Can one measure the weak phase of a penguin diagram?

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The $b\to d$ penguin amplitude receives contributions from internal u, c and t quarks. We show that it is impossible to measure the weak phase of any of these penguin contributions without theoretical input. However, it is possible to obtain the weak phase if one makes a single assumption involving the hadronic parameters. With such an assumption, one can test for the presence of new physics in the $b\to d$ flavor-changing neutral current by comparing the weak phase of B_d^0-$\overline{B}_d^0$ mixing with that of the t-quark contribution to the $b\to d$ penguin amplitude. [S0556-2821(99)00319-7]

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I. INTRODUCTION

In the near future, it is expected that experiments at B factories, DESY HERA-B, and hadron colliders will measure CP-violating rate asymmetries in B decays [1], thus yielding values of α, β and γ, the three interior angles of the unitarity triangle. What is particularly compelling about CP violation in the B system is that all three angles can be extracted *cleanly*, i.e. without theoretical hadronic uncertainties. If nature is kind, these measurements will reveal the presence of physics beyond the standard model (SM).

The most obvious way to detect new physics is to compare the unitarity triangle as constructed from these CP angles with the triangle constructed from independent measurements of the sides. Any inconsistency will be evidence for new physics. The potential problem with this approach is that there are large theoretical errors, all related to hadronic physics, in extracting the lengths of the sides of the unitarity triangle from the experimental data. Because of this, the presently allowed region for the unitarity triangle is still rather large [2]. Thus, it is conceivable that new physics might be present, but we would still not be certain due to the theoretical uncertainties. Furthermore, even if the presence of new physics were clearly established, this method would not tell us which of the measurements of the sides and angles were affected by the new physics.

In light of this, a more promising technique for searching for new physics is to consider two distinct decay modes which, in the SM, probe the same CP angle. If there is a discrepancy between the two values, this would be unequivocal, clean evidence for new physics. In addition, we would have a much better idea of where the new physics entered.

In fact, there are several decay modes which can be used in this way. For example, the angle γ can be measured using rate asymmetries in $B^\pm\to DK^\pm$ [3] or $B_s^0(t)\to D_s^\pm K^\mp$ [4]. The angle β can be measured via $B_d^0(t)\to J/\psi K_S$ or $B_d^0(t)\to\phi K_S$ [5]. In either case, a discrepancy in the values of the measured CP angles would be a smoking-gun signal for new physics. [A third possibility, which is similar in spirit to these two examples, is the CP asymmetry in $B_s^0(t)\to J/\psi\phi$. To a good approximation, in the SM this asymmetry is zero, so that a nonzero value would clearly point to new physics.]

If such a discrepancy were observed, what type of new physics could be responsible? Tree-level weak decays, being dominated by W-exchange, are essentially unaffected by new physics. Thus, new physics enters principally through new contributions to loop-level processes, such as B^0-$\overline{B}^0$ mixing [6] or penguin decays [7]. We can therefore conclude that a discrepancy in the value of γ as extracted from $B^\pm\to DK^\pm$ and $B_s^0(t)\to D_s^\pm K^\mp$ is due to the presence of new physics in B_s^0-$\overline{B}_s^0$ mixing. Similarly, since the decay $B_d^0(t)\to\phi K_S$ is a pure penguin process, new physics in the $b\to s$ penguin amplitude can lead to different values of β as measured in $B_d^0(t)\to J/\psi K_S$ and $B_d^0(t)\to\phi K_S$. (If there were new physics in B_d^0-$\overline{B}_d^0$ mixing, both of these decays would be equally affected, so that this could not be the cause of any discrepancy.) Thus, in both cases, not only would we be certain that new physics is present, we would also know exactly where it had entered. (Similarly, if the CP asymmetry in $B_s^0(t)\to J/\psi\phi$ were found to be nonzero, this would clearly indicate the presence of new physics in B_s^0-$\overline{B}_s^0$ mixing.)

In all of these examples we are able to probe new physics in the $b\to s$ flavor-changing neutral current (FCNC). The obvious question is then: is there a way to use this type of method to probe new physics in the $b\to d$ FCNC?

One possibility is to try to measure the weak phase of a $b\to d$ penguin diagram. In the (approximate) Wolfenstein parametrization [8] of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, only V_{td} and V_{ub} have significant non-zero phases. These phases are two of the angles in the unitarity triangle: $\beta=\arg(V_{td}^*)$ and $\gamma=\arg(V_{ub}^*)$(α is defined to be $\pi-\beta-\gamma$). The $b\to d$ penguin amplitude receives a contribution from an internal t-quark, and the product of CKM matrix elements found in this contribution is $V_{tb}^* V_{td}$, whose

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weak phase is $-\beta$. Thus, if one could compare the value of β as extracted from the t-quark contribution to the $b\to d$ penguin with that measured in some other decay [e.g. $B_d^0(t)\to J/\psi K_S$], one might be able to detect the presence of new physics in the $b\to d$ FCNC.

If the t-quark contribution to the $b\to d$ penguin amplitude were dominant, this would be straightforward. In this case, one could simply measure CP violation in a pure $b\to d$ penguin decay such as $B_d^0\to K^0\overline{K^0}$ or $B_s^0\to\phi K_S$. In the SM, the CP asymmetry in $B_d^0(t)\to K^0\overline{K^0}$ would be expected to vanish (the weak phase of B_d^0-$\overline{B}_d^0$ mixing cancels the weak phase of the t-quark penguin amplitude), while the measurement of $B_s^0(t)\to\phi K_S$ would allow one to extract $\sin 2\beta$ [9]. If a disagreement were found between these predictions and the experimental results, this would be a clear indication of new physics in the $b\to d$ FCNC.

Unfortunately, things are not so easy. Theoretical estimates suggest that the $b\to d$ penguin is *not* dominated by the internal t-quark. On the contrary, the u- and c-quark contributions can be substantial, perhaps even as large as 20%–50% of the t-quark contribution [10]. If this is the case, the CP asymmetries do not cleanly probe weak phases, and the SM predictions given above are altered. The asymmetries now depend on (unknown) hadronic quantities such as the strong phases and the relative sizes of the various penguin contributions, so that a discrepancy between the above predictions and the measurements does not necessarily imply new physics.

Still, if we could find a way to *isolate* the t-quark contribution to the $b\to d$ penguin, we could perhaps measure its weak phase, and thereby test for the presence of new physics in the $b\to d$ FCNC.

