Quark-composites approach to QCD: The nucleons

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We present a new perturbative approach to QCD based on the use of quark composites with hadronic quantum numbers as fundamental variables. We apply it to the case of the nucleons by performing a nonlinear change of variables in the Berezin integral which defines the partition function of QCD. The nucleon composites are thereby assumed as new integration variables. We evaluate the Jacobian and certain transformation functions which appear in the change of variables. We show that the free action of the nucleon composites is the Dirac action, and we evaluate the first perturbative contributions to their electroweak effective action, which turn out to be a pure renormalization. Our expansion is compatible with a perturbative as well as nonperturbative regime of the gluons and it has the characteristic feature that the confinement of the quarks is built in. [S0556-2821(99)02317-6]

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I. INTRODUCTION AND SUMMARY

''After nineteen years of study we still lack reliable, analytic tools for treating (the large distance region) of QCD. This remains one of the most important, and woefully neglected, areas of theoretical particle physics'' [1]. We can indeed divide the actual calculations in this field into two classes: Those which aim at clarifying the confining mechanism, and those devoted to the description of hadronic physics, and the difficulty stems from the fact that the former calculations are done in a framework where it would be very awkward, if not impossible, to perform the latter. In other words, the actual understanding of the theory requires a nonperturbative approach for low energy and perturbative methods for high energy, which until now has been impossible to unify in a unique scheme.

The direction which we take in the hope of overcoming this impasse is to use quark composites with hadronic quantum numbers as integration variables [2] in Berezin integrals. This possibility was considered a few years ago and the necessary formalism was developed [3]. The relations which define the physical hadronic fields in terms of the quarks cannot obviously be inverted, and therefore the quarks cannot be eliminated. But we will see that an ''effective inversion'' can be achieved in a precise way, allowing us to perform the physical calculations. One can then hope that this will help in connection with our problem, which turns out indeed to be the case.

The strategy we adopt is to add to the standard action an irrelevant operator containing the free action of the hadronic composites and perform a perturbative expansion by assuming these composites as new variables of integration. There is a fundamental difference between trilinear and bilinear composites. Surprisingly enough for some trilinear composites, in particular the nucleon fields, the change of variables preserves the form of the Berezin integral. As a consequence the free action of such composites is the Dirac action.

Also the mesonic composites can be introduced as integration variables, but the resulting integral does not reduce in general to a Berezin or to an ordinary integral, and it is such that the propagator *is not the inverse of the wave operator*. It then required some effort to construct an irrelevant operator containing the free action of the pions, but this was eventually done, along with the first investigation of the related perturbative expansion, which reproduced the interactions of the chiral models. Although identical in spirit and in its distinctive features, this expansion differs significantly in the technicalities from that for the barions, and it is the subject of separate works [4].

Our approach is compatible with a perturbative as well as nonperturbative regime of the gluons. Therefore, even though its applicability is in no way restricted to a specific regularization, we adopt a regularization on a Euclidean lattice, which is the only one suitable for both cases, a choice which seems also natural dealing with composites.

In this paper we consider the case of the nucleons. Our main results are the evaluation of the Jacobian and of certain transformation functions, necessary to perform the change of variables. Then we evaluate the first perturbative contribution to the partition function. We expect that, since the physics of the nucleons is dominated by the exchange of the pions, unless the fields of these particles are included, a realistic application in the field of strong interactions is not possible. A unified treatement of nucleons and pions will be presented in a forthcoming paper, where the pion-nucleon interaction is investigated [5]. Here we restrict ourselves to the electroweak interactions of the nucleons, evaluating only the first order terms originating from the quarks, where the pions cannot contribute. In the presence of these interactions, the free action of the nucleons must be gauge invariant, and it must therefore contain the appropriate electroweak couplings. The first order corrections turn out to be a mere renormalization of the electroweak action of the nucleons. This

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result is nontrivial and it provides a significant test of the consistency of our perturbative expansion.

A distinctive feature of our approach is that the quark confinement is built in both for the nucleons and the pions. This is due to the fact that our expansion is of weak coupling for the hadrons, but of strong coupling for the quarks. To any finite order, the quarks can then move only by a finite number of lattice spacings, and therefore they can never be produced in the continuum limit, so that our expansion is only compatible with quark confinement. In this connection one can wonder whether the Wilson term to avoid spurious quarks is necessary in our approach: Since the quarks have no poles whatsoever to finite order, should we worry about the spurious ones? The actual settlement of this issue requires the study of the anomaly, which is presently under way.

