Final state interaction phase in B decays

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From an estimate of the meson-meson inelastic scattering at 5 GeV, it is concluded that a typical strong phase in B decays to two mesons is of the order of 20°. For a particular final state, an estimate of the phase depends on whether that state is more or less probable as a final state compared to those states to which it is connected by the strong interaction S matrix. [S0556-2821(99)07317-8]

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I. INTRODUCTION

Strong final state interactions play an important role in the analysis of CP-violating effects in B decays. Direct CP violation such as the difference in rates for $B^+ \to F$ and $B^- \to \overline{F}$ vanishes in the limit at which there are no strong phase shifts. Final state phases play a critical role in amplitude analyses of a set of B^0 decay experimental results.

An approach to final state phases in inclusive decays was given by Bander, Silverman, and Soni [1]. For decays corresponding to the transition $b \to u\bar{u}s$, they considered at the quark level $b \to c\bar{c}s \to u\bar{u}s$, where the second transition on the mass shell yielded the final state phase. Whether or not this is reasonable for inclusive decays, its application [2] to exclusive decays such as $B \to \overline{K}\pi$ has been criticized [3], because the major final state interactions of $\overline{K}\pi$ are ''soft'' scattering to $\overline{K}n\pi$ and not to $c\bar{c}$ states.

A major difference of B decays from decays of lighter hadrons is the presence of a large number of final decay channels. The value of the strong phase for a given decay channel depends on the weak and strong amplitudes of all final states that communicate with it through strong interaction. While many theoretical attempts have been made, none of them have tackled the formidable complexity arising from the large number of final decay channels. To give quantitative predictions, one must in some way face a multichannel problem involving both weak and strong interactions.

We concentrate here on the decay of B to two mesons, referring to $B \to \pi\pi$ to be specific. Arguments have been given that the final state phase shifts should be small. For example, Bjorken [4] argued that there is little final state scattering in $B \to \pi\pi$ because B decays directly to colorless $q\bar{q}$ pairs that do not interact as they evolve into $\pi\pi$. Taken literally this is not correct, since the s-state scattering at 5 GeV is expected to be sizable [5], as we discuss below. In the present paper we seek to analyze the relations between the weak decay amplitude and the strong interaction S matrix of final states that might lead to large or small final state phases.

II. MULTICHANNEL FINAL-STATE INTERACTION

Consider the decay matrix element $\langle f^{\text{out}}|O_i|B\rangle$ for the B meson into the hadronic final state f, where O_i is a weak decay operator. The strong interaction S matrix is defined with the ''in'' and ''out'' states by

$$S_{ff'} = \langle f^{\text{out}}|f'^{\text{ in}}\rangle. \tag{1}$$

We choose states that are eigenstates of J, not of individual meson momenta. The phases of the *in* and *out* states are fixed by the time reversal transformation T:

$$\begin{aligned} T|f'^{\text{ in}}\rangle &\to \langle f'^{\text{ out}}|, \\ T|f^{\text{out}}\rangle &\to \langle f^{in}|. \end{aligned} \tag{2}$$

With this phase convention, time reversal invariance of strong interactions requires that $S_{ff'}$ be a symmetric matrix

$$S_{ff'} = S_{f'f}. \tag{3}$$

Applying time-reversal operation on $M_f = \langle f^{\text{out}}|O_i|B\rangle$, one obtains

$$M_f \xrightarrow{T} \langle B|TO_iT^{-1}|f^{\text{in}}\rangle. \tag{4}$$

If one inserts a complete set of *out* states and uses Eq. (3), this relation becomes $M_f = \Sigma_{f'} S_{ff'} M^*_{f'}$ for a T-even decay operator O_i. One can express it in the operator form as

$$M = SM^*, \tag{5}$$

where M is represented in a column vector. This matrix equation is formally solvable as

$$M = S^{1/2} M^0, \tag{6}$$

where M^0 is an arbitrary *real* vector of the same dimension as M. If one uses the eigenstates $|\alpha\rangle$ of the S matrix as a basis, Eq. (6) reduces to the Watson theorem $M_\alpha = M^0_\alpha e^{i\delta_\alpha}$. We thus may consider the vector M^0 as representing the decay amplitude in the absence of the final state phases due to the strong interaction. Since M and M_0 are related by a unitary matrix, it holds that $\Sigma_f|M_f|^2 = \Sigma_f|M^0_f|^2$.

