# Quark part of the nonforward BFKL kernel and the ''bootstrap'' for the gluon Reggeization

V. S. Fadin*
*Budker Institute for Nuclear Physics, Novosibirsk State University, 630090 Novosibirsk, Russia*

R. Fiore† and A. Papa‡
*Dipartimento di Fisica, Università della Calabria, Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Cosenza, Arcavacata di Rende, I-87036 Cosenza, Italy*

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We calculate the quark part of the kernel of the generalized nonforward BFKL equation at nonzero momentum transfer $t$ in the next-to-leading logarithmic approximation. Along with the quark contribution to the gluon Regge trajectory, this part includes pieces coming from the quark-antiquark production and from the quark contribution to the radiative corrections in one-gluon production in Reggeon-Reggeon collisions. The results obtained can be used for an arbitrary representation of the color group in the $t$ channel. Using the results for the adjoint representation, we demonstrate explicitly the fulfillment of the ''bootstrap'' condition for the gluon Reggeization in the next-to-leading logarithmic approximation in the part concerning the quark contribution. [S0556-2821(99)02817-9]

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## I. INTRODUCTION

The Balitskiĭ-Fadin-Kuraev-Lipatov (BFKL) equation [1] became very popular in recent years due to the experimental results on deep inelastic scattering obtained at the DESY $ep$ collider HERA [2]. These results show that the power of the growth of the cross section of the photon-proton interaction with the energy for a ''hard'' photon (the ''hardness'' is supplied either by the photon virtuality or by the masses of the quarks into which the photon is converted) is larger than the corresponding power for hadron processes. The idea arose that the rapid increase of the cross section of the ''hard'' photon interactions is the manifestation of the BFKL dynamics.

The BFKL equation was derived in the leading logarithmic approximation (LLA) of the QCD perturbation theory, which means the summation of all terms of the type $[\alpha_s \ln s]^n$; $\alpha_s$ is the QCD coupling constant and $s$ is the square of the c.m.s. energy. Unfortunately, in this approximation neither the scale of $s$ nor the argument of the running coupling constant $\alpha_s$ are fixed. So, in order to do accurate theoretical predictions, we have to know the radiative corrections to the LLA. The program of the calculation of the radiative corrections was formulated in Ref. [3] and satisfied in Refs. [4–9]. Recently, the calculation of the radiative corrections to the kernel of the BFKL equation was completed and the equation in the next-to-leading logarithmic approximation (NLLA) was obtained [10,11]. The corrections appear to be large and caused a series of papers [12] devoted to the problem how to deal with them and what they mean.

The BFKL equation is a particular case (for forward scattering, i.e., $t=0$ and vacuum quantum numbers in the $t$ channel) of the equation for the $t$-channel partial waves of the elastic amplitudes [1]. Independently from the value of $t$, we have in general a mixture of various irreducible representations $\mathcal{R}$ of the color group in the $t$ channel. The most interesting representations are the color singlet (Pomeron channel) and the antisymmetric color octet (gluon channel). For brevity, we use the term ''BFKL equation'' for the general case as well, adding the word ''nonforward'' when it is necessary to distinguish the general case from the particular ''forward'' case.

It is very important to find the corrections to the kernel of the nonforward BFKL, for the gluon channel as well as for the Pomeron channel. In the case of the Pomeron channel the equation can be applied directly for the description of experimental data. The importance of the correction in the gluon channel is determined by a remarkable property of QCD, the gluon Reggeization. We remind that the derivation of the BFKL equation was based [1] on this property. In fact, this equation is the equation for the Green function of two Reggeized gluons. In the color singlet state these Reggeized gluons create the Pomeron. The self-consistency requires that in the color octet case the two Reggeized gluons reproduce the Reggeized gluon itself (''bootstrap'' condition). The above statements are valid in the NLLA as well as in LLA. The ''bootstrap'' equations in the NLLA were recently derived [13]. Since the BFKL equation is very important for the theory of Regge processes at high energy $\sqrt{s}$ in perturbative QCD, these equations must be checked. Along with the stringent test of the gluon Reggeization, this check has another important meaning. The calculations of the radiative corrections to the kernel are very complicated. Therefore, they should be carefully verified. Up to now, only a small part of the calculations was independently performed [8] or checked [14]. The bootstrap equations contain all the values appearing in the calculations of the NLLA kernel, so that they provide a global test of the calculations. Beside this, the color octet state of two Reggeized gluons is necessary for the description of colorless compound states of more than two gluons, in particular, for the odderon.

*Email address: FADIN@INP.NSK.SU
†Email address: FIORE@FIS.UNICAL.IT
‡Email address: PAPA@FIS.UNICAL.IT

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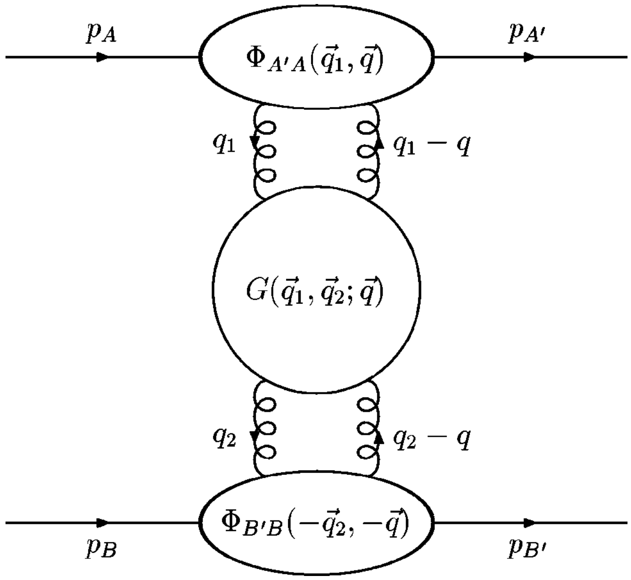


FIG. 1. Diagrammatic representation of the elastic scattering amplitude $A+B\to A'+B'$.

In this paper we consider the nonforward BFKL equation, calculate the quark contribution to the kernel of this equation and demonstrate explicitly the fulfillment of the ''bootstrap'' conditions in the NLLA in the part concerning the quark-antiquark contribution.

In the next section we present the general form of the quark contribution to the kernel. In Sec. III we give the explicit form of the quark piece of the gluon trajectory and derive the quark part of the contribution to the kernel from the one-gluon production. In Sec. IV we consider the quark-antiquark production in collisions of two Reggeized gluons. In Sec. V we obtain the contribution of this process to the kernel. In Sec. VI we demonstrate the fulfillment of the ''bootstrap'' condition for the trajectory. The results obtained are discussed in Sec. VII.

## II. DEFINITIONS AND BASIC EQUATIONS

As usual, in an analysis of processes at high energy particle collisions, we use the Sudakov decomposition for particle momenta. Admitting that the initial particles $A$ and $B$ have non-zero masses, we introduce the light-cone vectors $p_+$ and $p_-$ in terms of which the momenta of the initial particles are $p_A=p_+ +(m_A^2/s)p_-$ and $p_B=p_- +(m_B^2/s)p_+$ respectively, with $s=2(p_+p_-)$. In the NLLA, as well as in the LLA, the elastic scattering amplitudes with momentum transfer $q\simeq q_\perp$ are expressed in terms of the impact factors $\Phi$ of the scattering particles and of the Green function $G$ for the scattering of Reggeized gluons [13] (see Fig. 1). The Mellin transform of the Green function for Reggeized gluons with initial momenta in the $s$ channel $q_1\simeq\beta p_+ + q_{1\perp}$ and $-q_2\simeq\alpha p_- - q_{2\perp}$, momentum transfer $q\simeq q_\perp$ and irreducible representation $\mathcal{R}$ of the color group in the $t$ channel, obeys the equation [13]

$$\omega G_\omega^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q})=\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2\delta^{(D-2)}(\vec{q}_1-\vec{q}_2)+\int\frac{d^{D-2}r}{\vec{r}^{\,2}(\vec{r}-\vec{q})^2}\times\mathcal{K}^{(\mathcal{R})}(\vec{q}_1,\vec{r};\vec{q})G_\omega^{(\mathcal{R})}(\vec{r},\vec{q}_2;\vec{q}). \quad (1)$$

Here $\vec{q}_1$ and $-\vec{q}_2$ are the transverse momenta of the colliding gluons in the $s$ channel, $\vec{q}$ is the momentum transfer and $D=4+2\epsilon$ is the space-time dimension, taken different from 4 to regularize the infrared divergences. Let us note that we use a normalization which is different from the one used for the forward case [10]. The nonforward kernel, analogously to the forward one, is given as the sum of the ''virtual'' part, defined by the gluon trajectory $j(t)=1+\omega(t)$, and the ''real'' part $\mathcal{K}_r^{(\mathcal{R})}$, related to the real particle production in Reggeon-Reggeon collisions:

$$\mathcal{K}^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q})=[\omega(-\vec{q}_1^{\,2})+\omega(-(\vec{q}_1-\vec{q})^2)]\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2\times\delta^{(D-2)}(\vec{q}_1-\vec{q}_2)+\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q}). \quad (2)$$