The main purpose of this paper is to examine whether such a method is feasible. We will show that, in fact, it is impossible to *cleanly* measure the weak phase of the t-quark contribution to the $b\to d$ penguin, or indeed the phase of any of the penguin contributions. The reason is fundamentally very simple: due to the unitarity of the CKM matrix, we have

$$V_{ub}^*V_{ud}+V_{cb}^*V_{cd}+V_{tb}^*V_{td}=0. \tag{1}$$

This is the equation used to define the unitarity triangle. But the three terms in this equation are also the CKM matrix elements of the u-, c- and t-quark contributions to the $b\to d$ penguin. It is therefore impossible to isolate any one contribution—it is always possible to write a particular contribution in terms of the other two. In this paper we refer to this as the ''CKM ambiguity.'' Since one cannot isolate the t-quark contribution, it is clearly impossible to measure its weak phase cleanly.

However, all is not lost. If the CKM ambiguity could somehow be resolved, then it might be possible to measure the weak phase of the t-quark contribution to the $b\to d$ penguin. In fact, as we will show, this can be done, but it requires making a theoretical assumption regarding the hadronic parameters of the penguin amplitude. Since such an assumption holds only within a particular parametrization of the penguin amplitude, this resolves the CKM ambiguity, and allows us to extract the weak phase of the t-quark $b\to d$ penguin, albeit not cleanly.

We discuss the CKM ambiguity in more detail in Sec. II. We also show explicitly that several methods which potentially could be used to extract the weak phase of the $b\to d$ penguin in fact do not contain enough information. In Sec. III we show that the CKM ambiguity can be resolved by making a single assumption about the penguin parameters, and give some examples of such assumptions. We conclude in Sec. IV.

II. THE CKM AMBIGUITY

The full $b\to d$ penguin amplitude can be written as the sum of three contributions:

$$P=\sum_{q=u,c,t}V_{qb}^*V_{qd}P_q, \tag{2}$$

where we have explicitly separated out the dependence on the CKM matrix element. In the Wolfenstein parametrization, the weak phases of the $V_{ub}^*V_{ud}$, $V_{cb}^*V_{cd}$ and $V_{tb}^*V_{td}$ terms are γ, 0, and $-\beta$, respectively.

Now, any one of the $V_{qb}^*V_{qd}$ terms can be eliminated using Eq. (1). Thus, depending on which term is eliminated, there are several parametrizations one can use. For reasons which will become clear below, we call this freedom the ''CKM ambiguity.''

Suppose, for example, that we choose to eliminate the $V_{ub}^*V_{ud}$ piece. The penguin amplitude can then be written

$$\begin{aligned}P&=V_{cb}^*V_{cd}(P_c-P_u)+V_{tb}^*V_{td}(P_t-P_u)\\&\equiv\mathcal{P}_{cu}e^{i\delta_{cu}}+\mathcal{P}_{tu}e^{i\delta_{tu}}e^{-i\beta},\end{aligned} \tag{3}$$

where we have explicitly separated out the weak and strong phases and absorbed the magnitudes $|V_{cb}^*V_{cd}|$ and $|V_{tb}^*V_{td}|$ into the definitions of $\mathcal{P}_{cu}$ and $\mathcal{P}_{tu}$, respectively. We refer to this as parametrization 1.

Suppose further that there exists a technique which permits us to extract the weak phase $-\beta$ in the above expression *cleanly*, i.e. with no theoretical input regarding the remaining hadronic parameters. If such a technique existed, then it would be possible to express $-\beta$ (and perhaps the other parameters) entirely in terms of measured observables.

However, if we had instead chosen to eliminate the $V_{tb}^*V_{td}$ piece from Eq. (2), we would have found

$$\begin{aligned}P&=V_{cb}^*V_{cd}(P_c-P_t)+V_{ub}^*V_{ud}(P_u-P_t)\\&\equiv\mathcal{P}_{ct}e^{i\delta_{ct}}+\mathcal{P}_{ut}e^{i\delta_{ut}}e^{i\gamma},\end{aligned} \tag{4}$$

where $\mathcal{P}_{ct}$ and $\mathcal{P}_{ut}$ are defined in a similar fashion to $\mathcal{P}_{cu}$ and $\mathcal{P}_{tu}$ in Eq. (3) above. We call this parametrization 2.

The key point here is that parametrizations 1 and 2 are very similar in form. If there existed a technique which could be used to cleanly obtain $-\beta$ from parametrization 1, that same technique could be applied to parametrization 2 to obtain γ. Furthermore, the function of observables which gives

$-\beta$ would be the same function which yields γ, leading to the conclusion that $-\beta=\gamma$, which is clearly false in general.

This argument demonstrates that it is impossible to cleanly measure the weak phase of the t-quark contribution to the $b\to d$ penguin amplitude, or indeed the weak phase of any contribution. This is due specifically to the CKM ambiguity, i.e. the fact that the $b\to d$ penguin amplitude does not have a well-defined parametrization. (A similar conclusion also holds for the $b\to s$ penguin amplitude. However, in that case the situation is slightly different. For the $b\to s$ penguin amplitude, the c- and t-quark contributions are real in the Wolfenstein parametrization. And the u-quark contribution, which has a nonzero weak phase, is considerably suppressed relative to the others. Thus, to a good approximation, one can say that the $b\to s$ penguin amplitude is real, so that the CKM ambiguity is irrelevant.)

Even though this argument is quite conclusive, it is instructive to examine several methods which one could conceivably use to attempt to measure the weak phase of the t-quark contribution to the $b\to d$ penguin amplitude, and see exactly how they fail. In particular, we are interested in counting the number of independent measurements, and comparing this with the number of theoretical parameters. As we will see, in all cases, the number of parameters exceeds the number of measurements by one.

Before turning to specific examples, it is useful to establish some notation. Due to B_d^0-$\overline{B_d^0}$ mixing, a B_d^0 meson can evolve in time into a mixture of B_d^0 and $\overline{B_d^0}$. The time-dependent decay rate for a $B_d^0(t)$ to decay into a final state f is

$$\Gamma[B_d^0(t)\to f]=e^{-\Gamma t}\left[\frac{|A'|^2+|\overline{A'}|^2}{2}+\frac{|A'|^2-|\overline{A'}|^2}{2}\times\cos(\Delta M t)-\mathrm{Im}\left(\frac{q}{p}A'^{*}\overline{A'}\right)\sin(\Delta M t)\right], \tag{5}$$

where $B_d^0(t)$ is a B meson which at $t=0$ was a B_d^0, and A' and $\overline{A'}$ are $A(B_d^0\to f)$ and $A(\overline{B_d^0}\to f)$, respectively. In the Wolfenstein parametrization the mixing parameter q/p takes the form

$$\frac{q}{p}=e^{-2i\beta}. \tag{6}$$

It is convenient to remove this mixing phase by redefining the definitions of the decay amplitudes, i.e.

$$A\equiv e^{i\beta}A', \quad \overline{A}\equiv e^{-i\beta}\overline{A'}. \tag{7}$$

The time-dependent decay rate then allows us to extract $|A|$, $|\overline{A}|$, and $\mathrm{Im}(A^*\overline{A})$, i.e. the magnitudes of A and $\overline{A}$, as well as their relative phase.

