

**CP violation in a supersymmetric model with nondegenerate A terms**

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We study the  $CP$  phases of the soft supersymmetry breaking terms in string-inspired models with nonuniversal trilinear couplings. We show that such nonuniversality plays an important role on all  $CP$  violating processes. In particular these new supersymmetric sources of  $CP$  violation may significantly contribute to the observed  $CP$  phenomena in kaon physics while respecting the severe bound on the electric dipole moment of the neutron. [S0556-2821(99)03217-8]

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**I. INTRODUCTION**

$CP$  phenomenology is sensitive to new physics beyond the standard model. In supersymmetric models, there appear new  $CP$  violating phases which arise from the complexity of the soft supersymmetry (SUSY) breaking terms, i.e., the trilinear scalar  $A$  terms, the bilinear scalar  $B$  term, and the Majorana gaugino masses, as well as from the  $\mu$  parameter. The presence of these phases would give large contributions, e.g., to the electric dipole moment of the neutron (EDMN) [1–6] and to the  $CP$  violation parameters ( $\varepsilon$  and  $\varepsilon'$ ) of the  $K-\bar{K}$  system [8,9]. There has been a considerable amount of work concerning these phases in the minimal supersymmetric standard model (MSSM).<sup>1</sup> It was shown that to suppress the EDMN, either large scalar masses (approaching more than 1 TeV) or small  $CP$  phases (of order  $10^{-3}$ , when all SUSY masses are of order 100 GeV) are required. In the latter case, the MSSM SUSY phases generate  $CP$  violation in the  $K-\bar{K}$  system far below the experimental value. Thus, the Cabibbo-Kobayashi-Maskawa (CKM) phase must provide almost the whole contribution to the observed  $CP$  violation in the  $K$  system.

Recently the question of whether the EDMN actually forces the SUSY contributions to  $CP$  violation in the  $K$  system to be quite small has been vigorously readdressed. In particular this has been due to a change in the perspectives of the SUSY model building. While in the 1980s and early 1990s most emphasis was put on the minimal SUSY extension of the SM (i.e., the MSSM), more recently it has become clear that the MSSM represents a very particular choice of SUSY extension of the SM with drastic assumptions on the SUSY breaking terms. The advent of the superstring inspired model has stressed even more the particular nature of the MSSM and the difficulty, in general, to obtain

all the strict boundary conditions on which the MSSM relies. If one gives up the MSSM and goes for more general SUSY realizations it is possible to avoid the above obstruction on large SUSY contributions to  $\varepsilon$ .

Three ways out have been indicated so far. First, even remaining within the MSSM context, the complete computation of the SUSY contributions to the EDMN involves several contributions and possible destructive interferences can occur in some regions of the SUSY parameter space [5,6]. A second possibility occurs in the so-called models of effective supersymmetries where the sfermion of the first two generations are very heavy (in the tens of TeV range) while those of the third generation remain light. Here the SUSY contributions to the EDMN are suppressed even with the maximal SUSY phases either because the squarks in the loop are very heavy or because the mixing angles are very small [7]. Finally, we come to the way out which is of most immediate interest in our work. It relies on the nonuniversality of the trilinear  $A$  terms of the soft breaking sector of the SUSY Lagrangian [11]. Let us expand more on this latter possibility.

In most of analysis universal or degenerate  $A$  terms have been assumed, i.e.,  $(A_{U,D,L})_{ij}=A$  or  $(A_{U,D,L})_{ij}=A_{U,D,L}$ . This is certainly a nice simplifying assumption, but it removes some interesting degrees of freedom. For example, every  $A$  term would, in general have an independent  $CP$  phase, and in principle we would have  $27(=3\times3\times3)$  independent  $CP$  phases. However, in the universal assumption only one independent  $CP$  phase is allowed.

The situation drastically changes if we are to allow for nondegenerate  $A$  terms with different and independent  $CP$  phases. For example, the off-diagonal element of the squark (mass)<sup>2</sup> matrix, say  $(M_Q^2)_{12}$ , includes a term proportional to  $(A_U)_{1i}(A_U^\dagger)_{i2}$ . However, in the universal or the degenerate case this term is always real. Furthermore, these off-diagonal elements play an important role in  $\varepsilon_K$ , as shall be shown later. If these terms enlarge the imaginary part of  $(M_Q^2)_{12}$ ,

<sup>1</sup>For a review, see e.g. [10] and references therein.

$CP$  violation in the  $K$  system may be enhanced. That such a case may occur was recently shown in Ref. [11].

