$ associated production.

Let us now consider the Zh_i and h_ih_j processes. We wish to determine how many of the 2D1S neutral Higgs bosons must be light in order that we are guaranteed to find at least one in either Higgs-strahlung or Higgs pair production. The crucial ingredient for obtaining the necessary sum rule is the unitarity sum rule for $ZZ \to h_ih_j$ as given in the Appendix of Ref. [20]. In applying this sum rule it is crucial to note that the ZZ-Higgs-Higgs boson coupling only receives contributions from the fields in the doublet sector. Thus, in the basis defined by Eq. (24), these interactions have the form $ZZ(\varphi_1^2 + \varphi_2^2 + \varphi_3^2)$ times the standard $g^2/(2c_W^2)$ factor. There are no $ZZ\varphi_4^2$ or $ZZ\varphi_5^2$ interactions. After diagonalizing, the ZZh_ih_j coupling coefficient is given [see Eq. (13)] by $X_{ij} \equiv R_{i1}R_{j1} + R_{i2}R_{j2} + R_{i3}R_{j3}$. In particular, $X_{ii} = R_i^2$, the two-doublet content of h_i defined earlier. Using our present notation, Eq. (A18) of Ref. [20] becomes

$$C_iC_j + \sum_{k \neq i} C_{ik}C_{jk} = X_{ij}, \tag{28}$$

which for $i = j$ yields

$$C_i^2 + \sum_{k \neq i} C_{ik}^2 = R_i^2. \tag{29}$$

Let us define

$$W_{1234} \equiv C_1^2 + C_2^2 + C_3^2 + C_4^2 + C_{12}^2 + C_{13}^2 + C_{14}^2 + C_{23}^2 + C_{24}^2 + C_{34}^2. \tag{30}$$

Using Eq. (29) and summing over $i = 1,2,3,4$ and over $i = 1,2,3,4,5$, one obtains

$$W_{1234} + \sum_{i,j=1,\ldots,5:i>j} C_{ij}^2 = \sum_{i=1}^{4} R_i^2 = 3 - R_5^2, \tag{31}$$

$$\sum_{i=1}^{5} C_i^2 + 2 \sum_{i,j=1,\ldots,5:i>j} C_{ij}^2 = \sum_{i=1}^{5} R_i^2 = 3, \tag{32}$$

respectively, where we also used $C_{ik}^2 = C_{ki}^2$. Unitarity for $ZZ \to ZZ$ scattering and for other vector boson $VV \to VV$ processes requires that $\Sigma_{i=1,5} C_i^2 = 1$. Inserting this into Eq. (32) implies that $\Sigma_{i,j=1,\ldots,5:i>j} C_{ij}^2 = 1$. Inserting this latter result into Eq. (31) yields $W_{1234} = 2 - R_5^2$ which must be $\geqslant 1$ by virtue of the fact that $R_5^2 \leqslant 1$ is required by orthogonality of R. In words, $W_{1234} \geqslant 1$ implies that if there are four Higgs bosons that are sufficiently light that all the Zh_i and h_ih_j production processes are kinematically allowed (and not significantly phase-space suppressed), then at least one of these Higgs bosons must be seen in Higgs-strahlung or a pair of Higgs bosons must be seen in pair production. Three light Higgs bosons are not enough. In particular, analogous procedures to those sketched above yield the result

$$W_{123} \equiv C_1^2 + C_2^2 + C_3^2 + C_{12}^2 + C_{13}^2 + C_{23}^2 = \sum_{i=1}^{3} R_i^2 - 1 + C_{45}^2. \tag{33}$$

Since we are only guaranteed that $\Sigma_{i=1}^3 R_i^2 \geqslant 1$ and since C_{45} could be quite small even when $\Sigma_{i=1}^3 R_i^2 = 1$, there is no lower bound to W_{123} and we cannot be certain of finding at least one Higgs boson in Higgs-strahlung or Higgs pair production in the case that only three are light.[10] Thus, if only three neutral Higgs bosons of the 2D1S model are light, searching for the Yukawa radiation processes is required in order to guarantee that we will find at least one.

Once again, the generalization of the above considerations to a CP-violating Higgs sector with one-doublet and more than one singlet is obvious. At least $2 + 2N_{\text{singlet}}$ of the neutral Higgs bosons must be light in order to be certain that at least one of them will be produced at a significant rate in either Higgs-strahlung or Higgs pair production.