The gluon trajectory is known [6] in the NLLA. The ''real'' part for the nonforward case is known in the LLA only [1]:

$$\mathcal{K}_r^{(\mathcal{R})B}(\vec{q}_1,\vec{q}_2;\vec{q})=\frac{g^2c_R}{(2\pi)^{D-1}}\times\left(\frac{\vec{q}_1^{\,2}(\vec{q}_2-\vec{q})^2+\vec{q}_2^{\,2}(\vec{q}_1-\vec{q})^2}{(\vec{q}_1-\vec{q}_2)^2}-\vec{q}^{\,2}\right), \quad (3)$$

where the superscript $B$ means the LLA (Born) approximation and the coefficients $c_R$ for the singlet $(R=1)$ and octet $(R=8)$ cases are

$$c_1=N, \quad c_8=\frac{N}{2}, \quad (4)$$

$N$ being the number of colors ($N=3$ in QCD). In Eq. (3) and below $g$ is the bare coupling constant, connected with the renormalized coupling $g_\mu$ in the modified minimal subtraction ($\overline{\text{MS}}$) scheme by the relation

$$g=g_\mu\mu^{-\epsilon}\left[1+\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{N}\right)\frac{\overline{g}_\mu^2}{2\epsilon}\right], \quad (5)$$

where

$$\overline{g}_\mu^2=\frac{g_\mu^2N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}. \quad (6)$$

Let us stress that in this paper we will systematically use the perturbative expansion in terms of the bare coupling $g$.

In the NLLA the ''real'' part of the kernel can be presented as [13]

$$\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q}\,)=\int\frac{ds_{RR}}{(2\pi)^D}\mathrm{Im}\,\mathcal{A}_{RR}^{(\mathcal{R})}(q_1,q_2;\vec{q}\,)\theta(s_\Lambda-s_{RR})-\frac{1}{2}\int\frac{d^{D-2}r}{\vec{r}^{\,2}(\vec{r}-\vec{q}\,)^2}\mathcal{K}_r^{(\mathcal{R})B}(\vec{q}_1,\vec{r};\vec{q}\,)\times\mathcal{K}_r^{(\mathcal{R})B}(\vec{r},\vec{q}_2;\vec{q}\,)\ln\left(\frac{s_\Lambda^2}{(\vec{r}-\vec{q}_1)^2(\vec{r}-\vec{q}_2)^2}\right).\tag{7}$$

Here $\mathcal{A}_{RR}^{(\mathcal{R})}(q_1,q_2;\vec{q}\,)$ is the scattering amplitude of the Reggeons with initial momenta $q_1=\beta p_+ + q_{1\perp}$ and $-q_2=\alpha p_- - q_{2\perp}$ and momentum transfer $q=q_\perp$, for the representation $\mathcal{R}$ of the color group in the $t$ channel, $s_{RR}=(q_1-q_2)^2$ is the squared invariant mass of the Reggeons. The $s_{RR}$-channel imaginary part $\mathrm{Im}\,\mathcal{A}_{RR}^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q}\,)$ is expressed in terms of the effective vertices for the production of particles in the Reggeon-Reggeon collisions [13]. The second term in the right-hand side (RHS) of Eq. (7) serves for the subtraction of the contribution of the large $s_{RR}$ region in the first term, in order to avoid a double counting of this region in the BFKL equation. The intermediate parameter $s_\Lambda$ in Eq. (7) must be taken tending to infinity. At large $s_{RR}$ only the contribution of the two-gluon production does survive in the first integral, so that the dependence on $s_\Lambda$ disappears in Eq. (7) because of the factorization property of the two-gluon production vertex [13]. Since we are interested here in the quark contribution only, we can omit the subtraction term and perform the integration over $s_{RR}$ up to infinity.

The imaginary part of the Reggeon-Reggeon scattering amplitudes entering Eq. (7) can be expressed in terms of the production vertices, with the help of the operators $\hat{\mathcal{P}}_\mathcal{R}$ for the projection of two-gluon color states in the $t$ channel on the irreducible representations $\mathcal{R}$ of the color group. We have [13]

$$\mathrm{Im}\,\mathcal{A}_{RR}^{(\mathcal{R})}(q_1,q_2;\vec{q}\,)=\frac{\langle c_1c_1'|\hat{\mathcal{P}}_\mathcal{R}|c_2c_2'\rangle}{2n_\mathcal{R}}\sum_{\{f\}}\int\gamma_{c_1c_2}^{\{f\}}(q_1,q_2)\times(\gamma_{c_1'c_2'}^{\{f\}}(q_1',q_2'))^* d\rho_f.\tag{8}$$

Here and below $q_i'=q_i-q,\ i=1,2;$ $n_\mathcal{R}$ is the number of independent states in the representation $\mathcal{R}$, $\gamma_{c_1c_2}^{\{f\}}(q_1,q_2)$ is the effective vertex for the production of particles $\{f\}$ in collisions of Reggeons with momenta $q_1,\ -q_2$ and color indices $c_1,\ c_2$ respectively, $d\rho_f$ is their phase space element,

$$d\rho_f=(2\pi)^D\delta^{(D)}\left(q_1-q_2-\sum_{\{f\}}l_f\right)\prod_{\{f\}}\frac{d^{D-1}l_f}{(2\pi)^{D-1}2\epsilon_f}.\tag{9}$$

The sum over $\{f\}$ in Eq. (8) is performed over all the contributing particles $\{f\}$ and over all their discreet quantum numbers. In the LLA only the one-gluon production does contribute and Eq. (7) gives for the kernel its LLA value (3); in the NLLA the contributing states include also the two-gluon and the quark-antiquark states. The normalization of the corresponding vertices is defined in Ref. [13].

The most interesting representations $\mathcal{R}$ are the color singlet (Pomeron channel) and the antisymmetric color octet (gluon channel). We have for the singlet case

$$\langle c_1c_1'|\hat{\mathcal{P}}_1|c_2c_2'\rangle=\frac{\delta_{c_1c_1'}\delta_{c_2c_2'}}{N^2-1},\quad n_1=1,\tag{10}$$

and for the octet case

$$\langle c_1c_1'|\hat{\mathcal{P}}_8|c_2c_2'\rangle=\frac{f_{c_1c_1'c}f_{c_2c_2'c}}{N},\quad n_8=N^2-1,\tag{11}$$

where $f_{abc}$ are the structure constants of the color group.

## III. QUARK PART OF THE KERNEL FROM THE GLUON TRAJECTORY AND REAL GLUON PRODUCTION

The gluon trajectory is known [6] in the NLLA. The quark contribution $\omega_Q(t)$ to the trajectory appears at the two-loop level only. For the case of $n_f$ massless quark flavors it can be written as

$$\omega_Q^{(2)}(t)=\frac{g^2t}{(2\pi)^{D-1}}\int\frac{d^{(D-2)}q_1}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q}\,)^2}[F_Q(\vec{q}\,)-2F_Q(\vec{q}_1)],\tag{12}$$

where $t=q^2=-\vec{q}^{\,2}$ and

$$F_Q(\vec{q}\,)=\frac{2g^2Nn_f\Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}\right)}{(4\pi)^{D/2}\Gamma(D)}(\vec{q}^{\,2})^{(D/2-2)}.\tag{13}$$

The quark contribution to the Reggeon-Reggeon-gluon (RRG) vertex was calculated in Ref. [5]. We remind that beyond the LLA the vertex has a complicated analytical structure [15,4], but in the NLLA only the real parts of the production amplitudes do contribute, because only these parts interfere with the LLA amplitudes, which are real. Neglecting the imaginary parts, the quark contribution to the RRG vertex becomes

$$\gamma_{c_1c_2}^{G(Q)}(q_1,q_2)=\varepsilon_G^d T_{c_1c_2}^d \frac{g^3 n_f \Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}\Gamma(D)} e_G^{*\mu}\left\{2C_\mu(q_2,q_1)[f_1^{(Q)}+f_2^{(Q)}] + \left(\frac{p_A}{(kp_A)}-\frac{p_B}{(kp_B)}\right)_\mu [f_3^{(Q)}-(2\vec{k}^2-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})f_2^{(Q)}]\right\}. \tag{14}$$

Here $\varepsilon_G^d$ and $e_G^{*\mu}$ are the polarization vectors of the produced gluon in the color and Minkowskii spaces, respectively, $T^d$ is the color group generator in the adjoint representation, $k=q_1-q_2$ is the gluon momentum,

$$C(q_2,q_1)=-q_{1\perp}-q_{2\perp}+(q_1-q_{1\perp})\left(1-\frac{2\vec{q}_1^{\,2}}{\vec{k}^2}\right)+(q_2-q_{2\perp})\left(1-\frac{2\vec{q}_2^{\,2}}{\vec{k}^2}\right) \tag{15}$$

and

$$f_1^{(Q)}=\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\left(\frac{D}{2}-1\right)^2\phi_0, \quad f_2^{(Q)}=\frac{\vec{k}^2}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\left[\left(\frac{D}{2}-2\right)\phi_2-\vec{q}_1^{\,2}\vec{q}_2^{\,2}\frac{D}{2}\phi_0\right],$$

$$f_3^{(Q)}=\frac{1}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\left[\vec{q}_1^{\,2}\vec{q}_2^{\,2}(D-2)^2\phi_0+\left(\frac{D}{2}-2\right)\vec{k}^2\phi_1\right], \tag{16}$$

where

$$\phi_n=[(\vec{q}_1^{\,2})^{n+\epsilon}-(\vec{q}_2^{\,2})^{n+\epsilon}]. \tag{17}$$

