# Nonresonant semileptonic heavy quark decay

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In both the large $N_c$ limit and the valence quark model, semileptonic decays are dominated by resonant final states. Using Bjorken's sum rule in an ''unquenched'' version of the quark model, I demonstrate that in the heavy quark limit nonresonant final states should also be produced at a significant rate. By calculating the individual strengths of a large number of exclusive two-body nonresonant channels, I show that the total rate for such processes is highly fragmented. I also describe some very substantial duality-violating suppression factors which reduce the inclusive nonresonant rate to a few percent of the total semileptonic rate for the finite quark masses of $\bar{B}$ decay, and comment on the importance of nonresonant decays as testing grounds for very basic ideas on the structure, strength, and significance of the $q\bar{q}$ sea and on quark-hadron duality in QCD. [S0556-2821(99)03817-5]

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## I. INTRODUCTION

### A. Background

In both the large $N_c$ limit [1] and the valence quark model [2,3], semileptonic heavy quark decays are saturated by resonant final states. In nature this idealization is broken by light quark pair creation which gives these infinitely narrow resonances widths and allows nonresonant final states to appear. While it is clear that $q\bar{q}$ pairs play an important role in this and many other phenomena, it is also clear that they remain poorly understood:

(1) Although from their widths it is easy to show that hadrons are full of $q\bar{q}$ pairs, meson and baryon spectroscopies are characterized by the valence degrees of freedom. In particular there is no evidence for excitations of the $q\bar{q}$ sea with respect to the valence quarks.

(2) Even if one were to assume that the $q\bar{q}$ sea is frozen out of spectroscopy, it is easy to show that, unless there is a conspiracy between them, meson loop graphs ought to destroy the successes of quark model spectroscopy. For example, the relative shift of the $\rho$ and $a_1$ due to meson loops formed from their dominant decay modes ($\pi\pi$ and $\rho\pi$, respectively) is hundreds of MeV [4].

(3) Related to (2), but even more dramatic, is the relative shift of pairs of mesons such as the $\rho$ and $\omega$. They are degenerate in the quark model and in the large $N_c$ limit, as observed in nature, but meson loop diagrams would lead one to expect them to develop a mass difference of hundreds of MeV [5].

(4) Given that meson loop diagrams which generate the $q\bar{q}$ sea are very strong, it is surprising that the valence quarks seem to dominate low energy current matrix elements [6]. The extreme interest generated by the proton spin crisis may be attributed to the fact that it indicates that there are some current matrix elements where the valence quarks do not dominate.

While the questions $q\bar{q}$ pair creation raises are ubiquitous, heavy quark systems (e.g., $Q\bar{d}$ and $udQ$) are likely to be the most favorable systems in which to find answers: of all systems governed by strong QCD [7], they are arguably the simplest. Indeed, many of their properties can be rigorously derived directly from QCD using heavy quark symmetry [8,9]. Their simplicity also makes them more amenable to modelling than other hadrons. For example, as ''the hydrogen atoms of QCD'' with the heavy quark defining an origin of coordinates, the simplest relativistic constituent quark models can treat $Q\bar{d}$ systems using the Dirac equation [10] rather than a Bethe-Salpeter-type equation. Because the heavy quark is removed from consideration as $m_Q\to\infty$, such systems offer unique opportunities to study the ''brown muck'' one chunk at a time.

In this paper I expand the quark model treatment of the ''brown muck'' in heavy meson semileptonic decay from a simple valence $\bar{d}$ or $\bar{u}$ antiquark confined to $Q$ to include the leading effects of $q\bar{q}$ pair creation. See Fig. 1. Previous studies have examined low energy pion emission in the context

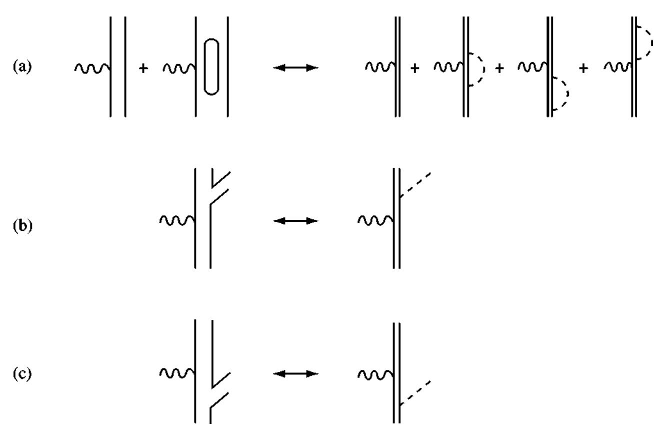


FIG. 1. Leading corrections to narrow resonance saturation of the rates for semileptonic heavy quark decay, with quark level diagrams on the left and their hadronic counterparts on the right. Internal hadronic lines are to be understood as summed over their full valence spectra. (a) The *valence→valence* graph with its leading vertex and external leg corrections, (b) the *valence→valence* graph followed by final state decay, and (c) decay from a nonvalence component of the initial state to a nonresonant two particle final state.

 

of heavy quark chiral perturbation theory [11]; this work is the first of which I am aware to address the full array of nonresonant processes. While the results presented here will be specific to heavy quark systems, I will draw lessons from them and suggest experimental consequences of much wider interest.

### B. Nonresonant final states in $\bar{B}$ and $D$ semileptonic decay

For $D$ decays induced by the underlying $c \to s \bar{l} \nu_l$ quark decay, $D \to \bar{K} \bar{l} \nu_l$ and $D \to \bar{K}^* \bar{l} \nu_l$ are clearly dominant, accounting for more than 90% of the inclusive $D \to \bar{X}_s \bar{l} \nu_l$ semileptonic rate [12]. For $\bar{B}$ decays induced by $b \to c l \bar{\nu}_l$, $\bar{B} \to D l \bar{\nu}_l$ and $\bar{B} \to D^* l \bar{\nu}_l$ account for $64 \pm 6\%$ of the inclusive $\bar{B} \to X_c l \bar{\nu}_l$ semileptonic rate [12]. Given the large energy release in $b \to c$ versus $c \to s$, it is not surprising that in the former the two ground state resonances of the $s_l^{\pi_l} = \frac{1}{2}^-$ multiplet [9] would account for less of the total semileptonic rate. Indeed, it has been argued [13] by assuming duality between the Isgur-Scora-Grinstein-Wise version 2 (ISGW2) valence quark model [3] and QCD-corrected inclusive calculations [14] that a *complete* ISGW2-based calculation would predict that $20 \pm 6\%$ of the $\bar{B} \to X_c l \bar{\nu}_l$ rate should go to resonant excited states above the $D$ and $D^*$. (The quoted error corresponds to an estimate of the theoretical uncertainties in the QCD-corrected inclusive rate calculations.) ISGW2 as published is in contrast not exhaustive: it explicitly computes the rates to the $L=1$ excited states with $s_l'^{\pi_l'} = \frac{1}{2}^+$ and $s_l'^{\pi_l'} = \frac{3}{2}^+$ and to the first radial excitations with $s_l'^{\pi_l'} = \frac{1}{2}^-$. These six lowest-lying excitations give $8 \pm 1\%$ of the QCD-corrected inclusive rate, implying that an additional $12 \pm 6\%$ of the rate of a complete ISGW2 calculation should be in yet more highly excited states (both ordinary mesons and hybrids). The ISGW2 model and next-to-leading-order QCD can be compared in this way because both are valence-quark-plus-glue calculations: $q\bar{q}$ pairs are ignored in ISGW2 as $1/N_c$ corrections and would enter the partonic level inclusive rates only at order $\alpha_s^2$ (*via* $1/N_c$-suppressed graphs).

Experimentally, the extent of resonance dominance of $\bar{B}$ and $D$ semileptonic decays remains unclear. In $D$ decay there are explicit measurements giving $D \to \bar{K} \pi \bar{l} \nu_l$ rates which are $3 \pm 1\%$ of the semileptonic rate. Here the $\bar{K}\pi$ signal is excluded from being the $\bar{K}^*$, but it is not excluded that it could arise from the tails of broad resonances, so even this small fraction cannot be unambiguously identified as nonresonant. In $\bar{B}$ decay, $36 \pm 6\%$ of the semileptonic rate is not in the $D$ or $D^*$. Since the dynamical part $\rho_{dyn}^2 \equiv \rho^2 - \frac{1}{4}$ of the slope of the Isgur-Wise function $\xi(w)$ could be as large as about $\frac{3}{4}$ [15], the *loss* of rate from these channels relative to $\rho_{dyn}^2 = 0$ in the approximation $\xi \simeq 1 - \rho^2(w-1)$ could indeed be as large as 36%, i.e., the observed non-$(D+D^*)$ rate is not inconsistent with that expected from Bjorken's sum rule [16,17] in the heavy quark limit. Since, as just explained, one expects $20 \pm 6\%$ of this non-$(D+D^*)$ rate to be in excited resonant states, there would be room for $16 \pm 8\%$ of $\bar{B}$ semileptonic decays to be nonresonant, i.e., for a large fraction of $\rho_{dyn}^2$ to be due to nonresonant channels. There is another closely related indication that nonresonant channels might be significant: ISGW2 overpredicts the $D$ and $D^*$ rates by amounts which are consistent with the observation that the Isgur-Wise function is falling about 25% more rapidly than expected from the opening of just excited resonance decay channels. While suggestive, both of these observations are also consistent with the ISGW2 model simply underpredicting decay rates to excited charm states [18]. An additional concern is that the suggestive loss of rate from the $D$ and $D^*$ channels calculated from Bjorken's sum rule is actually an upper bound in the heavy quark limit: expected quadratic terms in $\xi$ will dampen this loss, suggesting that perhaps the experimental non-$(D+D^*)$ fraction is high.

In summary, there is weak circumstantial evidence for nonresonant processes in heavy quark semileptonic decays. The most compelling case for the existence of such processes at some level is nevertheless the simple theoretical observation that in the real world we expect very strong nonvalence $Q\bar{q}q\bar{d}$ components in a $Q\bar{d}$ state and so expect some inclusive rate to be lost from resonances and transferred to continua. In what follows I will make these qualitative expectations, previously outlined in Ref. [13], concrete.

### C. The ISGW and ISGW2 models

In addition to providing a useful phenomenological guide to semileptonic decays, the ISGW model [2] was in many respects a stepping-stone to heavy quark symmetry, as it manifested this symmetry near zero recoil. ISGW2 [3] is an update of ISGW with many new features required by heavy quark symmetry: it includes constraints on the relations between form factors away from zero recoil and on the slopes of form factors near zero recoil [16,17], it relates the naive currents of the quark model to the full weak currents *via* the matching conditions of heavy quark effective theory (HQET) [19], and it modifies the *ad hoc* ISGW prescription for connecting quark model form factors to physical form factors to be consistent with the constraints of heavy-quark-symmetry-breaking at order $1/m_Q$. Several other improvements were also made, including the addition of heavy-quark-symmetry-breaking color magnetic interactions to the quark model's dynamics, the incorporation of relativistic corrections to the axial coupling constants (known to be important in the analogous coupling $g_A$ in neutron beta decay), and the use of more realistic form factor shapes, based on the measured pion form factor. For a more complete discussion of the foundations, strengths, and weaknesses of such models, see Refs. [2,3].

In this paper I focus on correcting these models' neglect of the nonvalence components of hadrons, but note that many of the results to be described here are anticipated in the extensive discussion of Ref. [2] leading to the conclusion that nonresonant states could be ignored as a first approximation to the dynamics of semileptonic decays.

### D. Unquenching the quark model

Some of the key puzzles associated with the nature and importance of $q\bar{q}$ pairs in low energy hadron structure were described above. These puzzles and potential solutions to them have been extensively discussed in a series of papers on ''unquenching'' the quark model [4–6]. In the following I briefly summarize these solutions, since they are the basis for the study presented here.

#### *1. The origin and resiliency of potential models*

A central puzzle in hadron spectroscopy is the apparent absence of low energy degrees of freedom beyond those which can be attributed to the valence quarks (e.g., gluonic or sea quark excitations). Very closely related to this puzzle is the apparent unimportance of strong meson loop corrections.

A simple resolution of this puzzle has been proposed [20]. In the flux tube model [21], the quark potential model arises from an adiabatic approximation to the gluonic and extra $q\bar{q}$ degrees of freedom embodied in the flux tube. This physics has an analog at short distances where perturbation theory applies. There $N_f$ types of light $q\bar{q}$ pairs shift (in lowest order) the coefficient of the Coulombic potential from $\alpha_s^{(0)}(Q^2)=12\pi/33\ln(Q^2/\Lambda_0^2)$ to $\alpha_s^{(N_f)}(Q^2)=12\pi/(33-2N_f)\ln(Q^2/\Lambda_{N_f}^2)$, the net effect of such pairs thus being to produce a *new* effective short distance $Q\bar{Q}$ potential. Similarly, when pairs bubble up in the flux tube (i.e., when the flux tube breaks to create a $Q\bar{q}$ plus $q\bar{Q}$ system and then ''heals'' back to $Q\bar{Q}$), their net effect is to cause a shift $\Delta E_{N_f}(r)$ in the ground state gluonic energy which in turn produces a new long-range effective $Q\bar{Q}$ potential.

It has indeed been shown [4] that the net long-distance effect of the bubbles is to create a new string tension $b_{N_f}$ (i.e., that the potential remains linear). Since this string tension is to be associated with the observed string tension, after renormalization *pair creation has no effect on the long-distance structure of the quark model in the adiabatic approximation*. Thus the net effect of mass shifts from pair creation is much smaller than one would naively expect from the magnitude of typical hadronic widths: such shifts can only arise from nonadiabatic effects [22].

It should be emphasized that no simple truncation of the set of all meson loop graphs can reproduce such results: to recover the adiabatic approximation requires summing over large towers of $Q\bar{q}$ plus $q\bar{Q}$ intermediate states to saturate their duality with $q\bar{q}$ loop diagrams which have strength at high energy.

#### *2. The survival of the OZI rule*

There is another puzzle of hadronic dynamics which is reminiscent of this one: the success of the Okubo-Zweig-Iizuka (OZI) rule [23]. A generic OZI-violating amplitude $A_{OZI}$ can be shown to vanish like $1/N_c$, and this is often quoted as a rationale for the OZI rule. However, there are several unsatisfactory features of this ''solution'' to the OZI mixing problem [24]. Consider $\omega$-$\phi$ mixing as an example. This mixing receives a contribution from the virtual hadronic loop process $\omega\to K\bar{K}\to\phi$, both steps of which are OZI-allowed, and each of which scales with $N_c$ like $\Gamma^{1/2}\sim N_c^{-1/2}$. The large $N_c$ result that this OZI-violating amplitude behaves like $N_c^{-1}$ is thus not peculiar to large $N_c$: it just arises from ''unitarity'' in the sense that the real and imaginary parts of a generic hadronic loop diagram will have the same dependence on $N_c$. The usual interpretation of the OZI rule in this case—that ''double hairpin graphs'' are dramatically suppressed—is untenable in the light of these OZI-allowed loop diagrams. They expose the deficiency of the large $N_c$ argument since $A_{OZI}\sim\Gamma$ is *not* a good representation of the OZI rule. (Continuing to use $\omega$-$\phi$ mixing as an example, we note that $m_\omega-m_\phi$ is numerically comparable to a typical hadronic width, so the large $N_c$ result would predict an $\omega$-$\phi$ mixing angle of order unity in contrast to the observed pattern of very weak mixing which implies that $A_{OZI}\ll\Gamma\ll m$.)

Unquenching the quark model thus endangers the naive quark model's agreement with the OZI rule. It has been shown [5] how this disaster is naturally averted in the flux tube model through a ''miraculous'' set of cancellations between mesonic loop diagrams consisting of apparently unrelated sets of mesons (e.g., the $K\bar{K}$, $K\bar{K}^*+K^*\bar{K}$, and $K^*\bar{K}^*$ loops tend to strongly cancel against loops containing a $K$ or $K^*$ plus one of the four strange mesons of the $L=1$ meson nonets). Of course the ''miracle'' occurs for a good reason: the sum of *all* hadronic loops is dual to a closed $q\bar{q}$ loop created and destroyed by a $^3P_0$ operator [25,26], but in the closure approximation such an operator cannot create mixing in other than a scalar channel. It can also be shown [6] that current matrix elements like $\bar{s}\gamma^\mu s$ vanish in this same approximation.

#### *3. A summary comment on modelling the effects of $q\bar{q}$ pairs*

The preceding discussion strongly suggests that models which have not addressed the effects of unquenching on spectroscopy and the OZI rule should be viewed very skeptically as models of the effects of the $q\bar{q}$ sea on hadron structure: large towers of mesonic loops are required to understand how quarkonium spectroscopy and the OZI rule survive once strong pair creation is turned on. In particular, while pion and kaon loops (which tend to break the closure approximation due to their exceptional masses) have a special role to play, they will not allow a satisfactory solution to these fundamental problems associated with unquenching the quark model and so cannot be expected to provide a reliable guide to the physics of $q\bar{q}$ pairs.

Indeed, I hope the reader can appreciate just on the basis of this lightning review that there are great dangers in drawing conclusions about the strength, structure, or significance of $q\bar{q}$ pairs in hadrons from any model that has not dealt with these issues.

## II. UNQUENCHING HEAVY QUARK DECAY

### A. Background

To unquench predictions of the quark model for semileptonic heavy quark decay, I will apply without alteration the

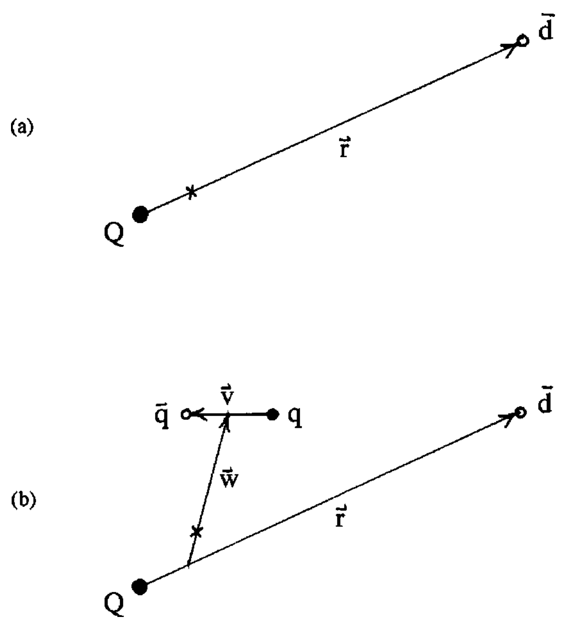


FIG. 2. The coordinates for (a) the $Q\bar{d}$ system, and (b) the $Q\bar{q}q\bar{d}$ system. In each diagram the cross $\times$ denotes the location of the center-of-mass; most results presented in this paper are in the heavy quark limit where the center-of-mass coincides with the position of $Q$.

model of Refs. [4–6] which solves the phenomenological problems associated with unquenching the quark model. In particular, I assume that the $q\bar{q}$ pair is created by the action of a pair creation Hamiltonian density $H_{pc}(x)$ in the $Q\bar{d}$ flux tube. I further assume that the pair is created with a nonlocality (corresponding to a finite constituent quark radius) in the coordinate $\vec{v}$. See Fig. 2.