In the examples which follow, we will adopt this notation, in which the mixing phase has automatically been absorbed into the decay amplitudes.

A. $B_d^0(t)\to K^0\overline{K^0}$

The decay $B_d^0\to K^0\overline{K^0}$ is a pure $b\to d$ penguin amplitude. The study of the time-dependent decay rate for this decay allows one to obtain the three quantities $|A|$, $|\overline{A}|$, and $\mathrm{Im}(A^*\overline{A})$, where $A\equiv e^{i\beta}A(B_d^0\to K^0\overline{K^0})$ and $\overline{A}\equiv e^{-i\beta}A(\overline{B_d^0}\to K^0\overline{K^0})$. However, it is straightforward to show that this information alone does not allow us to extract any of the theoretical parameters in the amplitudes.

Since the decay is pure penguin, the CKM ambiguity allows us to write the amplitude A in a variety of ways. Since we are interested in measuring the weak phase of the t-quark contribution to the $b\to d$ penguin amplitude, we will keep the $V_{tb}^*V_{td}$ piece of the amplitude. Suppose that we eliminate the $V_{ub}^*V_{ud}$ piece. The amplitude A can then be written

$$\begin{aligned}A&=e^{i\beta}[\mathcal{P}_{cu}e^{i\delta_{cu}}+\mathcal{P}_{tu}e^{i\delta_{tu}}e^{-i\beta'}]\\&=\mathcal{P}_{cu}e^{i\delta_{cu}}e^{i\beta}+\mathcal{P}_{tu}e^{i\delta_{tu}}e^{-i\theta_{NP}}.\end{aligned} \tag{8}$$

The quantities δ_{cu} and δ_{tu} are strong phases; only their difference is measurable. Also, in the presence of new physics, the phase of B_d^0-$\overline{B_d^0}$ mixing may not be the same as that of the t-quark contribution to the $b\to d$ penguin. We have allowed for this possibility by writing the weak phase of the penguin as β' in the first line. The new-physics phase is defined as $\theta_{NP}=\beta'-\beta$. Measuring the phase of the t-quark penguin contribution then is equivalent to measuring θ_{NP}. The $\overline{A}$ amplitude can be obtained from the above equation simply by changing the signs of the weak phases β and θ_{NP}.

From this expression we can count the number of theoretical parameters. There are five: β, θ_{NP}, $\mathcal{P}_{cu}$, $\mathcal{P}_{tu}$ and $\delta_{cu}-\delta_{tu}$. Since we have three measurements in five unknowns, it is impossible to solve for these parameters. In particular, one cannot obtain θ_{NP}. We can improve things slightly by noting that β, which is the phase of B_d^0-$\overline{B_d^0}$ mixing, can be independently measured in $B_d^0(t)\to J/\psi K_S$. However, this still gives us one more unknown than there are measurements.

If we had instead eliminated the $V_{cb}^*V_{cd}$ piece, this conclusion would not change. Including now the independent measurement of α [say in $B_d^0(t)\to\pi^+\pi^-$], we would still be left with four measurements in five unknowns.

In light of the CKM ambiguity it was to be expected that we would be unable to cleanly extract θ_{NP}. However, the point that we wish to stress here is that there is only one more unknown than there are measurements.

B. Isospin analysis of $B\to\pi\pi$

The decay mode which is usually associated with the measurement of the CP angle α is $B_d^0(t)\to\pi^+\pi^-$. A decade ago it was noticed that $b\to d$ penguin contributions, if large, can spoil the clean extraction of α [9,11]. This is often referred to as ‘‘penguin pollution.’’ Shortly thereafter, a method was proposed for removing the penguin pollution. This method was based on the fact that the amplitudes for the decays $B_d^0\to\pi^+\pi^-$, $B_d^0\to\pi^0\pi^0$ and $B^+\to\pi^+\pi^0$ form a triangle in isospin space [12].

In general, the decay $B_d^0 \to \pi^+\pi^-$ receives contributions from a tree diagram and a $b \to d$ penguin diagram. Using unitarity to eliminate the $V_{cb}^* V_{cd}$ piece of the penguin diagram, we can write

$$\frac{1}{\sqrt{2}} A^{+-} = e^{i\beta}[-T^{+-} e^{i\delta^{+-}} e^{i\gamma} + P e^{i\delta_P} e^{-i\beta'}]$$
$$= T^{+-} e^{i\delta^{+-}} e^{-i\alpha} + P e^{i\delta_P} e^{-i\theta_{NP}}. \tag{9}$$

In the above the $T^{+-} e^{i\delta^{+-}}$ term includes the u-quark piece of the penguin amplitude, and δ^{+-} and δ^{00} are strong phases.

The $\bar{A}^{+-}$ amplitude is obtained from the A^{+-} amplitude by changing the signs of the weak phases α and θ_{NP}. If there were no penguin contributions (i.e. $P=0$), then we would have $\mathrm{Im}(A^{+-*}\bar{A}^{+-}) \sim \sin 2\alpha$, so that we could obtain a clean measurement of α. However, if $P \neq 0$, then the phase probed in $\mathrm{Im}(A^{+-*}\bar{A}^{+-})$ is clearly a complicated function of α and the other parameters. Thus, α can no longer be extracted cleanly.

The situation can be improved by using an isospin analysis. Isospin relates the amplitude for $B_d^0 \to \pi^+\pi^-$ to the amplitudes for $B_d^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$:

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0} \tag{10}$$

with a similar triangle relation for the conjugate decays

$$\frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{-0}. \tag{11}$$

The amplitudes A^{00} and A^{+0} can be explicitly written as

$$A^{00} = T^{00} e^{i\delta^{00}} e^{-i\alpha} - P e^{i\delta_P} e^{-i\theta_{NP}},$$
$$A^{+0} = [T^{+-} e^{i\delta^{+-}} + T^{00} e^{i\delta^{00}}] e^{-i\alpha}, \tag{12}$$

where again δ^{00} is a strong phase, and only the difference of strong phases is measurable. The $\bar{A}$ amplitudes are again obtained from the above expressions by changing the signs of the weak phases.