In this paper we study more explicitly and concretely such aspects of the  $CP$  violation in the  $K$  system due to nondegenerate  $A$  terms, using soft SUSY breaking parameters derived from superstring models with certain assumptions [12,13]. We assume real Yukawa matrices in order to study the  $CP$  violation effect due to SUSY  $CP$  phases.<sup>2</sup> Also we use generically realistic Yukawa matrices. We investigate how much the  $CP$  violation parameter  $\varepsilon$  in the  $K$  system is enhanced by the effect due to nondegenerate  $A$  terms.

The paper is organized as follows. In Sec. II we review the soft SUSY breaking terms derived from superstring models. We assign family-dependent modular weights in order to have nondegenerate trilinear couplings. In Sec. III, we study the effect of these phases in the  $CP$  violating physics of kaons. We show that SUSY  $CP$  phases could contribute to the observed value of  $\varepsilon$  in the  $K$  system. Section IV deals with the EDMN. We give our conclusions in Sec. V.

## II. SOFT SUSY BREAKING TERMS

First we give a brief review on the soft SUSY breaking terms in string models,

$$\begin{aligned}
 -\mathcal{L}_{\text{SB}} = & \frac{1}{6} Y_{ijk}^A \phi_i \phi_j \phi_k + \frac{1}{2} (\mu B)^{ij} \phi_i \phi_j \\
 & + \frac{1}{2} (m^2)_i^j \phi^{*i} \phi_j + \frac{1}{2} M_a \lambda \lambda + \text{H.c.}, \quad (1)
 \end{aligned}$$

where  $Y_{ijk}^A = (YA)_{ijk}$ , the  $\phi_i$  are the scalar parts of the chiral superfields  $\Phi_i$ , and  $\lambda$  are the gauginos. We assume the string model to have the same massless matter content of the MSSM, i.e., three families of quark doublets  $Q_i$ , the up-type quark singlets  $U_i$ , the down-type quark singlets  $D_i$ , lepton doublets  $L_i$ , and lepton singlets  $E_i$  as well as two Higgs fields  $H_1$  and  $H_2$ . Here we consider orbifold models with the overall moduli field  $T$  as well as the dilaton field  $S$ . We assume that the dilaton and the moduli fields contribute to SUSY breaking and the vacuum energy vanishes.

In this case the soft scalar masses  $m_i$  and the gaugino masses  $M_a$  are written as [13]

$$m_i^2 = m_{3/2}^2 (1 + n_i \cos^2 \theta), \quad (2)$$

$$M_a = \sqrt{3} m_{3/2} \sin \theta e^{-i\alpha_S}, \quad (3)$$

where  $m_{3/2}$  is the gravitino mass,  $n_i$  is the modular weight of the chiral multiplet and  $\sin \theta$  corresponds to a ratio between  $F$  terms of  $S$  and  $T$ . For example, the limit  $\sin \theta \rightarrow 1$  corresponds to the dilaton-dominant SUSY breaking. Here the phase  $\alpha_S$  is originated from the  $F$  term of  $S$ . In the equation for  $M_a$  the  $T$ -dependent threshold corrections are neglected.

These latter are important only for the case where the tree level value is very small, i.e.,  $\sin \theta \rightarrow 0$ . Here we do not discuss such limit. Similarly the  $A$  terms are written as

$$\begin{aligned}
 A_{ijk} = & -\sqrt{3} m_{3/2} \sin \theta e^{-i\alpha_S} - m_{3/2} \\
 & \times \cos \theta (3 + n_i + n_j + n_k) e^{-i\alpha_T}, \quad (4)
 \end{aligned}$$

where  $n_i, n_j$ , and  $n_k$  are modular weights of fields in the corresponding Yukawa coupling  $Y_{ijk}$ . Here the phase  $\alpha_T$  is originated from the  $F$  term of  $T$ .<sup>3</sup> If  $Y_{ijk}$  depends on  $T$ , there appears another contribution. However, we do not take such a case.

Thus, the gaugino masses and the  $A$  terms as well as the  $B$  term are, in general, complex. We have a degree of freedom to rotate  $M_a$  and  $A_{ijk}$  at the same time [3]. Here we use the basis where  $M_a$  is real. In  $A$  terms of the above basis, there remains only one independent degree of freedom of the phase, i.e.,  $\alpha' \equiv \alpha_T - \alpha_S$ . However, note that in general  $A$  terms can have different phases with each other except the case with  $\cos \theta \sin \theta = 0$ . The case with  $\cos \theta = 0$  corresponds to the dilaton dominant SUSY breaking leading to the universal  $A$  term, while the case with  $\sin \theta = 0$  corresponds to the moduli-dominant SUSY breaking, where  $CP$  phases are universal, i.e.,  $A_{ijk} = |A_{ijk}| e^{i\alpha'}$ .