To conclude a few words about the parameter of expansion. This is the inverse of a dimensionless constant entering the definition of the nucleon composites, which is, however, accompanied by the inverse of large numerical factors related to the number of quark components, which is 24 for up and down quarks. A similar situation occurs with the pions, but in the latter case the role of the number of quark components is more transparent [4], being the asymptotic parameter of a saddle point expansion. Since we have not evaluated any physical quantity, we cannot fix the value of the expansion parameter, and therefore at the present stage there is nothing we can say about the rate of convergence.

One can wonder about the apparent absense of a scale characterizing the dynamical region where the use of composites as fundamental variables will be physically convenient. Such a scale is hidden in the dimensionless expansion parameter, which, to obtain sensible results, must scale in a proper way with the lattice spacing [5].

The paper is organized as follows. In Sec. II we report, for the convenience of the reader, the formalism for nonlinear change of variables in Berezin integrals. In Sec. III we define the quark composites with the quantum numbers of the nucleons, in Sec. IV we evaluate the Jacobian, and in Sec. V the transformation functions of the change of variables. In Sec. VI we show that the the free action for the nucleon composites is the Dirac action.

In Sec. VII we couple the nucleons to the electromagnetic field and evaluate the first perturbative contribution to their effective action by assuming as unperturbed action the Dirac action and the quark action S_q as the perturbation. As already stated, the first perturbative contribution, which comes from $(S_q)^3$, turns out to be simply a renormalization of the electromagnetic action. It would not be difficult to evaluate higher order terms. One can indeed deduce by inspection that we will get a nucleon-nucleon interaction, namely, a fourth order term in the composites, from $(S_q)^6$. In such a term there can be no contribution from the gluons. Such a contribution will appear from $(S_q)^{12}$ in a configuration where there are nucleon composites at the vertices of a plaquette. Whether such terms give a finite contribution in the continuum limit or not, depends on how the parameter k scales with the lattice spacing, but again this dependence cannot be fixed without taking into account the pions which will give comparable or even more important contributions. We conclude this section by showing that our expansion of the quark propagator is of strong coupling.

In Sec. VIII we consider the weak couplings of the nucleons. Because of the well known difficulties with chirality, our results can only be regarded as a further illustration of the potentiality of our formalism in the generation of the structures appearing in the composite action starting from the constituents. Again we find that the first order correction, where the pions cannot contribute, is a pure renormalization.

II. BARIONIC COMPOSITES AS INTEGRATION VARIABLES

In this section all the fields are defined at one and the same lattice site. We assume the convention of summation over repeated indices.

Let us consider cubic quark composites

$$\psi_I = c_I^{i_1,i_2,i_3} \lambda_{i_1} \lambda_{i_2} \lambda_{i_3}. \tag{2.1}$$

We want to assume these composites as new integration variables in the Berezin integral which appears in the partition function. The above equations define indeed a nonlinear change of variables. Of course they cannot be inverted, but we will specify below in which sense the quark fields can be expressed in terms of the barionic composites.

We want to define the integral of a function of the ψ in such a way that its value be equal to that obtained by expressing these variables in terms of the λ, and performing the Berezin integral over the latter

$$\int [d\lambda] g[\psi(\lambda)] = \int [d\psi] g(\psi). \tag{2.2}$$

We remind the reader that the above integral involves the field components at a given site. Let us restrict ourselves to functions g which have a Taylor expansion. The Berezin integral is a linear functional which associates to any function g the coefficient of the (unique) monomial containing all the λ in a given order in its expansion. Let us denote this monomial by

$$\Lambda = \lambda_1 \lambda_2 \cdots \lambda_{3N}. \tag{2.3}$$

To define the integral of $g(\psi)$ over ψ we must determine all the monomials of the ψ which, when expressed in terms of the λ, are proportional to Λ (with nonzero coefficient). To this end we introduce the generic monomials

$$\Psi_I = \psi_1^{I_1} \psi_2^{I_2} \cdots \psi_N^{I_N}, \tag{2.4}$$

with degree

$$d_I = \sum_k I_k, \tag{2.5}$$

where I is a vector with components $I_1, \ldots, I_N$. It is understood that if $d_I = 0$, $\chi_I = 1$. Since all the odd composites have index of nilpotency 1, $I_k = 0,1$. Notice that

$$\Psi_I=\psi_k, \quad \text{if } I_k=1, \quad I_h=0 \quad \text{for } h\neq k. \tag{2.6}$$