Using the expressions (14)–(16), with the help of Eqs. (8)–(11), we obtain from Eq. (7)

$$\mathcal{K}_{RRG}^{(\mathcal{R})Q}(\vec{q}_1,\vec{q}_2;\vec{q})=c_R\frac{g^4 n_f}{(2\pi)^{D-1}}\frac{\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{[\Gamma(1+\epsilon)]^2}{\Gamma(4+2\epsilon)}\left\{[\vec{k}^2(2\vec{k}^2-\vec{q}_1^{\,2}-\vec{q}_2^{\,2}-2\vec{q}_1^{\,\prime 2}-2\vec{q}_2^{\,\prime 2}+2\vec{q}^{\,2}) + (\vec{q}_1^{\,2}-\vec{q}_2^{\,2})(\vec{q}_1^{\,\prime 2}-\vec{q}_2^{\,\prime 2})]\frac{[2(1+\epsilon)\vec{q}_1^{\,2}\vec{q}_2^{\,2}\phi_0-\epsilon(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})\phi_1]}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}+\frac{(\vec{k}^2-\vec{q}_1^{\,\prime 2}-\vec{q}_2^{\,\prime 2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\epsilon\phi_1 + 2(1+\epsilon)^2\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime 2}}{\vec{k}^2}+\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\vec{q}^{\,2}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\right)\phi_0+(\vec{q}_1\leftrightarrow\vec{q}_1^{\,\prime},\vec{q}_2\leftrightarrow\vec{q}_2^{\,\prime})\right\}. \tag{18}$$

In the physical limit $\epsilon\to 0$ we have

$$\mathcal{K}_{RRG}^{(\mathcal{R})Q}(\vec{q}_1,\vec{q}_2;\vec{q})=-c_R\frac{g^4 n_f}{24(2\pi)^5}\left\{[\vec{k}^2(2\vec{k}^2-\vec{q}_1^{\,2}-\vec{q}_2^{\,2}-2\vec{q}_1^{\,\prime 2}-2\vec{q}_2^{\,\prime 2}+2\vec{q}^{\,2}) + (\vec{q}_1^{\,2}-\vec{q}_2^{\,2})(\vec{q}_1^{\,\prime 2}-\vec{q}_2^{\,\prime 2})]\frac{[2\vec{q}_1^{\,2}\vec{q}_2^{\,2}\ln(\vec{q}_1^{\,2}/\vec{q}_2^{\,2})-(\vec{q}_1^{\,4}-\vec{q}_2^{\,4})]}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}+(\vec{k}^2-\vec{q}_1^{\,\prime 2}-\vec{q}_2^{\,\prime 2}) + 2\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime 2}}{\vec{k}^2}+\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\vec{q}^{\,2}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+(\vec{q}_1\leftrightarrow\vec{q}_1^{\,\prime},\vec{q}_2\leftrightarrow\vec{q}_2^{\,\prime})\right\}. \tag{19}$$

A remarkable property of the kernel, which follows from the gauge invariance of the theory, is that it vanishes when any of the vectors $\vec{q}_i$ or $\vec{q}_i^{\,\prime}$ tends to zero. One can check that this property is fulfilled in Eqs. (18) and (19).

## IV. THE QUARK-ANTIQUARK PRODUCTION IN THE REGGEON-REGGEON COLLISIONS

Let us consider the production of a quark and an antiquark with momenta $l_1$ and $l_2$ respectively, in collisions of two

Reggeons with momenta $q_1$ and $-q_2$. We will use the Sudakov parametrization for the produced quark and antiquark momenta $l_1$ and $l_2$:

$$l_i=\beta_i p_A+\alpha_i p_B+l_{i\perp}, \quad s\alpha_i\beta_i=-l_{i\perp}^2 \;=\vec{l}_i^{\,2}, \quad i=1,2,$$

$$\beta_1+\beta_2=\beta, \quad \alpha_1+\alpha_2=\alpha, \quad l_{1\perp}+l_{2\perp}=q_{1\perp}-q_{2\perp}, \tag{20}$$

and the denotation

$$k=l_1+l_2=q_1-q_2, \quad s_{RR}=k^2. \tag{21}$$

For the effective vertex of the quark-antiquark production in the Reggeon-Reggeon collision we have [9]

$$\gamma_{c_1c_2}^{Q\bar{Q}}(q_1,q_2)=\frac{1}{2}g^2\bar{u}(l_1)[t^{c_1}t^{c_2}b(l_1,l_2) - t^{c_2}t^{c_1}\overline{b(l_2,l_1)}]v(l_2), \tag{22}$$

where $t^c$ are the color group generators in the fundamental representation. The expressions for $b(l_1,l_2)$ and $\overline{b(l_2,l_1)}$ can be presented in the following way:

$$b(l_1,l_2)=\frac{4\not{p}_A\not{Q}_1\not{p}_B}{s\tilde{t}_1}-\frac{1}{k^2}\not{\Gamma} \tag{23}$$

and

$$\overline{b(l_2,l_1)}=\frac{4\not{p}_B\not{Q}_2\not{p}_A}{s\tilde{t}_2}-\frac{1}{k^2}\not{\Gamma}, \tag{24}$$

where

$$\tilde{t}_1=(q_1-l_1)^2, \quad \tilde{t}_2=(q_1-l_2)^2,$$

$$Q_1=q_{1\perp}-l_{1\perp}, \quad Q_2=q_{1\perp}-l_{2\perp},$$

$$\Gamma=2\left[(q_1+q_2)_\perp-\beta p_A\left(1-2\frac{\vec{q}_1^{\,2}}{s\alpha\beta}\right)+\alpha p_B\left(1-2\frac{\vec{q}_2^{\,2}}{s\alpha\beta}\right)\right]. \tag{25}$$

According to Eqs. (7) and (8), the quark-antiquark contribution to the BFKL kernel can be presented in the form

$$\mathcal{K}_{RRQ\bar{Q}}^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q})=\frac{\langle c_1c_1'|\hat{\mathcal{P}}_{\mathcal{R}}|c_2c_2'\rangle}{2n_{\mathcal{R}}}\frac{1}{(2\pi)^D}\times\sum_{Q\bar{Q}}\int\gamma_{c_1c_2}^{Q\bar{Q}}(q_1,q_2)\times(\gamma_{c_1'c_2'}^{Q\bar{Q}}(q_1',q_2'))^* ds_{RR}d\rho_f, \tag{26}$$

where the sum is taken over spin, color and flavor states of the produced pair, $q_i'=q_i-q$, $s_{RR}=(q_1-q_2)^2$ is the squared invariant mass of the two Reggeons and the element $d\rho_f$ of the phase space is given by Eq. (9). The $\theta$-function in Eq. (7) is omitted since the integral over $s_{RR}$ is convergent at infinity. From the representation (22) we obtain

$$\frac{\langle c_1c_1'|\hat{\mathcal{P}}_{\mathcal{R}}|c_2c_2'\rangle}{2n_{\mathcal{R}}}\sum_{Q\bar{Q}}\gamma_{c_1c_2}^{Q\bar{Q}}(q_1,q_2)(\gamma_{c_1'c_2'}^{Q\bar{Q}}(q_1',q_2'))^* =\frac{g^4n_f}{32N}[a_RA+b_RB+(l_1\leftrightarrow l_2)]. \tag{27}$$

Here $n_f$ is the number of light quark flavors, the coefficients $a_R$ and $b_R$ for the interesting cases of singlet and octet representations are

$$a_0=N^2-1, \quad b_0=1; \quad a_8=\frac{N^2}{2}, \quad b_8=0 \tag{28}$$

and

$$A=\mathrm{tr}(\not{l}_1b(l_1,l_2)\not{l}_2\overline{b'(l_1,l_2)}),$$

$$B=\mathrm{tr}(\not{l}_1b(l_1,l_2)\not{l}_2b'(l_2,l_1)), \tag{29}$$

where $b'(l_1,l_2)$ is obtained from $b(l_1,l_2)$ by the substitution $q_{1,2}\to q_{1,2}'\equiv q_{1,2}-q$. In the following, for reasons evident from Eq. (28), we will call $A$ and $(B-A)$ ''non-Abelian'' and ''Abelian'' parts, respectively.