VII. IMPLICATIONS FOR THE FERMILAB TEVATRON AND THE CERN LHC

Determining the implications of our sum rules for future experiments at the Tevatron and the LHC is much more involved than the e^+e^- collider study we have focused on in previous sections. As above, consider a subset of 1

[10]Note: these results correct the erroneous result for this case given in Ref. [8].

$+2N_{\rm singlet}$ Higgs bosons coming from a Higgs sector consisting of 2 doublets plus $N_{\rm singlet}$ singlets. Further, consider parameters such that none of the Higgs in this subset has substantial WW/ZZ coupling, but assume that all have mass $\ll \sqrt{\hat{s}}-2m_t$ (where $\sqrt{\hat{s}}$ is the typically available sub-process energy). Then, the Yukawa coupling sum rules guarantee that at least $\underline{1}$ of the $1+2N_{\rm singlet}$ Higgs bosons will have a substantial $b\bar{b}+$Higgs or $t\bar{t}+$Higgs cross section. The greatest difficulty at a hadron collider arises if parameters are chosen so that the $t\bar{t}+$Higgs channel(s) are crucial.[11] At the Tevatron, the energy is marginal for a $t\bar{t}+$Higgs channel and even $L=30\,{\rm fb}^{-1}$ leads to an insufficient number of events for Higgs discovery in this channel. At the LHC, one typically finds a large number of $t\bar{t}+$Higgs events, but it is easy to find parameter regions such that the large backgrounds cannot be overcome even if $L=300\,{\rm fb}^{-1}$ is accumulated by the ATLAS and CMS detectors (each). Still, because of the sum rules, the obviously problematical parameter regions are fairly limited.

To be more specific, let us consider the general 2HDM. To make discovery difficult, one chooses parameters so that only the h_1 is light and yet the ZZh_1 and WWh_1 couplings are suppressed (something that is not possible for example in the more constrained MSSM two-Higgs-doublet sector). Then, it will be necessary to rely on $b\bar{b}h_1$ and $t\bar{t}h_1$ production for h_1 discovery. As for e^+e^- collisions, the general 2HDM model parameters can be chosen so that these cross sections are essentially the same as their respective $b\bar{b}A^0$ and $t\bar{t}A^0$ production counterparts in a CP-conserving model. Detection of these latter processes has been examined in the context of the MSSM, most recently as part of the Run2 Higgs/SUSY workshop [21] for the Tevatron and by the ATLAS [22] and CMS [23] Collaborations for the LHC. In these studies one finds the following. Detection of $b\bar{b}A^0$ production will not be possible at either the Tevatron or the LHC if $\tan\beta$ is such that the $b\bar{b}A^0$ coupling is not significantly enhanced. For example, even for $L=300$ fb^{-1} accumulated by ATLAS and CMS (each) at the LHC, $b\bar{b}A^0$ detection requires $\tan\beta \gtrsim 2$ for $m_{A^0}<100$ GeV rising to $\tan\beta \gtrsim 4.5$ for $m_{A^0}\sim 200$ GeV. Turning to $t\bar{t}A^0$ production, we have already noted that it is kinematically suppressed at the Tevatron. Even at the LHC, if $\tan\beta \gtrsim 1$ the current ATLAS and CMS analyses indicate that the $t\bar{t}A^0$ cross section will not be large enough for detection of the dominant $t\bar{t}b\bar{b}$ and $t\bar{t}\tau^+\tau^-$ final states above backgrounds unless further improvements in vertex tagging and top-identification efficiencies are possible. In combination, we are left with a clear gap of at least $1\lesssim\tan\beta\lesssim 2$ for which A^0 (and, hence, h_1) detection is not possible.

Of course, purely to avoid difficulty with unitarity limits for WW and ZZ production, there must be one or several neutral Higgs in the 2HDM with masses below ~ 800 GeV and with substantial WW and ZZ coupling. The LHC would search for a signal in the usual channels employed for the SM Higgs boson search (e.g., the $\gamma\gamma$, $WW\to 2\ell 2\nu$ or $ZZ\to 4\ell$ final states). The concern regarding such signals is that the WW and ZZ couplings could be shared among several Higgs bosons so that the branching ratios to these final states would be reduced and the resulting signal for any one Higgs boson would be too weak to be detectable. That this is possible is illustrated in the 2D1S supersymmetric model study of [24], where it was found that several moderately light Higgs bosons could share the SM WW/ZZ coupling strength in such a way that the signal for any one was not visible above background. A dedicated study is needed to determine whether this is also possible in the context of the general 2HDM without giving rise to an observable Higgs-pair production, $b\bar{b}+$Higgs or $t\bar{t}+$Higgs signal.