If one subtracts the complex conjugate of M from both sides in Eq. (5) and divides by $2i$, the familiar form Im$M = \mathbf{t}M^*$ emerges for the imaginary part of M, where $\mathbf{t} = (S-1)/2i$. In components, it reads

$$\mathrm{Im}M_f=\sum_{f'} t_{ff'}M^*_{f'}. \tag{7}$$

This form is commonly derived starting with Lehmann-Symanzik-Zimmermann's reduction formula. In the applications of interest, the weak decay Hamiltonian H_w is given in the form

$$H_w=\sum_i \lambda_i O_i, \tag{8}$$

where λ_i is a combination of the Cabibbo-Kobayashi-Maskawa matrix elements and O_i is a T-even operator. It is to be understood that M_f is to be evaluated separately for different operators O_i.

III. STRONG INTERACTION *S* MATRIX

When two mesons such as $\pi^+\pi^-$ interact in the s state, we believe that they will scatter into a large number of multiparticle final states. Indeed, we expect similar inelastic behavior for all partial waves of $l<kr$, where r is a characteristic hadron radius. The sum over these partial waves can be described by a diffractive scattering formula such as that given by Pomeron exchange. For the case of meson-meson scattering, we extrapolate from the analysis of meson-baryon and baryon-baryon scattering, and for the invariant elastic scattering amplitude we write

$$T(s,t)=i\sigma_{\mathrm{tot}}se^{bt}, \tag{9}$$

where the constant in front has been fixed by the optical theorem. Defining the Pomeron residue by $\beta(t)\equiv\sigma_{\mathrm{tot}}e^{bt}$, with the factorization relation $\beta(t)_{MM'}\beta(t)_{pp}=\beta(t)_{Mp}\beta(t)_{M'p}$[1] we obtain

$$\sigma^{\pi\pi}_{\mathrm{tot}}\simeq 12\ \mathrm{mb},\quad \sigma^{\pi K}_{\mathrm{tot}}\simeq 10\ \mathrm{mb},\quad \sigma^{K\bar{K}}_{\mathrm{tot}}\simeq 8\ \mathrm{mb}, \tag{10}$$

where $\sigma^{pp}_{\mathrm{tot}}=37$ mb, $\sigma^{\pi p}_{\mathrm{tot}}=21$ mb, and $\sigma^{Kp}_{tot}=17$ mb [6] have been used for the diffractive contribution of σ_{tot} at $\sqrt{s}\simeq m_B$. For the diffractive peak width, the factorization gives

$$b^{\pi\pi}\simeq 3.6\ \mathrm{GeV}^{-2},\quad b^{K\pi}\simeq 2.8\ \mathrm{GeV}^{-2},$$
$$b^{K\bar{K}}\simeq 2.0\ \mathrm{GeV}^{-2} \tag{11}$$

if we choose $b^{pp}\simeq 5.0\ \mathrm{GeV}^{-2}$, $b^{\pi p}\simeq 4.3\ \mathrm{GeV}^{-2}$, and $b^{Kp}\simeq 3.5\ \mathrm{GeV}^{-2}$ [7]. For $D\pi$ scattering, we use the quark counting rule for σ_{tot} and the assumption that the charmed quark interacts with the light quarks much more weakly. Then we obtain a crude estimate

$$\sigma^{D\pi}_{\mathrm{tot}}\approx \tfrac{1}{2}\sigma^{\pi\pi}_{\mathrm{tot}}, \tag{12}$$

and $b^{D\pi}$ is a little smaller than $b^{K\pi}$.

Projecting out the s wave from the amplitude in Eq. (9),

$$a_{l=0}(s)=\frac{1}{16\pi s}\int_{-s}^{0}T(s,t)dt, \tag{13}$$

yields

$$\mathrm{Im}a_{l=0}\simeq\begin{cases}0.16 & (\pi\pi)\\ 0.17 & (K\pi)\\ 0.18 & (K\bar{K})\\ 0.12 & (D\pi)\end{cases} \tag{14}$$

at $\sqrt{s}=5\sim 6$ GeV.