The angle α can then be found as follows. The magnitudes of the six amplitudes $|A^{+-}|$, $|A^{00}|$, $|A^{+0}|$, $|\bar{A}^{+-}|$, $|\bar{A}^{00}|$ and $|\bar{A}^{-0}|$, can be measured experimentally. We can therefore construct the A- and $\bar{A}$-triangles [Eqs. (10) and (11)]. In addition, $\mathrm{Im}(A^{+-*}\bar{A}^{+-})$ gives the relative phase between the A^{+-} and $\bar{A}^{+-}$ amplitudes, thereby fixing the relative orientations of the A- and $\bar{A}$-triangles. The key point is that this then fixes the relative orientations of the A^{+0} and $\bar{A}^{-0}$ amplitudes. But the relative phase of these two amplitudes is just 2α. Thus, the isospin analysis allows one to remove the penguin pollution and cleanly extract α. (In fact, there are discrete ambiguities in the above procedure, but they are not our concern here.)

Although it is nice to be able to obtain α cleanly, the question which we wish to explore in this paper is: can we get more? In particular, is there enough information to also extract θ_{NP}? It is straightforward to show that the answer is no.

First, we note that there are a total of seven parameters which appear in the theoretical expressions for the amplitudes: α, θ_{NP}, T^{+-}, T^{00}, P, $\Delta^{+-} \equiv \delta^{+-} - \delta^P$ and $\Delta^{00} \equiv \delta^{00} - \delta_P$. Experimentally, at best one can measure the magnitudes and relative phases of the six A and $\bar{A}$ amplitudes, giving 11 measurements. However, due to the A and $\bar{A}$ triangle relations, the four measurements involving the A^{00} and $\bar{A}^{00}$ amplitudes are not independent. Furthermore, we have $|A^{+0}| = |\bar{A}^{-0}|$. Thus, of the 11 measurements, only six are independent. With six measurements in seven unknowns, one cannot solve for θ_{NP}.

Again, given the discussion of the CKM ambiguity, this was to be expected. However, as before, we find that there is only one more unknown than there are measurements.

C. Dalitz plot analysis of $B \to 3\pi$

An alternative way to cleanly extract α in the presence of penguin contributions is to study the Dalitz plot of $B_d^0(t) \to \pi^+\pi^-\pi^0$ decays [13]. This final state can be reached via the intermediate states $\rho^+\pi^-$, $\rho^-\pi^+$ and $\rho^0\pi^0$. It is the interference between these intermediate states which allows one to remove the penguin pollution and cleanly obtain α.

In this method, it is the $B \to \rho\pi$ amplitudes which are the key ingredients. The isospin symmetry allows one to relate neutral $B \to \rho\pi$ decays to charged $B \to \rho\pi$ decays. Defining

$$S_1 \equiv e^{i\beta}\sqrt{2} A(B^+ \to \rho^+\pi^0),$$
$$S_2 \equiv e^{i\beta}\sqrt{2} A(B^+ \to \rho^0\pi^+),$$
$$S_3 \equiv e^{i\beta} A(B_d^0 \to \rho^+\pi^-),$$
$$S_4 \equiv e^{i\beta} A(B_d^0 \to \rho^-\pi^+),$$
$$S_5 \equiv e^{i\beta} 2A(B_d^0 \to \rho^0\pi^0), \tag{13}$$

one can form an isospin pentagon

$$S_1 + S_2 = S_3 + S_4 + S_5. \tag{14}$$

As in the $B \to \pi\pi$ case, there are in general both tree and $b \to d$ penguin contributions to $B \to \rho\pi$ decays. Eliminating again the $V_{cb}^* V_{cd}$ piece, the above amplitudes can be written explicitly as follows [13]:

$$S_1 = T^{+0} e^{i\delta^{+0}} e^{-i\alpha} + 2P_1 e^{i\delta_1} e^{-i\theta_{NP}}, \tag{15}$$
$$S_2 = T^{0+} e^{i\delta^{0+}} e^{-i\alpha} - 2P_1 e^{i\delta_1} e^{-i\theta_{NP}},$$
$$S_3 = T^{+-} e^{i\delta^{+-}} e^{-i\alpha} + P_1 e^{i\delta_1} e^{-i\theta_{NP}} + P_0 e^{i\delta_0} e^{-i\theta_{NP}},$$
$$S_4 = T^{-+} e^{i\delta^{-+}} e^{-i\alpha} - P_1 e^{i\delta_1} e^{-i\theta_{NP}} + P_0 e^{i\delta_0} e^{-i\theta_{NP}},$$

$$S_5=-T^{+-}e^{i\delta^{+-}}e^{-i\alpha}-T^{-+}e^{i\delta^{-+}}e^{-i\alpha}+T^{+0}e^{i\delta^{+0}}e^{-i\alpha}+T^{0+}e^{i\delta^{0+}}e^{-i\alpha}-2P_0e^{i\delta_0}e^{-i\theta_{NP}}.$$

There is a similar pentagon relation for the conjugate amplitudes:

$$\overline{S}_1+\overline{S}_2=\overline{S}_3+\overline{S}_4+\overline{S}_5, \tag{16}$$

in which the $\overline{S_i}$ amplitudes can again be obtained from the S amplitudes by changing the signs of the weak phases α and θ_{NP}.

The Dalitz plot of the $\pi^+\pi^-\pi^0$ final state contains enough information to determine the magnitudes and relative phases of the six amplitudes S_3, S_4, S_5, $\overline{S}_3$, $\overline{S}_4$ and $\overline{S}_5$. One can then obtain α via

$$\frac{S_3+S_4+S_5}{\overline{S}_3+\overline{S}_4+\overline{S}_5}=e^{-2i\alpha}. \tag{17}$$

As in the $B\to\pi\pi$ case, one can again show that there is not enough information to extract θ_{NP}. There are a total of 13 theoretical parameters: α, θ_{NP}, six T and P amplitudes, and five relative strong phases. Experimentally, one can determine the magnitudes and relative phases of all S and $\overline{S}$ amplitudes (S_1, S_2, $\overline{S_1}$ and $\overline{S_2}$ can be obtained from an analysis of the Dalitz plot of $\pi^+\pi^0\pi^0$). Thus, there are nominally 19 measurements.

Because of the S and $\overline{S}$ pentagon relations, the amplitudes S_5 and $\overline{S}_5$ are not independent. This removes four measurements.

We have the equality $|S_1+S_2|=|\overline{S_1}+\overline{S_2}|$. This removes one more measurement.