If one goes beyond the 2HDM to include one additional singlet, the study of the 2D1S supersymmetric model of Ref. [24] becomes directly relevant. Despite the strong constraints on the 2D1S model in [24] from supersymmetry and the assumption of no CP violation, it was found that all three of the CP-even and one (at least) of the two CP-odd Higgs bosons could be quite light (<200 GeV) without there being any observable signal at the LHC with $L=300$ fb^{-1} accumulated by CMS and ATLAS each.

The reason for our focus on e^+e^- collisions should now be apparent. Because backgrounds are smaller, the simple analyses of earlier sections were sufficient to show that the sum rules guarantee discovery of at least 1 of $1+2N_{\rm singlet}$ "light" Higgs bosons at an e^+e^- collider for anticipated integrated luminosities.

VIII. DISCUSSION AND CONCLUSIONS

We have derived a crucial new sum rule, Eq. (17), relating the Yukawa and Higgs-ZZ couplings of a general CP-violating two-Higgs-doublet model. This sum rule has two important implications. First, it says that if the ZZh coupling of a neutral Higgs boson is small, then its $t\bar{t}h$ or $b\bar{b}h$ Yukawa coupling must be substantial. This means that any one of the three neutral Higgs bosons that is light enough to be produced in $e^+e^-\to t\bar{t}h$ (implying that $e^+e^-\to Zh$ and $e^+e^-\to b\bar{b}h$ are also kinematically allowed) will be found at an e^+e^- linear collider of sufficient luminosity. In particular, if mixing angles and Higgs masses are such that a light Higgs boson cannot be observed via the Zh Higgs-strahlung process, then it is guaranteed to be found via Yukawa-coupling-induced radiation from top or bottom quarks. Second, for an h that is observed in the Zh final state but also light enough to be seen in $t\bar{t}h$ and, by implication, $b\bar{b}h$, this same sum rule can be used to show that measurement of at least one of its third-family Yukawa couplings will be pos-

[11]Large Yukawa couplings will also imply a large $gg\to$Higgs cross section. But, the absence of WW/ZZ coupling implies that the Higgs boson decays primarily to $b\bar{b}$ and $\tau^+\tau^-$, for which backgrounds are overwhelming: $\gamma\gamma$, $WW^\star\to 2\ell 2\nu$ and $ZZ^\star\to 4\ell$ decays (the usual light Higgs boson signals from gg fusion at the Tevatron and LHC) are negligible.

sible (the required luminosity depending on the amount of phase space suppression in the $t\bar{t}h$ channel). Of course, in the experimental analysis one must be careful to not exclude the Yukawa radiation processes by placing restrictive invariant mass constraints on the $f\bar{f}$ system, e.g., $M_{f\bar{f}}\sim m_Z$.

We have also extended to high energies the quantitative analysis of a previously derived sum rule, Eq. (14). This latter sum rule implies that if any two of the three neutral Higgs bosons of the CP-violating 2HDM are light enough that Zh_1, Zh_2 and h_1h_2 production are all kinematically allowed (and not phase space suppressed), then at least one of these processes will be observable, regardless of the mixing structure of the neutral Higgs sector. For planned luminosities, the predicted cross sections are such that discovery of one or both of the Higgs bosons will be possible even rather close to the relevant kinematic boundary in the m_{h_1}-m_{h_2} mass plane.