The calculation of the traces gives

$$A=32\frac{s\alpha_1\beta_2}{\tilde{t}_1\tilde{t}_1'}(\vec{Q}_1\vec{Q}_1')+8-\frac{16}{k^2}\left(\vec{q}^{\,2}-\frac{\vec{q}_1^{\,2}\vec{q}_2'^{\,2}+\vec{q}_2^{\,2}\vec{q}_1'^{\,2}}{s\alpha\beta}\right)-\frac{8}{(k^2)^2}\left[s\alpha\beta_1-s\alpha_1\beta+2\vec{q}_1^{\,2}\frac{\alpha_1}{\alpha}-2\vec{q}_2^{\,2}\frac{\beta_1}{\beta}-2\vec{l}_1(\vec{q}_1+\vec{q}_2)\right]$$
$$\times\left[s\alpha\beta_1-s\alpha_1\beta+2\vec{q}_1'^{\,2}\frac{\alpha_1}{\alpha}-2\vec{q}_2'^{\,2}\frac{\beta_1}{\beta}-2\vec{l}_1(\vec{q}_1'+\vec{q}_2')\right]+\frac{16}{k^2}\left\{\left[\frac{1}{\tilde{t}_1}\left(\left[(\vec{Q}_1\vec{q}_1')(s\alpha_1\beta_2-\vec{l}_1\vec{l}_2)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.+(\vec{Q}_1\vec{l}_2)\left(s\alpha_1\beta+\vec{l}_1(\vec{q}_1'+\vec{q}_2')-2\vec{q}_1'^{\,2}\frac{\alpha_1}{\alpha}\right)\right]+(\vec{l}_1\leftrightarrow\vec{l}_2,\alpha_1\leftrightarrow\beta_2,\alpha_2\leftrightarrow\beta_1,\vec{q}_1\leftrightarrow-\vec{q}_2,\vec{q}_1'\leftrightarrow-\vec{q}_2')\right)\right]+[\vec{q}_i\leftrightarrow\vec{q}_i']\right\}, \tag{30}$$

$$(A-B)+(\vec{l}_1\leftrightarrow\vec{l}_2)=32\left\{\frac{s\alpha_1\beta_2}{\tilde{t}_1\tilde{t}_1'}(\vec{Q}_1\vec{Q}_1')-\frac{(\vec{l}_1\vec{Q}_1)(\vec{l}_2\vec{Q}_2')+(\vec{l}_2\vec{Q}_1)(\vec{l}_1\vec{Q}_2')-(\vec{l}_1\vec{l}_2)(\vec{Q}_1\vec{Q}_2')}{\tilde{t}_1\tilde{t}_2'}\right\}+\{\vec{l}_1\leftrightarrow\vec{l}_2\}. \tag{31}$$

The Abelian and non-Abelian parts possess a ''nice'' behavior at large transverse momenta of the produced particles and at large values of their invariant mass, that guarantees the convergence of the integral in Eq. (26) at $\vec{l}_i^{\,2}\to\infty$ and $s_{RR}\to\infty$. The only region which leads to a singularity at the physical dimension $D=4$ is the infrared region $k^2\to 0$. This singularity is regularized by the nonzero $\epsilon=D/2-2$. To make the discussed behavior explicit, one has to take into account the relations (20) between longitudinal and transverse variables. The functions $A$ and $B$ can be expressed through the transverse momenta and one ratio of the longitudinal momenta. Choosing this ratio as

$$x=\frac{\beta_1}{\beta}, \tag{32}$$

we have

$$\frac{\beta_2}{\beta}=1-x, \quad \frac{\alpha_1}{\alpha}=\frac{(1-x)\vec{l}_1^{\,2}}{\Sigma}, \quad \frac{\alpha_2}{\alpha}=\frac{x\vec{l}_2^{\,2}}{\Sigma},$$

$$s\alpha\beta=\frac{\Sigma}{x(1-x)}, \quad s_{RR}=k^2=\frac{\vec{\Lambda}^2}{x(1-x)}, \tag{33}$$

where

$$\vec{\Lambda}=(1-x)\vec{l}_1-x\vec{l}_2, \quad \Sigma=\vec{\Lambda}^2+x(1-x)\vec{k}^2, \tag{34}$$

and

$$\tilde{t}_1=-\frac{\vec{l}_1^{\,2}}{x}-\vec{q}_1^{\,2}+2(\vec{l}_1\vec{q}_1), \quad \tilde{t}_2=-\frac{\vec{l}_2^{\,2}}{1-x}-\vec{q}_1^{\,2}+2(\vec{l}_2\vec{q}_1),$$

$$\tilde{t}_1'=-\frac{\vec{l}_1^{\,2}}{x}-\vec{q}_1^{\,\prime 2}+2(\vec{l}_1\vec{q}_1^{\,\prime}), \quad \tilde{t}_2'=-\frac{\vec{l}_2^{\,2}}{1-x}-\vec{q}_1^{\,\prime 2}+2(\vec{l}_2\vec{q}_1^{\,\prime}). \tag{35}$$

Using these relations, we obtain

$$\begin{aligned}
A=16x(1-x)\Bigg\{&-x(1-x)\left(2\frac{(\vec{\Lambda}\vec{q}_1)}{\vec{\Lambda}^2}+\frac{2(\vec{q}_1\vec{l}_1)-\vec{q}_1^{\,2}}{x\tilde{t}_1}\right)\left(2\frac{(\vec{\Lambda}\vec{q}_1^{\,\prime})}{\vec{\Lambda}^2}+\frac{2(\vec{q}_1^{\,\prime}\vec{l}_1)-\vec{q}_1^{\,\prime 2}}{x\tilde{t}_1'}\right)\\
&-\frac{\vec{q}^{\,2}}{2}\left(\frac{1}{\vec{\Lambda}^2}+\frac{1}{x\tilde{t}_1}+\frac{2(\vec{q}_1\vec{l}_1)-\vec{q}_1^{\,2}}{x\tilde{t}_1\tilde{t}_1'}\right)+\frac{2(\vec{\Lambda}\vec{k})}{\vec{\Lambda}^2}\,\frac{(\vec{q}_1\vec{q}_1^{\,\prime})-\vec{q}_1^{\,\prime 2}}{\tilde{t}_1}+\frac{2(\vec{\Lambda}\vec{q}_1)(\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime})}{\vec{\Lambda}^2}\left(\frac{1}{\tilde{t}_1}-\frac{1}{\tilde{t}_1'}\right)\\
&+\frac{\vec{q}_1^{\,\prime 2}}{\Sigma}\left(4x(1-x)\frac{(\vec{\Lambda}\vec{q}_1)}{\vec{\Lambda}^2}+\frac{2(1-x)\big(2(\vec{q}_1\vec{l}_1)-\vec{q}_1^{\,2}\big)+\vec{q}_2^{\,2}}{\tilde{t}_1}\right)\left(1-2x+2x(1-x)\frac{(\vec{\Lambda}\vec{k})}{\vec{\Lambda}^2}\right)\\
&+x(1-x)\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}}{\vec{\Lambda}^2\Sigma}-x(1-x)\frac{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime 2}}{\Sigma^2}\left(1-2x+2x(1-x)\frac{(\vec{\Lambda}\vec{k})}{\vec{\Lambda}^2}\right)^2\Bigg\}+\{\vec{q}_i\leftrightarrow\vec{q}_i^{\,\prime}\},
\end{aligned} \tag{36}$$

$$\begin{aligned}
(A-B)+(l_1\leftrightarrow l_2)=16x(1-x)\Bigg\{&-x(1-x)\left(\frac{2(\vec{q}_1\vec{l}_1)-\vec{q}_1^{\,2}}{x\tilde{t}_1}+\frac{2(\vec{q}_1\vec{l}_2)-\vec{q}_1^{\,2}}{(1-x)\tilde{t}_2}\right)\left(\frac{2(\vec{q}_1^{\,\prime}\vec{l}_1)-\vec{q}_1^{\,\prime 2}}{x\tilde{t}_1'}+\frac{2(\vec{q}_1^{\,\prime}\vec{l}_2)-\vec{q}_1^{\,\prime 2}}{(1-x)\tilde{t}_2'}\right)\\
&+\frac{\vec{q}^{\,2}\big(2(\vec{q}_1\vec{l}_1)-\vec{q}_1^{\,2}\big)}{2\tilde{t}_1}\left(\frac{1}{(1-x)\tilde{t}_2'}-\frac{1}{x\tilde{t}_1'}\right)+\frac{\vec{q}^{\,2}\big(2(\vec{q}_1^{\,\prime}\vec{l}_1)-\vec{q}_1^{\,\prime 2}\big)}{2\tilde{t}_1'}\left(\frac{1}{(1-x)\tilde{t}_2}-\frac{1}{x\tilde{t}_1}\right)\\
&+\frac{1}{x(1-x)\tilde{t}_1\tilde{t}_2'}\left(-2(\vec{q}_1\vec{l}_1)(\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime})+2(\vec{q}_1^{\,\prime}\vec{l}_1)(\vec{q}_1\vec{q}_2)+(\vec{q}_1^{\,\prime 2}-\vec{q}_1^{\,2})(\vec{l}_1\vec{k})+\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}-\frac{\vec{k}^2\vec{q}^{\,2}}{2}\right)\Bigg\}+\{l_1\leftrightarrow l_2\}.
\end{aligned} \tag{37}$$