We call fundamental, with weight J, those monomials which are proportional to Λ with nonzero coefficient

$$\Psi_I=J_I\Lambda, \quad J_I\neq 0. \tag{2.7}$$

We can expand any function $g(\psi)$ in terms of fundamental monomials plus irrelevant terms (in the sense that they do not contribute to the integral)

$$g=\sum_{d_I=N} g_I\Psi_I+\text{irrelevant terms}. \tag{2.8}$$

The definition of the integral over the ψ we are looking for is therefore

$$\int [d\psi]g(\psi)=\sum_{d_I=N} g_I J_I. \tag{2.9}$$

Note that, although in general different expansions

$$g(\psi)=\sum_{d_I=N} g_I\Psi_I+\text{irrelevant terms}=\sum_{d_I=N} g'_I\Psi_I + \text{irrelevant terms} \tag{2.10}$$

can exist, the above equality implies

$$\sum_{d_I=N} g_I J_I=\sum_{d_I=N} g'_I J_I, \tag{2.11}$$

since both the left and right hand sides are equal to the coefficient of Λ in the expansion of g in terms of the generating elements, so that the value of the integral does not depend on the particular expansion of g.

It is remarkable that if the composites are chosen in such a way that there is only one fundamental monomial, the integral becomes identical, apart from the weight, to the Berezin integral over the constituents. In this case it is convenient to define the integral over the composites exactly as a Berezin integral

$$\int [d\psi]\Psi=1, \tag{2.12}$$

and regard the weight J as the Jacobian of the transformation. Accordingly we will replace the definition (2.9) by

$$\int [d\lambda]g[\psi(\lambda)]=J\int [d\psi]g(\psi). \tag{2.13}$$

We will restrict ourselves to this case, which is also remarkable because more general integrals, depending on the ψ as well as the λ can be simply evaluated according to the equation

$$\int [d\lambda]g(\psi)\lambda_{i_1}\lambda_{i_2}\cdots\lambda_{i_{3n}}=f_{Ii_1i_2\cdots i_{3n}}J\int [d\psi]g(\psi)\Psi_I, \tag{2.14}$$

where the transformation functions f are defined below. The proof follows.

By hypotesis the product of our N composites is different from zero:

$$\psi_1\cdots\psi_N=\Psi. \tag{2.15}$$

We define the complementary monomials

$$C_I=\epsilon_I\Psi_{\hat{I}}, \tag{2.16}$$

where

$$\hat{I}_k=0,1 \quad \text{for } I_k=1,0, \tag{2.17}$$

and ϵ is chosen to be ± 1 in such a way that the equation

$$C_I\Psi_J=\delta_{I,J}\Psi, \quad \forall I,J, \quad d_I=d_J, \tag{2.18}$$

be satisfied. This equation implies that

$$\int [d\psi]C_I\Psi_J=\delta_{I,J}, \quad \forall I,J, \tag{2.19}$$

since, if $d_I\neq d_J$, $C_I\Psi_J$ is never a nonzero multiple of Ψ + irr. terms. Consider now the integral

$$\mathcal{I}=\int [d\lambda]g(\psi)\lambda_{i_1}\cdots\lambda_{i_m}. \tag{2.20}$$

It is obviously zero if $m>3N$, the total number of quarks, but also if $m\neq 3n$, with n integer. We then set, by definition

$$\int [d\psi]g(\psi)\lambda_{i_1}\cdots\lambda_{i_m}=0, \quad m\neq 3n. \tag{2.21}$$

When $m=3n$,

$$\mathcal{I}=\int [d\lambda]g^{(N-n)}(\psi)\lambda_{i_1}\cdots\lambda_{i_{3n}}, \tag{2.22}$$