In order to avoid any conflict with the experimental results on flavor changing neutral current processes, we assume that the soft scalar masses of the first and second families are degenerate, that is, the first and second families have the same modular weights. Under this assumption, we in general have the  $A$ -parameter matrix

$$A_{ij}^{u,d} = \begin{pmatrix} a_{u,d} & a_{u,d} & b_{u,d} \\ a_{u,d} & a_{u,d} & b_{u,d} \\ b'_{u,d} & b'_{u,d} & c_{u,d} \end{pmatrix}, \quad (5)$$

that is, all of the entries in the first  $2 \times 2$  block are degenerate, and the (1,3) and (2,3) [(3,1) and (3,2)] entries are degenerate each other. After assigning specific modular weights, we obtain explicit values for the entries in the  $A$ -parameter matrix.

In addition to the soft terms, we have to fix the Yukawa matrices to be able to perform an explicit computation. There are several types of Ansätze for realistic Yukawa matrices. Some typical Yukawa matrices leading to approximate values of quark masses and their mixing angles are enough for our purpose. Here we assume (1) every entry in the Yukawa matrix is real, (2) the Yukawa matrix is symmetric, and (3) the Yukawa matrix has the following hierarchical structure:

$$Y_{33} > Y_{ij}, \quad Y_{22} > Y_{mn}, \quad (6)$$

where  $Y_{i,j}$  is any entry except  $Y_{33}$ , and  $Y_{mn}$  denotes the (1,1), (1,2) and (2,1) entries.

<sup>2</sup>In higher-dimensional field theory and some type of string compactification  $CP$  is a nice symmetry, that is, Yukawa couplings are real [14].

<sup>3</sup>We treat  $\alpha_S$  and  $\alpha_T$  as free parameters. If we fix a form of the SUSY breaking superpotential, these magnitudes can be fixed [15].

Under these assumptions, we can write a generic form of the down-Yukawa matrix,

$$Y_{ij}^d = Y_{33}^d \begin{pmatrix} (m_d/m_b)\Theta_{11}^d & V_{12}^{\text{CKM}}(m_s/m_b) & V_{13}^{\text{CKM}}\Theta_{13}^d \\ V_{12}^{\text{CKM}}(m_s/m_b) & (m_s/m_b) & V_{23}^{\text{CKM}}\Theta_{23}^d \\ V_{13}^{\text{CKM}}\Theta_{13}^d & V_{23}^{\text{CKM}}\Theta_{23}^d & 1 \end{pmatrix}, \quad (7)$$

and the up-Yukawa matrix

$$Y_{ij}^u = Y_{33}^u \begin{pmatrix} (m_u/m_t)\Theta_{11}^u & \sqrt{(m_u m_c/m_t^2)}\Theta_{12}^u\Theta_{22}^u & V_{13}^{\text{CKM}}\Theta_{13}^u \\ \sqrt{(m_u m_c/m_t^2)}\Theta_{12}^u\Theta_{22}^u & (m_c/m_t)\Theta_{22}^u & V_{23}^{\text{CKM}}\Theta_{23}^u \\ V_{13}^{\text{CKM}}\Theta_{13}^u & V_{23}^{\text{CKM}}\Theta_{23}^u & 1 \end{pmatrix}, \quad (8)$$

in terms of the eight free parameters  $\Theta_{11}^{u,d}$ ,  $\Theta_{12}^{u,d}$ ,  $\Theta_{13}^{u,d}$ , and  $\Theta_{23}^{u,d}$ , while  $\Theta_{22}^u$  is of order one. In addition we have a constraint,  $\Theta_{23}^d - \Theta_{23}^u \approx 1$ . A detailed discussion of this parametrization will be given in Ref. [16].

Actually, most of symmetric and hierarchical Yukawa mass matrices which have been already proposed in the literature are included in the above textures (7) and (8). For example, five types of symmetric Yukawa matrices with five texture zeros have been obtained in Ref. [17]. The following parameter assignments correspond to four Ramond-Roberts-Ross (RRR) types:

$$(\Theta_{23}^d, \Theta_{13}^d, \Theta_{23}^u, \Theta_{13}^u, \Theta_{12}^u) = (1,0,0,0,1) \text{ in the first RRR type}, \quad (9)$$

$$= (1,0,0,1,0) \text{ in the third RRR type}, \quad (10)$$

$$= (0,0,1,0,1) \text{ in the fourth RRR type}, \quad (11)$$

$$= (0,0,1,1,0) \text{ in the fifth RRR type}, \quad (12)$$

with the other parameters  $\Theta_{ij}^{u,d}$  suppressed.<sup>4</sup> Moreover, in Ref. [18] string-inspired realistic quark mass matrices have been studied and the obtained matrices correspond to the case with  $\Theta_{23}^d = \Theta_{12}^u = 1$ , a small value of  $\Theta_{23}^u$  and the other  $\Theta_{ij}^{u,d}$  suppressed.