It is easy to see from Eqs. (36) and (37) that the integrand in Eq. (26) falls down as $|\vec{l}_i|^4$ when $|\vec{l}_i|\to\infty$. Taking into account that

$$\int \frac{dk^2}{(2\pi)} d\rho_f = \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}, \tag{38}$$

we see that the integration is well convergent at large $|\vec{l}_i|$. As for the regions of values of $x$ close to 0 or 1 [which correspond to large invariant masses $s_{RR}$—see Eqs. (33)], the convergence of the integration is guaranteed by the vanishing of the functions $A$ and $B$ as $x(1-x)$, as it is evident from Eqs. (36) and (37). The limit $k^2\to 0$ means $\vec{\Lambda}^2\to 0$, according to Eq. (33). The Abelian part is regular in this limit, as it can be seen from Eq. (37). As for the non-Abelian part, from Eq. (36) we have for its singular part

$$A_{sing} = 16x(1-x)\left\{ -8x(1-x)\left( \frac{(\vec{\Lambda}\vec{q}_1)}{\vec{\Lambda}^2} - \frac{\vec{q}_1^{\,2}(\vec{\Lambda}\vec{k})}{\vec{\Lambda}^2\vec{k}^2} \right)\left( \frac{(\vec{\Lambda}\vec{q}_1')}{\vec{\Lambda}^2} - \frac{\vec{q}_1'^{\,2}(\vec{\Lambda}\vec{k})}{\vec{\Lambda}^2\vec{k}^2} \right) - \frac{\vec{q}^{\,2}}{\vec{\Lambda}^2} + \frac{\vec{q}_1^{\,2}\vec{q}_2'^{\,2} + \vec{q}_2^{\,2}\vec{q}_1'^{\,2}}{\vec{\Lambda}^2\vec{k}^2} \right\}. \tag{39}$$

## V. THE QUARK CONTRIBUTION TO THE KERNEL

To obtain the contribution to the kernel $\mathcal{K}^{(\mathcal{R})}_{RRQ\bar{Q}}(\vec{q}_1,\vec{q}_2;\vec{q})$ from the real quark-antiquark production, we have to perform the integration in Eq. (26). Though the expressions (36) and (37) are convenient for analyzing the behavior of the functions $A$ and $B$, they are not suitable for the integration with the measure given in Eq. (38). For this purpose, it is better to use the representations (30) and (31) which are explicitly invariant under the ''left-right'' transformation

$$\vec{l}_1 \leftrightarrow \vec{l}_2, \quad \alpha_1 \leftrightarrow \beta_2, \quad \alpha_2 \leftrightarrow \beta_1, \quad \vec{q}_1 \leftrightarrow -\vec{q}_2, \quad \vec{q}_1{}' \leftrightarrow -\vec{q}_2{}', \tag{40}$$

and to exploit also the integration measure in the alternative form:

$$\int \frac{dk^2}{(2\pi)} d\rho_f = \int_0^1 \frac{dy}{2y(1-y)} \int \frac{d^{D-2}l_2}{(2\pi)^{(D-1)}}, \tag{41}$$

where

$$y = \frac{\alpha_2}{\alpha}. \tag{42}$$

The details of the integration are presented in the Appendix. The integration for the non-Abelian contribution can be performed for arbitrary space-time dimension. The result is

$$\begin{aligned}\int \frac{dk^2}{(2\pi)} d\rho_f A = {} & \frac{16}{(4\pi)^{2+\epsilon}} \frac{\Gamma(1-\epsilon)}{\epsilon} \frac{[\Gamma(1+\epsilon)]^2}{\Gamma(4+2\epsilon)} \left\{ 2(1+\epsilon)^2 \left( \frac{(\vec{k}^2)^\epsilon}{\vec{k}^2} (\vec{q}_1^{\,2}\vec{q}_2'^{\,2} + \vec{q}_2^{\,2}\vec{q}_1'^{\,2}) + (\vec{q}^{\,2})^{1+\epsilon} \right) \right. \\ & + [\vec{k}^2(2\vec{k}^2 - \vec{q}_1^{\,2} - \vec{q}_2^{\,2} - 2\vec{q}_1'^{\,2} - 2\vec{q}_2'^{\,2} + 2\vec{q}^{\,2}) + (\vec{q}_1^{\,2} - \vec{q}_2^{\,2})(\vec{q}_1'^{\,2} - \vec{q}_2'^{\,2})] \frac{[2(1+\epsilon)\vec{q}_1^{\,2}\vec{q}_2^{\,2}\phi_0 - \epsilon(\vec{q}_1^{\,2} + \vec{q}_2^{\,2})\phi_1]}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})^3} \\ & \left. + \frac{\epsilon(\vec{k}^2 - \vec{q}_1'^{\,2} - \vec{q}_2'^{\,2}) - 4(1+\epsilon)^2\vec{q}^{\,2}}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})} \phi_1 + 4(1+\epsilon)^2 \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})} \phi_0 + (\vec{q}_1 \leftrightarrow \vec{q}_1', \quad \vec{q}_2 \leftrightarrow \vec{q}_2') \right\},\end{aligned} \tag{43}$$

where the functions $\phi_n$ are given in Eq. (17). Considering the physical limit $\epsilon\to 0$, we must take into account the subsequent integration of the kernel over $\vec{k}$, which leads to contributions $\sim\epsilon^{-1}$ from the terms having the singularity at $\vec{k}^2=0$. Conserving all the terms giving nonzero contributions in the limit $\epsilon\to 0$ after integration over $\vec{k}$, the result (43) in this limit reads

$$\int \frac{dk^2}{(2\pi)} d\rho_f A = \frac{2}{3(2\pi)^2}\Bigg\{ -\frac{12}{(4\pi)^\epsilon}\Gamma(-\epsilon)\frac{[\Gamma(2+\epsilon)]^2}{\Gamma(4+2\epsilon)}\left(\frac{(\vec{k}^2)^\epsilon}{\vec{k}^2}(\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}+\vec{q}_2^{\,2}\vec{q}_1^{\,\prime 2})-(\vec{q}^{\,2})^{1+\epsilon}\right)$$
$$+[\vec{k}^2(2\vec{k}^2-\vec{q}_1^{\,2}-\vec{q}_2^{\,2}-2\vec{q}_1^{\,\prime 2}-2\vec{q}_2^{\,\prime 2}+2\vec{q}^{\,2})+(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})(\vec{q}_1^{\,\prime 2}-\vec{q}_2^{\,\prime 2})]\frac{[2\vec{q}_1^{\,2}\vec{q}_2^{\,2}\ln(\vec{q}_1^{\,2}/\vec{q}_2^{\,2})-(\vec{q}_1^{\,4}-\vec{q}_2^{\,4})]}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}$$
$$+(\vec{k}^2-\vec{q}_1^{\,\prime 2}-\vec{q}_2^{\,\prime 2})+2\left(\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\vec{q}^{\,2}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-2\vec{q}^{\,2}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}^{\,4}}\right)+(\vec{q}_1\leftrightarrow\vec{q}_1^{\,\prime},\vec{q}_2\leftrightarrow\vec{q}_2^{\,\prime})\Bigg\}. \tag{44}$$

The Abelian contribution is not singular at all, so we can consider it from the beginning in the physical space-time dimension, $\epsilon=0$. Nevertheless, this contribution has a form much more complicated than the non-Abelian one. Evidently, the circumstance that the latter contribution is simpler must be related to the special role played by the gluon channel in presence of gluon Reggeization. In fact, the Abelian contribution was calculated many years ago [16] in the framework of quantum electrodynamics and we can use the results obtained there. We have

$$\int \frac{dk^2}{(2\pi)} d\rho_f(A-B+l_1\leftrightarrow l_2) = \frac{32}{(2\pi)^2}K_1\left(\vec{q}_1-\frac{\vec{q}}{2},\vec{q}_2-\frac{\vec{q}}{2}\right), \tag{45}$$

with the function $K_1$ given by Eq. (A39) of Ref. [16], where in the RHS we have to make the substitutions

$$\vec{q}\rightarrow\vec{q}_1-\frac{\vec{q}}{2},\quad \vec{q}^{\,\prime}\rightarrow\vec{q}_2-\frac{\vec{q}}{2},\quad \vec{r}\rightarrow\frac{\vec{q}}{2},\quad \vec{Q}\rightarrow\vec{q}_1-\vec{q}(1-y),\quad \vec{Q}^{\,\prime}\rightarrow\vec{q}_2-\vec{q}(1-x).$$

It is worthwhile to say that Eq. (45) contains a nonzero fermion mass and, at first sight, has a logarithmic singularity when the mass tends to zero; but the singularity is spurious because of cancellations among various terms.