It is easy to verify the complex equality

$$\frac{S_3-S_4-S_1}{\overline{S}_3-\overline{S}_4-\overline{S}_1}=\frac{S_1+S_2}{\overline{S}_1+\overline{S}_2}. \tag{18}$$

This removes two more measurements.

Thus, of the 19 measurements, in fact only 12 are independent. Since there are 13 unknowns, we cannot solve for θ_{NP}, as per the CKM ambiguity.[1] And, as in the $B\to\pi\pi$ case there is one more unknown than there are measurements.

D. Angular analysis of $B\to VV$ decays

Consider the case where a neutral B meson decays to a final state consisting of two vector mesons V. Because of the fact that this final state does not have a well-defined orbital angular momentum, it cannot be a CP eigenstate. However, it is possible to disentangle the CP-even and CP-odd components of the VV state through a helicity analysis of the decay products of the V mesons [14].

The amplitudes for a neutral B or $\overline{B}$ meson to decay into a pair of vector mesons can be written as

$$A(B\to V_1V_2)=\sum_{\lambda=\{0,\perp,\parallel\}}\mathcal{A}_\lambda\zeta_\lambda f_\lambda,$$

$$\overline{A}(\overline{B}\to V_1V_2)=\eta_{CP}\sum_{\lambda=\{0,\perp,\parallel\}}\overline{\mathcal{A}}_\lambda\zeta_\lambda^* f_\lambda, \tag{19}$$

where the f_λ are the coefficients of the helicity amplitudes written in the linear polarization basis; the f_λ depend only on the angles describing the kinematics. The factor ζ_λ has been introduced to account for the fact that the P-wave amplitude $A_\perp$ is CP-odd, while A_0 and $A_\parallel$ are CP-even. Thus, $\zeta_\lambda=1$ for $\lambda=\{0,\parallel\}$ and $\zeta_\lambda=i$ for $\lambda=\perp$. The intrinsic CP parity of the A_0 final state is defined as η_{CP}.

The quantities which appear in the time-dependent decay rate [Eq. (5)] are

$$|A|^2=\sum_{\lambda,\sigma}\mathcal{A}_\lambda\mathcal{A}_\sigma^*\zeta_\lambda\zeta_\sigma^* f_\lambda f_\sigma, \tag{20}$$

$$|\overline{A}|^2=\sum_{\lambda,\sigma}\overline{\mathcal{A}}_\lambda\overline{\mathcal{A}}_\sigma^*\zeta_\lambda^*\zeta_\sigma f_\lambda f_\sigma, \tag{21}$$

and

$$A^*\overline{A}=\sum_{\lambda,\sigma}\mathcal{A}_\lambda^*\overline{\mathcal{A}}_\sigma\zeta_\lambda^*\zeta_\sigma^* f_\lambda f_\sigma. \tag{22}$$

Using Eqs. (20)–(22), Eq. (5) can be rewritten as

$$\Gamma[B_d^0(t)\to f]=e^{-\Gamma t}\sum_{\lambda\leqslant\sigma}[\Lambda_{\lambda\sigma}+\Sigma_{\lambda\sigma}\cos(\Delta Mt)-\rho_{\lambda\sigma}\sin(\Delta Mt)]f_\lambda f_\sigma, \tag{23}$$

where the summation is done realizing the fact that $f_\lambda f_\sigma$ cannot be distinguished from $f_\sigma f_\lambda$ in an angular analysis. The quantities appearing in the above equation are defined as

$$\Lambda_{\lambda\lambda}=\frac{|\mathcal{A}_\lambda|^2+|\overline{\mathcal{A}}_\lambda|^2}{2}, \tag{24}$$

$$\Lambda_{\lambda\sigma}=\mathrm{Re}(\mathcal{A}_\lambda\mathcal{A}_\sigma^*\zeta_\lambda\zeta_\sigma^*+\overline{\mathcal{A}}_\lambda\overline{\mathcal{A}}_\sigma^*\zeta_\lambda^*\zeta_\sigma),\quad \lambda\neq\sigma, \tag{25}$$

$$\Sigma_{\lambda\lambda}=\frac{|\mathcal{A}_\lambda|^2-|\overline{\mathcal{A}}_\lambda|^2}{2}, \tag{26}$$

$$\Sigma_{\lambda\sigma}=\mathrm{Re}(\mathcal{A}_\lambda\mathcal{A}_\sigma^*\zeta_\lambda\zeta_\sigma^*-\overline{\mathcal{A}}_\lambda\overline{\mathcal{A}}_\sigma^*\zeta_\lambda^*\zeta_\sigma),\quad \lambda\neq\sigma, \tag{27}$$

$$\rho_{\lambda\lambda}=\mathrm{Im}(\mathcal{A}_\lambda^*\overline{\mathcal{A}}_\lambda\zeta_\lambda^{*2}), \tag{28}$$

[1]We note that this contradicts one of the conclusions of Ref. [13]. The authors of Ref. [13] concede that this particular point is in error in their paper. We thank Helen Quinn for discussions of this matter.

$$\rho_{\lambda\sigma}=\mathrm{Im}((\mathcal{A}_\lambda^*\overline{\mathcal{A}}_\sigma+\mathcal{A}_\sigma^*\overline{\mathcal{A}}_\lambda)\zeta_\lambda^*\zeta_\sigma^*), \quad \lambda\neq\sigma. \tag{29}$$

We remind the reader that we have adopted a notation in which the mixing phase q/p has been absorbed into the decay amplitudes $\mathcal{A}_\lambda$ and $\overline{\mathcal{A}}_\lambda$.

It is clear from Eqs. (23)–(29) that 18 quantities can be measured. However, it is equally clear that only 11 of these observables are independent. The fundamental quantities are the six amplitudes $\mathcal{A}_\lambda$ and $\overline{\mathcal{A}}_\lambda$, $\lambda=0,\perp,\|$. The most one can measure is their magnitudes and relative phases, for a total of 11 independent measurements. And in fact, it is straightforward to show that the observables in Eqs. (24)–(29) suffice to measure these 11 quantities, up to a twofold discrete ambiguity in the relative phases.

What can we learn from this information? For definitiveness, let us consider the pure $b\to d$ penguin decay $B_d^0\to K^*\overline{K^*}$, which is quite similar to the previous decay $B_d^0\to K^0\overline{K}^0$. Using CKM unitarity to eliminate the $V_{ub}^*V_{ud}$ piece, the helicity amplitudes can be written

$$\mathcal{A}_\lambda=\mathcal{P}_{cu}^\lambda e^{i\delta_{cu}^\lambda}e^{i\beta}+\mathcal{P}_{tu}^\lambda e^{i\delta_{tu}^\lambda}e^{-i\theta_{NP}}. \tag{30}$$

As usual, the $\overline{\mathcal{A}}_\lambda$ amplitudes are obtained by changing the signs of β and θ_{NP} in the above expression.