### III. CP VIOLATION

As an example of nonuniversal cases, we take  $n_i = -1$  for the third family and  $n_i = -2$  for the first and second families. Also we assume that  $n_{H_1} = -1$  and  $n_{H_2} = -2$ . In this case, we find the following texture for the  $A$ -parameter matrix at the string scale

$$A^d = \begin{pmatrix} a_d & a_d & b_d \\ a_d & a_d & b_d \\ b_d & b_d & c_d \end{pmatrix}, \quad A^u = \begin{pmatrix} a_u & a_u & b_u \\ a_u & a_u & b_u \\ b_u & b_u & c_u \end{pmatrix}, \quad (13)$$

where

$$a_u = m_{3/2}(-\sqrt{3} \sin \theta + 3e^{-i\alpha'} \cos \theta), \quad (14)$$

$$a_d = b_u = m_{3/2}(-\sqrt{3} \sin \theta + 2e^{-i\alpha'} \cos \theta), \quad (15)$$

$$b_d = c_u = m_{3/2}(-\sqrt{3} \sin \theta + e^{-i\alpha'} \cos \theta), \quad (16)$$

$$c_d = -\sqrt{3}m_{3/2} \sin \theta. \quad (17)$$

In this paper we take  $\Theta_{23}^d = \Theta_{13}^d = \Theta_{13}^u = 1$  and  $\Theta_{11}^d = \Theta_{11}^u = \Theta_{12}^u = \Theta_{23}^u = 0$  as an example. Having specified the values of the soft terms at the string scale, we can use the electroweak breaking conditions at  $M_Z$ , which at tree level can be expressed as

$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - M_Z^2/2, \quad (18)$$

$$\sin 2\beta = \frac{-2|B \cdot \mu|}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2}, \quad (19)$$

where  $\tan \beta$  is the ratio of the vacuum expectation values of the Higgs fields. Using Eqs. (18) and (19) we can determine the value of  $|\mu|$  and  $|B|$  as functions on  $m_{3/2}$ ,  $\theta$ , and  $\alpha'$ . We impose  $\phi_B = 0$  to avoid large EDMs. The origin of this latter phase is linked to the way the  $\mu$  term is produced in effective supergravities. Given the focus of our paper on the role of nonuniversal  $A$  terms for  $CP$  violation, we are not going to discuss the different mechanisms to originate  $\mu$  and we simply consider a vanishing  $\phi_B$ . We also assume a low value of  $\tan \beta$ , namely, we consider it to be of order 3. Then all the

<sup>4</sup>The second RRR type corresponds to the suppressed value of  $\Theta_{22}^u$ .

supersymmetric particle spectrum is completely determined in terms of  $m_{3/2}$ ,  $\theta$ , and  $\alpha'$ . Apart from the key role of the nonuniversal  $A$  terms and of their phases in  $CP$  violation, it is worth mentioning that they have also important effects on the SUSY spectrum and on the non- $CP$  violating processes [6] and [19].

Before computing the contribution to  $CP$  violation in the kaon system from the phases in the above matrices  $A^d$  and  $A^u$ , one important remark is in order. The fact of having nondegenerate  $A$  terms is essential to obtain nonvanishing  $CP$  violating contributions from SUSY loops when the CKM matrix is taken to be real. Namely, it can be shown that, independently from how large one takes the SUSY  $CP$  phases  $\phi_A$  and  $\phi_B$ , in the presence of degenerate  $A$  terms and real CKM there is no way of generating a phenomenologically viable amount of  $CP$  violation in  $K$  physics. This point was already emphasized by Abel and Frere [11] who showed explicitly that  $\varepsilon$  turns out to be extremely tiny in box-diagrams with chargino–up-squark exchange and external left-handed quarks when the matrices  $(YA)$  in our notation are symmetric (notice that the symmetric form of the trilinear matrices  $A^u$  and  $A^d$  and the Yukawas matrices  $Y^u$  and  $Y^d$  in our case is at the GUT scale and indeed at weak scale our trilinear scalar terms are not symmetric in generation space since they have different running). Their proof can be readily extended to SUSY loops with gluino exchange. This can be seen as follow.