$g^{(N-n)}(\psi)$ being the homogeneous portion of g of degree $N-n$ in the expansion of g in products of the ψ'. Since

$$g^{(N-n)}(\psi)=\sum_{d_I=n} g_I^{(N-n)}C_I \tag{2.23}$$

with uniquely determined coefficients

$$\mathcal{I}=J\sum_{d_I=n} g_I^{(N-n)}f_{Ii_1\cdots i_{3n}}, \tag{2.24}$$

which requires the following definition of the transformation functions:

$$C_I\lambda_{i_1}\cdots\lambda_{i_{3n}}=f_{Ii_1\cdots i_{3n}}\Psi. \tag{2.25}$$

But, of course,

$$\int [d\psi]g(\psi)\sum_{d_I=n} f_{Ii_1\cdots i_{3n}}\Psi_I=\sum_{d_I=n} g_I^{(N-n)}f_{Ii_1\cdots i_{3n}}. \tag{2.26}$$

Hence

$$\int [d\lambda] g(\psi)\lambda_{i_1}\cdots\lambda_{i_m} = J\int [d\psi] g(\psi)\lambda_{i_1}\cdots\lambda_{i_m} = \delta_{m,3n}\sum_{d_I=n} f_{Ii_1\cdots i_{3n}} J\Psi_I, \tag{2.27}$$

namely,

$$\lambda_{i_1}\cdots\lambda_{i_m} \sim \delta_{m,3n}\sum_{d_I=n} f_{Ii_1\cdots i_{3n}}\int [d\psi] g(\psi)\Psi_I \tag{2.28}$$

as far as the Berezin integral is concerned. It is perhaps worthwhile recalling that in the above equation all the fields are defined at the same site.

In our physical applications the quark variables appear in two sets λ_i and $\overline{\lambda}_i$, and in the Grassmann algebra they generate we have the antilinear conjugation $\lambda_i \to \overline{\lambda}_i$, $\overline{\lambda}_i \to \lambda_i$ satisfying, for generic elements $\overline{\xi\eta} = \overline{\eta}\,\overline{\xi}$. So we have to extend the previous formulas to include the conjugate variables. The basic Berezin integrals are

$$\int [d\lambda][d\overline{\lambda}]\overline{\Lambda}\Lambda = 1, \quad \int [d\psi][d\overline{\psi}]\overline{\Psi}\Psi = 1 \tag{2.29}$$

the ordering of $\overline{\Lambda},\Lambda$ and $\overline{\Psi},\Psi$ being immaterial if, as is the case in our physical applications, $N = 4\times$integer, since these monomials contain an even number of quark fields. The composites ψ_I are now accompanied by the composites $\overline{\psi}_I$ obtained by conjugation. Corresponding to Eqs. (2.18) and (2.25) we have by conjugation

$$\overline{\Psi_K\, C_I} = \delta_{K,I}\overline{\Psi}, \quad d_I = d_K, \tag{2.30}$$

$$\overline{\lambda}_{i_{3m}}\cdots\overline{\lambda}_{i_1}\overline{C}_I = \overline{f_{Ii_1\cdots i_{3m}}\,\Psi}, \tag{2.31}$$

and from $\Psi = J\Lambda$, $\overline{\Psi} = J\overline{\Lambda}$, if J is real, as it turns out to be the case. It can then be easily checked that, if we set by definition

$$\int [d\overline{\psi}][d\psi] g(\overline{\psi},\psi)\overline{\lambda}_{i_1}\cdots\overline{\lambda}_{i_m}\lambda_{k_1}\cdots\lambda_{k_{m'}} = 0, \quad m \neq 3\times\text{integer}, \quad m' \neq 3\times\text{integer}, \tag{2.32}$$

we have

$$\begin{aligned}\int [d\overline{\lambda}][d\lambda] g(\overline{\psi},\psi)\overline{\lambda}_{i_1}\cdots\overline{\lambda}_{i_m}\lambda_{k_1}\cdots\lambda_{k_{m'}} &= J^2\int [d\overline{\psi}][d\psi] g(\overline{\psi},\psi)\overline{\lambda}_{i_1}\cdots\overline{\lambda}_{i_{m'}}\lambda_{k_1}\cdots\lambda_{k_{m'}} \\ &= \delta_{m,3n}\delta_{m',3n'}\sum_{d_I=n,d_K=n'}\overline{f_{Ii_{3n}\cdots i_1}f_{Kk_1\cdots k_{3n'}}}\,J^2 \\ &\quad\times\int [d\overline{\psi}][d\psi] g(\overline{\psi},\psi)\overline{\Psi}_I\Psi_K.\end{aligned} \tag{2.33}$$