Extraction of the s-wave amplitude from the diffractive formula may arouse suspicion since one thinks of diffraction as a peripheral process [8]. It would be better to consider $T(s,t)$ as describing the scattering from an absorbing gray sphere of radius r. The values of a_l up to $l\sim kr$ vary gradually with l, thus adding up to a large forward peak. As a result, about 90% of the contribution to the integral in Eq. (13) comes from $|t|<1\ \mathrm{GeV}^2$ even though the $l=0$ amplitude itself is, of course, independent of t.

In what follows we use the estimate

$$S_{ff}\simeq 0.7 \tag{15}$$

corresponding to

$$a_{l=0}\equiv t_{ff}=\frac{S_{ff}-1}{2i}=0.15i. \tag{16}$$

This corresponds to the case of a gray sphere with an inelasticity of 0.85. In the limiting case of a black sphere S_{ff} goes to zero and the inelasticity goes to 0.5. If one goes beyond the diffractive scattering approximation, S_{ff} is not purely real. In the Regge theory, the real part arises from exchange of the non-Pomeron trajectories such as ρ and f_2. In $\pi^{\pm}p$ scattering, the real-to-imaginary ratio of 10–20% was observed in the forward scattering amplitudes at $\sqrt{s}=5-6$ GeV [9]. We can make an estimate of the real part for meson-meson scattering by using the factorization. We first determine the Regge parameters at $t=0$ from the total cross-section differences [6], and then extract their t dependences from the angular dependence of the differential cross sections [10]. The analysis is simpler if exchange degeneracy is imposed. The smaller σ_{tot} and the larger ρ-f_2 residues tend to enhance the real-to-imaginary amplitude ratio for $\pi\pi$ scattering over πp scattering, while the smaller $b^{\pi\pi}$ partially compensates the trend. Particularly for $\pi^+\pi^0$, the real parts of the ρ and f_2 terms add up close to 30% of the imaginary part. However, our major goal is to understand the implications of the sizable inelastic scattering; for this purpose, we use the simplifying approximation that S_{ff} is real.

IV. TWO CHANNELS

Relation (5) was studied in the case of two channels [5] assuming that the diagonal S matrix elements S_{ff} are purely real. This requirement on the S matrix turns out to be so

[1]The factorization can be proved only for simple l-plane singularities. It is an assumption for the Pomeron.

strong in the case of two channels that there is only a single parameter left:

$$S=\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} e^{2i\theta} & 0 \\ 0 & e^{-2i\theta} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$=\begin{pmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{pmatrix}. \tag{17}$$

When $S^{1/2}$ is computed from S and substituted into Eq. (6), a simple relation results:

$$M_1=M_{01}\cos\theta+iM_{02}\sin\theta,$$
$$M_2=iM_{01}\sin\theta+M_{02}\cos\theta. \tag{18}$$

The phase of the decay amplitude in channel 1 is large if the particle decays preferrentially to channel 2, while it is small if channel 1 is the dominant decay channel for small values of θ. Though this is an interesting conclusion, this picture turns out to be specific to the two-channel case. When we add one more channel in the final state, there are three real parameters even after $\mathrm{Im}S_{ff}=0$ is assumed. The nice simple relation of Eq. (18) does not hold any longer. If we go to N final channels, there are $N(N-1)/2$ real parameters even with $\mathrm{Im}S_{ff}=0$, and no meaningful prediction results. Therefore we must change our strategy in studying the case of $N \gg 1$ such as the B decay.