We can now count the number of theoretical parameters in the decay amplitudes. There are 13: β, θ_{NP}, six $\mathcal{P}$ magnitudes, and five relative strong phases. Assuming that β is independently measured, this still leaves 12 measurements in 13 unknowns. Once again, we cannot obtain θ_{NP} cleanly. And once again, there is one more theoretical unknown than there are measurements.

E. Isospin + angular analysis of $B\to\rho\rho$ decays

As a final example, one can imagine combining isospin and angular analyses. Consider the decay $B_d^0\to\rho^+\rho^-$. The $\rho^+\rho^-$ final state is not a CP eigenstate. An angular analysis can distinguish the CP-even piece from the CP-odd piece by separating out the three helicities. That is, we could obtain the magnitudes and relative phases of the amplitudes A_λ^{+-} ($\lambda=0,\perp,\|$), along with the corresponding conjugate amplitudes $\overline{A}_\lambda^{+-}$, where

$$\frac{1}{\sqrt{2}}A_\lambda^{+-}=T_\lambda^{+-}e^{i\delta_\lambda^{+-}}e^{-i\alpha}+P_\lambda e^{i\delta_P^\lambda}e^{-i\theta_{NP}}. \tag{31}$$

However, as in the $B_d^0\to\pi^+\pi^-$ case, in the presence of penguin contributions, this is not enough to obtain α cleanly—-an isospin analysis is also necessary.

Imagine, then, that an angular analysis were also performed on the decays $B_d^0(t)\to\rho^0\rho^0$ and $B^+\to\rho^+\rho^0$. We could then also obtain the amplitudes A_λ^{00}, A_λ^{+0}, along with their conjugate amplitudes. These amplitudes can be written as

$$A_\lambda^{00}=T_\lambda^{00}e^{i\delta_\lambda^{00}}e^{-i\alpha}-P_\lambda e^{i\delta_P^\lambda}e^{-i\theta_{NP}},$$

$$A_\lambda^{+0}=[T_\lambda^{+-}e^{i\delta_\lambda^{+-}}+T_\lambda^{00}e^{i\delta_\lambda^{00}}]e^{-i\alpha}. \tag{32}$$

The $\overline{A}_\lambda$ amplitudes are obtained by changing the signs of the weak phases. The helicity amplitudes form isospin triangles:

$$\frac{1}{\sqrt{2}}A_\lambda^{+-}+A_\lambda^{00}=A_\lambda^{+0}, \quad \frac{1}{\sqrt{2}}\overline{A}_\lambda^{+-}+\overline{A}_\lambda^{00}=\overline{A}_\lambda^{-0}. \tag{33}$$

There are thus six isospin triangles involving 18 amplitudes.

From the above, we see that there are a total of 19 theoretical parameters: α, θ_{NP}, nine magnitudes ($T_\lambda^{+-}, T_\lambda^{00}, P_\lambda$), and eight strong-phase differences. Experimentally, the magnitudes and relative phases of all 18 amplitudes can be obtained, giving a total of 35 measurements. However, not all measurements are independent:

Because of the isospin triangles, the amplitudes A_λ^{00} and $\overline{A}_\lambda^{00}$ are not independent. This removes 12 measurements.

We have

$$\frac{A_\lambda^{+0}}{\overline{A}_\lambda^{-0}}=e^{-2i\alpha}, \quad \lambda=0,\perp,\|. \tag{34}$$

Thus, the magnitudes of A_λ^{+0} and $\overline{A}_\lambda^{-0}$ are equal, as are their relative phases, for the helicities $\lambda=0,\perp,\|$. This removes an additional five measurements.

We therefore find that we have a total of 18 independent experimental measurements, but 19 theoretical unknowns. Although we can cleanly find the CP-phase α, we cannot determine any of the remaining parameters, including θ_{NP}.

III. RESOLVING THE CKM AMBIGUITY

In the previous section, we showed that, due to the CKM ambiguity, it is not possible to cleanly measure the weak phase of a penguin amplitude. Indeed, in all the examples considered, we found that there were more theoretical parameters than measurements, in agreement with this result. However, what is interesting about the study of these examples is that in all cases there was only one more unknown than there were measurements. Although we did not present a proof, the result appears to be very general.

This result indicates something quite useful: if we wish to test for the presence of new physics in the $b\to d$ FCNC by comparing the weak phase of B_d^0-$\overline{B}_d^0$ mixing with that of the t-quark contribution to the $b\to d$ penguin amplitude, it is necessary to make a single assumption about the theoretical (hadronic) parameters describing the decay. This assumption will hold in only one parametrization of the decay amplitude, and will therefore resolve the CKM ambiguity. Furthermore, the requirement of a single theoretical assumption holds regardless of which type of method is used.

In this section, we present several examples which show explicitly how a theoretical assumption allows one to extract the weak phase of the t-quark contribution to the $b\to d$ penguin, thereby enabling one to test for the presence of new physics in the $b\to d$ FCNC.

A. $B_d^0(t)\to K^0\overline{K^0}$ and $B_s^0(t)\to \phi K_S$

As mentioned in the introduction, $B_d^0\to K^0\overline{K^0}$ and $B_s^0\to\phi K_S$ are pure $b\to d$ penguin decays. It was recently shown in Ref. [15] that, by measuring the time-dependent decay rates for these processes, and adding a theoretical assumption, together these decays can be used to measure the weak phase of the t-quark contribution to the $b\to d$ penguin amplitude.

The amplitude for $B_d^0\to K^0\overline{K^0}$ was given in Eq. (8) and is repeated for convenience below:

$$A_d^{KK}=\mathcal{P}_{cu}e^{i\delta_{cu}}e^{i\beta}+\mathcal{P}_{tu}e^{i\delta_{tu}}e^{-i(\beta'-\beta)}, \tag{35}$$

where β is the weak phase of B_d^0-$\overline{B}_d^0$ mixing and β' is the weak phase of the t-quark contribution to the $b\to d$ penguin amplitude.