The value of the indirect  $CP$  violation in the kaon decays  $\varepsilon$  is defined as

$$\varepsilon = \frac{e^{i(\pi/4)} \text{Im} M_{12}}{\sqrt{2} \Delta m_K}, \quad (20)$$

where  $\Delta m_K = 2 \text{Re} \langle K^0 | H_{\text{eff}} | \bar{K}^0 \rangle = 3.52 \times 10^{-15} \text{ GeV}$ . The amplitude  $M_{12} = \langle K^0 | H_{\text{eff}} | \bar{K}^0 \rangle$  is given in Ref. [9] in terms of the mass insertion  $\delta_{AB}$  defined by  $\delta_{AB} = \Delta_{AB} / \tilde{m}^2$  where  $\tilde{m}$  is an average sfermion mass and the  $\Delta$ 's denote off-diagonal, flavor changing terms in the sfermion mass matrices.

We consider gluino exchange contributions with all external left-handed quarks. In this case the relevant flavor changing mass insertions that appear on the internal squark propagator lines accomplish the transition from  $\tilde{d}_{1L}$  to  $\tilde{d}_{2L}$  (1 and 2 are flavor indices):

$$(\Delta_{LL}^d)_{12} = [KM_Q^2 K^\dagger]_{12}, \quad (21)$$

where  $K$  is the Kobayashi-Maskawa matrix.

In the case of degenerate  $A$  terms, i.e.,  $A_{ij} = A \delta_{ij}$ , one obtains [8]

$$(\Delta_{LL}^d)_{12} \simeq -\frac{1}{8\pi^2} (K^\dagger h_U^2 K)_{12} (3m_0^2 + |A|^2), \quad (22)$$

i.e., the flavor changing mass insertions remain real (for a real CKM matrix) independently from the phase  $\phi_A$ . In such diagonal or degenerate cases the SUSY contribution to  $CP$  violation relies on the  $(\delta_{12})_{LR}$  and  $(\delta_{12})_{RL}$  mass insertions and it turns out to be very small [11]. Obviously this is no

longer true if we switch on the  $CP$  violating phase of the CKM matrix. In that case SUSY loops can give a non-negligible contribution to  $\varepsilon_K$ , although such contribution cannot be the major source of  $CP$  violation.

To obtain a large SUSY contribution to  $\varepsilon$ , it is necessary to enhance the values of  $\text{Im}(\delta_{12})_{LR}$  and  $\text{Im}(\delta_{12})_{RL}$ . The non-degenerate  $A$  terms is an interesting example for enhancing these quantities since the off-diagonal terms, namely,  $A_{12}^d$ , lead to nonvanishing values of  $(\delta_{12})_{LR}$  and  $(\delta_{12})_{RL}$  at the tree level [see Eq. (1)].

We consider the box diagrams which are responsible for the  $K^0$ - $\bar{K}^0$  transition and we focus on the contribution coming from gluino exchange in the loop (as we shall see below this turns out to be the dominant contribution to  $\varepsilon$ ) [9]

$$\begin{aligned} M_{12}^{\text{gluino}} = & -\frac{\alpha_S^2}{216m_q^2} \frac{1}{3} m_K f_K^2 \left\{ (\delta_{12}^d)_{LL}^2 [24x f_6(x) + 66\tilde{f}_6(x)] \right. \\ & + (\delta_{12}^d)_{RR}^2 [24x f_6(x) + 66\tilde{f}_6(x)] \\ & + (\delta_{12}^d)_{LL} (\delta_{12}^d)_{RR} \left\{ \left[ 384 \left( \frac{m_K}{m_s + m_d} \right)^2 + 72 \right] x f_6(x) \right. \\ & \left. + \left[ -24 \left( \frac{m_K}{m_s + m_d} \right)^2 + 36 \right] \tilde{f}_6(x) \right\} \\ & + (\delta_{12}^d)_{LR}^2 \left[ -132 \left( \frac{m_K}{m_s + m_d} \right)^2 x f_6(x) \right] \\ & + (\delta_{12}^d)_{RL}^2 \left[ -132 \left( \frac{m_K}{m_s + m_d} \right)^2 x f_6(x) \right] \\ & \left. + (\delta_{12}^d)_{LR} (\delta_{12}^d)_{RL} \left[ -144 \left( \frac{m_K}{m_s + m_d} \right)^2 - 84 \right] \tilde{f}_6(x) \right\}. \end{aligned} \quad (23)$$

Here,  $x = (m_g / M_{\tilde{q}})^2$  and the functions  $f_6(x), \tilde{f}_6(x)$  are given in Ref. [9]. The above result is obtained in the so-called super-KM basis [20] by making use of the mass insertion approximation method.