The conclusion is that

$$\overline{\lambda}_{i_1}\cdots\overline{\lambda}_{i_m}\lambda_{k_1}\cdots\lambda_{k_{m'}} \sim \delta_{m,3n}\delta_{m',3n'}\sum_{d_I=n,d_K=n'}\overline{f_{Ii_{3n}\cdots i_1}f_{Kk_1\cdots k_{3n'}}}\,\overline{\Psi}_I\Psi_K \tag{2.34}$$

is the substitution rule in the Berezin integral in the general case.

It should now be clear in which sense we can talk of a change of variables. Even though the constituents cannot be expressed in terms of the composites, we only need to invert trilinear expressions in the quark and antiquark fields, and this can be done according to Eq. (2.34).

III. THE NUCLEON COMPOSITES

We assume the nucleon composites to be [6]

$$\psi_{\tau\alpha} = -\frac{2}{3}k^{1/2}a^3\delta_{\tau\tau_2}\epsilon_{\tau_1\tau_3}(\gamma_5\gamma_\mu)_{\alpha\alpha_1} \times (C\gamma_\mu)_{\alpha_2\alpha_3}(\lambda_{\tau_1\alpha_1}\lambda_{\tau_2\alpha_2}\lambda_{\tau_3\alpha_3}). \tag{3.1}$$

In the above equation and in the following the summation over repeated indices is understood, C is the charge conjugation matrix, and

$$(\lambda_{\tau_1\alpha_1}\lambda_{\tau_2\alpha_2}\lambda_{\tau_3\alpha_3}) = \epsilon_{a_1a_2a_3}\lambda^{a_1}_{\tau_1\alpha_1}\lambda^{a_2}_{\tau_2\alpha_2}\lambda^{a_3}_{\tau_3\alpha_3}. \tag{3.2}$$

The fields $\lambda^a_{\tau\alpha}$ have color, isospin and Dirac indices a, τ, and α, respectively, and are related to the up and down quarks according to

$$\lambda^a_{1\alpha} = u^a_\alpha, \quad \lambda^a_{2\alpha} = d^a_\alpha. \tag{3.3}$$

Correspondingly $\psi_{1\alpha},\psi_{2\alpha}$ are the proton, neutron fields. It is easy to check that with the above definition they transform like the quarks under isospin and O(4) transformations.

Finally we should explain why we have included in the definition the cubic power of the lattice spacing a and the parameter k. The cubic power of a parameter with the dimension of a length, say l^3, is necessary to give the nucleon field the canonical dimension of a fermion field. At the same time a power of the lattice spacing at least cubic is necessary to make the kinetic term irrelevant. We have written for later

convenience l^3 in the form $k^{1/2}a^3$, where k is dimensionless and it must not diverge with vanishing lattice spacing.

There are altogether 8 ψ. Since monomials of less than 8 ψ cannot obviously contain all the λ, there can be at most one fundamental monomial Ψ

$$\Psi=\psi_{1,1}\cdots\psi_{1,4}\psi_{2,1}\cdots\psi_{2,4}=J\Lambda. \tag{3.4}$$

We have chosen for the product Λ of all the quark field components at a given site the following ordering

$$\Lambda=P(\lambda_{11})\cdots P(\lambda_{14})P(\lambda_{21})\cdots P(\lambda_{24}), \tag{3.5}$$

where

$$P(\lambda_{\tau\alpha})=\lambda^1_{\tau\alpha}\lambda^2_{\tau\alpha}\lambda^3_{\tau\alpha}. \tag{3.6}$$

The Jacobian is obviously proportional to k^4a^{24}. The nontrivial factor of proportionality will be evaluated in the next section.