V. RANDOMNESS OF WEAK DECAY AMPLITUDES

In the presence of many decay channels, strong interactions are so complicated that it is beyond our ability to predict final state interactions accurately. We must substitute lack of our knowledge with reasonable dynamical assumptions and/or approximations. In search of such an assumption, we notice that since $M_{f'}$ and $S_{ff'}$ come from two different sources, weak and strong interactions, the phase of product $S_{f'f}M^*_{f'}$ for $f'\neq f$ takes equally likely a positive or a negative value as f' is varied with f fixed. While $M_{f'}$ is related to M_f $(f'\neq f)$ by rescattering, there exist so many states that the influence of f on f' can be disregarded. We therefore introduce the postulate that the phase of $S_{ff'}M^*_{f'}$ takes random values as f' varies. It should be noted that randomness is postulated here for the relative phase and sign of the decay matrix element to the S-matrix element, not for the dynamical phases and mixing of S matrix, as was introduced in the random S matrix theory of nuclear physics [11].

We start our analysis with Eqs. (7) and (15). Taking the $f'=f$ term in the sum to the left-hand side in Eq. (7) and using $t_{ff}\simeq i\mathrm{Im}t_{ff}$, we write Eq. (7) in the form

$$(1+it_{ff})\mathrm{Im}M_f-t_{ff}\mathrm{Re}M_f=\sum_{f'\neq f}t_{ff'}M^*_{f'}. \tag{19}$$

The first and second terms of the left-hand side are real and imaginary, respectively, for $\mathrm{Re}t_{ff}=0$. Given the estimate of Eq. (16) the coefficient of the first term is much larger than that of the second term, and we consider this primarily as an equation for $\mathrm{Im}M_f$. With the randomness postulate, the phase of M_f is equally often positive or negative if we consider a large ensemble of final states f. It is some kind of average of the magnitude of the phase of M_f, not values for individual M_f, that we can study with our randomness postulate. For this purpose, we take the absolute value squared for both sides of Eq. (19). Then the right-hand side is

$$\mathcal{R}=\frac{1}{4}\sum_{f',f''\neq f}S_{ff'}M^*_{f'}S^*_{ff''}M_{f''}, \tag{20}$$

where $t_{ff'}=S_{ff'}/2i$ has been used for $f'\neq f$. The random phase postulate allows us to retain only the terms of $f'=f''$ and to reduce the double sum to a single sum:

$$\mathcal{R}\simeq\frac{1}{4}\sum_{f'\neq f}S_{ff'}S^\dagger_{f'f}|M^2_{f'}| \tag{21}$$

$$\equiv\frac{1}{4}\overline{|M^2_{f'}|}\sum_{f\neq f'}|S_{ff'}|^2, \tag{22}$$

where the second line defines $\overline{|M^2_{f'}|}$ as the weighted average of the decay amplitudes into states f'. Then, using the unitarity of the S matrix, we reach

$$\mathcal{R}\simeq\frac{1}{4}(1-S^2_{ff})\overline{|M^2_{f'}|}. \tag{23}$$

While our estimate of S_{ff} is made on the basis of the Pomeron exchange, it should be noted that contributions to $S_{ff'}$ from quantum number exchange may be important in determining $\overline{|M^2_{f'}|}$ from Eq. (22) if they correspond to states f' with large values of $|M^2_{f'}|$. Identifying Eq. (23) with the absolute value squared of the left-hand side of Eq. (19), we obtain the prediction of our random phase approximation:

$$(1+S_{ff})^2(\mathrm{Im}M_f)^2+(1-S_{ff})^2(\mathrm{Re}M_f)^2=(1-S^2_{ff})\overline{|M^2_{f'}|}. \tag{24}$$

Defining ρ by

$$\rho\equiv\overline{|M^2_{f'}|}^{1/2}/|M_f|,$$
$$|M_f|^2=(\mathrm{Im}M_f)^2+(\mathrm{Re}M_f)^2, \tag{25}$$

the ratio of the imaginary-to-real part of M_f is solved from Eq. (24) as

$$\frac{(\mathrm{Im}M_f)^2}{(\mathrm{Re}M_f)^2}\equiv\tan^2\delta_f=\frac{\tau^2(\rho^2-\tau^2)}{1-\rho^2\tau^2}, \tag{26}$$

where

$$\tau = \left(\frac{1 - S_{ff}}{1 + S_{ff}} \right)^{1/2}. \tag{27}$$

Note that τ^2 is equal to the ratio of elastic to inelastic scattering cross section $\sigma_{\mathrm{el}}/\sigma_{\mathrm{inel}}$ of the relevant partial wave. Since the left-hand side of Eq. (26) is nonnegative, τ and ρ are constrained for $S_{ff} > 0$ by