We can now consider the quark contribution $\mathcal{K}_r^{(\mathcal{R})Q}(\vec{q}_1,\vec{q}_2;\vec{q})$ to the ''real'' part of the nonforward kernel of the BFKL equation. It was explained already that in the NLLA this contribution is determined by the quark correction to the one-gluon production and by the quark-antiquark production in the Reggeon-Reggeon collisions:

$$\mathcal{K}_r^{(\mathcal{R})Q}(\vec{q}_1,\vec{q}_2;\vec{q}) = \mathcal{K}_{RRG}^{(\mathcal{R})Q}(\vec{q}_1,\vec{q}_2;\vec{q}) + \mathcal{K}_{RRQ\bar{Q}}^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q}). \tag{46}$$

The first term in the RHS of this equation is given by Eqs. (4) and (17)–(19); the second, by Eqs. (26)–(29) and (43)–(45). For the octet case, as it can be seen from Eqs. (26)–(29), the contribution from the quark-antiquark production contains only the non-Abelian part, which was calculated for arbitrary space-time dimension. Since the quark correction to the one-gluon production was also calculated for arbitrary $D$, the quark contribution to the ''real'' part of the kernel in the gluon channel for arbitrary $\epsilon$ turns out to be

$$\mathcal{K}_r^{(8)Q}(\vec{q}_1,\vec{q}_2;\vec{q}) = g^4 n_f N\frac{1}{(2\pi)^{D-1}}\frac{1}{(4\pi)^{2+\epsilon}}\frac{\Gamma(1-\epsilon)}{\epsilon}\frac{[\Gamma(2+\epsilon)]^2}{\Gamma(4+2\epsilon)}\Bigg\{\frac{(\vec{k}^2)^\epsilon}{\vec{k}^2}(\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}+\vec{q}_2^{\,2}\vec{q}_1^{\,\prime 2})$$
$$+\vec{q}^{\,2}((\vec{q}^{\,2})^\epsilon-(\vec{q}_1^{\,2})^\epsilon+(\vec{q}_2^{\,2})^\epsilon)-\frac{(\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime 2})}{\vec{k}^2}((\vec{q}_1^{\,2})^\epsilon-(\vec{q}_2^{\,2})^\epsilon)+(\vec{q}_1\leftrightarrow\vec{q}_1^{\,\prime},\vec{q}_2\leftrightarrow\vec{q}_2^{\,\prime})\Bigg\}. \tag{47}$$

It is easy to see that the expression in the curly brackets vanishes when any of the $\vec{q}_i$'s or $\vec{q}_i^{\,\prime}$'s tends to zero, as it should be. In the physical limit $\epsilon\rightarrow 0$, keeping all the terms giving nonzero contributions in this limit after integration over $\vec{k}$, we obtain

$$\mathcal{K}_r^{(8)Q}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{g^4 n_f N}{24(2\pi)^5}\Bigg\{-\frac{1}{\pi^\epsilon}\frac{6}{(4\pi)^{2\epsilon}}\Gamma(-\epsilon)\frac{[\Gamma(2+\epsilon)]^2}{\Gamma(4+2\epsilon)}\left[\frac{(\vec{k}^2)^\epsilon}{\vec{k}^2}(\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}+\vec{q}_2^{\,2}\vec{q}_1^{\,\prime 2})\right.$$
$$\left.+\vec{q}^{\,2}((\vec{q}^{\,2})^\epsilon-(\vec{q}_1^{\,2})^\epsilon-(\vec{q}_2^{\,2})^\epsilon)\right]-\frac{(\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime 2})}{\vec{k}^2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+(\vec{q}_1\leftrightarrow\vec{q}_1^{\,\prime},\vec{q}_2\leftrightarrow\vec{q}_2^{\,\prime})\Bigg\}. \tag{48}$$

The quark contribution to the ''real'' part of the kernel in the Pomeron channel, according to Eqs. (4) and (27)–(29) and (49), can be presented as

$$\mathcal{K}_r^{(1)Q}(\vec{q}_1,\vec{q}_2;\vec{q}\,)=2\mathcal{K}_r^{(8)Q}(\vec{q}_1,\vec{q}_2;\vec{q})-\frac{g^4 n_f}{(2\pi)^5 N}K_1\left(\vec{q}_1-\frac{\vec{q}}{2},\vec{q}_2-\frac{\vec{q}}{2}\right). \tag{49}$$

Let us mention the properties of the kernel:

$$\mathcal{K}_r^{(\mathcal{R})}(0,\vec{q}_2;\vec{q})=\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,0;\vec{q})=\mathcal{K}_r^{(\mathcal{R})}(\vec{q},\vec{q}_2;\vec{q})=\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q};\vec{q})=0,$$

which follow from the gauge invariance, and

$$\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q})=\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_2,\vec{q}_1;-\vec{q})=\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1{}',\vec{q}_2{}';-\vec{q}),$$

which follow from the symmetry of the imaginary part of the Reggeon-Reggeon scattering amplitude (8), entering the expression (7) for the ''real'' part of the kernel. Let us stress here that the above properties, descending from very general arguments, are valid also for the gluon part of the kernel; we omitted indeed the superscript $Q$ in the above equations.

Another important property of the kernel is its infrared finiteness at fixed $\vec{k}=\vec{q}_1-\vec{q}_2$. The $1/\epsilon$ singularity in Eq. (47) is the ultraviolet one and disappears when we expand the kernel in terms of the renormalized coupling. Indeed, in this case we have to add to the RHS of Eq. (47) the piece coming from the coupling constant renormalization [see Eqs. (5) and (6)] in the LLA kernel (3). For the expansion in terms of the renormalized coupling we obtain in the limit $\epsilon\to 0$

$$\mathcal{K}_r^{(8)Q}(\vec{q}_1,\vec{q}_2;\vec{q})_{renorm}=\frac{\bar{g}_\mu^4\mu^{-2\epsilon}n_f}{\pi^{1+\epsilon}\Gamma(1-\epsilon)N}\left\{\frac{1}{\epsilon}\left(\frac{2[\Gamma(2+\epsilon)]^2}{\Gamma(4+2\epsilon)}\left(\frac{\vec{k}^2}{\mu^2}\right)^{\epsilon}-\frac{1}{3}\right)\left[\frac{(\vec{q}_1^{\,2}\vec{q}_2{}'^2+\vec{q}_2^{\,2}\vec{q}_1{}'^2)}{\vec{k}^2}-\vec{q}^{\,2}\right]\right.$$
$$\left.+\frac{1}{3}\left[\vec{q}^{\,2}\ln\left(\frac{\vec{q}^{\,2}\vec{k}^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right)-\frac{(\vec{q}_1^{\,2}\vec{q}_2{}'^2-\vec{q}_2^{\,2}\vec{q}_1{}'^2)}{\vec{k}^2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right]+(\vec{q}_1\leftrightarrow\vec{q}_1{}',\vec{q}_2\leftrightarrow\vec{q}_2{}')\right\}. \tag{50}$$

As it can be seen from this expression, the $1/\epsilon$ singularity at fixed $\vec{k}^2$ disappear after expanding $(\vec{k}^2/\mu^2)^\epsilon$ in powers of $\epsilon$. The expansion is not performed here, since in the integral over $\vec{k}$ of Eq. (1) the region of small $\vec{k}^2$, for which $\epsilon\ln(\vec{k}^2/\mu^2)\sim 1$, does contribute. This region contributes to the integral with terms singular in $1/\epsilon$. For the vacuum channel, these terms cancel the singularity in the ''virtual'' contribution to the kernel, related to the gluon trajectory. The cancellation occurs quite similarly to the forward case, so it does not need a special treatment. For the octet case such cancellation is evidently absent because of the different coefficients (4) between vacuum and gluon channels. We observe, however, that the cancellation is recovered in the case of the colourless compound state of three Reggeized gluons, i.e., the odderon. In this case, indeed, the ''real'' part of the kernel involves the three combinations with a different pair of Reggeized gluons in the octet channel, while the ''virtual'' part of the kernel involves three gluon trajectories. The cancellation of the infrared singularities follows then quite simply from the singular part of Eq. (49).

## VI. THE CHECK OF THE ''BOOTSTRAP'' CONDITION

The ''bootstrap'' condition derived in Ref. [13] has the form

$$\frac{g^2Nt}{2(2\pi)^{D-1}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2}\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2-\vec{q})^2}\mathcal{K}^{(8)}(\vec{q}_1,\vec{q}_2;\vec{q})=\omega^{(1)}(t)(\omega^{(1)}(t)+\omega^{(2)}(t)). \tag{51}$$

Here $\mathcal{K}^{(8)}(\vec{q}_1,\vec{q}_2;\vec{q})$ is the kernel of the nonforward BFKL equation, $\omega(t)=\omega^{(1)}(t)+\omega^{(2)}(t)$ is the deviation of the gluon Regge trajectory from unity in the two-loop approximation and $t=-\vec{q}^{\,2}$. In the one-loop approximation (LLA) the trajectory

$$\omega^{(1)}(t)=\frac{g^2Nt}{2(2\pi)^{D-1}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2} \tag{52}$$

is purely gluonic. The quark contribution to the trajectory appears at the two-loop level (NLLA) and is given by Eqs. (12) and (13). The kernel $\mathcal{K}^{(8)}(\vec{q}_1,\vec{q}_2;\vec{q})$, according to Eq. (2), is expressed through the trajectory and the ''real'' part. The quark piece of the latter is given by Eq. (47). Using this equation together with Eqs. (12) and (13) for the quark contribution to the trajectory, we arrive at

$$\int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2-\vec{q})^2}\mathcal{K}^{(8)Q}(\vec{q}_1,\vec{q}_2;\vec{q}) = g^4 n_f N \frac{1}{(2\pi)^{D-1}}\frac{1}{(4\pi)^{2+\epsilon}}\frac{\Gamma(1-\epsilon)}{\epsilon}\frac{[\Gamma(2+\epsilon)]^2}{\Gamma(4+2\epsilon)}\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}} \times\left\{\frac{\vec{q}_1^{\,2}}{\vec{k}^2}((\vec{q}_1^{\,2})^\epsilon+(\vec{q}_1^{\,\prime 2})^\epsilon-(\vec{q}_2^{\,2})^\epsilon-(\vec{q}_2^{\,\prime 2})^\epsilon)+\frac{\vec{q}^{\,2}}{\vec{q}_2^{\,\prime 2}}((\vec{q}^{\,2})^\epsilon-(\vec{q}_1^{\,2})^\epsilon-(\vec{q}_2^{\,2})^\epsilon)+(\vec{q}_1\leftrightarrow\vec{q}_1^{\,\prime})\right\}, \tag{53}$$

where $\vec{k}=\vec{q}_1-\vec{q}_2$, $\vec{q}_1^{\,\prime}=\vec{q}_1-\vec{q}$ and $\vec{q}_2^{\,\prime}=\vec{q}_2-\vec{q}$. Putting the RHS expression into Eq. (51) and using Eqs. (52), (12) and (13), it is easy to verify that the ''bootstrap'' equation (51) is satisfied.