As we mentioned, we assume that the CKM matrix is real and the soft SUSY breaking terms are the only source for the complexity of the amplitude  $M_{12}$ . The relevant contribution to  $CP$  violation comes from the terms proportional to  $(\delta_{12})_{LR}$  and  $(\delta_{12})_{RL}$  in the above expression. Going to the basis where the down quark mass matrix is diagonal, the mass insertion  $(\delta_{12})_{LR}$  is given by

$$(\delta_{12}^d)_{LR} = U_{1i} (Y_d^A)_{ij} U_{j2}^T, \quad (24)$$

where  $U$  is the matrix diagonalizing the symmetric down quark mass matrix. The most relevant contributions in the above equation are

$$(\delta_{12}^d)_{LR} \simeq U_{11} (Y_d^A)_{12} U_{22}^T + U_{12} (Y_d^A)_{22} U_{22}^T + U_{13} (Y_d^A)_{33} U_{23}^T, \quad (25)$$

which implies that  $\text{Im}(\delta_{12}^d)_{LR}$  is of the same order as  $\text{Im}(Y_d^A)_{12}$  and indeed it is found to be of order  $10^{-4}$  and the

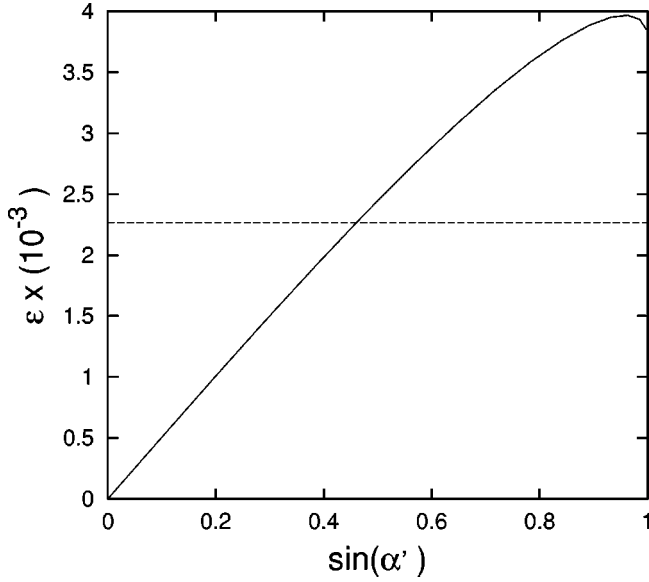


FIG. 1. The values of  $\varepsilon$  versus  $\sin \alpha'$  where the goldstino angle  $\theta \approx 0.8$  rad. and  $m_{3/2} \approx 100$  GeV.

same for  $(\delta_{12})_{RL}$ . Moreover the values of  $\text{Im}(\delta_{12})_{LL}$  and  $\text{Im}(\delta_{12})_{RR}$  are nonzero unlike in the universal case, but are smaller than  $(\delta_{12})_{LR}$ .

Also we estimate the chargino contribution to  $\varepsilon$ . It is found that it is proportional to  $(\delta_{13}^u)_{LR}(\delta_{23}^u)_{LR}$ . The amplitude for the chargino box contribution to  $K^0$ - $\bar{K}^0$  mixing can be written as

$$M_{12}^{\text{chargino}} \simeq -\frac{K_{13}K_{23}\alpha_W}{216m_q^2} \frac{1}{3} m_K f_K^2 (\delta_{13}^u)_{LR} (\delta_{23}^u)_{LR} \times \left[ -132 \left( \frac{m_K}{m_s + m_d} \right)^2 x f_6(x) \right], \quad (26)$$

where  $x = (m_{\chi^\pm} / M_{\tilde{q}})^2$ . The values of  $(\delta_{13}^u)_{LR}(\delta_{23}^u)_{LR}$  are two orders of magnitude larger than the values of  $(\delta_{12})_{LR}^2$  but because of the smallness of the coupling it is found that the amplitude of the chargino contribution is one order of magnitude less than the gluino amplitude. Thus the main contribution to  $\varepsilon$  is due to the gluino exchange. However, this is not necessarily true in the case of nonvanishing  $\phi_B$ .

Using the above values of the mass insertions we can determine the SUSY contribution to  $\varepsilon$ . Figure 1 shows  $\varepsilon$  in terms of  $\sin \alpha'$  for  $\theta \approx 0.8$  rad and  $m_{3/2} \approx 100$  GeV.

Also we give the values of  $\varepsilon$  in terms of  $\theta$  in Fig. 2, which shows that the nonuniversality between the soft supersymmetry breaking terms is preferred to enhance the SUSY  $CP$  violating contribution. Finally, we present the values of  $\varepsilon$  as a function of the gravitino mass in Fig. 3.

It is interesting to note that for  $m_{3/2} \sim 100$  GeV we obtain large values of  $\varepsilon$ , which even exceed the experimental limit  $2.2 \times 10^{-3}$ . Thus we have a constraint on  $(m_{3/2}, \alpha')$  from the experimental limit on  $\varepsilon$ . For instance, in case of  $\alpha' = \pi/2$  we find that  $m_{3/2} > 120$  GeV.