The coordinates used here in $Q\bar{d}$ are the standard center-of-mass and relative coordinates

$$\vec{R}_{\rm cm}=\frac{m_Q\vec{r}_Q+m_d\vec{r}_{\bar{d}}}{m_{Q\bar{d}}}\simeq\vec{r}_Q \tag{1}$$

$$\vec{r}=\vec{r}_{\bar{d}}-\vec{r}_Q \tag{2}$$

while in $Q\bar{q}q\bar{d}$ the choice is

$$\vec{R}'_{\rm cm}=\frac{m_Q\vec{r}_Q+m_q(\vec{r}_q+\vec{r}_{\bar{q}})+m_d\vec{r}_{\bar{d}}}{m_{Q\bar{q}q\bar{d}}}\simeq\vec{r}_Q \tag{3}$$

$$\vec{r}=\vec{r}_{\bar{d}}-\vec{r}_Q \tag{4}$$

$$\vec{v}=\vec{r}_{\bar{q}}-\vec{r}_q \tag{5}$$

$$\vec{w}=\frac{1}{2}(\vec{r}_q+\vec{r}_{\bar{q}})-\frac{m_Q\vec{r}_Q+m_d\vec{r}_{\bar{d}}}{m_{Q\bar{d}}}\simeq\frac{1}{2}(\vec{r}_q+\vec{r}_{\bar{q}})-\vec{r}_Q, \tag{6}$$

where $m_{ij\dots k}\equiv m_i+m_j+\cdots+m_k$. [Note that the three-vector coordinate $\vec{w}$ should not be confused with the Lorentz invariant heavy quark scalar product $w=v'\cdot v$ called "double $u$" (or, in many European countries, "double $v$") used in heavy quark form factors.] Inverting, we have in $Q\bar{d}$

$$\vec{r}_Q=\vec{R}_{cm}-\epsilon_{d/Q\bar{d}}\vec{r}\simeq\vec{R}_{cm} \tag{7}$$

$$\vec{r}_{\bar{d}}=\vec{R}_{cm}+\epsilon_{Q/Q\bar{d}}\vec{r}\simeq\vec{R}_{cm}+\vec{r} \tag{8}$$

where $\epsilon_{\alpha/\beta}\equiv m_\alpha/m_\beta$. In $Q\bar{q}q\bar{d}$ we have

$$\vec{r}_Q=\vec{R}'_{cm}-\epsilon_{q\bar{q}/Q\bar{q}q\bar{d}}\vec{w}-\epsilon_{d/Q\bar{d}}\vec{r}\simeq\vec{R}'_{cm} \tag{9}$$

$$\vec{r}_{\bar{q}}=\vec{R}'_{cm}+\epsilon_{Q\bar{d}/Q\bar{q}q\bar{d}}\vec{w}+\frac{1}{2}\vec{v}\simeq\vec{R}'_{cm}+\vec{w}+\frac{1}{2}\vec{v} \tag{10}$$

$$\vec{r}_q=\vec{R}'_{cm}+\epsilon_{Q\bar{d}/Q\bar{q}q\bar{d}}\vec{w}-\frac{1}{2}\vec{v}\simeq\vec{R}'_{cm}+\vec{w}-\frac{1}{2}\vec{v} \tag{11}$$

$$\vec{r}_{\bar{d}}=\vec{R}'_{cm}-\epsilon_{q\bar{q}/Q\bar{q}q\bar{d}}\vec{w}+\epsilon_{Q/Q\bar{d}}\vec{r}\simeq\vec{R}'_{cm}+\vec{r}. \tag{12}$$

Note that most of the results of this paper are presented in the heavy quark limit where the approximations shown in these formulas will often be used.

Thus for a $0^-$ state

$$\Psi_{Q\bar{d}}=\frac{1}{(2\pi)^{3/2}}e^{i\vec{P}_{cm}\cdot\vec{R}_{\rm cm}}\psi_{Q\bar{d}}(\vec{r})\chi^0_{s_Qs_{\bar{d}}} \tag{13}$$

where $\chi^0_{s_Qs_{\bar{d}}}$ is the spin zero wave function, so that

$$\Phi_{Q\bar{d}}=\delta^3(\vec{P}-\vec{P}_{cm})\phi_{Q\bar{d}}(\vec{p})\chi^0_{s_Qs_{\bar{d}}} \tag{14}$$

is the momentum space $Q\bar{d}$ wave function with

$$\phi_{Q\bar{d}}(\vec{p})\equiv\frac{1}{(2\pi)^{3/2}}\int d^3r e^{-i\vec{p}\cdot\vec{r}}\psi_{Q\bar{d}}(\vec{r}) \tag{15}$$

and accordingly

$$|P_{Q\bar{d}}(\vec{P}_{cm})\rangle=\sqrt{2m_{Q\bar{d}}}\int d^3p\,\phi_{Q\bar{d}}(\vec{p})\chi^0_{s_Qs_{\bar{d}}}\times|Q(\epsilon_{Q/Q\bar{d}}\vec{P}_{cm}-\vec{p},s_Q)\bar{d}(\epsilon_{d/Q\bar{d}}\vec{P}_{cm}+\vec{p},s_{\bar{d}})\rangle. \tag{16}$$

Note that in the limit $\vec{P}_{cm}\to 0$ relevant here, the factor $\sqrt{2m_{Q\bar{d}}}\simeq\sqrt{2E_{P_{Q\bar{d}}}}$ is purely conventional as $m_Q\to\infty$.

When the flux-tube-breaking pair creation Hamiltonian $\mathbf{H}^{q\bar{q}}_{pc}\equiv\int d^3x H^{q\bar{q}}_{pc}(0,\vec{x})$ acts,

$$\mathbf{H}^{q\bar{q}}_{pc}|P_{Q\bar{d}}(\vec{P}_{cm})\rangle=\eta_{q\bar{q}}|P_{Q\bar{q}q\bar{d}}(\vec{P}_{cm})\rangle \tag{17}$$

where, according to the flux tube model,

(1) since the $q\bar{q}$ pair is created in the flux tube, its center-of-mass is found in a wave function $\psi_{ft}(\vec{w},\vec{r})$ defined by the flux tube spatial profile,

(2) the *internal* wave function of the $q\bar{q}$ pair has $J^{PC}=0^{++}$ and, independent of $\vec{w}$ and $\vec{r}$, is of the form

$\psi^m_{pc}(\vec{v})\cdot\chi^{-m}_{s_q s_{\bar q}}$ where $\psi^m_{pc}(\vec{v})$ has $L=1$, $\chi^{-m}_{s_q s_{\bar q}}$ has $S=1$, and $\psi^m\cdot\chi^{-m}\equiv(1/\sqrt{3})(\psi^1\chi^{-1}-\psi^0\chi^0+\psi^{-1}\chi^1)$, and

(3) the amplitude to find the $Q\bar{d}$ subsystem inside the $Q\bar{q}q\bar{d}$ system at relative separation $\vec{r}$ is identical to that in the ground state, namely $\psi_{Q\bar{d}}(\vec{r})$, since the pair creation Hamiltonian density acts locally and instantaneously on the flux tube. In this formulation, $\eta_{q\bar{q}}$ defines the strength of the pair creation, and the normalized state $|P_{Q\bar{q}q\bar{d}}(\vec{P}_{cm})\rangle$ is determined by the wave function just described:

$$\Psi_{Q\bar{q}q\bar{d}} = \frac{1}{(2\pi)^{3/2}}e^{i\vec{P}_{cm}\cdot\vec{R}'_{\rm cm}}\psi_{ft}(\vec{w},\vec{r})\psi^m_{pc}(\vec{v})\cdot\chi^{-m}_{s_q s_{\bar q}}\psi_{Q\bar{d}}(\vec{r})\chi^0_{s_Q s_{\bar d}}. \tag{18}$$

The component parts of this wave function and of Eq. (13) are defined by previous studies. From ISGW2 [3],

$$\psi_{Q\bar{d}}\simeq\frac{\beta^{3/2}_{Q\bar{d}}}{\pi^{3/4}}e^{-(1/2)\beta^2_{Q\bar{d}}r^2} \tag{19}$$

where $\beta_{Q\bar{d}}=0.41$ GeV as $m_Q\to\infty$ as determined variationally from a Coulomb-plus-linear-plus-hyperfine Schrödinger equation. The pair creation wave function $\vec{\psi}_{pc}$ is constrained in Ref. [4] by fitting decay data assuming the form

$$\vec{\psi}_{pc}\sim\vec{v}e^{-3v^2/8r_q^2}\equiv\vec{v}e^{-(1/2)\beta^2_{pc}v^2} \tag{20}$$

to have a quark radius $0<r_q<0.4$ fm. Given this constraint, I will take $r_q=0.3$ fm as a ''canonical'' value, corresponding to $\beta_{pc}\simeq0.58$ GeV, but will consider deviations of $\pm0.1$ fm from this value as plausible. (This central value and range are guided by the difficulty of inventing a mechanism which could lead to a constituent quark radius $r_q<0.2$ fm.) Ideally [26], $\psi_{ft}(\vec{w},\vec{r})$ should have a probability profile which is a tube around the $Q\bar{d}$ axis with ''caps'' at $Q$ and $\bar{d}$. This structure is probably very significant for the decays of highly excited states, but since all our decays will emerge from the $Q\bar{d}$ ground state, I adopt a simpler and more heuristic model which simply takes

$$\psi_{ft}(\vec{w})=\frac{\beta^{3/2}_{ft}}{\pi^{3/4}}e^{-(1/2)\beta^2_{ft}w^2} \tag{21}$$

with $\beta^2_{ft}=fb$, $b$ being the string tension and $f$ a coefficient with ''canonical'' value 2 and an uncertainty estimated to be $\pm1$ based on the calculations of the properties of the ground state wave function of the flux tube presented in Appendix A of Ref. [26]. Some defects in this simplification and some subtleties associated with both the alternative of using a flux-tube shape and nonrelativistic kinematics are discussed in Appendix A of this paper.

Since strong decay amplitudes are determined by matrix elements of $H^{qq}_{pc}(t,\vec{x})$ between the decaying particle and the continuum, $\eta_{q\bar{q}}$ can be determined empirically. In Appendix B, I extract from $D^*\to D\pi$ and $K^*\to K\pi$ decays the coefficients $\eta_{q\bar{q}}$ for $q=u$ or $d$. For concreteness, I will assume following Refs. [26,4], that $\eta_{s\bar{s}}$ is identical, but that the $\eta_{Q\bar{Q}}$ for $Q=c,b,t$ are all zero. Finally, for ease of exposition, I will treat explicitly the case of a single $q\bar{q}$ flavor in what follows, but take into account $q=u,d,s$ in numerical results.

Within these approximations,

$$\Phi_{Q\bar{q}q\bar{d}}=\delta^3(\vec{P}-\vec{P}_{cm})\phi_{ft}(\vec{\omega})\phi^m_{pc}(\vec{\pi})\cdot\chi^{-m}_{s_q s_{\bar q}}\phi_{Q\bar{d}}(\vec{p})\chi^0_{s_Q s_{\bar d}} \tag{22}$$

where $\phi_{ft}$, $\phi^m_{pc}$, and $\phi_{Q\bar{d}}$ are the momentum space wave functions corresponding to $\psi_{ft}$, $\psi^m_{pc}$, and $\psi_{Q\bar{d}}$, respectively, so that

$$\begin{aligned}|P_{Q\bar{q}q\bar{d}}(\vec{P}_{cm})\rangle &= \sqrt{2m_{Q\bar{q}q\bar{d}}}\int d^3\omega\int d^3\pi\int d^3p\,\phi_{ft}(\vec{\omega})\\ &\times\phi^m_{pc}(\vec{\pi})\cdot\chi^{-m}_{s_q s_{\bar q}}\phi_{Q\bar{d}}(\vec{p})\chi^0_{s_Q s_{\bar d}}\Big|Q(\epsilon_{Q/Q\bar{q}q\bar{d}}\vec{P}_{cm}\\ &-\epsilon_{Q/Q\bar{d}}\vec{\omega}-\vec{p},s_Q)\bar{q}\left(\epsilon_{q/Q\bar{q}q\bar{d}}\vec{P}_{cm}+\frac{1}{2}\vec{\omega}+\pi,s_{\bar q}\right)\\ &\times q\left(\epsilon_{q/Q\bar{q}q\bar{d}}\vec{P}_{cm}+\frac{1}{2}\vec{\omega}-\pi,s_q\right)\bar{d}(\epsilon_{d/Q\bar{q}q\bar{d}}\vec{P}_{cm}\\ &-\epsilon_{d/Q\bar{d}}\vec{\omega}+\vec{p},s_{\bar d})\Big\rangle.\end{aligned} \tag{23}$$

(Up to this point, I have retained the exact kinematics of the nonrelativistic limit for finite $m_Q$, but from now on I will generally simplify results by taking the heavy quark symmetry limit $m_Q\to\infty$ with $\vec{V}_{cm}\equiv\vec{P}_{cm}/m_Q$ fixed.)

While Eq. (17) defines the action of $\mathbf{H}^{\mathbf{q\bar{q}}}_{\mathbf{pc}}$ on the $Q\bar{d}$ sector, it does not of course provide us with the $\mathbf{H}^{\mathbf{q\bar{q}}}_{\mathbf{pc}}$-perturbed ground state. This state is of the form

$$|P_Q(\vec{P}_{cm})\rangle=\frac{1}{\sqrt{1+c^2_{q\bar{q}}}}[|P_{Q\bar{d}}(\vec{P}_{cm})\rangle+c_{q\bar{q}}|\tilde{P}_{Q\bar{q}q\bar{d}}(\vec{P}_{cm})\rangle] \tag{24}$$

where $|\tilde{P}_{Q\bar{q}q\bar{d}}(\vec{P}_{cm})\rangle$ is a normalized $Q\bar{q}q\bar{d}$ state of the same general form as Eq. (23) but with a wave function $\tilde{\Phi}_{Q\bar{q}q\bar{d}}$ to be specified below and determined via

$$c_{q\bar{q}}|\tilde{P}_{Qq\bar{q}\bar{d}}(\vec{P}_{cm})\rangle = \sum_{ab} \int \frac{d^3q}{2m_{Qq\bar{q}\bar{d}}} \times \frac{|\vec{P}_{cm};ab(\vec{q})\rangle\langle\vec{P}_{cm};ab(\vec{q})|\mathbf{H}_{\mathbf{pc}}^{\mathbf{q\bar{q}}}|P_{Q\bar{d}}(\vec{P}_{cm})\rangle}{E_{ab}(\vec{P}_{cm},\vec{q})-E_{P_{Q\bar{d}}}(\vec{P}_{cm})}. \tag{25}$$

Here $|\vec{P}_{cm};ab(\vec{q})\rangle$ is the two meson eigenstate with $Q\bar{q}$ in internal state $a$, $q\bar{d}$ in internal state $b$, and with $(Q\bar{q})_a$ and $(q\bar{d})_b$ having relative momentum $\vec{q}$ and total momentum $\vec{P}_{cm}$; $E_{ab}(\vec{P}_{cm},\vec{q})$ and $E_{P_{Q\bar{d}}}(\vec{P}_{cm})$ are the total energies of the respective $Qq\bar{q}\bar{d}$ and $Q\bar{d}$ states with fixed total center-of-mass momentum $\vec{P}_{cm}$. To obtain a rough expression for $|\tilde{P}_{Qq\bar{q}\bar{d}}\rangle$, I make use of the approximate duality between each of the towers of states $a$ and $b$ and their corresponding free particle spectra in the internal relative momenta $\vec{p}_{Q\bar{q}}$ and $\vec{p}_{q\bar{d}}$, respectively. For example, I use

$$\sum_a \langle (Q\bar{q})_a| \simeq \sum_{s_Q s_{\bar{q}}} \int d^3p_{Q\bar{q}} \langle Q(-\vec{p}_{Q\bar{q}},s_Q)\bar{q}(\vec{p}_{Q\bar{q}},s_{\bar{q}})| \tag{26}$$

where for simplicity I have illustrated the duality equation in the $Q\bar{q}$ center of mass frame. While this replacement is imperfect for low $a$ (corresponding to small $\vec{p}_{Q\bar{q}}$), since $\mathbf{H}_{\mathbf{pc}}^{\mathbf{q\bar{q}}}$ is quite pointlike, Eq. (25) has most of its strength for relatively massive states $a$ and $b$ and the use of duality should be satisfactory for our purposes.