We have also considered the general CP violating two-doublet+one-singlet Higgs sector model. In this case, we find that if only one or two of the neutral Higgs bosons are light then both could be primarily singlet and, therefore, undetectable in Higgs-strahlung, Higgs pair production and Yukawa radiation processes. However, there are two important guarantees. (a) If there are three light neutral Higgs bosons, then we are guaranteed to detect at least one in Yukawa radiation processes. (b) If there are four light neutral Higgs bosons we are guaranteed to detect one or two in Higgs-strahlung or Higgs pair production; but, there is no such guarantee for just three light Higgs bosons. Guarantee (a) requires that all $t\bar{t}h_i$ $(i=1,2,3)$ (and by implication all $b\bar{b}h_i$) processes have substantial phase space. Guarantee (b) requires that all four h_i be light enough that the Zh_i and h_ih_j $(i,j=1,2,3,4, i\neq j)$ processes all have substantial phase space. Thus, for extensions of the two-doublet Higgs sector that include one or more singlet Higgs fields, it could happen that observation of a Higgs boson at an e^+e^- collider of limited energy will only be possible by looking for Higgs production in association with bottom and top quarks.

Finally, we noted that these same guarantees for the 2HDM and 2D1S models do not apply to the Tevatron and LHC hadron colliders. In the case of the Tevatron, the small rate for $t\bar{t}+$Higgs production is a clear problem. In the case of the LHC, a detailed study would be appropriate. However, existing studies in the context of supersymmetric models can be used to point to parameter regions that are problematical because of large backgrounds and/or signal dilution due to sharing of available coupling strength. Still, it is clear that the sum rules are sufficiently powerful to imply that such parameter regions are of fairly limited extent.

ACKNOWLEDGMENTS

This work was supported in part by the Committee for Scientific Research (Poland) under grants No. 2 P03B 014 14, No. 2 P03B 030 14, by Maria Sklodowska-Curie Joint Fund II (Poland-USA) under grant No. MEN/NSF-96-252, by the U.S. Department of Energy under grant No. DE-FG03-91ER40674 and by the U.C. Davis Institute for High Energy Physics.

APPENDIX

Consider production of the Higgs boson h_i in association with a fermion pair $f\bar{f}$ in e^+e^- collisions, i.e., $e^+e^-\to f\bar{f}h_i$. Note that the diagram with Higgs pair production requires summation over virtual Higgs bosons h_j and h_k, where i,j,k are permutations of 1,2,3. The differential cross section is given by Eq. (21) with F_i and G_i as given below.

For a short hand notation, we introduce $h_j=m_{h_j}^2/s$, $\gamma_j=\Gamma_{h_j}^2/s$, $(j=1,2,3)$ and $f=m_f^2/s$. The reduced energy of the observed Higgs boson h_i is denoted by $x=2E_{h_i}/\sqrt{s}=2-x_1-x_2$; we also define $x_{12}=(1-x_1)(1-x_2)$. In the formulas below, Z and h_j widths are included in terms corresponding to Z and h_j decay to the $f\bar{f}$ pair.

The functions G_1 and G_2 describe the h_i Higgs boson radiation off the fermions due to the scalar couplings,

$$G_1=\frac{(S_i^f)^2}{4\pi x_{12}}\left[x^2-h_i\left(\frac{x^2}{x_{12}}+2(x-1-h_i)\right)\right.$$
$$\left.+2f\left(4(x-h_i)+\frac{x^2}{x_{12}}(4f-h_i+2)\right)\right],$$

$$G_2=-\frac{2(S_i^f)^2}{4\pi x_{12}}\left[x_{12}(1+x)-h_i(x_{12}+8f+2x-2h_i)\right.$$
$$\left.+3fx\left(\frac{x}{3}+4+\frac{x}{x_{12}}(4f-h_i)\right)\right] \tag{A1}$$

whereas the F_1 and F_2 terms arise from the pseudoscalar couplings,

$$F_1=\frac{(P_i^f)^2}{4\pi x_{12}}\left[x^2-h_i\left(\frac{x^2}{x_{12}}(1+2f)+2x-2-2h_i\right)\right],$$

$$F_2=\frac{2(P_i^f)^2}{4\pi x_{12}}\left[(2h_i-x_{12})(1+x-h_i)-2h_i(1+2f)\right.$$
$$\left.-\frac{fx^2}{x_{12}}(x_{12}-3h_i)\right]. \tag{A2}$$

The terms G_3 and G_4 account for the emission of the Higgs boson (only its $CP=1$ component) from the Z-boson line:

$$G_3=\frac{2g_{ZZh_i}^2}{4\pi(p^2+z\gamma_z)}\left[f(4h_i-x^2-12z)+\frac{f}{z}(4h_i-x^2)(x-1-h_i+z)\right],$$

$$G_4=\frac{2zg_{ZZh_i}^2}{4\pi(p^2+z\gamma_z)}[h_i+x_{12}+2-2x+4f], \tag{A3}$$

where the reduced propagator of the off-shell Z-boson has been denoted by $p=x-1-h_i+z$.