$$\tau^2 \leqslant \rho^2 \leqslant 1/\tau^2. \tag{28}$$

For $S_{ff} = 0.7$,

$$\tau^2 = 0.18, \tag{29}$$

so that rescattering among the final states does not allow $\overline{|M_{f'}|^2}$ and $|M_f|^2$ to differ too greatly in magnitude. In the weak limit of rescattering ($\tau \to 0$), of course, Eq. (28) allows any value for ρ. In the black sphere limit ($\tau \to 1$) Eq. (26) is useless, and Eq. (28) constrains $\rho = 1$. Our approach is only useful to the extent that inelastic scattering dominates very much over elastic for the final state f. It should be noticed that Eq. (26) reduces to the two-channel case [Eq. (18)] with

$$\left| \frac{M_2^0}{M_1^0} \right|^2 = \frac{\rho^2 - \tau^2}{1 - \rho^2 \tau^2}. \tag{30}$$

Our random approximation amounts to lumping all inelastic channels together as if they were a single inelastic channel with an ''average'' decay amplitude. However, we now interpret this as something like the standard deviation of the phase for an ensemble of independent final states f with a given value of ρ.

If the relevant states f' were similar to the state f, then we might expect ρ to be close to unity. For $\rho = 1$, Eq. (26) reduces to

$$\tan^2 \delta_f = \tau^2 = \frac{1 - S_{ff}}{1 + S_{ff}}, \qquad \sin \delta_f = \sqrt{\frac{1 - S_{ff}}{2}}. \tag{31}$$

With $S_{ff} = 0.7$, this gives $|\delta_f| \sim 23°$. Thus a typical value of the final strong interaction phase in this case is not small. This result for a typical state has a simple heuristic interpretation. The original real decay amplitude M_1^0 is reduced as a result of absorption by a factor a, but an imaginary term arises due to rescattering from other states. Since the total decay rate is not changed by final state scattering, the final value of $|M_f|$ for a typical state will be equal to $|M_1^0|$. Thus M_f takes the forms

$$M_f = M_1^0 [a + i\sqrt{1 - a^2}], \qquad \frac{\mathrm{Im} M_f}{\mathrm{Re} M_f} = \sqrt{1 - a^2}/a. \tag{32}$$

This agrees with the result above if the absorption factor is identified as

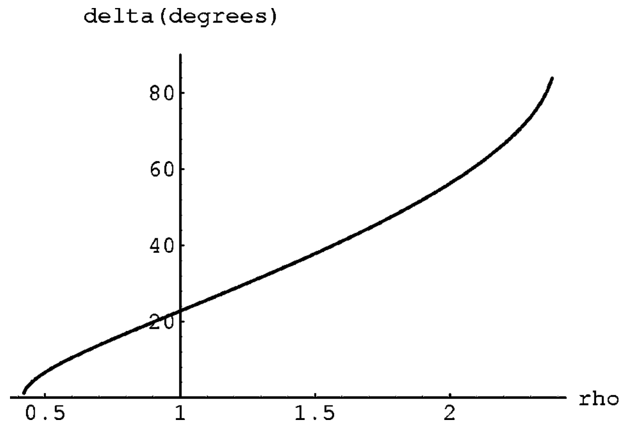

FIG. 1. The strong phase δ_f defined in Eq. (26) plotted against the ratio ρ for $S_{ff} = 0.7$. δ_f is chosen between 0° and 90°.

$$a = \sqrt{(1 + S_{ff})/2}. \tag{33}$$

Any argument that a final state phase is small must be an argument that ρ is small. It should be noted that ρ depends on the particular final state f and on the weak interaction operator O_i. The quantity $\overline{|M_{f'}^2|}$ is an average of the square of the decay amplitude to state f' via O_i weighted by the square of the scattering amplitude from f to f' [cf. Eqs. (22) and (23)]. Thus a value of ρ much smaller than unity means that on average the states to which f scatters are much less likely than f to be final states in the decay due to operator O_i. Conversely, if f is a particularly unfavored final state ρ may well be above unity. Figure 1 shows the dependence of the phase on ρ for the choice of $S_{ff} = 0.7$.