With these approximations we can change variables from $a,b,\vec{q}$ to $\vec{p}_{Q\bar{q}}$, $\vec{p}_{q\bar{d}},\vec{q}$, and then convert both sides of Eq. (25) to the variables $\vec{\omega}$, $\vec{\pi}$, and $\vec{p}$ to identify

$$c_{q\bar{q}}\tilde{\Phi}_{Qq\bar{q}\bar{d}} \simeq \frac{\eta_{q\bar{q}}\Phi_{Qq\bar{q}\bar{d}}}{\Delta E} \tag{27}$$

where in the rest frame $\vec{P}_{cm}=\vec{0}$ and with $m_Q\to\infty$,

$$\Delta E = 2m_q + \frac{\pi^2}{m_q} + \frac{\omega^2}{4m_q} + \delta \tag{28}$$

with

$$\delta = m_Q + m_d + \frac{p^2}{2m_d} - m_{P_{Q\bar{d}}}. \tag{29}$$

In the duality approximation we have adopted, and with the wave function of the $Q\bar{d}$ system identical in $Qq\bar{q}\bar{d}$ and $Q\bar{d}$, $\delta\simeq 0$. Moreover, using the wave functions (20) and (21) and the parameters given earlier

$$\frac{\left\langle \dfrac{\omega^2}{4m_q} \right\rangle}{\left\langle \dfrac{\pi^2}{m_q} \right\rangle} \simeq \frac{3\beta_{ft}^2}{20\beta_{pc}^2} \tag{30}$$

is small so that

$$\Delta E \simeq 2m_q + \frac{\pi^2}{m_q} \tag{31}$$

and we may deduce that

$$\tilde{\Phi}_{Qq\bar{q}\bar{d}} \simeq \delta^3(\vec{P}-\vec{P}_{cm})\phi_{ft}(\vec{\omega})\tilde{\phi}_{pc}^m(\vec{\pi})\cdot\chi_{s_q s_{\bar{q}}}^{-m}\phi_{Q\bar{d}}(\vec{p})\chi_{s_Q s_{\bar{d}}}^0 \tag{32}$$

i.e., that $\tilde{\Phi}_{Qq\bar{q}\bar{d}}$ differs from $\Phi_{Qq\bar{q}\bar{d}}$ simply by the replacement $\phi_{pc}\to\tilde{\phi}_{pc}$ where

$$\tilde{\phi}_{pc}^m(\vec{\pi}) \equiv n^{-1/2}\frac{\phi_{pc}^m(\vec{\pi})}{1+\pi^2/2m_q^2}. \tag{33}$$

The normalization factor $n$ is given by

$$n = \int d^3\pi \left| \frac{\phi_{pc}^m(\vec{\pi})}{1+\pi^2/2m_q^2} \right|^2 \tag{34}$$

which may be quite well approximated by the formula $n=(1+y)/(1+21y+12y^3)$ with $y=\beta_{pc}^2/8m_q^2$. From Eqs. (27) and (33) follows the key relation that

$$c_{q\bar{q}} = \frac{n^{1/2}\eta_{q\bar{q}}}{2m_q}. \tag{35}$$

Note that $\tilde{\phi}_{pc}^m(\vec{\pi})$ is softer than $\phi_{pc}^m(\vec{\pi})$. We will often use a harmonic approximation

$$\tilde{\phi}_{pc}^m(\vec{\pi}) = \frac{\pi^m}{\pi^{3/4}\tilde{\beta}_{pc}^{5/2}} e^{-\pi^2/2\tilde{\beta}_{pc}^2} \tag{36}$$

to $\tilde{\phi}_{pc}^m$. It is of course the softer shape of $\tilde{\phi}_{pc}^m$ that makes $n<1$, and thus $\tilde{\beta}_{pc}$ can be determined by requiring that Eqs. (33) and (36) match near $\vec{\pi}=0$, i.e., that

$$\tilde{\beta}_{pc} = n^{1/5}\beta_{pc}. \tag{37}$$

With realistic parameter values, $n^{1/5}\sim 0.7$, so the softening is not dramatic: with our canonical value for $\beta_{pc}$, $\tilde{\beta}_{pc}\simeq 0.4$ GeV.

### B. The unquenched Isgur-Wise function

We are now in a position to calculate the unquenched quark model contribution to the Isgur-Wise function [8]. By heavy quark symmetry, the form factors for a general $Q_1 \to Q_2$ transition can be calculated as matrix elements for the simpler $Q \to Q$ transition with an arbitrary current $\bar{Q}\Gamma Q$. We therefore focus on the matrix elements of the scalar current $\bar{Q}Q$ between $Q$-containing states:

$$\xi^{QM}(w) = \frac{1}{2m_Q}\left\langle P_Q\left(\frac{\vec{P}_{cm}}{2}\right)\middle|\bar{Q}Q\middle|P_Q\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \quad (38)$$

$$= \frac{1}{1+c_{q\bar{q}}^2}[\xi_{Q\bar{d}}^{QM}(w) + c_{q\bar{q}}^2 \xi_{Qq\bar{q}\bar{d}}^{QM}(w)] \quad (39)$$

where $w \equiv v' \cdot v \simeq 1 + P_{cm}^2/2m_Q^2 = 1 + V_{cm}^2/2$ and where

$$\xi_{Q\bar{d}}^{QM}(w) = \frac{1}{2m_Q}\left\langle P_{Q\bar{d}}\left(\frac{\vec{P}_{cm}}{2}\right)\middle|\bar{Q}Q\middle|P_{Q\bar{d}}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \quad (40)$$

$$= \int d^3r\, \psi_{Q\bar{d}}^*(\vec{r}) e^{-im_d\vec{V}_{cm}\cdot\vec{r}} \psi_{Q\bar{d}}(\vec{r}) \quad (41)$$

and

$$\xi_{Qq\bar{q}\bar{d}}^{QM}(w) = \frac{1}{2m_Q}\left\langle P_{Qq\bar{q}\bar{d}}\left(\frac{\vec{P}_{cm}}{2}\right)\middle|\bar{Q}Q\middle|P_{Qq\bar{q}\bar{d}}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \quad (42)$$

$$= \xi_{Q\bar{d}}^{QM}(w)\xi_{ft}^{QM}(w) \quad (43)$$

with

$$\xi_{ft}^{QM}(w) \equiv \int d^3w\, \psi_{ft}^*(\vec{w}) e^{-2im_q\vec{V}_{cm}\cdot\vec{w}} \psi_{ft}(\vec{w}). \quad (44)$$

(In these equations the notation ''$QM$'' reminds us that we are calculating in the quark model so that the perturbative matching of these HQET matrix elements to field theory must be done at the quark model scale $\mu_{QM} \sim 1$ GeV.)

*We see from Eq. (44) that* $\xi_{Qq\bar{q}\bar{d}}^{QM}$ *does not depend on the poorly known* $q\bar{q}$ *wave function* $\psi_{pc}$. This simplification arises because this $q\bar{q}$ wave function defines the relative position of the $q$ and $\bar{q}$, while $\xi_{ft}^{QM}$ is sensitive only to the $q\bar{q}$ system's wave function relative to $Q$. Defining for $w-1 \ll 1$

$$\xi^{QM} \simeq 1 - \rho_{wf}^2(w-1) \quad (45)$$

$$\xi_{Q\bar{d}}^{QM} \simeq 1 - \rho_{Q\bar{d}}^2(w-1) \quad (46)$$

$$\xi_{ft}^{QM} \simeq 1 - \rho_{q\bar{q}}^2(w-1) \quad (47)$$

and recalling the conventional definition

$$\xi \simeq 1 - \rho^2(w-1) \quad (48)$$

we therefore have (displaying now explicitly the effects of summing over $q=u$, $d$, and $s$) simply

$$\rho_{wf}^2 = \rho_{Q\bar{d}}^2 + \frac{\sum_q c_{q\bar{q}}^2 \rho_{q\bar{q}}^2}{1+\sum_q c_{q\bar{q}}^2} \quad (49)$$

where $\rho_{wf}^2$ is the nonrelativistic wave function contribution to $\rho^2$, i.e., it excludes the relativistic $\frac{1}{4}$ and the contribution $\Delta\rho_{pert}^2$ from matching to the low energy effective theory [3]. Using Eqs. (41) and (43) we then have the old result

$$\rho_{Q\bar{d}}^2 = \frac{m_d^2\langle r^2\rangle}{3} = \frac{m_d^2}{2\beta_{Q\bar{d}}^2} \quad (50)$$

and the new correction from $q\bar{q}$ pairs

$$\rho_{q\bar{q}}^2 = \frac{4m_q^2\langle w^2\rangle}{3} = \frac{2m_q^2}{\beta_{ft}^2} \quad (51)$$

so that in the $SU(3)$ limit where $m_u = m_d = m_s = m_q$

$$\rho_{wf}^2 = \frac{m_d^2}{2\beta_{Q\bar{d}}^2} + \frac{\sum_q c_{q\bar{q}}^2 \frac{2m_q^2}{\beta_{ft}^2}}{1+\sum_q c_{q\bar{q}}^2} = \frac{m_d^2}{2\beta_{Q\bar{d}}^2} + \frac{c_{q\bar{q}}^2}{1+3c_{q\bar{q}}^2}\left(\frac{6m_q^2}{\beta_{ft}^2}\right) \quad (52)$$

$$\equiv \frac{m_d^2}{2\beta_{Q\bar{d}}^2} + \Delta\rho_{sea}^2. \quad (53)$$

This is one of our main new results. It shows that even if the $c_{q\bar{q}}^2$ are large, the contribution of pairs to $\rho^2$ will be small in the adiabatic limit where they are highly localized in the flux tube (i.e., as $\beta_{ft}^2 \to \infty$). See Appendix A for a discussion of this result for a more general flux-tube shape. We will see next that to the extent that pairs contribute to the exclusive ''elastic'' slope $\rho^2$, they will also contribute to the *inclusive* nonresonant semileptonic rate.

### C. A duality interpretation of $\Delta\rho_{sea}^2$ via Bjorken's sum rule

#### *1. Motivation*

We have just seen that even if $P_{Q_1}$ is full of $q\bar{q}$ pairs, they may not contribute to $\rho^2$. We will now see that it is incorrect to associate the relative probabilities of $Q_1\bar{d}$ and $Q_1 q\bar{q}\bar{d}$ in $P_{Q_1}$ with the resonant and nonresonant parts, respectively, of $Q_1 \to Q_2$ semileptonic decay. As heavy quark symmetry requires, at $w=1$ the $Q_1 \to Q_2$ transition creates only $P_{Q_2}$ and $V_{Q_2}$ of the ground state $s'^{\pi'_l}_l = \frac{1}{2}^-$ multiplet *independent of the structure of the ''brown muck.''* That is, in this limit the $Q_2\bar{d}$ and $Q_2 q\bar{q}\bar{d}$ components of the hadronic final state, no matter what their relative strengths, form perfectly into the resonant states $P_{Q_2}$ and $V_{Q_2}$. For $w-1$ small but nonzero, Bjorken's sum rule [16,17] tells us that the loss of rate from the ''elastic'' transitions $P_{Q_1} \to P_{Q_2}$ and $P_{Q_1} \to V_{Q_2}$ relative to structureless hadrons with $\rho^2 = 1/4$ will be exactly compensated by the production of $s'^{\pi'_l}_l=\frac{1}{2}^+$ and $\frac{3}{2}^+$ states. In the valence quark model, this rate must appear in $Q_2\bar{d}$ excited states. In Ref. [17] this valence quark model duality to the quark level semileptonic decay was explicitly demonstrated. Here I will show that the $Q_1\bar{q}q\bar{d}$ content of $P_{Q_1}$ leads in general to the production of both resonant and nonresonant final states, with the latter rates proportional to $\Delta\rho^2_{sea}$. In particular, in the adiabatic limit there will be *no* nonresonant production.

To the extent that $\Delta\rho^2_{sea}$ is nonzero, nonresonant $(Q_2\bar{q})_a(q\bar{d})_b$ final states with $s'^{\pi'_l}_l=\frac{1}{2}^+$ and $\frac{3}{2}^+$ will be produced to compensate for the additional loss of rate from the elastic channels which it causes. It is natural to expect the compensation to occur in these channels. The softening of the elastic form factors which depletes the rate to $P_{Q_2}$ and $V_{Q_2}$ will have its analog in inelastic resonance excitation form factors, so these rates will also be diminished and cannot compensate for the additional loss of rate from $P_{Q_2}$ and $V_{Q_2}$. The population of inelastic channels is thus the only avenue available for satisfying Bjorken's sum rule. Of course this is also intuitively appealing: the loss of rate to the elastic channels occurs because after the recoil from $-\vec{P}_{cm}/2$ to $+\vec{P}_{cm}/2$, the $q\bar{q}$ parts of the ground state wave functions of the initial and final states fail to overlap, and in so doing they must ''by conservation of probability'' find themselves in *their* excited states, namely as $(Q_2\bar{q})_a(q\bar{d})_b$ continua. We will now make these heuristic observations precise.

### 2. Production of nonresonant states at low $w-1$

The $n^{th}$ valence state perturbed by $\mathbf{H}^{\mathbf{q\bar{q}}}_{\mathbf{pc}}$ [the generalization of Eq. (24)] may be written

$$|M_Q^{(n)}(\vec{P}_{cm})\rangle=\cos\theta|M_{Q\bar{d}}^{(n)}(\vec{P}_{cm})\rangle+\sin\theta|X_{Q\bar{q}q\bar{d}}^{(n00)}(\vec{P}_{cm})\rangle \tag{54}$$

where $|M_Q^{(n)}\rangle$, $|M_{Q\bar{d}}^{(n)}\rangle$, and $|X_{Q\bar{q}q\bar{d}}^{(n00)}\rangle$ are the generalizations of the states $|P_Q\rangle\equiv|M_Q^{(0)}\rangle$, $|P_{Q\bar{d}}\rangle\equiv|M_{Q\bar{d}}^{(0)}\rangle$, and $|\tilde{P}_{Q\bar{q}q\bar{d}}\rangle\equiv|X_{Q\bar{q}q\bar{d}}^{(000)}\rangle$ of Eqs. (24), (16), and (25), respectively, and where under our assumptions that the state of the flux tube is independent of *n* (the adiabatic approximation) and that $H_{pc}^{q\bar{q}}$ does not affect the coordinate $\vec{r}$, $\theta$ is independent of *n*. [The rationale for the notation $(n00)$ will become apparent below.] From this expression one can immediately obtain the generalization of our result for the elastic transition that

$$\xi^{QM}(w)^{n'n}\equiv\frac{1}{2m_Q}\left\langle M_Q^{(n')}\left(+\frac{\vec{P}_{cm}}{2}\right)\middle|\bar{Q}Q\middle|M_Q^{(n)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \tag{55}$$

$$=\xi_{Q\bar{d}}^{QM}(w)^{n'n}[\cos^2\theta+\sin^2\theta\xi_{ft}^{QM}(w)] \tag{56}$$

where $\xi_{Q\bar{d}}^{QM}(w)^{n'n}$ is the valence quark model generalization of the Isgur-Wise function for $n\to n'$ transitions and $\xi_{ft}^{QM}(w)$ is exactly the same $q\bar{q}$ overlap form factor that appears in the elastic $n=0\to n'=0$ transition. Thus the unquenched result for small $w-1$ is

$$\xi^{QM}(w)^{n'n}=\xi_{Q\bar{d}}^{QM}(w)^{n'n}[1-\sin^2\theta\rho^2_{q\bar{q}}(w-1)] \tag{57}$$

as for the Isgur-Wise function with, once again, $\sin^2\theta=\Sigma_q c^2_{q\bar{q}}/(1+\Sigma_q c^2_{q\bar{q}})$. Since the production of each inelastic resonant channel occurs with strength proportional to $w-1$ to a positive integral power, the $q\bar{q}$ modification of $\xi^{QM}(w)^{n'n}$ for $n'>0$ has no effect on the saturation of Bjorken's sum rule for $\xi(w)$ to order $w-1$ since it produces effects which are at least of order $(w-1)^2$. This is in accord with the expectations outlined above that the additional depletion of elastic rate by $q\bar{q}$ pairs must be compensated by the explicit production of a $(Q\bar{q})_a(q\bar{d})_b$ continuum.

To see this we must introduce the states in the continuum orthogonal to $|M_Q^{(n)}\rangle$. To this end we define a complete set of states $|X_{Q\bar{q}q\bar{d}}^{(n\alpha\beta)}(\vec{P}_{cm})\rangle$ in the $Q\bar{q}q\bar{d}$ sector. Here *n*, $\alpha$, and $\beta$ are excitation quantum numbers associated with the $\vec{r}$, $\vec{w}$, and $\vec{v}$ coordinates, respectively. These states are *not* the eigenstates of this sector in the absence of $\mathbf{H}^{\mathbf{q\bar{q}}}_{\mathbf{pc}}$: the eigenstates are the $|\vec{P}_{cm};ab(\vec{q})\rangle$ defined above. However, we can expand

$$|\vec{P}_{cm};ab(\vec{q})\rangle=\sum_{n\alpha\beta}\phi_{ab}^{(n\alpha\beta)}(\vec{q})^*|X_{Q\bar{q}q\bar{d}}^{(n\alpha\beta)}(\vec{P}_{cm})\rangle. \tag{58}$$

It follows that to lowest order in $\theta$ (or equivalently the pair creation operator) we can form an orthogonal set of $\mathbf{H}^{\mathbf{q\bar{q}}}_{\mathbf{pc}}$-perturbed states

$$|M_Q^{(n)}(\vec{P}_{cm})\rangle\simeq|M_{Q\bar{d}}^{(n)}(\vec{P}_{cm})\rangle+\theta\sum_{ab}\int d^3q\,\phi_{ab}^{(n00)}(\vec{q})|\vec{P}_{cm};ab(\vec{q})\rangle \tag{59}$$

$$|X_{ab}(\vec{P}_{cm},\vec{q})\rangle\simeq|\vec{P}_{cm};ab(\vec{q})\rangle-\theta\sum_n\phi_{ab}^{(n00)}(\vec{q})^*|M_{Q\bar{d}}^{(n)}(\vec{P}_{cm})\rangle \tag{60}$$

where $(\alpha,\beta)=(0,0)$ define the universal state of the $q\bar{q}$ pair created in a flux tube by the action of $\mathbf{H}^{\mathbf{q\bar{q}}}_{\mathbf{pc}}$. Let us now compute the transition amplitude to the continuum:

$$\left\langle X_{ab}\left(+\frac{\vec{P}_{cm}}{2},\vec{q}\right)\middle|\bar{Q}Q\middle|M_Q^{(0)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle$$
$$\simeq\theta\left[\left\langle+\frac{\vec{P}_{cm}}{2};ab(\vec{q})\middle|\bar{Q}Q\middle|\tilde{P}_{Q\bar{q}q\bar{d}}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle-\sum_n\phi_{ab}^{(n00)}(\vec{q})\left\langle M_{Q\bar{d}}^{(n)}\left(+\frac{\vec{P}_{cm}}{2}\right)\middle|\bar{Q}Q\middle|M_{Q\bar{d}}^{(0)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle\right] \tag{61}$$
$$\simeq\theta\left[\sum_{n\alpha\beta}\phi_{ab}^{(n\alpha\beta)}(\vec{q})\left\langle X_{Q\bar{q}q\bar{d}}^{(n\alpha\beta)}\left(+\frac{\vec{P}_{cm}}{2}\right)\middle|\bar{Q}Q\middle|X_{Q\bar{q}q\bar{d}}^{(000)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle\right.$$
$$\left.-\sum_n\phi_{ab}^{(n00)}(\vec{q})\left\langle M_{Q\bar{d}}^{(n)}\left(+\frac{\vec{P}_{cm}}{2}\right)\middle|\bar{Q}Q\middle|M_{Q\bar{d}}^{(0)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle\right]. \tag{62}$$