The interference between the radiation amplitudes off the fermion and the Z-boson lines is included in the G_5 and G_6 terms:[12]

$$G_5=\frac{S_i^f g_{ZZh_i}}{4\pi}\frac{4xm_f}{x_{12}m_Z}\frac{p}{p^2+z\gamma_z}[(x_{12}-h_i)(x-1-h_i)+f(12z-4h_i+x^2)-3zh_i+6zx_{12}/x],$$

$$G_6=\frac{S_i^f g_{ZZh_i}}{4\pi}\frac{4zm_f}{x_{12}m_Z}\frac{p}{p^2+z\gamma_z}[x(h_i-4f-2)-2x_{12}+x^2]. \tag{A4}$$

Finally, the contributions from the Higgs pair h_ih_j and h_ih_k production diagrams (with subsequent h_j and h_k decays to fermion pairs) and from their interference with the h_i radiation off the fermion and the Z-boson lines are collected in G_7, G_8, F_3 and F_4 as follows:[13]

$$F_3=\frac{1}{2\pi}(x-1-h_i+4f)(4h_i-x^2)\frac{(P_k^fC_{ik}u_j+P_j^fC_{ij}u_k)^2}{(u_j^2+h_j\gamma_j)(u_k^2+h_k\gamma_k)},$$

$$F_4=-\frac{P_i^f}{\pi}\left[\frac{S_j^fC_{ij}u_j}{u_j^2+h_j\gamma_j}+\frac{S_k^fC_{ik}u_k}{u_k^2+h_k\gamma_k}\right]\times\frac{x}{x_{12}}[(x_{12}-h_i)(1-x+h_i)+f(4h_i-x^2)], \tag{A5}$$

$$G_7=\frac{4h_i-x^2}{2\pi}\left[(x-1-h_i)\frac{(P_k^fC_{ik}u_j+P_j^fC_{ij}u_k)^2}{(u_j^2+h_j\gamma_j)(u_k^2+h_k\gamma_k)}+4s_Wc_W\frac{2m_f}{m_Z}a_fg_{ZZh_i}\left(\frac{P_j^fC_{ij}u_j}{u_j^2+h_j\gamma_j}+\frac{P_k^fC_{ik}u_k}{u_k^2+h_k\gamma_k}\right)\right],$$

$$G_8=\frac{S_i^f}{\pi}\left[\frac{P_j^fC_{ij}u_j}{u_j^2+h_j\gamma_j}+\frac{P_k^fC_{ik}u_k}{u_k^2+h_k\gamma_k}\right]\times\frac{x}{x_{12}}[(x_{12}-h_i)(x-1-h_i)-f(4h_i-x^2)]. \tag{A6}$$

In the above expressions, terms of order $\gamma_i,(i=1,2,3)$ in the numerator have been neglected. The scaled propagator of the virtual Higgs boson h_j has been abbreviated by (for the virtual h_k boson, replace $j\to k$)

$$u_j=x-1-h_i+h_j. \tag{A7}$$

If the Higgs and Z boson widths are neglected then the above expressions reduce to those given in Ref. [16] with the exception that our $G_7+4s_Wc_Wa_fG_8$ becomes G_7 of [16].

[12]Due to a different convention regarding the sign of the g_{ZZH} coupling, our G_5 and G_6 have opposite signs to those in Eq. (9) of [16].

[13]We correct some typos in [16]. The last term for G_7 in Eq. (16) of [16] should have the opposite sign, i.e. $-2a_fg_{ffH^i}$ should read $+2a_fg_{ffH^i}$. In Eq. (18), an overall factor of 4 multiplying F_3 is missing and g_{ffA} should read g_{ffH^i}, and for F_4 a factor of 2 is missing. We thank S. Dawson and M. Spira for help in clarifying this point.

[1] J. H. Christenson *et al.*, Phys. Rev. Lett. **13**, 138 (1964).

[2] K. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[3] T. D. Lee, Phys. Rev. D **8**, 1226 (1973); S. Weinberg, *ibid.* **42**, 860 (1990).