VI. POSSIBILITY OF SMALL OR LARGE STRONG PHASES FOR TWO-BODY CHANNELS

While our randomness postulate leads to sizable strong phases for ''typical'' decay channels, dynamical arguments are necessary to estimate the parameter ρ for a specific channel and a specific decay operator. We ask what dynamical arguments could let our prediction agree with Bjorken's argument, which favors small phases for decays such as $B^0 \to \pi^+\pi^-$. The observed branching fraction of $B^0 \to \pi^+\pi^-$ is about 10^{-3} of the inclusive branching fraction for $b \to d\overline{q}q$ within large uncertainties. This might seem to indicate that $\pi^+\pi^-$ is not a favored final state, and that the strong phase of $\pi^+\pi^-$ might be of order of 20°. However, this conclusion can be evaded if certain conditions are satisfied.

It should be noted that we are only interested in whether $\pi^+\pi^-$ is a favored decay channel relative to those to which it is connected by $S_{f'f}$. Most of the states f' are multimeson states with little jetlike character. It could be argued that these states are not favored as B decay products because they are not likely to develop from three quark jets into which B naturally decays. It is not clear whether this distinction is really operative for the energy m_B.

If the above argument were true, it could be considered as an interpretation of the Bjorken argument. In order to produce a $\pi\pi$ final state, the final quarks must emerge as colorless pairs; alternative quark configurations are unlikely to hadronize into $\pi\pi$. Thus ρ would be close to its minimum value and $\mathrm{Im}M_f$ would be small. It is not true, however, that the final $\pi\pi$ state has little interaction, but the effect would primarily be a moderate absorption correction to the real part. In the two-channel reduction this corresponds to M_{20}/M_{10} close to zero, and the change of the real part of M_1 from M_{10} to $M_{10}\cos\theta$ would be considered as the absorption correction.

An alternative possibility in which one might expect a large final state phase shift has been emphasized in a number of recent papers [12]. These are cases in which M_f vanishes in the naive factorization approximation. An example is the ''tree'' amplitude proportional to $\lambda_u = V_{ub}V_{us}^*$ for the decay $B^- \to \bar{K}^0\pi^-$. In this case the major contribution to M_f may arise from rescattering from favored states, so that ρ might be larger than unity. It can be argued [12] that, in this case, even though the final state scattering mainly goes to multiparticle states, the main contribution to $\mathrm{Im}M_f$ arises from the scattering of quantum number exchange involving two-particle to two-particle transitions. Of course, the real part of M_f obtains a contribution from the dispersive part of such diagrams, which is hard to calculate, so that quantitative results for the phase are not possible.

VII. CONCLUSION

For a ''typical'' final state, the strong final state phase shift is not small; a typical magnitude is 20°. We understand this to be the magnitude averaged over the states that are interconnected by the final state S matrix. We expect there to be sizable fluctuations about the average; in fact, since we cannot predict the sign of the phases, our analysis suggests that the algebraic average phase may be zero.

A simple heuristic understanding of this phase is that the final state absorption reduces the value of the original real decay amplitudes, whereas rescattering from other states provides an imaginary amplitude. For a ''typical'' state the absolute value of the amplitude is not changed since the final state interaction does not change the total decay rate. Our magnitude estimate of $\sim$20° is derived from the expected inelasticity of the meson-meson scattering.

It may be possible to argue for a particular final state f that the phase is small. Any such argument must show that the states to which f is connected by the S matrix are generally less likely to be a decay product of B than f. Thus it might be argued that the four-quark operator leads easily to $\pi^+\pi^-$, whereas there are many states to which $\pi^+\pi^-$ scatters that are not easily reached directly via B decay. Conversely it might be argued that for states which are not easily reached via the four-quark operator, the strong phase is large.

It should be emphasized that at best one may give qualitative arguments why a particular phase may be large or small. We do not believe any quantitative calculations are reliable because of the complexity of the multichannel problem.

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