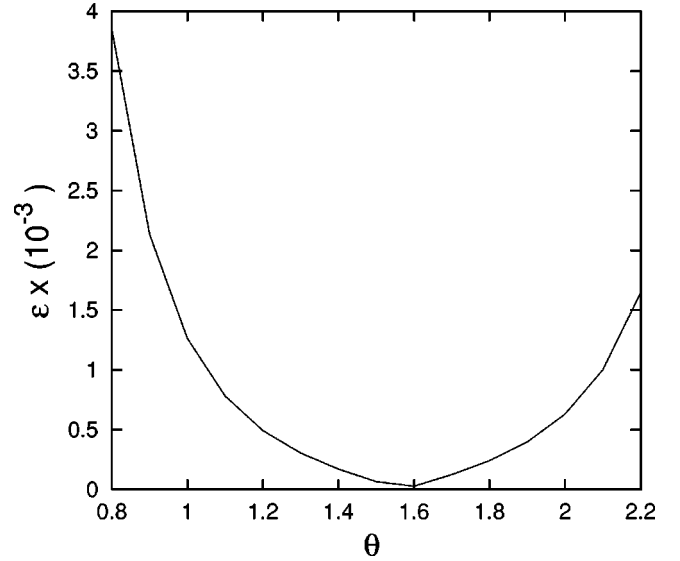


FIG. 2. The values of  $\varepsilon$  versus  $\theta$  where  $\alpha' \approx \pi/2$  and  $m_{3/2} \approx 100$  GeV.

We can proceed analogously for other values of  $\Theta_{ij}^{u,d}$ . For example, the Ramond-Roberts-Ross textures lead to very similar results. As another example, we can take the case with  $n_{H1} = -1$ ,  $n_{H2} = -2$ , and  $n_i = -1$  for the other matter fields. This case leads to a degenerate  $A$  matrix, i.e.,  $A_{ij}^u = A^u \delta_{ij}$  and  $A_{ij}^d = A^d \delta_{ij}$ . However, note that these  $A$  parameters in general have  $CP$  phases independent of the gaugino mass unlike the case where every field has the same modular weight  $n_i = -1$ . Consistently with our previous general considerations on the necessity of nondegeneracy of the  $A$  matrices to have sizable  $CP$  violating contributions, in this case  $\varepsilon$  turns out to be smaller than  $O(10^{-3})$ .

#### IV. ELECTRIC DIPOLE MOMENT OF THE NEUTRON

The supersymmetric contributions to the EDMN include gluino, chargino, and neutralino loops. Since we are consid-

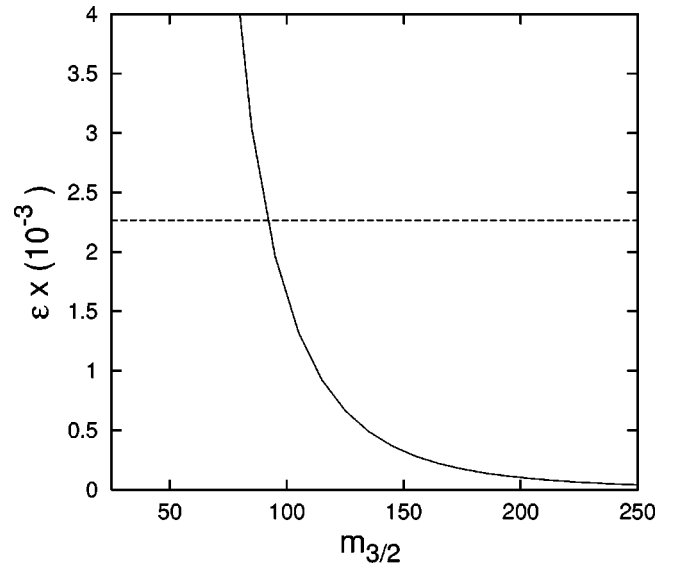


FIG. 3. The values of  $\varepsilon$  versus  $m_{3/2}$  where the Goldstino angle  $\theta \approx 0.8$  rad. and  $\alpha' \approx \pi/2$ .

ering the case with vanishing  $\phi_B$ , the gluino contribution is dominant. For the EDM of the quark  $u$  and  $d$  it amounts to [9]

$$d_d^g/e = -\frac{2}{9} \frac{\alpha_S}{\pi} \frac{m_{\tilde{g}}}{M_{\tilde{q}}^2} M_1(x) \text{Im}(\delta_{11}^d)_{LR}, \quad (27)$$

$$d_u^g/e = \frac{4}{9} \frac{\alpha_S}{\pi} \frac{m_{\tilde{g}}}{M_{\tilde{q}}^2} M_1(x) \text{Im}(\delta_{11}^u)_{LR}, \quad (28)$$

where  $m_{\tilde{g}}$  is the gluino mass,  $M_{\tilde{q}}^2$  is the average squark mass. The function  $M_1(x)$  is given by

$$M_1(x) = \frac{1 + 4x - 5x^2 + 4x \ln(x) + 2x^2 \ln(x)}{2(1-x)^4}. \quad (29)$$