The amplitude for $B_s^0\to\phi K_S$ can be written

$$A_s^{\phi K}=\tilde{\mathcal{P}}_{cu}e^{i\tilde{\delta}_{cu}}+\tilde{\mathcal{P}}_{tu}e^{i\tilde{\delta}_{tu}}e^{-i\beta'}, \tag{36}$$

where we have assumed that there is no new physics in B_s^0-$\overline{B}_s^0$ mixing. (As discussed previously, it is possible to directly test for the presence of such new physics. If it turns out that there is new physics in B_s^0-$\overline{B}_s^0$ mixing, it can be included straightforwardly in the above equation.)

Comparing the above two equations, one immediately notes that the parameters in Eq. (36) are written with tildes compared to those in Eq. (35). There are several reasons. First, there is a different spectator quark in the two decays. Second, the decay $B_s^0\to\phi K_S$ receives additional contributions from electroweak penguins and Zweig-suppressed gluonic penguins. And third, the decay $B_d^0\to K^0\overline{K^0}$ has two pseudoscalars in the final state, while $B_s^0\to\phi K_S$ has a vector and a pseudoscalar. Because of these differences, we expect that the parameters in Eq. (35) are not equal to their counterparts with tildes in Eq. (36).

There are thus a total of eight unknowns in the two amplitudes: β, β', $\mathcal{P}_{cu}$, $\tilde{\mathcal{P}}_{cu}$, $\mathcal{P}_{tu}$, $\tilde{\mathcal{P}}_{tu}$, $\delta_{cu}-\delta_{tu}$, and $\tilde{\delta}_{cu}-\tilde{\delta}_{tu}$. However, there are (as usual) only seven measurements: the magnitudes and relative phase of A_d^{KK} and $\overline{A}_d^{KK}$, the magnitudes and relative phase of $A_s^{\phi K}$ and $\overline{A}_s^{\phi K}$, and the weak phase in B_d^0-$\overline{B}_d^0$ mixing, β.

The number of theoretical unknowns can be made equal to the number of measurements by making an assumption. In Ref. [15] it is assumed that $r=\tilde{r}$, where $r\equiv\mathcal{P}_{cu}/\mathcal{P}_{tu}$ and $\tilde{r}\equiv\tilde{\mathcal{P}}_{cu}/\tilde{\mathcal{P}}_{tu}$. The uncertainty on this assumption is estimated to be fairly small:

$$\frac{r-\tilde{r}}{r}\simeq 20\%. \tag{37}$$

Thus, taking $r\simeq\tilde{r}$ is a reasonably good approximation. With this assumption, we now have seven measurements in seven unknowns, and we can therefore solve for β and β' separately, up to discrete ambiguities. (The explicit solution is given in Ref. [15].) A comparison of β and β' may then reveal the presence of new physics in the $b\to d$ FCNC.

B. Isospin analysis of $B\to\pi\pi$

In Sec. II B we discussed how an isospin analysis of $B\to\pi\pi$ decays allows one to remove the penguin pollution and cleanly extract α. However, there are not enough measurements to obtain further information about the remaining theoretical parameters. We had

$$\frac{1}{\sqrt{2}}A^{+-}=T^{+-}e^{i\delta^{+-}}e^{-i\alpha}+Pe^{i\delta_P}e^{-i\theta_{NP}},$$

$$\frac{1}{\sqrt{2}}\overline{A}^{+-}=T^{+-}e^{i\delta^{+-}}e^{i\alpha}+Pe^{i\delta_P}e^{i\theta_{NP}}. \tag{38}$$

Defining

$$2\alpha_{eff}\equiv\arg(A^{+-*}\overline{A}^{+-}), \tag{39}$$

we therefore have three measurements ($|A^{+-}|$, $|\overline{A}^{+-}|$, $2\alpha_{eff}$) in four unknowns ($T^{+-},P,\delta^{+-}-\delta_P,\theta_{NP}$).

Suppose that we knew the value of P/T^{+-}. This could come, for example, from a theoretical estimate or a lattice calculation. In this case, we can solve for θ_{NP}. One can derive the following expressions [16]:

$$P^2=\frac{|A^{+-}|^2+|\overline{A}^{+-}|^2-2|A^{+-}||\overline{A}^{+-}|\cos(2\alpha-2\alpha_{eff})}{8\sin^2(\alpha-\theta_{NP})},$$

$$(T^{+-})^2=\frac{|A^{+-}|^2+|\overline{A}^{+-}|^2-2|A^{+-}||\overline{A}^{+-}|\cos(2\theta_{NP}-2\alpha_{eff})}{8\sin^2(\alpha-\theta_{NP})}. \tag{40}$$

Thus, if we assume a particular value of the ratio P/T^{+-}, these expressions allow us to derive θ_{NP} in terms of known quantities. And we again stress that the assumption about the value of P/T^{+-} holds only within a particular parametrization, thus lifting the CKM ambiguity.

C. Angular analysis of $B_d^0(t)\to K^*\overline{K}^*$

In Sec. II D, the decay $B_d^0\to K^*\overline{K}^*$ was discussed in the context of an angular analysis. The helicity amplitudes were

$$\mathcal{A}_\lambda=\mathcal{P}_{cu}^\lambda e^{i\delta_{cu}^\lambda}e^{i\beta}+\mathcal{P}_{tu}^\lambda e^{i\delta_{tu}^\lambda}e^{-i\theta_{NP}},$$

$$\overline{\mathcal{A}}_\lambda=\mathcal{P}_{cu}^\lambda e^{i\delta_{cu}^\lambda}e^{-i\beta}+\mathcal{P}_{tu}^\lambda e^{i\delta_{tu}^\lambda}e^{i\theta_{NP}}, \tag{41}$$

where $\lambda=0,\perp,\|$. The helicity analysis allows the extraction of the magnitudes and relative phases of all the $\mathcal{A}_\lambda$ and $\overline{\mathcal{A}}_\lambda$ amplitudes.

In order to extract θ_{NP}, it is possible to use one of the techniques described in the previous subsections. However, one might also consider an alternative method involving the strong phases. Defining $\delta^{\lambda} \equiv \delta^{\lambda}_{cu} - \delta^{\lambda}_{tu}$, suppose we assume that the δ^{λ}'s are the same for all helicities. This assumption is somewhat in the spirit of Bander, Silverman, and Soni (BSS) [17], in which the strong phases arise principally from the absorptive parts of the loop diagrams, and so are independent of the helicity of the final state.