IV. EVALUATION OF THE JACOBIAN

The weight of nonrelativistic nucleon composites was evaluated in the second of Ref. [3]. In this section we consider the relativistic case. For simplicity we will omit the trivial factor k^4a^{24} which will be reinstated at the end of the section.

The function Ψ is a homogeneous polynomial of degree 24 in the quark fields. But many terms vanish because they contain some component of the quark field to a power higher than 1, and the remaining terms are proportional to Λ. In order to identify the vanishing terms and to get rid of them, we find convenient the following representation of the γ matrices:

$$\gamma_k=i\begin{pmatrix}0 & \sigma_k\\ -\sigma_k & 0\end{pmatrix},\quad \gamma_4=-\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\quad \gamma_5=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{4.1}$$

and of the charge conjugation matrix

$$C=-i\begin{pmatrix}\sigma_2 & 0\\ 0 & -\sigma_2\end{pmatrix} \tag{4.2}$$

which satisfy the equations

$$C^{-1}\gamma_\mu^TC=-\gamma_\mu. \tag{4.3}$$

In such a representation the nucleon composites take the simpler form

$$\psi_{1i}=-4\delta_{ii_2}\epsilon_{i_1i_2}(u_{+i_2}u_{-i_3}d_{-i_1}),$$

$$\psi_{1i+2}=-4\delta_{ii_2}\epsilon_{i_1i_2}(u_{-i_2}u_{+i_3}d_{+i_1}),\quad i=1,2,$$

$$\psi_{2\alpha}=-\psi_{1\alpha}(u\leftrightarrow d), \tag{4.4}$$

where the indices + and − refer to the upper and lower components of Dirac spinors.

To make the following formulas more readable we introduce the notations

$$u^a_{+i}=x^a_i,\quad u^a_{-i}=y^a_i,$$

$$d^a_{+i}=z^a_i,\quad d^a_{-i}=w^a_i, \tag{4.5}$$

$$[x^a,y^b]=x^a_1y^b_2-x^a_2y^b_1. \tag{4.6}$$

Therefore, if we remember the convention (3.2)

$$(x_i[y,z])=\epsilon^{abc}[x^a_iy^b_1z^c_2-x^a_iy^b_2z^c_1] \tag{4.7}$$

the nucleon composites become

$$\psi_{1,i}=4(x_i[y,w]),\quad \psi_{1,i+2}=4(y_i[x,z]),$$

$$\psi_{2,i}=4(z_i[y,w]),\quad \psi_{2,i+2}=4(w_i[x,z]). \tag{4.8}$$

The calculation of Ψ is now straightforward and it is based on the following identities:

$$(u\pi)(uAB)(uCD)=P(u)[(\pi C)(ABD)+(\pi D)(ABC)],$$

$$(uuA)(u\pi)=-2P(u)(A\pi),\quad \pi=ab,[a,b], \tag{4.9}$$

u,a,b,A,B,C,D being any of the quark fields x_i,y_i,z_i,w_i, and

$$P(u)=\frac{1}{6}(xxx). \tag{4.10}$$

To avoid unnecessary proliferation of terms, as a guiding rule, it is convenient to express the $[a,b]$ in terms of a and b in the various ψ only as far as to be able to use the previous identities. Let $\psi_{11}\cdots\psi_{14}=4^4P$, $\psi_{21}\cdots\psi_{24}=4^4N$, so that $\Psi=2^{16}PN$, and

$$P=(x_1[y,w])(x_2[y,w])(y_1[x,z])(y_2[x,z]). \tag{4.11}$$

If we explicitate $[x,z]$, and use Eq. (4.9) we get

$$\begin{aligned}P=&\frac{1}{4}P(x_1)\{(y_2[y,w])(y_1z_2z_2)+c_1\psi_{22}\}\psi_{12}-\frac{1}{4}P(x_2)\\&\times\{(y_2[y,w])(y_1z_1z_1)+c_2\psi_{21}\}\psi_{11}\\&-\frac{1}{16}\psi_{11}\psi_{12}\{(x_1y_1z_2)(x_2y_2z_1)-(x_1y_2z_2)(x_2y_1z_1)\}.\end{aligned} \tag{4.12}$$