[4] For a review and references, see J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, *The Higgs Hunters Guide* (Addison-Wesley Publishing Company, Redwood City, CA, 1990); J. F. Gunion, A. Stange, and S. Willenbrock, in *Electroweak Symmetry Breaking and New Physics at the TeV Scale*, edited by T. Barklow, S. Dawson, H. E. Haber, and J. Siegrist (World Scientific Publishing Co., Singapore, in press, p. 23).

[5] A. Pilaftsis, Phys. Rev. D **58**, 096010 (1998); Phys. Lett. B **435**, 88 (1998); D. A. Demir, Phys. Rev. D **60**, 055006 (1999).

[6] Results presented at LEP Committee, 1998, CERN/LEPC 98-9.

[7] A. Mendez and A. Pomarol, Phys. Lett. B **272**, 313 (1991).

[8] J. F. Gunion, B. Grzadkowski, H. E. Haber, and J. Kalinowski, Phys. Rev. Lett. **79**, 982 (1997).

[9] Delphi Collaboration, C. Martinez-Rivero, DELPHI 98-117, CONF-179.

[10] LEP Electroweak Working Group, http://www.cern.ch/LEPEWWG.

[11] J. R. Espinosa and J. F. Gunion, Phys. Rev. Lett. **82**, 1084 (1999).

[12] K. Symanzik, in *Fundamental Interactions at High-Energies*,

edited by A. Perlmutter *et al.* (Gordon and Breach, New York, 1970).

[13] G. C. Branco, A. J. Buras, and J. M. Gérard, Nucl. Phys. **B259**, 273 (1985).

[14] See the first paper of Ref. [3].

[15] J. Kalinowski and S. Pokorski, Phys. Lett. B **219**, 116 (1989); J. Kalinowski and H.-P. Nilles, *ibid.* **255**, 134 (1991); A. Djouadi, P. M. Zerwas, and J. Zunft, *ibid.* **259**, 175 (1991); J. Kalinowski and M. Krawczyk, *ibid.* **361**, 66 (1995); Acta Phys. Pol. B **27**, 961 (1996); M. Krawczyk, in *Proceedings of the XXVIIIth International Conference on High Energy Physics*, Warsaw, Poland, 1996, edited by Z. Ajduk and A. K. Wroblewski (World Scientific, Singapore, 1997), p. 1460.

[16] A. Djouadi, J. Kalinowski, and P. M. Zerwas, Z. Phys. C **54**, 255 (1992).

[17] R. Brinkmann, Talk at ECFA/DESY Linear Collider Workshop, 2nd ECFA/DESY Study on Physics and Detectors for a Linear Electron-Positron Collider, LAL, Orsay, France, 1998.

[18] OPAL Collaboration, G. Abbiendi *et al.*, Eur. Phys. J. C **7**, 407 (1999).

[19] OPAL Collaboration, K. Ackerstaff *et al.*, Eur. Phys. J. C **5**, 19 (1999).

[20] J. G. Gunion, H. E. Haber, and J. Wudka, Phys. Rev. D **43**, 904 (1991).

[21] The latest results are available on the home page (fnth37.fnal.gov/susy.html) for the Physics at Run II—Supersymmetry/Higgs workshop, and will appear in the proceedings [edited by M. Carena and J. Lykken (World Scientific, Singapore, in press)] thereof.

[22] For a recent discussion, see E. Richter-Was, D. Froidevaux, F. Gianotti, L. Poggioli, D. Cavalli, and S. Resconi, Int. J. Mod. Phys. A **13**, 1371 (1998). See also ATLAS Collaboration, W. W. Armstrong *et al.*, ATLAS Technical Proposal, CERN-LHCC-94-43 (1994).

[23] See, for example, CMS Collaboration, D. Denegri, in *Tegernsee 1997, Beyond the Desert*, 1997, p. 434. See also CMS Collaboration, G. L. Bayatian *et al.*, CMS Technical Proposal, CERN-LHCC-94-38 (1994).

[24] J. F. Gunion, H. E. Haber, and T. Moroi, in Proceedings of the 1996 DPF/DPB Summer Study on New Directions in High-Energy Physics (Snowmass96), Snowmass, CO, 1996 (hep-ph/9610337), p. 598.