As $m_Q\to\infty$, $\vec{P}_{cm}=m_Q\vec{V}_{cm}$ is much larger than any internal momentum so the matrix elements $\langle Q|\bar{Q}Q|Q\rangle$ appearing in the $Q\bar{d}\to Q\bar{d}$ and $Q\bar{q}q\bar{d}\to Q\bar{q}q\bar{d}$ transitions here are identical. Moreover, since the $\bar{Q}Q$ current does not affect the internal state of the $q\bar{q}$ pair, $\beta$ is required to be zero. Thus Eq. (62) becomes

$$\left\langle X_{ab}\left(+\frac{\vec{P}_{cm}}{2},\vec{q}\right)\middle|\bar{Q}Q\middle|M_Q^{(0)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \simeq\theta\sum_n\xi_{Q\bar{d}}^{QM}(w)^{n0}\left[\sum_\alpha\xi_{ft}^{QM}(w)^{\alpha0}\phi_{ab}^{(n\alpha0)}(\vec{q})-\phi_{ab}^{(n00)}(\vec{q})\right] \tag{63}$$

where $\xi_{ft}^{QM}(w)^{\alpha0}$ is the generalization of $\xi_{ft}^{QM}(w)$ encountered above, namely

$$\xi_{ft}^{QM}(w)^{\alpha0}\equiv\int d^3w\,\psi_{ft}^{(\alpha)}(\vec{w})^*e^{-2im_q\vec{V}_{cm}\cdot\vec{w}}\psi_{ft}^{(0)}(\vec{w}) \tag{64}$$

where $\psi_{ft}^{(\alpha)}$ is the $\alpha$th basis state for the expansion of the $q\bar{q}$ center of mass coordinate $\vec{w}$. Expanding the exponential in powers of $\vec{V}_{cm}$ as is appropriate for small $w-1$, we obtain Eq. (47) of Sec. II for $\alpha=0$ and

$$\xi_{ft}^{QM}(w)^{\alpha\neq0,0}\simeq-2im_q\vec{V}_{cm}\cdot\int d^3w\,\psi_{ft}^{(\alpha)}(\vec{w})^*\vec{w}\psi_{ft}^{(0)}(\vec{w}). \tag{65}$$

Since $\xi_{ft}^{QM}(w)^{00}\simeq1-\rho_{qq}^2(w-1)$ with $w-1=V_{cm}^2/2$, to leading order in $V_{cm}$ the $\alpha=0$ term in Eq. (63) cancels with $\phi_{ab}^{(n00)}(\vec{q})$ to leave

$$\left\langle X_{ab}\left(+\frac{\vec{P}_{cm}}{2},\vec{q}\right)\middle|\bar{Q}Q\middle|M_Q^{(0)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \simeq\theta\sum_{n,\alpha\neq0}\xi_{Q\bar{d}}^{QM}(w)^{n0}\xi_{ft}^{QM}(w)^{\alpha0}\phi_{ab}^{(n\alpha0)}(\vec{q}). \tag{66}$$

An immediate consequence of this relation is that in the adiabatic limit ($\beta_{ft}\to\infty$), $\xi_{ft}^{QM}(w)^{\alpha0}\sim\beta_{ft}^{-1}\to0$ for $\alpha\neq0$ so we have explicitly demonstrated that there is no nonresonant production in this limit.

Since we have for our discussion assumed that $\psi_{ft}^{(0)}(\vec{w})$ has the form of a ground state harmonic oscillator wave function, it is natural to use a harmonic oscillator basis as the orthonormal expansion functions for the variable $\vec{w}$. Doing so, it follows that only three basis states give nonzero contributions to Eq. (66) to leading order in $\vec{V}_{cm}$ since $\vec{w}\psi_{ft}^{(0)}(\vec{w})$ is proportional to the three $n_w=0$, $l_w=1$ harmonic oscillator wave functions:

$$\psi_{ft}^{[n_w=0,l_w=1,i]}(\vec{w})=\sqrt{2}\frac{\beta_{ft}^{5/2}}{\pi^{3/4}}w^ie^{-(1/2)\beta_{ft}^2w^2} \tag{67}$$

in a Cartesian basis, giving

$$\xi_{ft}^{QM}(\vec{w})^{[n_w=0,l_w=1,i]0}=-\frac{i\sqrt{2}m_qV_{cm}^i}{\beta_{ft}} \tag{68}$$

and thence

$$\left\langle X_{ab}\left(+\frac{\vec{P}_{cm}}{2},\vec{q}\right)\middle|\bar{Q}Q\middle|M_Q^{(0)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \simeq\theta\sum_{n,i}\xi_{Q\bar{d}}^{QM}(w)^{n0}\xi_{ft}^{QM}(\vec{w})^{[n_w=0,l_w=1,i]0}\times\phi_{ab}^{(n[n_w=0,l_w=1,i]0)}(\vec{q}). \tag{69}$$

Next we note that

$$\xi_{Q\bar{d}}^{QM}(w)^{n\neq0,0}\sim(w-1)^k \tag{70}$$

where $k$ is a positive integer by Luke's theorem [27] so that to order $\vec{V}_{cm}$

$$\left\langle X_{ab}\left(+\frac{\vec{P}_{cm}}{2},\vec{q}\right)\middle|\bar{Q}Q\middle|M_Q^{(0)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \simeq\theta\xi_{Q\bar{d}}^{QM}(w)\sum_i\xi_{ft}^{QM}(\vec{w})^{[n_w=0,l_w=1,i]0}\phi_{ab}^{(0[n_w=0,l_w=1,i]0)}(\vec{q}) \tag{71}$$

$$\simeq-\frac{i\theta\sqrt{2}m_q\vec{V}_{cm}}{\beta_{ft}}\cdot\vec{\phi}_{ab}(\vec{q}) \tag{72}$$

where I have introduced the notation $\vec{\phi}_{ab}(\vec{q})$ for the three-vector $\phi_{ab}^{(0[n_w=0,l_w=1,i]0)}(\vec{q})$.

This key result has a simple interpretation. Nonresonant production at order $w-1$ requires that the current $\bar{Q}Q$ acting on $Q\bar{q}q\bar{d}$ create neither the ground state nor the $P$-wave resonances. However, it cannot excite the $q\bar{q}$ internal coordinate $\vec{v}$ and, if it excites $\vec{r}$ then to order $w-1$ it has just produced the $Q\bar{q}q\bar{d}$ component of either the ground state or the $P$-wave resonances. *Hence nonresonant production to order* $w-1$ *occurs purely by excitation of the* $q\bar{q}$ *coordinate* $\vec{w}$ *to* $l_w=1$. The factors $\vec{\phi}_{ab}(\vec{q})$ are simply the projections of these $l_w=1$ states onto the continuum eigenstates consisting of mesons $a$ and $b$ with relative momentum $\vec{q}$.

We are now in a position to verify that the $\Delta\rho^2_{sea}$ contribution to the slope of the Isgur-Wise function is indeed compensated by the production of these continuum $(Q\bar{q})_a(q\bar{d})_b$ states. The probability for their production at $w$ is, up to $(w-1)^2$ corrections,

$$dP(P_Q\rightarrow\text{continuum})$$
$$\simeq\sum_{ab}\int d^3q\left|\left\langle X_{ab}\left(+\frac{\vec{P}_{cm}}{2},\vec{q}\right)\right|\bar{Q}Q\left|M_Q^{(0)}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle\right|^2 \tag{73}$$

which since $\Sigma_{ab}\int d^3q\,\phi^i_{ab}(\vec{q})\phi^j_{ab}(\vec{q})=\delta^{ij}$, gives

$$dP(P_Q\rightarrow\text{continuum})\simeq\frac{2\theta^2m_q^2V_{cm}^2}{\beta_{ft}^2} \tag{74}$$

$$\simeq\frac{4m_q^2}{\beta_{ft}^2}\theta^2(w-1) \tag{75}$$

for the flavor $q$. On the other hand, according to Eq. (53), the contribution of flavor $q$ to the loss of elastic rate is

$$dP(P_Q\rightarrow P_Q+V_Q)_{Q\bar{q}q\bar{d}}\simeq-\frac{4m_q^2}{\beta_{ft}^2}\theta^2(w-1). \tag{76}$$

The two rates match, explicitly demonstrating the connection of $\Delta\rho^2_{sea}$ to the nonresonant continuum.

#### *3. Production of exclusive nonresonant states at low* $w-1$

It remains to assess the fractional population of individual continuum channels inside of the total given by Eq. (75). To do this we must calculate $\vec{\phi}_{ab}(\vec{q})$, which from its definition is

$$\phi_{ab}^{(n\alpha\beta)}(\vec{q})\,\delta^3(\vec{P}'_{cm}-\vec{P}_{cm})=\langle\vec{P}'_{cm};ab(\vec{q})|X^{(n\alpha\beta)}_{Q\bar{q}q\bar{d}}(\vec{P}_{cm})\rangle, \tag{77}$$

for the case $(n\alpha\beta)=(0[n_w=0,l_w=1,i]0)$. These calculations are straightforward, but would be quite tedious without the introduction of several tricks described in Appendix C. Results of the calculations of $\tau^{(m)}_{1/2}(w)$ and $\tau^{(p)}_{3/2}(w)$ for a number of low-lying nonresonant channels are given in Table I.

## III. UNQUENCHING HEAVY QUARK DECAY: RESULTS

We now turn to the quantitative evaluation of $\Delta\rho^2_{sea}$ (which is a reflection of total nonresonant production) and then to the distribution of these decays into exclusive nonresonant channels.

### A. The total nonresonant rate

As shown in Appendix B, the light quark amplitudes $\eta_{u\bar{u}}=\eta_{d\bar{d}}$ can be determined from strong decays to be

$$\eta_{q\bar{q}}\simeq0.9\ \text{GeV} \tag{78}$$

with an uncertainty of a factor of two mainly arising from a strong dependence on the poorly known quantity $\beta_{pc}$. It follows from Eq. (35) that

$$c_{q\bar{q}}\simeq0.5. \tag{79}$$

Assuming $SU(3)$ symmetry for the contributions to $\Delta\rho^2_{sea}$, one then obtains from Eq. (53)

$$\Delta\rho^2_{sea}\simeq\frac{1}{4}, \tag{80}$$

corresponding to an increase of $\rho^2$ from the value $0.74\pm0.05$ quoted in ISGW2 to a value near unity. Either value would be in reasonable agreement with measurements [15]. *Via* the Bjorken sum rule, such a $\rho^2_{sea}$ would be consistent with the possibility discussed in the Introduction that $16\pm8\%$ of the inclusive semileptonic $\bar{B}$ rate is in nonresonant channels.

This reasonable quantitative correspondence between our calculated $\Delta\rho^2_{sea}$ and a possible experimental anomaly should not be taken too seriously. The missing non-$(D+D^*)$ rate attributed to nonresonant production might be due to an ISGW2 underestimate of excited resonance production [18], or to an experimental overestimate of non-$(D+D^*)$ production. Moreover, while it is a ‘‘canonical estimate,’’ $\Delta\rho^2_{sea}$ is subject to very substantial uncertainties: see Table II.

### B. The rate to low-lying exclusive nonresonant channels

Using the amplitudes of Table I and the formulas of Appendix C, one can easily calculate the *fractions* of $\Delta\rho^2_{sea}$ due to the individual low-mass nonresonant channels shown in Table III. Note that the nonresonant rate is highly fragmented: none of the many channels tabulated account for more than 4% of the inclusive nonresonant rate (and thus no more than about 1% of the total semileptonic rate). These results are consistent with previous studies of single low energy pion emission using heavy quark chiral perturbation theory [11]. Note also that the thirty final states considered

TABLE I. $\tau_{1/2}^{(m)}$ and $\tau_{3/2}^{(p)}$ for nonresonant semileptonic decays to low-lying exclusive channels in units of $\alpha_S \equiv \sqrt{4\pi}(3c^{00} + c_{ij}^{00}q^2/\tilde{\beta}_v^2)I^{00}e^{-q^2/8\tilde{\beta}_v^2}$, $\alpha_D \equiv \sqrt{4\pi}(c_{ij}^{00}q^2/\tilde{\beta}_v^2)I^{00}e^{-q^2/8\tilde{\beta}_v^2}$, $\beta_n^{10} \equiv \sqrt{4\pi}(c_n^{10}q)I^{10}e^{-q^2/8\tilde{\beta}_{**}^2}$, $\gamma_n^{10} \equiv \sqrt{4\pi}(c_{ijk}^{10}q^3/\tilde{\beta}_{v,**}^2)I^{10}e^{-q^2/8\tilde{\beta}_{**}^2}$, $\beta_n^{01} \equiv \sqrt{4\pi}(c_n^{01}q)I^{01}e^{-q^2/8\tilde{\beta}_a^2}$, and $\gamma_n^{01} \equiv \sqrt{4\pi}(c_{ijk}^{01}q^3/\tilde{\beta}_{v,a}^2)I^{01}e^{-q^2/8\tilde{\beta}_a^2}$, as defined in Appendix C. Shown explicitly are the amplitudes for emission of a $(D+D^*)$, $D_{3/2}^{**}$, or $D_{1/2}^{**}$ and a *positively charged light hadron* ($\pi^+$, $\rho^+$, $a_2^+$, $a_1^+$, $b_1^+$, or $a_0^+$) from $\bar{B}^0$ by $\bar{u}u$ pair creation. [Here $(D+D^*)$, $D_{3/2}^{**}$, and $D_{1/2}^{**}$ are the lowest-lying $s_l'^{\pi_l'} = \frac{1}{2}^-$, $\frac{3}{2}^+$, and $\frac{1}{2}^+$ heavy quark spin multiplets, respectively.] The subscripts on a channel in the first column are the total spin (the $s_a^{\pi_a}$ of the charmed meson $a$ plus the spin of the light meson $b$), and the $ab$ relative orbital angular momentum, respectively; the second and third columns define the total $s_l'^{\pi_l'}$ as either $\frac{1}{2}^+$ or $\frac{3}{2}^+$. Note that since partial wave amplitudes with respect to the direction of the vector $\vec{q}$ are given, full rates to a channel are obtained by integrating over $q^2dq$ and not $d^3q$, i.e., a factor $\sqrt{4\pi}$ is included in each amplitude. Shown in parentheses under each allowed amplitude is the fraction of the $\bar{u}u$ rate going into this channel in percent, based on Eqs. (C12) and (C13).

| Channel | $\sqrt{3}\tau_{1/2}^{(channel)}/\sqrt{\Delta\rho_{sea}^2/3}$ [fraction of $\bar{u}u$ rate (%)] | $\sqrt{3}\tau_{3/2}^{(channel)}/\sqrt{\Delta\rho_{sea}^2/3}$ [fraction of $\bar{u}u$ rate (%)] |
|---|---|---|
| $[(D+D^*)\pi^+]_{(1/2)S}$ | $-\alpha_S$ (0.4) | - |
| $[(D+D^*)\pi^+]_{(1/2)D}$ | - | $-\alpha_D$ (1.5) |
| $[(D+D^*)\rho^+]_{(3/2)S}$ | - | $-\sqrt{\frac{4}{3}}\alpha_S$ (1.0) |
| $[(D+D^*)\rho^+]_{(3/2)D}$ | $+\sqrt{\frac{8}{3}}\alpha_D$ (2.0) | $+\sqrt{\frac{4}{3}}\alpha_D$ (2.0) |
| $[(D+D^*)\rho^+]_{(1/2)S}$ | $+\sqrt{\frac{1}{3}}\alpha_S$ (0.1) | - |
| $[(D+D^*)\rho^+]_{(1/2)D}$ | - | $+\sqrt{\frac{1}{3}}\alpha_D$ (0.5) |
| $[D_{3/2}^{**}\pi^+]_{(3/2)P}$ | $+\sqrt{\frac{4}{3}}[2\beta_j^{10}+3\beta_i^{10}]+\sqrt{\frac{4}{3}}\gamma^{10}$ (0.5) | $-\sqrt{\frac{10}{3}}\beta_j^{10}-\sqrt{\frac{2}{15}}\gamma^{10}$ (0.8) |
| $[D_{3/2}^{**}\pi^+]_{(3/2)F}$ | - | $+\sqrt{\frac{6}{5}}\gamma^{10}$ (0.1) |
| $[D_{1/2}^{**}\pi^+]_{(1/2)P}$ | $+\sqrt{\frac{2}{3}}[\beta_j^{10}-3\beta_i^{10}-2\beta_k^{10}]-\sqrt{\frac{2}{3}}\gamma^{10}$ (1.1) | $-\sqrt{\frac{2}{3}}[2\beta_j^{10}+3\beta_k^{10}]-\sqrt{\frac{2}{3}}\gamma^{10}$ (0.4) |
| $[D_{3/2}^{**}\rho^+]_{(5/2)P}$ | - | $+\sqrt{12}[\beta_j^{10}+\beta_i^{10}]+\sqrt{\frac{48}{25}}\gamma^{10}$ (2.6) |
| $[D_{3/2}^{**}\rho^+]_{(5/2)F}$ | $-\sqrt{\frac{16}{5}}\gamma^{10}$ (0.1) | $-\sqrt{\frac{32}{25}}\gamma^{10}$ (0.1) |
| $[D_{3/2}^{**}\rho^+]_{(3/2)P}$ | $+\sqrt{\frac{20}{9}}[2\beta_j^{10}+\beta_i^{10}]+\sqrt{\frac{4}{5}}\gamma^{10}$ (1.0) | $+\sqrt{\frac{2}{9}}[\beta_j^{10}-\beta_i^{10}]-\sqrt{\frac{2}{25}}\gamma^{10}$ (0.1) |
| $[D_{3/2}^{**}\rho^+]_{(3/2)F}$ | - | $+\sqrt{\frac{18}{25}}\gamma^{10}$ (0.1) |