Since $\psi_{2\alpha}=-\psi_{1\alpha}(x\leftrightarrow z,y\leftrightarrow w)$ it follows that

$$\begin{aligned}N=&\frac{1}{4}P(z_1)\{(w_2[y,w])(w_1x_2x_2)+c_1'\psi_{12}\}\psi_{22}-\frac{1}{4}P(z_2)\\&\times\{(w_2[y,w])(w_1x_1x_1)+c_2'\psi_{11}\}\psi_{21}\\&-\frac{1}{16}\psi_{21}\psi_{22}\{(z_1w_1x_2)(z_2w_2x_1)-(z_1w_2x_2)(z_2w_1x_1)\}.\end{aligned} \tag{4.13}$$

Observe now that in the product PN the terms with coefficients $c_{1,2}, c'_{1,2}$ do not contribute. Consider, for example, the one with coefficient c_1. Its factor ψ_{22}, in PN, will be multiplied either with itself or with $P(z_2)$ giving zero, in the second case, because $P(z_2)$ is the monomial of maximum degree in z_2, and ψ_{22} is linear in z_2. For this same reason it is evident that nonzero contribution can arise only from the products $P(x_i)\{\cdots\}P(z_i)\{\cdots\}$, $i=1,2$, (no summation), and from the product of the last two terms in P and N. Working out the details with the help of Eq. (4.9), we obtain

$$PN = 24P(x_1)P(x_2)P(z_1)P(z_2)R, \tag{4.14}$$

where

$$R = (y_1[y,w])(y_2[y,w])(w_1[y,w])(w_2[y,w]). \tag{4.15}$$

Using the same procedure, explicitating, say, first the variables in the ''links'' with y_1, y_2, we get

$$R = 2^3 \times 3^2 \times 5 P(y_1)P(y_2)P(w_1)P(w_2), \tag{4.16}$$

so that, if we reinstate the factor $k^4 a^{24}$, we get

$$J = k^4 a^{24} 2^{22} \times 3^3 \times 5. \tag{4.17}$$

V. EVALUATION OF THE TRANSFORMATION FUNCTIONS

Also in this section we omit the factors k and a which will be reinstated at the end. According to Eqs. (2.18) (2.25) the monomial $C_{\tau\alpha}$ complementary to $\Psi_{\tau\alpha}$ must satisfy

$$C_{\tau\alpha}\psi_{\tau'\alpha'} = \delta_{\alpha\alpha'}\delta_{\tau\tau'}\Psi, \tag{5.1}$$

so that

$$C_{\tau\alpha} = (-1)^{\alpha}\psi_{11}\cdots\hat{\psi}_{\tau\alpha}\cdots\psi_{24}, \tag{5.2}$$

and the transformation functions are defined by

$$C_{\tau\alpha}\lambda^{a_1}_{\tau_1\alpha_1}\lambda^{a_2}_{\tau_2\alpha_2}\lambda^{a_3}_{\tau_3\alpha_3} = f_{\tau\alpha,\tau_1\alpha_1,\tau_2\alpha_2,\tau_3\alpha_3,a_1,a_2,a_3}\Psi. \tag{5.3}$$

As a first step to evaluate the transformation function f we exploit the invariance of Ψ under color, isospin and O(4) transformations. We will find in this way that f is constructed in terms of invariant tensors. Let us start by a transformation in color space

$$C_{\tau\alpha}U^{a_1b_1}U^{a_2b_2}U^{a_3b_3}\lambda^{b_1}_{\tau_1\alpha_1}\lambda^{b_2}_{\tau_2\alpha_2}\lambda^{b_3}_{\tau_3\alpha_3} = f_{\tau\alpha,\tau_1\alpha_1,\tau_2\alpha_2,\tau_3\alpha_3,a_1,a_2,a_3}\Psi. \tag{5.4}$$

The above equation can be rewritten

$$U^{a_1b_1}U^{a_2b_2}U^{a_3b_3}f_{\tau\alpha,\tau_1\alpha_1,\tau_2\alpha_2,\tau_3\alpha_3,b_1,b_2,b_3}\Psi = f_{\tau\alpha,\tau_1\alpha_1,\tau_2\alpha_2,\tau_3\alpha_3,a_1,a_2,a_3}\Psi, \tag{5.5}$$

showing that f is an invariant tensor in color, and therefore factorizes according to

$$f_{\tau\alpha,\tau_1\alpha_1,\tau_2\alpha_2,\tau_3\alpha_3,a_1,a_2,a_3} = \epsilon_{a_1a_2a_3}h_{\tau\alpha,\tau_1\alpha_1,\tau_2\alpha_2,\tau_3\alpha_3}. \tag{5.6}$$