We can then solve for θ_{NP} as follows. We define the measurable quantity φ^{λ}_{eff} as

$$2\varphi^{\lambda}_{eff} \equiv \arg(\mathcal{A}^{*}_{\lambda}\overline{\mathcal{A}}_{\lambda}\zeta^{*2}_{\lambda}). \tag{42}$$

Then we can solve for δ^{λ} as a function of θ_{NP} [16]:

$$\begin{aligned}\tan\delta^{\lambda} &= \frac{\sin(\beta+\theta_{NP})(|\mathcal{A}_{\lambda}|^2 - |\overline{\mathcal{A}}_{\lambda}|^2)}{\cos(\beta+\theta_{NP})(|\mathcal{A}_{\lambda}|^2 + |\overline{\mathcal{A}}_{\lambda}|^2) - 2|\mathcal{A}_{\lambda}||\overline{\mathcal{A}}_{\lambda}|\cos(2\varphi^{\lambda}_{eff}+\beta-\theta_{NP})} \\ &= \frac{\tan(\beta+\theta_{NP})\Sigma_{\lambda\lambda}}{\Lambda_{\lambda\lambda} - \sqrt{\Lambda^2_{\lambda\lambda} - \Sigma^2_{\lambda\lambda}}[\cos(2\varphi^{\lambda}_{eff}+2\beta) + \tan(\beta+\theta_{NP})\sin(2\varphi^{\lambda}_{eff}+2\beta)]},\end{aligned} \tag{43}$$

where $\Lambda_{\lambda\lambda}$ and $\Sigma_{\lambda\lambda}$ are defined in Eqs. (24) and (26). Assuming that $\delta^{\lambda} = \delta^{\sigma}$, where λ and σ represent different helicities, the above equation allows one to solve for θ_{NP} in terms of measurable quantities only. Of course, the solution will contain discrete ambiguities, but it will still be possible to establish whether θ_{NP} is nonzero.

We must point out here that, although the assumption of a common δ^{λ} for all helicity amplitudes does allow us to obtain θ_{NP}, its theoretical justification is problematic. From Eqs. (3) and (30), recall that

$$\begin{aligned}\mathcal{P}^{\lambda}_{cu} e^{i\delta^{\lambda}_{cu}} &= (|P^{\lambda}_{c}| e^{i\delta^{\lambda}_{c}} - |P^{\lambda}_{u}| e^{i\delta^{\lambda}_{u}})|V^{*}_{cb}V_{cd}|, \\ \mathcal{P}^{\lambda}_{tu} e^{i\delta^{\lambda}_{tu}} &= (|P^{\lambda}_{t}| e^{i\delta^{\lambda}_{t}} - |P^{\lambda}_{u}| e^{i\delta^{\lambda}_{u}})|V^{*}_{tb}V_{td}|.\end{aligned} \tag{44}$$

In the BSS calculation [17], the details of the calculations of the penguin diagrams are independent of the helicity of the final state. In particular, the strong phases δ^{λ}_{i} $(i=u,c,t)$ are in fact λ-independent. Furthermore, the factors $|P^{\lambda}_{i}|$ $(i=u,c,t)$ can each be written as a λ-independent penguin piece multiplied by a *common* λ-dependent matrix element. In this case, it is straightforward to verify that, indeed, the strong phase δ^{λ} is the same for all helicity states.

However, there is also a problem: the only λ dependence of the amplitudes $\mathcal{A}_{\lambda}$ and $\overline{\mathcal{A}}_{\lambda}$ in Eqs. (41) is the presence of this overall multiplicative common matrix element. And this matrix element cancels in all ratios involving $\mathcal{A}_{\lambda}$ and/or $\overline{\mathcal{A}}_{\lambda}$. In particular, ratios such as $\Sigma_{\lambda\lambda}/\Lambda_{\lambda\lambda}$ are in fact λ-*independent*. But this implies that the right-hand side of Eq. (43) is actually independent of λ, so that the technique does not work.

The only possible loophole in the above argument is that the BSS calculation makes use of factorization. If nonfactorizable effects are large — and they may well be for penguin diagrams — then there may not be a common λ-dependent matrix element for each of the $|P^{\lambda}_{i}|$ factors. In this case, the helicity dependence of $\mathcal{A}_{\lambda}$ and $\overline{\mathcal{A}}_{\lambda}$ is considerably more complicated than a simple multiplicative factor. On the other hand, in general the strong phases δ^{λ} will then not all be equal.

Thus, in order to justify the assumption of a common δ^{λ} for all helicity amplitudes, one must hope that nonfactorizable effects exist which give different matrix elements for the various internal-quark contributions, but nevertheless give the same δ^{λ} for all helicities. Although this is a logical possibility, it must be admitted that it seems implausible.

IV. CONCLUSIONS

In the coming years, many measurements will be made of CP violation in the B system. Hopefully these measurements will reveal the presence of new physics. Although this in itself will be very exciting, we will then want to know what kind of new physics it is, and how it has affected the CP asymmetries.

New physics generally can affect CP-violating asymmetries through its effects on loop-level processes, such as B^0-$\overline{B}^0$ mixing or penguin decays. It is then useful to categorize these effects as belonging to the $b\to s$ or the $b\to d$ flavor-changing neutral current (FCNC). Although there are several ways to cleanly test for the presence of new physics in the $b\to s$ FCNC, it is not so easy to do this for the $b\to d$ FCNC.

In the SM, the weak phase in B^0_d-$\overline{B}^0_d$ mixing is the same as that found in the t-quark contribution to the $b\to d$ penguin amplitude. In the presence of new physics, these two phases may be different. The question then is: can one measure the phase of the penguin amplitude? In this paper we have shown that it is *not* possible to cleanly measure this phase. The reason is essentially the following. There are three contributions to the $b\to d$ penguin, coming from internal u, c and t-quarks. However, due to the unitarity of the CKM matrix, it is always possible to write any one of these contributions in terms of the other two. We call this the ''CKM ambiguity.'' It is therefore not possible to isolate the t-quark contribution, and so one cannot cleanly measure its weak phase.

We have explicitly analyzed several methods which could conceivably have been used to try to obtain the weak phase of the t-quark contribution to the $b \to d$ penguin, and found that, indeed, there is not enough information to extract the phase of the t-quark penguin.

However, in performing this analysis, we have also obtained an interesting result: in all cases there is one more theoretical (hadronic) unknown than there are measurements. Thus, the addition of a single assumption about the hadronic parameters, which removes the CKM ambiguity, allows the extraction of the weak phase of the t-quark penguin. This can then be used to test for the presence of new physics in the $b \to d$ FCNC. We have given several examples of methods, along with the corresponding assumptions, in which this can be done.

Note added. While we were writing this paper, we received a paper by Fleischer [18] which discusses some of these same issues for the specific case of the angular analysis in $B \to VV$ decays.

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