TABLE I. (*Continued*).

| channel | $\sqrt{3}\,\tau_{1/2}^{(channel)}/\sqrt{\Delta\rho_{sea}^2/3}$ [fraction of $\overline{u}u$ rate (%)] | $\sqrt{3}\,\tau_{3/2}^{(channel)}/\sqrt{\Delta\rho_{sea}^2/3}$ [fraction of $\overline{u}u$ rate (%)] |
|---|---|---|
| $[D_{3/2}^{**}\rho^+]_{(1/2)P}$ | $+\sqrt{\frac{16}{9}}[\beta_j^{10}-\beta_i^{10}]$ (0.3) | $-\sqrt{\frac{4}{9}}[\beta_j^{10}-\beta_i^{10}]$ (0.1) |
| $[D_{1/2}^{**}\rho^+]_{(3/2)P}$ | $+\sqrt{\frac{16}{9}}[\beta_j^{10}-\beta_i^{10}]$ (0.3) | $+\sqrt{\frac{40}{9}}[\beta_j^{10}-\beta_i^{10}]$ (1.3) |
| $[D_{1/2}^{**}\rho^+]_{(3/2)F}$ | - | 0 (0.0) |
| $[D_{1/2}^{**}\rho^+]_{(1/2)P}$ | $+\sqrt{\frac{2}{9}}[5\beta_j^{10}+\beta_i^{10}+9\beta_k^{10}]+\sqrt{2}\gamma^{10}$ (0.8) | $+\sqrt{\frac{2}{9}}[2\beta_j^{10}-4\beta_i^{10}-9\beta_k^{10}]+\sqrt{2}\gamma^{10}$ (5.6) |
| $[(D+D^*)a_0^+]_{(1/2)P}$ | $+\sqrt{\frac{2}{3}}[3\beta_j^{01}-\beta_i^{01}+3\beta_k^{01}]+\sqrt{\frac{2}{3}}\gamma^{01}$ (2.4) | $+\sqrt{\frac{2}{3}}[2\beta_i^{01}+3\beta_k^{01}]+\sqrt{\frac{2}{3}}\gamma^{01}$ (1.0) |
| $[(D+D^*)a_1^+]_{(3/2)P}$ | $+\sqrt{\frac{2}{3}}[4\beta_j^{01}+\beta_i^{01}]+\sqrt{\frac{2}{3}}\gamma^{01}$ (1.1) | $+\sqrt{\frac{5}{3}}[\beta_j^{01}-2\beta_i^{01}]-\sqrt{\frac{1}{15}}\gamma^{01}$ (0.3) |
| $[(D+D^*)a_1^+]_{(3/2)F}$ | - | $+\sqrt{\frac{3}{5}}\gamma^{01}$ (0.0) |
| $[(D+D^*)a_1^+]_{(1/2)P}$ | $+\sqrt{\frac{4}{3}}[\beta_j^{01}+\beta_i^{01}+3\beta_k^{01}]+\sqrt{\frac{4}{3}}\gamma^{01}$ (2.1) | $+\sqrt{\frac{4}{9}}[2\beta_j^{01}+2\beta_i^{01}+3\beta_k^{01}]+\sqrt{\frac{4}{9}}\gamma^{01}$ (2.1) |
| $[(D+D^*)b_1^+]_{(3/2)P}$ | $+\sqrt{\frac{4}{3}}[2\beta_j^{01}+3\beta_i^{01}]+\sqrt{\frac{4}{3}}\gamma^{01}$ (0.6) | $-\sqrt{\frac{10}{3}}\beta_j^{01}-\sqrt{\frac{2}{15}}\gamma^{01}$ (0.7) |
| $[(D+D^*)b_1^+]_{(3/2)F}$ | - | $+\sqrt{\frac{6}{5}}\gamma^{01}$ (0.1) |
| $[(D+D^*)b_1^+]_{(1/2)P}$ | $+\sqrt{\frac{2}{3}}[\beta_j^{01}-3\beta_i^{01}-3\beta_k^{01}]-\sqrt{\frac{2}{3}}\gamma^{01}$ (0.3) | $-\sqrt{\frac{2}{3}}[2\beta_j^{01}+3\beta_k^{01}]-\sqrt{\frac{2}{3}}\gamma^{01}$ (3.3) |
| $[(D+D^*)a_2^+]_{(5/2)P}$ | - | $+\sqrt{12}[\beta_j^{01}+\beta_i^{01}]+\sqrt{\frac{48}{25}}\gamma^{01}$ (2.5) |
| $[(D+D^*)a_2^+]_{(5/2)F}$ | $+\sqrt{\frac{16}{5}}\gamma^{01}$ (0.1) | $-\sqrt{\frac{32}{25}}\gamma^{01}$ (0.1) |
| $[(D+D^*)a_2^+]_{(3/2)P}$ | $-\sqrt{\frac{10}{3}}\beta_i^{01}-\sqrt{\frac{2}{15}}\gamma^{01}$ (0.0) | $+\sqrt{\frac{1}{3}}[3\beta_j^{01}-2\beta_i^{01}]+\sqrt{\frac{1}{75}}\gamma^{01}$ (0.6) |
| $[(D+D^*)a_2^+]_{(3/2)F}$ | - | $-\sqrt{\frac{3}{25}}\gamma^{01}$ (0.0) |

TABLE II. Assessing the uncertainties in $c_{q\bar{q}}$ and $\Delta\rho^2_{sea}$.

| | $\beta_{ft}$ (GeV) | $\beta_{pc}$ (GeV) [a] | $\eta_{q\bar{q}}$ (GeV) | $c_{u\bar{u}}$ | $\Delta\rho^2_{sea}$ |
|---|---|---|---|---|---|
| canonical | 0.60 | 0.58 | $\sim 1$ | $\sim 1/2$ | $\sim 1/4$ |
| harder pair creation [b] | 0.60 | 0.87 | $\sim 3$ | $\sim 1$ | $\sim 1/2$ |
| softer pair creation [c] | 0.60 | 0.43 | $\sim 1/2$ | $\sim 1/3$ | $\sim 1/6$ |
| larger flux tube diameter [d] | 0.42 | 0.58 | $\sim 1/2$ | $\sim 1/2$ | $\sim 1/2$ |

[a]Recall $\tilde{\beta}_{pc} \simeq 0.7\beta_{pc}$ so that the canonical value of $\tilde{\beta}_{pc}$ is $\sim 0.4$ GeV.
[b]$r_q = 0.3$ fm $\rightarrow r_q = 0.2$ fm.
[c]$r_q = 0.3$ fm $\rightarrow r_q = 0.4$ fm.
[d]$f = 2 \rightarrow f = 1$.

here account for only about 40% of the nonresonant rate, so that most of this rate resides in states $ab$ composed of highly excited resonances.

There are several reasons why the results given in Table III must be interpreted with caution. The most prominent is simply that they depend roughly on $\tilde{\beta}_{pc}^{-5}$. Recalling that $\tilde{\beta}_{pc} = n^{1/5}\beta_{pc}$, our canonical value for $\tilde{\beta}_{pc}$ is 0.4 GeV, but over the range of $\beta_{pc}$ allowed by Table II, it varies from its canonical value by $\pm 0.1$ GeV. Although the factor $n^{1/5}$ makes this range narrower than that of $\beta_{pc}$ itself, it still leaves us with an uncertainty of more than a factor of two on the basic unit for production of exclusive nonresonant channels.

The second word of caution concerns large corrections to the $m_Q \rightarrow \infty$ limit studied here which arise for real $\bar{B}$ decay. The problem is phase space: in the heavy quark limit, all final states occur at the $w$ of the underlying quark production process, i.e., the full mass range of the final state spectrum of states is negligible compared to the heavy quark energies even at low $w-1$. In actual $\bar{B} \rightarrow X_c l \bar{\nu}_l$ decays, each final state $X_c$ will have a Dalitz plot that is a shrinking fraction of the $b \rightarrow c l \bar{\nu}_l$ Dalitz plot as $m_{X_c} \rightarrow m_B$. Thus in the heavy quark limit the loss of rate from the elastic channels $D + D^*$ would be *locally compensated* in the variable $w$. For finite $m_b$ and $m_c$, however, a loss of elastic rate will still occur *via* $\Delta\rho^2_{sea}$, but the compensating channels will experience a delayed turn-on because of their thresholds; indeed, some processes which would have helped to compensate $\Delta\rho^2_{sea}$ will be kinematically forbidden. These phase space suppressions lead to $\Lambda_{QCD}/m_Q$-type corrections to the inclusive rate, and therefore also corrections to the accuracy of quark-hadron duality. However, it is known on general grounds from heavy quark symmetry and the operator product expansion (OPE) that, as the energy release $m_b - m_c \rightarrow \infty$, the leading corrections to the inclusive rate must be of order $\Lambda^2_{QCD}/m_Q^2$ [28,29]. The resolution of this apparent paradox has been discussed in Ref. [30]: the OPE has a radius of convergence which does not include the region in which (significant) hadronic thresholds are turning on, so for real $b \rightarrow c$ transitions there can be $\Lambda_{QCD}/m_Q$ corrections, and they can be substantial. However, associated with these $\Lambda_{QCD}/m_Q$ threshold effects, which would diminish the integrated contribution of individual hadronic channels below the level required for perfect duality, are $\Lambda_{QCD}/m_Q$ corrections to the rate for the production of such channels which *enhance* their production once they are above threshold. These counterbalancing effects soften the breaking of duality: they are the precursors of the perfect cancellation of $\Lambda_{QCD}/m_Q$ effects that must occur as $m_b - m_c \rightarrow \infty$.

How then should one make use of Table III for real $b \rightarrow c$ transitions? The amplitudes $\tau^{(m)}_{1/2}$ and $\tau^{(p)}_{3/2}$ shown are the leading order predictions for their respective channels. For the reasons just outlined, we can expect that they will be enhanced by $\Lambda_{QCD}/m_Q$ corrections which are expected to be of the order of $25-50\%$ [30]. However, despite this effect, I believe that the dramatically reduced size of their Dalitz plots (relative to that for the underlying quark process $b \rightarrow c l \bar{\nu}_l$) will reduce the actual population of all nonresonant states well below that expected from $\Delta\rho^2_{sea}$. Indeed, each channel shown in Table III has a continuous spectrum of masses from its threshold up to masses exceeding $m_B$. Consider, for example, the $D+\rho$ channel. Its mass is given by $m_{D\rho}(q^2) = \sqrt{m_D^2 + q^2} + \sqrt{m_\rho^2 + q^2}$, while its dominant $D$-wave production rate is proportional to $q^4 e^{-q^2/4\tilde{\tilde{\beta}}^2}$, where $2\tilde{\tilde{\beta}} \sim 1$ GeV. The production is therefore very weak at low masses where the available phase space is generous, and peaks at $m_{D\rho} \sim m_B$ where it vanishes. Given these basic kinematic facts, it is clear that even this simple exclusive channel will be produced at a rate far less than in the heavy quark limit, and that most of $\Delta\rho^2_{sea}$ will be uncompensated. The third column of Table III gives the phase space factors by which individual channels will be reduced in real $\bar{B} \rightarrow X_c l \bar{\nu}_l$ decays relative to the heavy quark limit; the fourth column gives a very rough estimate for the *net* suppression of each channel as a product of this phase space factor and a generous guess that, after $\Lambda_{QCD}/m_Q$ corrections, each channel has a compensatory increase of 50%. Considering that in general the untabulated channels of yet more highly excited states $ab$ will suffer even greater phase space suppressions, an overall diminution of the nonresonant rate by at least a factor of four seems likely.

## IV. CONCLUSIONS

Although the successes of the valence quark model and the arguments of the large $N_c$ limit provide indications that sea quarks play a relatively minor role in hadronic physics, this hope is far from being justified by our current under-

TABLE III. The fractions of the total nonresonant semileptonic decay rate predicted in low-lying exclusive channels. These fractions are obtained from Table I by taking $(|\tau_{1/2}|^2+2|\tau_{3/2}|^2)/\Delta\rho^2_{sea}$ (which may be obtained by summing the appropriate channels from Table I and dividing by 3) times flavor factors of $\frac{3}{2}$ for $I=1$, $\frac{1}{2}$ for $I=0$, and unity for $\bar{K}$ emission. Dominant subchannels can be read off from the single flavor rates quoted in Table I. For the separate emission of $\eta$ and $\eta'$, or consideration of OZI-violating mixing in other nonets, mixing angles between the $\omega$- and $\phi$-like components may easily be introduced; note that with the neglect of OZI mixing, emission of the $\phi$-like meson is forbidden.

| Channel | Percent of inclusive nonresonant rate in the heavy quark limit | Approximate phase space suppression factor in $b\to c$ decays | Estimated net suppression after $\Lambda_{QCD}/m_Q$ corrections |
|---|---|---|---|
| $(D+D^*)\pi$ | 0.9 | 1/3 | 0.5 |
| $(D+D^*)\eta+(D+D^*)\eta'$ | 0.3 | 1/4 | 0.4 |
| $(D+D^*)\bar{K}$ | 0.6 | 1/3 | 0.5 |
| $(D+D^*)\rho$ | 2.8 | 1/4 | 0.4 |
| $(D+D^*)\omega$ | 0.9 | 1/4 | 0.4 |
| $(D+D^*)\bar{K}^*$ | 1.8 | 1/4 | 0.4 |
| $D^{**}_{3/2}\pi$ | 0.7 | 1/4 | 0.4 |
| $D^{**}_{3/2}\eta+D^{**}_{3/2}\eta'$ | 0.2 | 1/5 | 0.3 |
| $D^{**}_{s\ 3/2}\bar{K}$ | 0.5 | 1/4 | 0.4 |
| $D^{**}_{1/2}\pi$ | 0.7 | 1/4 | 0.4 |
| $D^{**}_{1/2}\eta+D^{**}_{1/2}\eta'$ | 0.2 | 1/5 | 0.3 |
| $D^{**}_{s\ 1/2}\bar{K}$ | 0.5 | 1/4 | 0.4 |
| $D^{**}_{3/2}\rho$ | 2.2 | 1/10 | 0.15 |
| $D^{**}_{3/2}\omega$ | 0.7 | 1/10 | 0.15 |
| $D^{**}_{s\ 3/2}\bar{K}^*$ | 1.5 | 1/10 | 0.15 |
| $D^{**}_{1/2}\rho$ | 4.0 | 1/5 | 0.3 |
| $D^{**}_{1/2}\omega$ | 1.3 | 1/5 | 0.3 |
| $D^{**}_{s\ 1/2}\bar{K}^*$ | 2.7 | 1/5 | 0.3 |
| $(D+D^*)a_0$ | 1.7 | 1/5 | 0.3 |
| $(D+D^*)f_0$ | 0.6 | 1/5 | 0.3 |
| $(D+D^*)\bar{K}^*_0$ | 1.1 | 1/5 | 0.3 |
| $(D+D^*)a_1$ | 2.8 | 1/6 | 0.25 |
| $(D+D^*)f_1$ | 0.9 | 1/6 | 0.25 |
| $(D+D^*)\bar{K}_{1a}$ | 1.9 | 1/6 | 0.25 |
| $(D+D^*)b_1$ | 2.4 | 1/5 | 0.3 |
| $(D+D^*)h_1$ | 0.8 | 1/5 | 0.3 |
| $(D+D^*)\bar{K}_{1b}$ | 1.6 | 1/5 | 0.3 |
| $(D+D^*)a_2$ | 1.6 | 1/6 | 0.25 |
| $(D+D^*)f_2$ | 0.5 | 1/6 | 0.25 |
| $(D+D^*)\bar{K}^*_2$ | 1.1 | 1/6 | 0.25 |

standing. Some failures of the quark model (e.g., the proton spin crisis) and the known existence of strong real and virtual decay channel couplings indeed make blithely ignoring the role of $q\bar{q}$ pairs both phenomenologically and theoretically untenable. In this work I have examined the influence of $q\bar{q}$ pairs on the simplest ''real'' hadrons: heavy quark mesons like the $\bar{B}$.

This study has led to a number of qualitative insights which I believe are quite general in nature. In earlier work on ''unquenching the quark model,'' the success of the valence quark model in spectroscopy was shown to have a possible basis in the validity of an adiabatic approximation. In this approximation, both the confining flux tube and the many $q\bar{q}$ pairs it generates remain in their adiabatically evolving ground state as the valence quarks move. In this work I have shown that the same approximation leads to valence quark and therefore resonance dominance of the simplest current matrix elements: $\bar{Q}_2\Gamma Q_1$ matrix elements of heavy quark mesons. The physical picture behind our results is simple and appealing. According to heavy quark symmetry, at small recoil the $Q_1\to Q_2$ decay of the $Q_1$ ground state $P_{Q_1}$ will lead with unit probability to $P_{Q_2}$ and $V_{Q_2}$ *no matter how complicated the QCD ''brown muck'' might be*. This simple observation makes it clear that for $\bar{Q}_2\Gamma Q_1$ matrix elements the issue is not the probability of $q\bar{q}$ pairs in $P_{Q_1}$ but rather how rapidly as $w-1$ increases these pairs fail to overlap with those in $P_{Q_2}$, $V_{Q_2}$, and the $Q_2\bar{d}$ excited states. I have shown explicitly that in the adiabatic limit these overlaps are perfect so that only valence states (the resonances) are produced. Moreover, I showed that violations of the adiabatic approximation can be directly associated with the production of nonresonant states. Thus this study leads to a way of understanding how the valence quark model can be so successful *even though* hadrons are full of $q\bar{q}$ pairs.

While they are quantitatively very crude, these calculations also have interesting consequences for real $\bar{B}$ semileptonic decays. First of all, they suggest that $\rho^2_{dyn} \equiv \rho^2 - \frac{1}{4}$ is composed of two comparable parts: $\rho^2_{resonant} \simeq \frac{1}{2}$ and $\rho^2_{nonresonant} \simeq \frac{1}{4}$, though we have stressed that this split of $\rho^2_{dyn}$ is very model-dependent. In the heavy quark limit, Bjorken's sum rule would then lead one to expect [using the central experimental value of the $(D+D^*)$ fraction] that roughly 24% (with a 50% error) of semileptonic decays go into resonances (both ordinary $Q\bar{d}$ mesons and $Q\bar{d}$ hybrids), and roughly 12% (with an error of a factor of two) go into nonresonant states. Since the former states will have most of their strength in the 2.4−3.0 GeV region, they will suffer, relative to the heavy quark limit, phase space suppression factors varying from only 0.75 to 0.50 over this range which may be fully compensated by the $\Lambda_{QCD}/m_Q$ enhancements required by asymptotic duality [29,30]. In contrast, we have seen that nonresonant states are expected to populate very high masses peaking in strength near $m_B$ and so to suffer a substantial *net* suppression factor of at least $\frac{1}{4}$. From this study I therefore expect that $\lesssim 5\%$ of $\bar{B}$ semileptonic decays will be nonresonant.