## VII. DISCUSSION

In this paper we have calculated the quark part of the kernel of the generalized nonforward BFKL equation at nonzero momentum transfer $t$ in the next-to-leading logarithmic approximation. Along with the quark contribution to the gluon Regge trajectory, which is the same as for the case of zero momentum transfer $t$ and therefore is known already, this part includes pieces coming from the quark contribution to the radiative corrections for the one-gluon production and from the quark-antiquark production in the Reggeon-Reggeon collisions. The results obtained can be used for an arbitrary representation of the color group in the $t$ channel.

For all such representations, the part of the kernel related with the real particle production is infrared finite, in the sense that there are no singularities at fixed transverse momentum $\vec{k}$ of the produced particles. The integration over $\vec{k}$ in the generalized BFKL equation leads to terms singular in the limit $\epsilon\to 0$. For the vacuum channel, these terms cancel the singularity in the ''virtual'' contribution related to the gluon trajectory. For the octet case such cancellation is evidently absent, although it is recovered in the case of the colourless compound state of three Reggeized gluons, i.e., for the odderon.

The kernel for the octet case enters the ''bootstrap'' equation for the gluon Reggeization in QCD. The fulfillment of this equation is necessary for the self-consistency of the derivation of the BFKL equation. We demonstrate explicitly the fulfillment of the ''bootstrap'' condition in the next-to-leading logarithmic approximation in the part concerning the quark contribution. The check performed serves simultaneously as a stringent examination of the correctness of the calculations of the quark contribution to the kernel of the BFKL equation.

Recently a paper by Braun and Vacca [17] appeared, devoted to the NLLA kernel of the nonforward BFKL equation in the octet case. In this paper the kernel was obtained using as a basis the bootstrap relation and a specific ansatz to solve it. In Ref. [17] the kernel was defined as the kernel of the equation for the amputated Reggeized gluon-target amplitude [see Eqs. (1)–(3) of Ref. [18] and Eq. (2) of Ref. [17]], while our kernel is the kernel for the nonamputated amplitude [see our Eq. (1)]. This implies that the relation between our kernel, $\mathcal{K}_r$, and their kernel, $V$, should be (in the denotations used in Ref. [17]) $\mathcal{K}_r(q,q_1,q_1')=q_1'^2 q_2'^2 V(q,q_1,q_1')$. With this correspondence our results disagree with those obtained in Ref. [17].

However, in a revised version of their paper [17] (see also Ref. [19]), which appeared after the present work was published as a report, the authors of Ref. [17] have added an Appendix where they use a different relation between their and our kernel [see Eq. (51) of Ref. [17]]. This relation follows from requiring that their ansatz for the kernel satisfies the correct symmetry properties. With this new correspondence there is indeed agreement between our results and theirs in the case of the quark contribution to the kernel. It would be interesting to check if the symmetrized ansatz leads to the correct results also for the gluon contribution to the kernel.

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## APPENDIX

In this Appendix we present the details of the calculation of

$$\int \frac{dk^2}{(2\pi)}d\rho_f A=\int_0^1\frac{dx}{2x(1-x)}\int\frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}A, \tag{A1}$$

where $A$ is given by Eq. (30). We group the terms contributing to $A$ in four different classes according to their behavior under the integration. The first class contains only the first term in the RHS of Eq. (30):

$$A_1=32\frac{s\alpha_1\beta_2}{\tilde{t}_1\tilde{t}_1'}(\vec{Q}_1\vec{Q}_1'). \tag{A2}$$

It can be rewritten using Eqs. (20), (32), (33), and (35) in the equivalent form

$$A_1=32\frac{1-x}{x}\frac{\vec{l}_1^{\,2}(\vec{Q}_1\vec{Q}_1')}{\tilde{t}_1\tilde{t}_1'}=\frac{32x(1-x)\vec{l}_1^{\,2}(\vec{Q}_1\vec{Q}_1')}{[\vec{l}_1^{\,2}+x(\vec{q}_1^{\,2}-2(\vec{l}_1\vec{q}_1))][\vec{l}_1^{\,2}+x(\vec{q}_1'^{\,2}-2(\vec{l}_1\vec{q}_1'))]}. \tag{A3}$$

Taking first the integral over $x$, we have

$$\int_0^1\frac{dx}{2x(1-x)}A_1=\frac{16(\vec{Q}_1\vec{Q}_1')}{(\vec{q}_1'-\vec{l}_1)^2-(\vec{q}_1-\vec{l}_1)^2}\ln\frac{(\vec{q}_1'-\vec{l}_1)^2}{(\vec{q}_1-\vec{l}_1)^2}.$$

Using now the following simple trick:

$$\frac{1}{(\vec{q}_1'-\vec{l}_1)^2-(\vec{q}_1-\vec{l}_1)^2}\ln\frac{(\vec{q}_1'-\vec{l}_1)^2}{(\vec{q}_1-\vec{l}_1)^2}=\int_0^1 dz\frac{1}{z(\vec{q}_1'-\vec{l}_1)^2+(1-z)(\vec{q}_1-\vec{l}_1)^2},$$

the integration over $l_1$ and the subsequent integration over $z$ become trivial and give

$$-\frac{64\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{[\Gamma(2+\epsilon)]^2}{\Gamma(4+2\epsilon)}(\vec{q}^{\,2})^{1+\epsilon}+16\int\frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}. \tag{A4}$$

The last term in the above expression vanishes in dimensional regularization. We stress once more, however, that independently from the regularization scheme, the integrals which diverge at large $|\vec{l}_i|$ in $D=4$ cancel in Eq. (A1).

The second class of terms in the RHS of Eq. (30) is

$$\begin{aligned}A_2=8-\frac{16}{k^2}\left(\vec{q}^{\,2}-\frac{\vec{q}_1^{\,2}\vec{q}_2'^{\,2}+\vec{q}_2^{\,2}\vec{q}_1'^{\,2}}{s\alpha\beta}\right)-\frac{8}{(k^2)^2}\bigg[s\alpha\beta_1-s\alpha_1\beta+2\vec{q}_1^{\,2}\frac{\alpha_1}{\alpha}\\ -2\vec{q}_2^{\,2}\frac{\beta_1}{\beta}-2\vec{l}_1(\vec{q}_1+\vec{q}_2)\bigg]\left[s\alpha\beta_1-s\alpha_1\beta+2\vec{q}_1'^{\,2}\frac{\alpha_1}{\alpha}-2\vec{q}_2'^{\,2}\frac{\beta_1}{\beta}-2\vec{l}_1(\vec{q}_1'+\vec{q}_2')\right].\end{aligned} \tag{A5}$$

Using again Eqs. (20), (32), (33) and (35), the above expression can be recast in the following form:

$$\begin{aligned}A_2=&16x(1-x)\left[2+(1-2x)^2\frac{\vec{q}_1^{\,2}+\vec{q}_1'^{\,2}}{\Sigma}-2x(1-x)(1-2x)^2\frac{\vec{q}_1^{\,2}\vec{q}_1'^{\,2}}{\Sigma^2}\right]\\ &-\frac{16x^2(1-x)^2}{\vec{\Lambda}^2}\bigg[\frac{2(1-2x)}{x(1-x)}\left((\vec{\Lambda}\vec{q}_1')-x(1-x)\frac{\vec{q}_1'^{\,2}(\vec{\Lambda}\vec{k})}{\Sigma}\right)\left(1-2x(1-x)\frac{\vec{q}_1^{\,2}}{\Sigma}\right)\\ &+\frac{2(1-2x)}{x(1-x)}\left((\vec{\Lambda}\vec{q}_1)-x(1-x)\frac{\vec{q}_1^{\,2}(\vec{\Lambda}\vec{k})}{\Sigma}\right)\left(1-2x(1-x)\frac{\vec{q}_1'^{\,2}}{\Sigma}\right)-\frac{\vec{q}_1'^{\,2}\vec{q}_2^{\,2}+\vec{q}_1^{\,2}\vec{q}_2'^{\,2}}{\Sigma}+\frac{\vec{q}^{\,2}}{x(1-x)}\bigg]\\ &-\frac{128x^2(1-x)^2}{(\vec{\Lambda}^2)^2}\left((\vec{\Lambda}\vec{q}_1)-x(1-x)\frac{\vec{q}_1^{\,2}(\vec{\Lambda}\vec{k})}{\Sigma}\right)\left((\vec{\Lambda}\vec{q}_1')-x(1-x)\frac{\vec{q}_1'^{\,2}(\vec{\Lambda}\vec{k})}{\Sigma}\right).\end{aligned} \tag{A6}$$