As we explained in the last section, by using the electroweak breaking condition we can write all the spectrum in terms of  $m_{3/2}$ ,  $\theta$ ,  $\alpha_S$ , and  $\alpha_T$ . Then the EDM  $d_q^g/e$  is given in terms of these parameters. We use the nonrelativistic quark model approximation of the EDMN:

$$d_n = \frac{1}{3}(4d_d - d_u). \quad (30)$$

The mass insertion  $(\delta_{11})_{LR}$  is given by

$$(\delta_{11}^d)_{LR} = (U^T Y_d^A U)_{11} \simeq U_{21}(Y_{21} A_{21}^d) U_{11} + U_{21}(Y_{22} A_{22}^d) U_{21}.$$

In our case, we find that the  $\delta_{11}$  is only one order of magnitude less than the  $\delta_{12}$ . Unless the phases appearing in  $(\delta_{11}^d)_{LR}$  are small, we would expect the imaginary part of this latter quantity to be of order  $10^{-5} - 10^{-6}$ . This implies that the gluino contribution to the EDMN in this model is of order of  $10^{-25} e$  cm. Recently it has been shown that the above EDMN contributions can interfere destructively with other contributions in some regions of the SUSY parameter space [5,6]. In Sec. III, we have shown that there exists a relatively large region of the parameter space to enhance  $\varepsilon$  with nonuniversal  $A$  parameters. Thus, it may be possible to find some cancellation with other contributions allowing for the EDMN to be suppressed to values below  $10^{-25} e$  cm. To find such a parameter space, a detailed analysis is needed in particular with the inclusion of the effects of  $\phi_B$  (we plan to provide it elsewhere [16]). Other than such a cancellation, as we said, the nonuniversal cases could include the very

specially fine-tuned case where only  $CP$  phases of  $A$  elements contributing to the EDMN are suppressed.

Finally we comment on the  $\Delta S=1$   $CP$  violating parameter  $\varepsilon'/\varepsilon$ . In our case, where  $(\delta_{12})_{LR}$  and  $(\delta_{12})_{RL}$  give the important contributions to the  $CP$  violation processes in kaon physics, the relevant part of the effective Hamiltonian  $\mathcal{H}_{\text{Meff}}$  for  $\Delta S=1$   $CP$  violation is

$$\mathcal{H}_{\text{eff}} = C_8 O_8 + \tilde{C}_8 \tilde{O}_8, \quad (31)$$

where  $C_8$  and  $O_8$  are given in Ref. [9] and  $\tilde{C}_8$  can be obtained from  $C_8$  by exchange  $L \leftrightarrow R$  and the matrix element of the operator  $\tilde{O}_8$  is obtained from the matrix element of  $O_8$  multiplying them by  $(-1)$ . Since we have  $(\delta_{12})_{LR}$  approximately equal to  $(\delta_{12})_{RL}$ , then  $C_8$  is very close to  $\tilde{C}_8$  and hence, the value of  $\varepsilon'$  is very small. This cancellation between the different contributions to  $\varepsilon'$  is mainly due to the symmetric nature of the trilinear and/or Yukawa matrices we adopted. It has recently been shown that in the absence of such cancellation it is possible to obtain large values of  $\varepsilon'/\varepsilon$  (compatible with the experimental results of NA31 and KTeV) using the flavor changing trilinear scalar terms of the soft breaking sector [22].

## V. CONCLUSIONS

We have studied  $CP$  violation in the SUSY model with the nondegenerate  $A$  terms derived from superstring theory. This type of nonuniversality has a significant effect in the  $CP$  violation. We have shown the region of the parameter space where we have SUSY contribution for  $\varepsilon$  of order  $10^{-3}$ .

It is interesting to investigate effects of nonuniversality on other  $CP$  aspects, e.g., detailed analysis of the EDMN. Also the effect of the nondegeneracy of the  $A$  terms is important in studying the  $B$  physics.

We have considered the case where the dilaton field and only the overall moduli field contribute to SUSY breaking and in this case only one independent  $CP$  phase  $\alpha' = \alpha_T - \alpha_S$  appears in the  $A$  parameters. It would be interesting to discuss multimoduli cases [21], where several independent  $CP$  phases appear.

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