As a corollary of this last observation, I note that if a 12% nonresonant semileptonic fraction is required for duality but only a quarter of this is realized, then duality will fail from this effect alone by $\sim 10\%$, as anticipated in Ref. [30]. There is, however, a minor inconsistency associated with this conclusion. Experiment requires that $36\pm 6\%$ of $\bar{B}$ semileptonic decays be non-$(D+D^*)$ decays, in contrast to the $\sim 25\%$ we would have estimated from the preceeding. As mentioned earlier, this could simply mean that the ISGW2 model underpredicts the production of excited charm mesons [18] or that experiment has overestimated non-$(D+D^*)$ production. Determining whether this discrepancy is real will require a more quantitative calculation than this one (and probably additional experimental measurements as well).

Detailed experimental studies of the structure of the hadronic final state in semileptonic $b\to c$ decays can therefore answer some fundamental questions about the role of $q\bar{q}$ pairs and about duality in strong QCD. A vital feature of these systems is that duality is underwritten by Bjorken's sum rule, requiring an exact and local duality between quark- and hadronic-level decays in the heavy quark limit. In particular, the experimental determination of the strength and structure of these nonresonant contributions would immediately test the conclusions reached here that these $q\bar{q}$ effects are highly suppressed in real $b\to c$ decays, that such decays extend to very high masses, and that they are highly fragmented into many small channels. Independent of the outcome, examining this problem in Nature's simplest hadronic system under the action of its simplest current (a heavy-to-heavy nonsinglet transition) should prove to be an excellent starting point for eventually understanding the $q\bar{q}$ sea in all strongly interacting matter. In particular, given the complexity of QCD, this seems an essential first step before tackling the problems of duality and nonresonant production in ordinary deep inelastic scattering.

## APPENDIX A: FLUX TUBES AND A CRITIQUE OF NONRELATIVISTIC KINEMATICS

It is not difficult to make the simplification $\psi_{ft}(\vec{w},\vec{r}) = \psi_{ft}(\vec{w})$ of Eq. (21) more flux-tube-like. In the case where $\psi_{ft}$ depends on $\vec{w}$ and $\vec{r}$, Eqs. (42)–(44) become

$$\xi^{QM}_{Qq\bar{q}\bar{d}}(w) = \frac{1}{2m_Q}\left\langle P_{Qq\bar{q}\bar{d}}\left(\frac{\vec{P}_{cm}}{2}\right)\middle|\bar{Q}Q\middle|P_{Qq\bar{q}\bar{d}}\left(-\frac{\vec{P}_{cm}}{2}\right)\right\rangle \tag{A1}$$

$$= \int d^3w\, d^3r\, \psi^*_{Q\bar{d}}(\vec{r})\,\psi^*_{ft}(\vec{w},\vec{r}) \times e^{-2im_q\vec{V}_{cm}\cdot\vec{w}} e^{-im_d\vec{V}_{cm}\cdot\vec{r}}\,\psi_{Q\bar{d}}(\vec{r})\,\psi_{ft}(\vec{w},\vec{r}) \tag{A2}$$

or, defining

$$\xi^{QM}_{Qq\bar{q}\bar{d}}(w) = 1 - \rho^2_{Qq\bar{q}\bar{d}}(w-1) \tag{A3}$$

we have

$$\rho^2_{Qq\bar{q}\bar{d}} = \frac{1}{3}\int d^3w\, d^3r\,(2m_q\vec{w} + m_d\vec{r})^2 |\psi_{Q\bar{d}}(\vec{r})\,\psi_{ft}(\vec{w},\vec{r})|^2 \tag{A4}$$

$$= \frac{1}{3}\langle (2m_q\vec{w} + m_d\vec{r})^2\rangle \tag{A5}$$

which reduces to the simplified results of the text in the appropriate limits.

Now consider a generic example of a more realistic $\psi_{ft}$ that has a flux tube's shape:

$$\psi_{ft}(\vec{w},\vec{r}) = \frac{\beta_{ft}}{\pi^{1/2}} e^{-(1/2)\beta^2_{ft} w^2_\perp}\, t(\vec{w}\cdot\hat{r}) \tag{A6}$$

where $\hat{r} = \vec{r}/r$, $\vec{w}_\perp = \vec{w} - (\vec{w}\cdot\hat{r})\hat{r}$, and $t(\vec{w}\cdot\hat{r})$, which depends only on the longitudinal variable $\vec{w}\cdot\hat{r}$, is a normalized tube-like function. [For example, one might have $t = (1/\sqrt{r})\,\theta(\vec{w}\cdot\hat{r})\,\theta(r - \vec{w}\cdot\hat{r})$ to create a cylindrical wave function that is Gaussian transverse to $\vec{r}$ and constant between $Q$ and $\bar{d}$.] With such a wave function

$$\rho^2_{Qq\bar{q}\bar{d}}=\frac{1}{3}[4m_q^2\langle w_\perp^2\rangle+\langle(2m_q\vec{w}\cdot\hat{r}+m_d r)^2\rangle]. \quad (A7)$$

The first term is as expected intuitively: it is the unchanged $\vec{w}_\perp$ part of the result of the text. One might also naively interpret the second term as the old $\vec{r}$ term plus a new longitudinal contribution due to the assumed spatial distribution of $\vec{w}\cdot\hat{r}$ in a tube-like configuration along $\vec{r}$.

I believe that the physics is more subtle than this. Consider the origin of $\rho^2_{wf}$ in the nonrelativistic kinematics of our model. In the $Q\bar{d}$ sector [see Eq. (1)], $\vec{r}_Q=-m_d\vec{r}/m_Q$ in the center of mass frame as $m_Q\to\infty$, while in $Qq\bar{q}\bar{d}$ [see Eq. (3)], $\vec{r}_Q\to-(2m_q\vec{w}+m_d\vec{r})/m_Q$. Since nonrelativistically $\rho^2=\frac{1}{3}m_Q^2\langle r_Q^2\rangle$, we see that Eq. (50) for $\rho^2_{Q\bar{d}}$ and Eq. (A5) for $\rho^2_{Qq\bar{q}\bar{d}}$ are simply consequences of these nonrelativistic relations.

To see the dangers of this approximation when the string tension and its renormalization are large compared to $m_q$, consider the calculation of the mass of a system of heavy quarks $Q$ and $\bar{d}$ at separation $\vec{r}$ connected by a renormalized flux tube, i.e., of the state (24) of the text which has string tension $b_{N_f}$ since it has the appropriate admixture of $q\bar{q}$ pairs. If its mass were determined nonrelativistically one would obtain

$$M^{nr}-m_Q=m_d+\frac{2m_q c^2_{q\bar{q}}}{1+c^2_{q\bar{q}}} \quad (A8)$$

i.e., the effective mass opposite $Q$ (against which it must recoil to conserve the position of the center of mass) would be the probability-weighted masses of the pure $\bar{d}$ state and the $\bar{q}q\bar{d}$ admixture. On adding interactions (both the diagonal potential $b_0 r$ and the off-diagonal perturbation $H_{pc}^{q\bar{q}}$ which mixes $Q\bar{d}$ and $Q\bar{q}q\bar{d}$), we obtain the correct answer

$$M^{adiabatic}-m_Q=m_d+b_{N_f}r. \quad (A9)$$

Thus the mass $2m_q$ does not in this circumstance have an independent reality as indicated by the nonrelativistic kinematics just described: it is subsumed into the properly renormalized string tension. Indeed, it is an implicit assumption of the model for the $q\bar{q}$ pairs described in the text that the unquenched flux tube also behaves like a relativistic string: it should support (only) transverse waves moving with the speed of light. Thus while the $q\bar{q}$ pairs change the longitudinal distribution of $\vec{r}_Q$, *this effect is already described in the flux tube model by the mass $b_{N_f}r$ residing in the flux tube*: when $r$ increases, not only does $\bar{d}$ move, but so does the center of mass of the flux tube. It would therefore be double-counting to include the effect on $\langle r_Q^2\rangle$ of the $2m_q\vec{w}\cdot\hat{r}$ term of Eq. (A7). I should hasten to add that the nonrelativistic quark model used in this paper does not *explicitly* take this effect into account. To do so would require a full treatment of the flux tube degrees of freedom (*versus* the adiabatic, potential-model approximation used here). Nevertheless, even the nonrelativistic quark model has undoubtedly already taken some of this effect into account implicitly by its choice of such free parameters as $m_d$. (For example, in many applications the constituent quark mass is effectively $m_d=m_d^0+\frac{1}{2}b\langle r\rangle$). In contrast, there is no mechanism in the quark potential model to take into account transverse displacements of $Q$ relative to $\vec{r}$ [31]. These transverse displacements are the true degrees of freedom of the (quenched and unquenched) flux tubes and the reaction of $Q$ to them makes a non-potential-model-type transverse contribution to $\rho^2$.

By renormalizing the string tension from $b_0\to b_{N_f}<b_0$, $q\bar{q}$ pairs increase the longitudinal contribution to $\rho^2$ (at least in the nonrelativistic approximation $\frac{1}{2}br\ll m_d$). However, this increase is *not* compensated by nonresonant production, since it is this same string tension $b_{N_f}<b_0$ which controls the structure and thereby the production of excited resonances. That is, the longitudinal effect of the pairs is real, but it simply renormalizes resonance physics. The dynamics behind this balancing act, characteristic of the adiabatic approximation, can be seen by calculating the contribution of the flux tube to the Isgur-Wise function [31]: since the flux tube has only transverse internal degrees of freedom, it has no impact on longitudinal overlap integrals over the total separation $\vec{r}$. Based on the preceding arguments, the transverse contributions to $\langle r_Q^2\rangle$ would also be those of a relativistic string with string tension $b_0$ or $b_{N_f}$ in the quenched and unquenched flux tubes, respectively. In contrast to the longitudinal contribution to $\langle r_Q^2\rangle$, the transverse contributions correspond to the reaction of $Q$ to real internal degrees of freedom, and these degrees of freedom (both gluonic and $q\bar{q}$) can be excited by the action of the $\bar{Q}\Gamma Q$ current. When acting on the pure $Q\bar{d}$ piece of the state, the current excites hybrid mesons which in the quenched limit exactly compensate for the loss of rate from the elastic channel due to transverse contributions to $\langle r_Q^2\rangle$ [31]. In the $Q\bar{q}q\bar{d}$ sector, the current could in principle excite either of the strings internal to mesons $(Q\bar{q})_a$ or $(q\bar{d})_b$, or it could excite the center of mass of the $q\bar{q}$ pair. In the quark potential model approximation to this latter process, which is of course the one of interest for this paper, one would recover the result shown in Eq. (A7) less the longitudinal part of $\vec{w}$:

$$\rho^2_{Qq\bar{q}\bar{d}}=\frac{4m_q^2}{3}\langle w_\perp^2\rangle+\frac{m_d^2}{3}\langle r^2\rangle \quad (A10)$$

$$=\frac{4m_q^2}{3\beta_{ft}^2}+\frac{m_d^2}{3\beta^2_{Q\bar{d}}}. \quad (A11)$$

The first term of this formula differs from the expression of the text by the factor $\frac{2}{3}$ corresponding to two of the three degrees of freedom of $\vec{w}$ being active. Note that for this picture to be consistent, the total transverse contribution to $\langle r_Q^2\rangle$ must be that of a relativistic string with string tension $b_{N_f}$; the decomposition into $Q\bar{d}$ and $Q\bar{q}q\bar{d}$ components is only useful in identifying the compensating channels required by Bjorken's sum rule. However, in the renormalized string picture $\langle r_Q^2 \rangle$ of course depends just on $b_{N_f}$, while Eq. (A11) shows a contribution proportional to $2m_q$. It would be interesting to examine how the dynamics of pair creation in the flux tube leads to such a term. I speculate that the mechanism is the "consumption" of a piece of flux tube of length $\Delta r \sim 2m_q/b$ in a nonlocal pair creation process.

In summary, in this Appendix I have described several subtleties in the description of $q\bar{q}$ pair creation in the flux tube model, and pointed out some interesting issues which arise in the physics of the renormalized flux tube. While I believe these matters are important conceptually and are worthy of further study, I am convinced that other uncertainties described in the text are of far greater impact numerically on our results. Given this and the great convenience of the spherical approximation, I therefore chose this simpler if less basic framework for the discussion of the text.

## APPENDIX B: DETERMINING $\eta_{q\bar{q}}$

Equation (17) defines the action of the pair creation Hamiltonian on $|P_{Q\bar{d}}\rangle$. This perturbation not only produces pairs to make the eigenstate $|P_Q\rangle$ of Eq. (24), but also leads to strong decays. In particular, the projection of the state (23) onto the continuum states $|\vec{P}_{cm};ab(\vec{q})\rangle$ determines the $P_Q \to (Q\bar{q})_a(q\bar{d})_b$ coupling constants. By heavy quark symmetry [9], the same dynamics determine the $P_Q^* \to (Q\bar{q})_a(q\bar{d})_b$ coupling constants, where $P_Q^*$ is the vector partner of the pseudoscalar state $P_Q$. In this Appendix I use these facts to determine the strength parameter $\eta_{q\bar{q}}$ of Eq. (17) by comparing to the decays $P_Q^* \to P_Q \pi$.

I begin with a practical matter. As $m_Q \to \infty$, the decays $P_Q^* \to P_Q \pi$ are forbidden since $P_Q^*$ and $P_Q$ become degenerate heavy quark spin partners. However, $\Gamma(K^* \to K\pi)$ is known, and $\Gamma(D^* \to D\pi)$ can be deduced if one assumes that the successful phenomenology of magnetic dipole decays can be extended to $\Gamma(D^* \to D\gamma)$. (The total width of the $D^*$ is so small that only decay branching fractions and not decay widths are known. If one takes the $K^* \to K\pi$ and $K^* \to K\gamma$ decays and scales them appropriately in $m_Q$ assuming that heavy quark scaling works all the way down to $m_s$, the observed $D^* \to D\pi$ and $D^* \to D\gamma$ branching ratios are explained nearly perfectly. This is another example of the often-noted fact that in many circumstances a strange quark behaves like a heavy quark.) Since the branching ratio for $D^{*0} \to D^0\pi^0$ is well-determined experimentally, I will use the value $\Gamma(D^{*0} \to D^0\pi^0) \simeq 30$ keV deduced in this manner as input "data."

The calculations themselves are simple. If $g_0 D^{0\dagger} \partial_\mu \pi^0 D^{0*\mu}$ is the effective Lagrangian density for the decay leading to $\Gamma(D^{*0} \to D^0\pi^0) = g^2 q^3/48\pi m_{D^*}^2$, then in the center of mass, with the pion emitted with momentum $\vec{q}$ from a $D^*$ with polarization $+1$ along $\hat{z}$,

$$-\frac{ig_0 q_+}{\sqrt{2}(2\pi)^{9/2}} = \eta_{q\bar{q}} \sqrt{\frac{1}{3}} \frac{\tilde{m}_D \tilde{m}_\pi^{1/2} \beta_D^3 \beta_\pi^{3/2} \beta_{ft}^{3/2}}{8\pi^6} \int d^3r \int d^3v \int d^3w\, e^{-(1/2)\beta_D^2(\vec{w}+(1/2)\vec{v})^2 - (1/2)\beta_\pi^2(\vec{r}-\vec{w}+(1/2)\vec{v})^2 - (1/2)\beta_{ft}^2 w^2 - (1/2)\beta_D^2 r^2}$$
$$\times \frac{1}{(2\pi)^{3/2}} e^{-i(\vec{q}/2)\cdot(\vec{r}+\vec{w}-(1/2)\vec{v})} \psi_{pc}(\vec{v})_+ \tag{B1}$$

in which the $1/\sqrt{2}$ for the pure $\pi^0$ decay via $\eta_{u\bar{u}}$ has been explicitly included, $\tilde{m}_X$ is the "mock meson" mass given by the sum of the constituent quark masses, and $\psi_{pc}(\vec{v})_+ = (\beta_{pc}^{5/2}/\pi^{3/4}) v_+ e^{-(1/2)\beta_{pc}^2 v^2}$. The integrals are straightforward and give

$$g_0 = -\eta_{q\bar{q}} \frac{16\pi^{3/4} \tilde{m}_D \tilde{m}_\pi^{1/2} \beta_D^3 \beta_\pi^{3/2} \beta_{ft}^{3/2} \beta_{pc}^{5/2} \left(\frac{1}{2} - a_{DD} - b_{DD}\right)}{\sqrt{3}\beta_{v_{DD}}^5 \beta_{y_{DD}}^3 \beta_{w_{DD}}^3} e^{-q^2/8\bar{\beta}_{DD}^2} \tag{B2}$$

where

$$\frac{1}{\bar{\beta}_{DD}^2} = \frac{1}{\beta_y^2} + \frac{1}{\beta_w^2}\left(\frac{\beta_D^2 + 2\beta_\pi^2}{\beta_D^2 + \beta_\pi^2}\right)^2 + \frac{1}{\beta_v^2}\left(\frac{1}{2} - a - b\right)^2 \tag{B3}$$

$$\beta_{v_{DD}}^2 = \beta_{pc}^2 + a^2(\beta_D^2 + \beta_\pi^2 + \beta_{ft}^2) + \left(b^2 + \frac{1}{4}\right)(\beta_D^2 + \beta_\pi^2)$$
$$+ a\beta_D^2 + (b - 2ab - a)\beta_\pi^2 \tag{B4}$$

$$\beta_{y_{DD}}^2 = \beta_D^2 + \beta_\pi^2 \tag{B5}$$

$$\beta_{w_{DD}}^2 = \beta_D^2 + \beta_{ft}^2 + \frac{\beta_\pi^2 \beta_D^2}{\beta_D^2 + \beta_\pi^2} \tag{B6}$$

$$a_{DD} = -\frac{\beta_D^4}{2[(\beta_D^2 + \beta_\pi^2)(\beta_D^2 + \beta_\pi^2 + \beta_{ft}^2) - \beta_\pi^4]} \tag{B7}$$

$$b_{DD}=-\frac{\beta_\pi^2(2\beta_D^2+\beta_{ft}^2)}{2[(\beta_D^2+\beta_\pi^2)(\beta_D^2+\beta_\pi^2+\beta_{ft}^2)-\beta_\pi^4]}. \tag{B8}$$

From the ''measured'' $D^{0*}\to D^0\pi^0$ width, we can deduce that $g_0\simeq 11$. With our canonical parameters it follows that, with $\beta_{pc}$ expressed in GeV,

$$\eta_{q\bar{q}}\simeq 0.32\left[\frac{3+2f}{f^{1/2}}\right]^{3/2}\left[\frac{\beta_{pc}^2+0.06}{\beta_{pc}}\right]^{5/2} \tag{B9}$$

where since $\beta_D\simeq\beta_\pi\simeq\beta_{ft}/f^{1/2}$ I have been able to explicitly display the dependence on $f$ as well as $\beta_{pc}$. We see that for a variation of $\pm 1$ around the canonical value $f=2$, $\eta_{q\bar{q}}$ varies by less than 10%. Thus we conclude that (since $\beta_{pc}^2 \gg 0.06\ \mathrm{GeV}^2$)

$$\eta_{q\bar{q}}\simeq 0.9\left[\frac{\beta_{pc}}{0.58\ \ \mathrm{GeV}}\right]^{5/2}\mathrm{GeV} \tag{B10}$$

i.e., $\eta_{q\bar{q}}\simeq 0.9$ GeV for the canonical value $\beta_{pc}=0.58$ GeV. We also see from this formula the expected result that as the pair creation operator becomes more pointlike, $\eta_{q\bar{q}}\to\infty$.