The nonzero integrals over $l_1$ are the following:

$$\begin{aligned}I_1&=\int\frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}\frac{1}{\Sigma}=\frac{2\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}[x(1-x)\vec{k}^{\,2}]^{\epsilon},\\ I_2&=\int\frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}\frac{1}{\Sigma^2}=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}[x(1-x)\vec{k}^{\,2}]^{\epsilon-1},\\ I_3&=\int\frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}\frac{1}{\vec{\Lambda}^2\Sigma}=-\frac{2\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}[x(1-x)\vec{k}^{\,2}]^{\epsilon-1},\end{aligned} \tag{A7}$$

$$I_4=\int \frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}\frac{1}{(\vec{\Lambda}^2)^2}\left((\vec{\Lambda}\vec{q}_1)-x(1-x)\frac{\vec{q}_1^{\,2}(\vec{\Lambda}\vec{k})}{\Sigma}\right)\left((\vec{\Lambda}\vec{q}_1^{\,\prime})-x(1-x)\frac{\vec{q}_1^{\,\prime 2}(\vec{\Lambda}\vec{k})}{\Sigma}\right)$$

$$=-\frac{\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{(1-\epsilon)\vec{q}_1^{\,2}\vec{q}_1^{\,\prime 2}-\vec{q}_1^{\,\prime 2}(\vec{q}_1\vec{k})-\vec{q}_1^{\,2}(\vec{q}_1^{\,\prime}\vec{k})}{(1+\epsilon)}x(1-x)[x(1-x)\vec{k}^2]^{\epsilon-1}.$$

Using the above result and integrating also over $x$ , we finally obtain

$$\int_0^1\frac{dx}{2x(1-x)}\int \frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}A_2=-\frac{64\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{[\Gamma(2+\epsilon)]^2}{\Gamma(4+2\epsilon)}\frac{(\vec{k}^2)^{\epsilon}}{\vec{k}^2}(\vec{q}_1^{\,\prime 2}\vec{q}_2^{\,2}+\vec{q}_1^{\,2}\vec{q}_2^{\,\prime 2}). \tag{A8}$$

The remaining terms in the RHS of Eq. (30) are

$$A_3=\frac{16}{k^2}\left[\frac{1}{\tilde{t}_1}\left(\left[(\vec{Q}_1\vec{q}_1^{\,\prime})(s\alpha_1\beta_2-\vec{l}_1\vec{l}_2)+(\vec{Q}_1\vec{l}_2)\left(s\alpha_1\beta+\vec{l}_1(\vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime})-2\vec{q}_1^{\,\prime 2}\frac{\alpha_1}{\alpha}\right)\right]\right.\right.$$
$$\left.\left.+(\vec{l}_1\leftrightarrow\vec{l}_2,\alpha_1\leftrightarrow\beta_2,\alpha_2\leftrightarrow\beta_1,\vec{q}_1\leftrightarrow-\vec{q}_2,\vec{q}_1^{\,\prime}\leftrightarrow-\vec{q}_2^{\,\prime})\right)\right] \tag{A9}$$

and

$$A_4=A_3(\vec{q}_1\leftrightarrow\vec{q}_1^{\,\prime},\quad \vec{q}_2\leftrightarrow\vec{q}_2^{\,\prime}). \tag{A10}$$

It is not necessary to calculate explicitly the integral of $A_4$, since it can be obtained by simple substitutions from that of $A_3$. Concerning $A_3$, it can be rewritten equivalently as

$$A_3=\frac{16x(1-x)}{\vec{\Lambda}^2\tilde{t}_1}\left[(\vec{l}_1\vec{Q}_1)\vec{l}_2(\vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime})+(\vec{l}_2\vec{Q}_1)\vec{l}_1(\vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime})\right.$$
$$\left.+\frac{(\vec{\Lambda}\vec{l}_1)\vec{Q}_1(\vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime})}{x}+\alpha_1\beta s\left(1-\frac{2\vec{q}_1^{\,\prime 2}}{\alpha\beta s}\right)(\vec{l}_2\vec{Q}_1)-\alpha\beta_2 s\left(1-\frac{2\vec{q}_2^{\,\prime 2}}{\alpha\beta s}\right)(\vec{l}_1\vec{Q}_1)\right]. \tag{A11}$$

Since $A_3$ is manifestly invariant under the ‘‘left-right’’ transformation (40), we can separate in the above expression two set of terms, related each other by the ‘‘left-right’’ transformation. One possible separation is

$$A_3=\frac{16x(1-x)}{\vec{\Lambda}^2\tilde{t}_1}\left[(\vec{l}_1\vec{Q}_1)\vec{l}_2(\vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime})+\frac{(\vec{\Lambda}\vec{l}_1)\,\vec{Q}_1(\vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime})}{2x}-\alpha\beta_2 s\left(1-\frac{2\vec{q}_2^{\,\prime 2}}{\alpha\beta s}\right)(\vec{l}_1\vec{Q}_1)\right]+[\text{‘‘left-right’’}]\equiv f_3+f_3^{(L/R)}, \tag{A12}$$

with obvious notation. Since the integration measure can be put in the two equivalent forms (38) and (41) connected by the ‘‘left-right’’ transformation, the result of the integration of $f_3^{(L/R)}$ can be obtained from that of $f_3$ by the change $(\vec{q}_1\leftrightarrow -\vec{q}_2,\ \vec{q}_1^{\,\prime}\leftrightarrow-\vec{q}_2^{\,\prime})$. Therefore the integration of $f_3^{(L/R)}$ can be avoided. This allows to escape those integrands with $\vec{\Lambda}^2\tilde{t}_1\Sigma$ at the denominator which come from the term proportional to $\alpha_1/\alpha$ in Eq. (A11) and would be very nasty to integrate with the measure (38). Let us focus then our attention on $f_3$ which can be written as

$$f_3=-\frac{8x(1-x)}{\vec{\Lambda}^2\tilde{t}_1}\left\{\frac{2}{x}\vec{\Lambda}^2(\vec{l}_1\vec{Q}_1)-\frac{(\vec{\Lambda}\vec{l}_1)\vec{Q}_1(\vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime})}{x}+2(\vec{l}_1\vec{Q}_1)[-2(1-x)(\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime})+x(\vec{q}_2^{\,\prime 2}-\vec{q}_1^{\,\prime 2})+\vec{l}_1(\vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime})]\right\}. \tag{A13}$$

The integration of the first term is trivial and gives

$$I_5=-\int_0^1\frac{dx}{2x(1-x)}\int \frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}\left[\frac{16(1-x)\vec{\Lambda}^2(\vec{l}_1\vec{Q}_1)}{\vec{\Lambda}^2\tilde{t}_1}\right]=\frac{32\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{[\Gamma(2+\epsilon)]^2}{\Gamma(4+2\epsilon)}(\vec{q}_1^{\,2})^{1+\epsilon}. \tag{A14}$$

For the remaining terms we limit ourselves to illustrate the strategy of the integration, since presenting all the intermediate results would be too lengthy. The basic integrals to be calculated are of the form[1]

$$I=\int_0^1 dx\int \frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}\frac{x^{n+1}}{(\vec{l}_1-x\vec{k})^2[(\vec{l}_1-x\vec{q}_1)^2+x(1-x)\vec{q}_1^{\,2}]},\tag{A15}$$

with $n$ natural number. Using the Feynman parametrization and integrating in $d^{D-2}l_1$, one obtains

$$I=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\int_0^1 dx\int_0^1 dz\frac{x^{n+1}}{\{xz[x(1-z)\vec{q}_2^{\,2}+(1-x)\vec{q}_1^{\,2}]\}^{1-\epsilon}}=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\int_0^1 dy\, y^{\epsilon-1}\int_y^1 dx\frac{x^n}{[x(\vec{q}_2^{\,2}-\vec{q}_1^{\,2})+\vec{q}_1^{\,2}-y\vec{q}_2^{\,2}]^{1-\epsilon}},\tag{A16}$$

where the change of variable $y=xz$ has been performed in the last equality. This integral can be now calculated integrating first over $x$ and then over $y$. The complete calculation for all the terms in Eq. (A13) except the first is long, but straightforward. The final result for

$$\int_0^1\frac{dx}{2x(1-x)}\int\frac{d^{D-2}l_1}{(2\pi)^{(D-1)}}(A_3+A_4)\tag{A17}$$

is given by the last three rows in the RHS of Eq. (43).

[1]Strictly speaking, there are also integrals with $(\vec{l}_1\vec{p})$, $\vec{l}_1^{\,2}$ or $(\vec{l}_1\vec{p})\vec{l}_1^{\,2}$ at the numerator, where $\vec{p}$ is a generic momentum in the transverse space, but they can be treated similarly.


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