## APPENDIX C: CALCULATING $\tau_{1/2}^{(m)}$ AND $\tau_{3/2}^{(p)}$ FOR SELECTED LOW-LYING EXCLUSIVE NONRESONANT CHANNELS

As discussed in Ref. [17], the semileptonic decays $\bar{B}\to D^{(n)}_{s_l'^{\pi_l'}} l\bar{\nu}_l$ are governed in the heavy quark limit by generalized Isgur-Wise functions which determine all of the form factors for the decay of the $\bar{B}$ with $s_l^{\pi_l}=\frac{1}{2}^-$ to both of the states of a heavy quark spin multiplet with quantum numbers $s_l'^{\pi_l'}$. As described in the text and elsewhere [17],

$$\rho^2=\frac{1}{4}+\Delta\rho^2_{pert}+\rho^2_{Q\bar{d}}+\Delta\rho^2_{sea} \tag{C1}$$

where the $\frac{1}{4}$ is Bjorken's relativistic correction [16], $\Delta\rho^2_{pert}$ is a perturbative QCD radiative correction, and $\rho^2_{Q\bar{d}}$ and $\Delta\rho^2_{sea}$ are the contributions to the slope of $\xi(w)$ from the valence and sea quarks, respectively. As we have seen, these latter two contributions may be related to the rates of decay into inelastic channels by

$$\rho^2_{Q\bar{d}}=\sum_{m=1/2^+\ Q\bar{d}\ \ resonances}|\tau_{1/2}^{(m)}|^2 +2\sum_{p=3/2^+\ Q\bar{d}\ \ resonances}|\tau_{3/2}^{(p)}|^2 \tag{C2}$$

$$\Delta\rho^2_{sea}=\sum_{m=1/2^+ continuum}|\tau_{1/2}^{(m)}|^2 +2\sum_{p=3/2^+ continuum}|\tau_{3/2}^{(p)}|^2 \tag{C3}$$

where the $\tau$'s are the appropriate Isgur-Wise functions. Once the $\tau$'s are specified, all transition form factors to the states of a heavy quark spin multiplet may be determined from symmetry considerations. For this reason, it is useful to calculate the $\tau$'s in the simplest possible setting, namely for the case where $b$ and $c$ are *spinless*. This is possible since the $\tau$'s depend only on the dynamics of the light degrees of freedom, i.e., on the transition $s_l^{\pi_l}=\frac{1}{2}^-\to s_l'^{\pi_l'}=\frac{1}{2}^+$ or $\frac{3}{2}^+$.

Another key simplification arises from the dynamics of the pair creation process. As demonstrated in the text, when $b(\vec{v})\to c(\vec{v}\,')$ in a $Q\bar{q}q\bar{d}$ state, excitation of the variable $\vec{r}$ cannot lead to a contribution of order $(w-1)$. Furthermore, the variable $\vec{v}$ cannot be excited since this is a $q\bar{q}$ internal coordinate. Thus to contribute at order $(w-1)$, $b(\vec{v})\to c(\vec{v}\,')$ *must* kick the $\vec{w}$ coordinate into an $l_w=1$ state: *such a state is the parent of all nonresonant production to order* $(w-1)$. Thus $\Delta\rho^2_{sea}$ arises entirely from the ''decay'' of the lowest $l_w=1$ excited state of $c\bar{q}q\bar{d}$ arising from the $b\to c$ transition from $b\bar{q}q\bar{d}$. With the $q\bar{q}$ pair in $J^P=0^+$, the decay can thus occur from the six states $w_+\uparrow$, $w_+\downarrow$, $w_0\uparrow$, $w_0\downarrow$, $w_-\uparrow$, and $w_-\downarrow$, depending on which component of $\vec{w}$ is excited by the recoil. Here $w_+,w_0,w_-$ represent the components of the $l_w=1$ state, and $\uparrow,\downarrow$ represent the spin state of the $\bar{d}$ spectator quark. Since the total decay rate of $b\bar{q}q\bar{d}\uparrow$ and $b\bar{q}q\bar{d}\downarrow$ must be the same, we can simplify if we average rates over the initial $\bar{d}$ spin and over directions of $\vec{P}_{cm}$ (or equivalently, over directions of $\vec{w}$). Then since the average over the six states just listed will be the same as the average over the two $j=\frac{1}{2}^+$ and four $j=\frac{3}{2}^+$ states formed from them, we can deal with ''parent'' states that are states with good angular momenta and which therefore uniquely feed the $s_l'^{\pi_l'}=\frac{1}{2}^+$ and $\frac{3}{2}^+$ states, respectively.

Let me provide an example: the production of the $(D+D^*)\pi$ nonresonant states. Since we know that all of $\Delta\rho^2_{sea}$ arises from the ''parent'' state, the fraction of $\Delta\rho^2_{sea}$ coming from a given channel can just be obtained as the $jm_j$ average of the square of an overlap between a given $jm_j$ in $Q\bar{q}q\bar{d}$ and the two particle continuum state of interest. Thus from the $jm_j=\frac{1}{2}\frac{1}{2}$ state we can extract $\langle(D$

$+D^*)_{1/2^-\,1/2}\pi(\vec{q})|c\bar{q}q\bar{d};\frac{1}{2}^+\frac{1}{2}\rangle$, which ought to leave the $(D+D^*)\pi$ system in an $S$-wave. Explicitly, as $m_Q\to\infty$,

$$\left\langle (D+D^*)_{1/2^-1/2}\pi(\vec{q})\middle| c\bar{q}q\bar{d};\frac{1}{2}^+\frac{1}{2}\right\rangle$$
$$=\int d^3w\int d^3v\int d^3r\frac{\beta_D^{3/2}}{\pi^{3/4}}e^{-(1/2)\beta_D^2(\vec{w}+(1/2)\vec{v})^2}\frac{\beta_\pi^{3/2}}{\pi^{3/4}}$$
$$\times e^{-(1/2)\beta_\pi^2(\vec{r}-\vec{w}+(1/2)\vec{v})^2}\frac{1}{(2\pi)^{3/2}}$$
$$\times e^{-i(\vec{q}/2)\cdot(\vec{r}+\vec{w}-(1/2)\vec{v})}\frac{\tilde{\beta}_{pc}^{5/2}}{\pi^{3/4}}$$
$$\times e^{-(1/2)\beta_{pc}^2v^2}\frac{\beta_{ft}^{5/2}}{\pi^{3/4}}e^{-(1/2)\beta_{ft}^2w^2}\frac{\beta_B^{3/2}}{\pi^{3/4}}e^{-(1/2)\beta_B^2r^2}\Sigma(\vec{w},\vec{v}) \tag{C4}$$

where

$$\Sigma\equiv\left\langle \uparrow\sqrt{\frac{1}{2}}(\uparrow\downarrow-\downarrow\uparrow)\middle|\sqrt{\frac{1}{3}}(\uparrow\uparrow v_- -[\uparrow\downarrow+\downarrow\uparrow]v_z\right.$$
$$\left.-\downarrow\downarrow v_+)\sqrt{\frac{2}{3}}(-w_+\downarrow-w_z\uparrow)\right\rangle \tag{C5}$$

is the spin overlap matrix element of the three quarks $\bar{q}q\bar{d}$, respectively. We get

$$\left\langle (D+D^*)_{1/2^-1/2}\pi(\vec{q})\middle| c\bar{q}q\bar{d};\frac{1}{2}^+\frac{1}{2}\right\rangle=-I_{+-}^{00}-I_{zz}^{00} \tag{C6}$$

where

$$I_{ij}^{00}\equiv\frac{1}{3}\frac{\beta_D^{3/2}\beta_\pi^{3/2}\beta_{ft}^{5/2}\tilde{\beta}_{pc}^{5/2}\beta_B^{3/2}}{\pi^{15/4}}$$
$$\times\int d^3w\int d^3v\int d^3r\frac{v_iw_j}{(2\pi)^{3/2}}$$
$$\times e^{-i(\vec{q}/2)\cdot(\vec{r}+\vec{w}-(1/2)\vec{v})-(1/2)E^2} \tag{C7}$$

with

$$E^2=\beta_D^2\left(\vec{w}+\frac{1}{2}\vec{v}\right)^2+\beta_\pi^2\left(\vec{r}-\vec{w}+\frac{1}{2}\vec{v}\right)^2+\beta_{ft}^2w^2+\tilde{\beta}_{pc}^2v^2$$
$$+\beta_B^2r^2. \tag{C8}$$

In these formulas I have distinguished between $\beta_D^2$ and $\beta_B^2$ to allow the ''ancestry'' of terms to be traced, even though $\beta_B=\beta_D$ from heavy quark symmetry. This integral is easily done, giving

$$I_{ij}^{00}=I^{00}\left[c^{00}\delta_{ij}+c_{ij}^{00}\frac{q_iq_j}{\beta_v^2}\right]e^{-q^2/8\tilde{\bar{\beta}}^2} \tag{C9}$$

where $I^{00}$, $c_{ij}^{00}$, $c^{00}$, and $\tilde{\bar{\beta}}$ are given below. For the problem at hand, we get

$$\left\langle (D+D^*)_{1/2^-1/2}\pi(\vec{q})\middle| c\bar{q}q\bar{d};\frac{1}{2}^+\frac{1}{2}\right\rangle$$
$$=-\left[3c^{00}+c_{ij}^{00}\frac{q_+q_-}{\beta_v^2}+c_{ij}^{00}\frac{q_z^2}{\beta_v^2}\right]I^{00}e^{-q^2/8\tilde{\bar{\beta}}^2}$$
$$=-\left[3c^{00}+c_{ij}^{00}\frac{q^2}{\beta_v^2}\right]I^{00}e^{-q^2/8\tilde{\bar{\beta}}^2}$$
$$=-\sqrt{4\pi}\left[3c^{00}+c_{ij}^{00}\frac{q^2}{\beta_v^2}\right]I^{00}e^{-q^2/8\tilde{\bar{\beta}}^2}Y_{00}(\Omega_q), \tag{C10}$$

a pure $S$-wave decay as required, with partial wave amplitude $A_{1/2}\equiv-\sqrt{4\pi}[3c^{00}+c_{ij}^{00}(q^2/\beta_v^2)]I^{00}e^{-q^2/8\tilde{\bar{\beta}}^2}$. Note that with $Y_{00}$ factored out, $|A_{1/2}|^2$ is already the probability for this channel integrated over angles $\Omega_q$, leaving only an integral $\int dq q^2|A_{1/2}|^2$ to be done to sum over all $(D+D^*)\pi$ states with quantum numbers $\frac{1}{2}^+\frac{1}{2}$ at any fixed value of $(w-1)$.

Next consider $\langle(D+D^*)_{1/2^-1/2}\pi(\vec{q})|c\bar{q}q\bar{d};\frac{3}{2}^+\frac{3}{2}\rangle$, which proceeds by replacing $\sqrt{2/3}(-w_+\downarrow-w_z\uparrow)\rangle$ by $-w_+\uparrow$ in Eq. (C5). With this change one gets

$$\left\langle (D+D^*)_{1/2^-1/2}\pi(\vec{q})\middle| c\bar{q}q\bar{d};\frac{3}{2}^+\frac{3}{2}\right\rangle$$
$$=-\sqrt{\frac{3}{2}}I_{z+}^{00}$$
$$=-\sqrt{\frac{3}{2}}c_{ij}^{00}\frac{q_zq_+}{\beta_v^2}I^{00}e^{-q^2/8\tilde{\bar{\beta}}^2}$$
$$=-\sqrt{\frac{1}{5}}\left[\sqrt{4\pi}c_{ij}^{00}\frac{q^2}{\beta_v^2}I^{00}e^{-q^2/8\tilde{\bar{\beta}}^2}\right]Y_{21}(\Omega_q), \tag{C11}$$

a pure $D$-wave decay as required. Moreover, since $-\sqrt{1/5}$ is the Clebsch-Gordan coefficient for coupling $(D+D^*)_{1/2^-1/2}$ and an $l=2$, $m=1$ pion into a $\frac{3}{2}\frac{3}{2}$ state, we can deduce that decays to this whole angular momentum multiplet with $(D+D^*)\pi$ in a $D$-wave coupled to $s_l'^{\pi_l'}=\frac{3}{2}^+$ are controlled by a $D$-wave amplitude $A_{3/2}\equiv-\sqrt{4\pi}c_{ij}^{00}(q^2/\beta_v^2)I^{00}e^{-q^2/8\tilde{\bar{\beta}}^2}$.

To complete this pedagogical example, I note that since the partial wave decay amplitudes are independent of the magnetic substate $m$,

$$\frac{1}{6}\sum_m |A_{1/2}|^2=\frac{1}{3}|A_{1/2}|^2 \tag{C12}$$

$$\frac{1}{6}\sum_m |A_{3/2}|^2=\frac{2}{3}|A_{3/2}|^2, \tag{C13}$$

where the $\frac{1}{6}$ arises from averaging over the six $jm_j$ states. On comparison with Eq. (C3), we see that $\tau_{1/2}=\sqrt{1/3}A_{1/2}\sqrt{(1/3)\Delta\rho^2_{sea}}$ and $\tau_{3/2}=\sqrt{(1/3)}A_{3/2}\sqrt{(1/3)\Delta\rho^2_{sea}}$. Note that $\frac{1}{3}\Delta\rho^2_{sea}$ appears since the overlap amplitudes $A_{1/2}$ and $A_{3/2}$ as calculated are for a single flavor, while the factor $\sqrt{1/3}$ arises as a residue of the angular and spin averaging.

The tricks outlined here are more powerful for more complex decays. I will give one partial illustration: $\langle[(D+D^*)\rho]_{3/2^+3/2}|c\overline{q}q\overline{d};\frac{3}{2}^+\frac{3}{2}\rangle$, where the subscripts on the bracket are total spin quantum numbers, but do not include relative orbital angular momentum. The overlap integral for this decay is obtained by replacing the pion spin wave function $\sqrt{1/2}(\uparrow\downarrow-\downarrow\uparrow)$ by $\uparrow\uparrow$, $\sqrt{2/3}(-w_+\downarrow-w_z\uparrow)$ by $-w_+\uparrow$, and $\beta_\pi$ by $\beta_\rho$ in Eq. (C5) to give

$$\begin{aligned}\left\langle[(D+D^*)\rho]_{3/2^+3/2}\middle|c\overline{q}q\overline{d};\frac{3}{2}^+\frac{3}{2}\right\rangle &= -\sqrt{3}I^{00}_{+-}\\ &= -\sqrt{3}\left[2c^{00}+c^{00}_{ij}\frac{q_+q_-}{\beta_v^2}\right]I^{00}e^{-q^2/8\tilde{\beta}^2}\\ &= -\sqrt{\frac{4}{3}}\sqrt{4\pi}\left(\left[3c^{00}+c^{00}_{ij}\frac{q^2}{\beta_v^2}\right]Y_{00}\right.\\ &\quad\left.-c^{00}_{ij}\frac{q^2}{\beta_v^2}\sqrt{\frac{1}{5}}Y_{20}\right)I^{00}e^{-q^2/8\tilde{\beta}^2}.\end{aligned} \tag{C14}$$

By examining the Clebsch-Gordan coefficients for coupling the spin state $[(D+D^*)\rho]_{3/2^+3/2}$ to a relative orbital $S$-wave or a $D$-wave to get a $\frac{3}{2}\frac{3}{2}$ state, we can deduce that the $S$- and $D$-wave amplitudes for this decay are $-\sqrt{4/3}[\sqrt{4\pi}[3c^{00}+c^{00}_{ij}(q^2/\beta_v^2)]I^{00}e^{-q^2/8\tilde{\beta}^2}]$ and $\sqrt{4/3}[\sqrt{4\pi}c^{00}_{ij}(q^2/\beta_v^2)I^{00}e^{-q^2/8\tilde{\beta}^2}]$, respectively. One very simple overlap integral thus gives two partial wave amplitudes simultaneously.

A complete set of results are given in the text in Table I in terms of the following basic integrals:

$$I^{00}_{ij}=I^{00}\left[c^{00}\delta_{ij}+c^{00}_{ij}\frac{q_iq_j}{\tilde{\beta}_v^2}\right]e^{-q^2/8\tilde{\beta}^2} \tag{C15}$$

$$\begin{aligned}I^{10}_{ijk}=I^{10}\Big[&c^{10}_iq_i\delta_{jk}+c^{10}_jq_j\delta_{ik}+c^{10}_kq_k\delta_{ij}\\ &+c^{10}_{ijk}\frac{q_iq_jq_k}{\tilde{\beta}^2_{v**}}\Big]e^{-q^2/8\tilde{\beta}^2_{**}}\end{aligned} \tag{C16}$$

$$\begin{aligned}I^{01}_{ijk}=I^{01}\Big[&c^{01}_iq_i\delta_{jk}+c^{01}_jq_j\delta_{ik}+c^{01}_kq_k\delta_{ij}\\ &+c^{01}_{ijk}\frac{q_iq_jq_k}{\tilde{\beta}^2_{va}}\Big]e^{-q^2/8\tilde{\beta}^2_a}\end{aligned} \tag{C17}$$

where $I^{00}_{ij}$ is specified in terms of

$$I^{00}=\frac{2\beta_D^{3/2}\beta_B^{3/2}\beta_\pi^{3/2}\beta_{ft}^{5/2}\tilde{\beta}_{pc}^{5/2}}{3\pi^{3/4}\tilde{\beta}_v^5\beta_w^5\beta_y^3} \tag{C18}$$

$$c^{00}=+4a\beta_w^2 \tag{C19}$$

$$c^{00}_{ij}=\left[\left(\frac{1}{2}-a-b\right)\left(\frac{\beta_B^2+2\beta_\pi^2}{\beta_B^2+\beta_\pi^2}\right)-\frac{a\beta_w^2}{\tilde{\beta}_v^2}\left(\frac{1}{2}-a-b\right)^2\right] \tag{C20}$$

$$\begin{aligned}\tilde{\beta}_v^2=\tilde{\beta}_{pc}^2&+a^2(\beta_D^2+\beta_\pi^2+\beta_{ft}^2)+b^2(\beta_B^2+\beta_\pi^2)\\ &+\frac{1}{4}(\beta_D^2+\beta_\pi^2)+a\beta_D^2+(b-2ab-a)\beta_\pi^2\end{aligned} \tag{C21}$$

$$\beta_y^2=\beta_B^2+\beta_\pi^2 \tag{C22}$$

$$\beta_w^2=\beta_D^2+\beta_{ft}^2+\frac{\beta_B^2\beta_\pi^2}{\beta_B^2+\beta_\pi^2} \tag{C23}$$

$$a=-\frac{(\beta_B^2+\beta_\pi^2)(\beta_D^2-\beta_\pi^2)+\beta_\pi^4}{2[(\beta_B^2+\beta_\pi^2)(\beta_D^2+\beta_\pi^2+\beta_{ft}^2)-\beta_\pi^4]} \tag{C24}$$

$$b=-\frac{\beta_\pi^2(2\beta_D^2+\beta_{ft}^2)}{2[(\beta_B^2+\beta_\pi^2)(\beta_D^2+\beta_\pi^2+\beta_{ft}^2)-\beta_\pi^4]} \tag{C25}$$

$$\frac{1}{\tilde{\tilde{\beta}}^2}=\frac{1}{\beta_y^2}+\frac{1}{\beta_w^2}\left(\frac{\beta_B^2+2\beta_\pi^2}{\beta_B^2+\beta_\pi^2}\right)^2+\frac{1}{\tilde{\beta}_v^2}\left(\frac{1}{2}-a-b\right)^2. \tag{C26}$$

Similarly, $I^{10}_{ijk}$ is specified in terms of

$$I^{10}=-\frac{4i\beta_{D**}^{5/2}\beta_B^{3/2}\beta_\pi^{3/2}\beta_{ft}^{5/2}\tilde{\beta}_{pc}^{5/2}}{3\pi^{3/4}\tilde{\beta}^5_{v_{**}}\beta^5_{w_{**}}\beta^3_{y_{**}}} \tag{C27}$$

$$c_i^{10}=\left[+a_{**}\left(\frac{\beta_B^2+2\beta_\pi^2}{\beta_B^2+\beta_\pi^2}\right)-a_{**}\left(\frac{1}{2}+a_{**}\right)\times\left(\frac{1}{2}-a_{**}-b_{**}\right)\frac{\beta_{w_{**}}^2}{\tilde{\beta}_{v_{**}}^2}\right] \tag{C28}$$

$$c_j^{10}=\left[-\left(\frac{1}{2}-a_{**}-b_{**}\right)-a_{**}\left(\frac{1}{2}+a_{**}\right)\times\left(\frac{1}{2}-a_{**}-b_{**}\right)\frac{\beta_{w_{**}}^2}{\tilde{\beta}_{v_{**}}^2}\right] \tag{C29}$$

$$c_k^{10}=\left[+\left(\frac{1}{2}+a_{**}\right)\left(\frac{\beta_B^2+2\beta_\pi^2}{\beta_B^2+\beta_\pi^2}\right)-a_{**}\left(\frac{1}{2}+a_{**}\right)\times\left(\frac{1}{2}-a_{**}-b_{**}\right)\frac{\beta_{w_{**}}^2}{\tilde{\beta}_{v_{**}}^2}\right] \tag{C30}$$

$$c_{ijk}^{10}=+\frac{1}{4}\left[\left(\frac{1}{2}-a_{**}-b_{**}\right)\frac{\tilde{\beta}_{v_{**}}^2}{\beta_{w_{**}}^2}\left(\frac{\beta_B^2+2\beta_\pi^2}{\beta_B^2+\beta_\pi^2}\right)^2-\left(\frac{1}{2}+2a_{**}\right)\left(\frac{1}{2}-a_{**}-b_{**}\right)^2\left(\frac{\beta_B^2+2\beta_\pi^2}{\beta_B^2+\beta_\pi^2}\right)+a_{**}\left(\frac{1}{2}+a_{**}\right)\left(\frac{1}{2}-a_{**}-b_{**}\right)^3\frac{\beta_{w_{**}}^2}{\tilde{\beta}_{v_{**}}^2}\right] \tag{C31}$$

where $\tilde{\beta}_{v_{**}}^2$, $\beta_{y_{**}}^2$, $\beta_{w_{**}}^2$, $a_{**}$, $b_{**}$, and $1/\tilde{\tilde{\beta}}_{**}^2$ are given by the formulas for the $I_{ij}^{00}$ variables $\tilde{\beta}_v^2$, $\beta_y^2$, $\beta_w^2$, $a$, $b$, and $1/\tilde{\tilde{\beta}}^2$, respectively, with $\beta_D\to\beta_{D**}$ everywhere.

Finally, $I_{ijk}^{01}$ is specified in terms of

$$I^{01}=-\frac{4i\beta_D^{3/2}\beta_B^{3/2}\beta_a^{5/2}\beta_{ft}^{5/2}\tilde{\beta}_{pc}^{5/2}}{3\pi^{3/4}\tilde{\beta}_{v_a}^5\beta_{w_a}^5\beta_{y_a}^3} \tag{C32}$$

$$c_i^{01}=\left[+\frac{a_a\beta_{w_a}^2}{\tilde{\beta}_{v_a}^2}-\frac{a_a\beta_B^2}{\beta_B^2+\beta_a^2}\left(\frac{\beta_B^2+2\beta_a^2}{\beta_B^2+\beta_a^2}\right)-a_a\left(\frac{1}{2}-a_a+b_a\right)\times\left(\frac{1}{2}-a_a-b_a\right)\frac{\beta_{w_a}^2}{\tilde{\beta}_{v_a}^2}\right] \tag{C33}$$

$$c_j^{01}=\left[+\frac{\beta_B^2}{\beta_B^2+\beta_a^2}\left(\frac{1}{2}-a_a-b_a\right)-a_a\left(\frac{1}{2}-a_a+b_a\right)\times\left(\frac{1}{2}-a_a-b_a\right)\frac{\beta_{w_a}^2}{\tilde{\beta}_{v_a}^2}\right] \tag{C34}$$

$$c_k^{01}=\left[+\left(\frac{\beta_B^2+2\beta_a^2}{\beta_B^2+\beta_a^2}\right)\left(\frac{1}{2}-a_a+b_a\right)-a_a\left(\frac{1}{2}-a_a+b_a\right)\times\left(\frac{1}{2}-a_a-b_a\right)\frac{\beta_{w_a}^2}{\tilde{\beta}_{v_a}^2}\right] \tag{C35}$$

$$c_{ijk}^{01}=+\frac{1}{4}\left[+\left(\frac{1}{2}-a_a-b_a\right)\frac{\tilde{\beta}_{v_a}^2}{\beta_{y_a}^2}\left(\frac{\beta_B^2+2\beta_a^2}{\beta_B^2+\beta_a^2}\right)-\frac{a_a\beta_{w_a}^2}{\beta_{y_a}^2}\times\left(\frac{1}{2}-a_a-b_a\right)^2-\frac{\tilde{\beta}_{v_a}^2}{\beta_{w_a}^2}\left(\frac{\beta_B^2+2\beta_a^2}{\beta_B^2+\beta_a^2}\right)^2\frac{\beta_B^2}{\beta_B^2+\beta_a^2}\times\left(\frac{1}{2}-a_a-b_a\right)+\frac{a_a\beta_B^2}{\beta_B^2+\beta_a^2}\left(\frac{\beta_B^2+2\beta_a^2}{\beta_B^2+\beta_a^2}\right)\left(\frac{1}{2}-a_a-b_a\right)^2-\left(\frac{\beta_B^2+2\beta_a^2}{\beta_B^2+\beta_a^2}\right)\left(\frac{1}{2}-a_a+b_a\right)\left(\frac{1}{2}-a_a-b_a\right)^2+\frac{a_a\beta_{w_a}^2}{\tilde{\beta}_{v_a}^2}\left(\frac{1}{2}-a_a+b_a\right)\left(\frac{1}{2}-a_a-b_a\right)^3\right] \tag{C36}$$

where $\tilde{\beta}_{v_a}^2$, $\beta_{y_a}^2$, $\beta_{w_a}^2$, $a_a$, $b_a$, and $1/\tilde{\tilde{\beta}}_a^2$ are given by the formulas for the $I_{ij}^{00}$ variables $\tilde{\beta}_v^2$, $\beta_y^2$, $\beta_w^2$, $a$, $b$, and $1/\tilde{\tilde{\beta}}^2$, respectively, with $\beta_\pi\to\beta_a$ everywhere.


[1] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974); E. Witten, *ibid.* **B160**, 57 (1979).

[2] N. Isgur, D. Scora, B. Grinstein, and M. B.Wise, Phys. Rev. D **39**, 799 (1989); B. Grinstein, M.B. Wise, and N. Isgur, Caltech Report No. CALT-68-1311 and University of Toronto Report No. UTPT-85-37, 1985; B. Grinstein, M. B. Wise, and N. Isgur, Phys. Rev. Lett. **56**, 298 (1986).

[3] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995) present an updated version of the ISGW model of Ref. [2] called ISGW2.

[4] P. Geiger and N. Isgur, Phys. Rev. D **41**, 1595 (1990).

[5] P. Geiger and N. Isgur, Phys. Rev. D **44**, 799 (1991); Phys. Rev. Lett. **67**, 1066 (1991); Phys. Rev. D **47**, 5050 (1993); P. Geiger, *ibid.* **49**, 6003 (1993).

[6] P. Geiger and N. Isgur, Phys. Rev. D **55**, 299 (1997).

[7] F.E. Close (private communication) cogently advocates using

the term ''strong QCD'' instead of the negative ''nonperturbative QCD.''

[8] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990). For an overview of heavy quark symmetry, see N. Isgur and M.B. Wise, in *B Decays*, edited by S. Stone (World Scientific, Singapore, 1991), p. 158; and in *Heavy Flavors*, edited by A.J. Buras and M. Lindner (World Scientific, Singapore, 1992), p. 234.

[9] N. Isgur and M.B. Wise, Phys. Rev. Lett. **66**, 1130 (1991).

[10] See, e.g., J. Zeng, J.W. Van Orden, and W. Roberts, Phys. Rev. D **52**, 5229 (1995) and references therein.

[11] See, e.g., J.L. Goity and W. Roberts, Phys. Rev. D **51**, 3459 (1995) and references therein.

[12] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[13] N. Isgur, Phys. Rev. D **54**, 5896 (1996).

[14] A.F. Falk, M. Luke, and M.J. Savage, Phys. Rev. D **53**, 6316 (1996).

[15] Experiments report a wide range of values for $\rho^2$. Reference [12] gives a ''world average'' value of $\rho^2=0.7\pm0.1$ in excellent agreement with ISGW2 [3], but experiments quote inconsistent values from $\rho^2=0.4\pm0.2$ [ALEPH Collaboration, D. Buskuvic *et al.*, Phys. Lett. B **395**, 373 (1997)] to $\rho^2=1.2\pm0.2$ [ARGUS Collaboration, H. Albrecht *et al.*, Z. Phys. C **57**, 533 (1993)]. I will use the round number $\rho^2=1$, which is within one standard deviation of the most accurate reported measurement of $\rho^2=0.84\pm0.13\pm0.08$ [CLEO Collaboration, A. Anastassov *et al.* (CLEO), CLEO CONF 96-8 (1996)], but conveys the considerable uncertainty remaining in this quantity.

[16] J.D. Bjorken, in *Proceedings of the 4th Rencontre de Physique de la Vallee d'Aoste*, La Thuile, Italy, 1990, edited by M. Greco (Editions Frontières, Gif-sur-Yvette, France, 1990); J. D. Bjorken, *Recent Developments in Heavy Flavor Theory*, in Proceedings of the XXVth International Conference on High Energy Physics, Singapore (World Scientific, Singapore, 1992).

[17] N. Isgur and M.B. Wise, Phys. Rev. D **43**, 819 (1991).

[18] L. Wolfenstein, Phys. Rev. D **54**, 5894 (1996).

[19] For an overview of heavy quark effective theory, see the review cited in Ref. [4] and the references therein, especially H. Georgi, Phys. Lett. B **240**, 447 (1990); E. Eichten and B. Hill, *ibid.* **234**, 511 (1990); M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45**, 743 (1987) [Sov. J. Nucl. Phys. **45**, 463 (1987)]; H.D. Politzer and M.B. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988); A.F. Falk, H. Georgi, B. Grinstein, and M.B. Wise, Nucl. Phys. **B343**, 1 (1990); B. Grinstein, *ibid.* **B339**, 253 (1990); M.B. Wise, in *Particles and Fields 3: Proceedings of the Banff Summer Institute (CAP)*, 1988, edited by N. Kamal and F. Khanna (World Scientific, Singapore, 1989), p. 124.

[20] For a review of the adiabatic approximation in the context of the flux tube model (based on the results of Refs. [21] and [4–6]), see N. Isgur, in Proceedings of *Few Body Problems in Physics*, edited by F. Gross, AIP Conf. Proc. No. 334 (AIP, New York, 1995), p. 3. For some early discussions of incorporating the nonperturbative effects of $q\bar{q}$ loops (which were all in the context of $c\bar{c}$ spectroscopy and decays), see J. Kogut and L. Susskind, Phys. Rev. Lett. **34**, 767 (1975); E. Eichten *et al.*, *ibid.* **36**, 500 (1976); and E.C. Poggio and H.J. Schnitzer, *ibid.* **41**, 1344 (1978). These discussions are all different, and each introduces essential elements of the adiabatic approximation adopted in Refs. [21] and [4–6].

[21] N. Isgur and J. Paton, Phys. Rev. D **31**, 2910 (1985).

[22] N. Isgur, ''Beyond the Adiabatic Approximation: the impact of thresholds on the hadronic spectrum,'' JLAB-THY-99-01, nucl-th/9901032, 1999.

[23] S. Okubo, Phys. Lett. **5**, 165 (1963); Phys. Rev. D **16**, 2336 (1977); G. Zweig, CERN Report No. 8419 TH 412, 1964; reprinted in *Developments in the Quark Theory of Hadrons*, edited by D. B. Lichtenberg and S. P. Rosen (Hadronic Press, Massachusetts, 1980); J. Iizuka, K. Okada, and O. Shito, Prog. Theor. Phys. **35**, 1061 (1966); J. Iizuka, Prog. Theor. Phys. Suppl. **37-38**, 21 (1966).

[24] H.J. Lipkin, Nucl. Phys. **B291**, 720 (1987); Phys. Lett. B **179**, 278 (1986); Nucl. Phys. **B244**, 147 (1984); Phys. Lett. **124B**, 509 (1983).

[25] L. Micu, Nucl. Phys. **B10**, 521 (1969); A. Le Yaouanc, L. Oliver, O. Pene, and J.-C. Raynal, Phys. Rev. D **8**, 2233 (1973); Phys. Lett. **71B**, 397 (1977); **72B**, 57 (1977); W. Roberts and B. Silvestre-Brac, Few-Body Syst. **11**, 171 (1992); P. Geiger and E.S. Swanson, Phys. Rev. D **50**, 6855 (1994).

[26] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).

[27] M.E. Luke, Phys. Lett. **25B**, 447 (1990).

[28] J. Chay, H. Georgi, and B. Grinstein, Phys. Lett. B **247**, 399 (1990).

[29] I.I. Bigi, N.G. Uraltsev, and A.I. Vainshtein, Phys. Lett. B **293**, 430 (1992); **297**, 477(E) (1993); I.I. Bigi, M.A. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993); Int. J. Mod. Phys. A **9**, 2467 (1994); B. Blok, L. Koyrakh, M.A. Shifman, and A.I. Vainshtein, Phys. Rev. D **49**, 3356 (1994); **50**, 3572(E) (1994); I.I. Bigi and N.G. Uraltsev, Z. Phys. C **62**, 623 (1994); M.A. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Phys. Rev. D **51**, 2217 (1995); **51**, 3149(E) (1995); B. Chibisov, R.D. Dikeman, M.A. Shifman, and N.G. Uraltsev, Int. J. Mod. Phys. A **12**, 2075 (1997); R.D. Dikeman and N.G. Uraltsev, Nucl. Phys. **B509**, 378 (1998).

[30] N. Isgur, Phys. Lett. B **448**, 111 (1999).

[31] In the full flux tube model (in contradistinction to the quark potential model derived from it in the adiabatic approximation) the motion of the flux tube center-of-mass can make both longitudinal and transverse contributions to a hadron's charge radius. See N. Isgur, ''Flux Tube Zero-Point Motion, Hadronic Charge Radii, and Hybrid Meson Production Cross Sections,'' JLAB-THY-99-09, hep-ph/9904494, 1999 [Phys. Rev. D (to be published)].