Let us now come to isospin and O(4) transformations. To ease the notation we represent the pair of indices $\tau\alpha$ by the single index i. If under isospin, O(4) and parity transformations, $S = \gamma_4$

$$\psi'_i = S_{ij}\psi_j \tag{5.7}$$

then

$$C'_i = S^c_{ij}C_j, \tag{5.8}$$

where S^c is the contragradient representation

$$S^c = (S^T)^{-1}. \tag{5.9}$$

Performing such transformations in Eq. (5.3) we then find that h must satisfy the equation

$$S^c_{kj}S_{i_1j_1}S_{i_2j_2}S_{i_3j_3}h_{jj_1j_2j_3} = h_{ki_1i_2i_3} \tag{5.10}$$

showing that it is an invariant tensor for the representation $S^c \otimes S^{\otimes 3}$.

For isospin transformations $S = D^{1/2}$, since the quarks are in the defining representation, and $S^c \sim S$. We must therefore look for the invariant tensors for the representation $(D^{1/2})^{\otimes 4}$. As it is well known, the number of these invariant tensors is equal to the number of times the identity representation is contained in the reduction of $(D^{1/2})^{\otimes 4}$ into a sum of irreps, hence it is equal to 2. We choose the following linearly independent tensors:

$$E^{(1)}_{\tau\tau_1\tau_2\tau_3} = \delta_{\tau\tau_1}\epsilon_{\tau_2\tau_3}, \quad E^{(2)}_{\tau\tau_1\tau_2\tau_3} = \delta_{\tau\tau_2}\epsilon_{\tau_1\tau_3}. \tag{5.11}$$

In the following developments we will also need the tensor

$$E^{(3)}_{\tau\tau_1\tau_2\tau_3} = \delta_{\tau\tau_3}\epsilon_{\tau_1\tau_2} \tag{5.12}$$

which is related to the previous ones by

$$E^{(1)} - E^{(2)} + E^{(3)} = 0. \tag{5.13}$$

The tensor h can then be decomposed according to

$$h = E^{(1)} \otimes t^{(1)} + E^{(2)} \otimes t. \tag{5.14}$$

The corresponding calculation for O(4) is more complicated, since for the subgroup $O(4)_c$ connected to the identity, $S \sim S^c$ is the representation $D^{(1/2,0)} \oplus D^{(0,1/2)}$, and $S^{\otimes 4}$ admits 10 invariant tensors. It is then convenient to determine the form of t directly, rather than going through the intermediate step of finding these 10 invariant tensors. For this purpose we write it in the form

$$t_{ii_1i_2i_3} = \sum_{AB} c_{AB}(\Gamma_A)_{i_1i}(\Gamma_B C^{-1})_{i_2i_3}, \tag{5.15}$$

where

$$\Gamma_A = I, \gamma_5, \gamma_5\gamma_\mu, \gamma_\mu, \gamma_\mu\gamma_\nu, \quad \mu < \nu. \tag{5.16}$$

No confusion should arise between the charge conjugation matrix C and the complementary monomials C_I. Now we proceed to enforce various conditions. First, as it is seen by inspection, h must be totally symmetric under the exchange of the numbered indices. For this it is sufficient to impose the symmetry under the permutations (12),(23)

$$(12)h = h, \quad (23)h = h. \tag{5.17}$$

The first of these equations yields

$$t^{(1)} = (12)t, \tag{5.18}$$

which allows us to write

$$h = (12)E^{(2)} \otimes (12)t + E^{(2)} \otimes t. \tag{5.19}$$

The second one, using Eq. (5.13), gives two conditions on t

$$(23)t = t, \quad [I + (12) + (13)]t = 0. \tag{5.20}$$

Since

$$(\Gamma_B C^{-1})^T = \epsilon_B \Gamma_B C^{-1}, \tag{5.21}$$

where

$$\epsilon_B = -1 \quad \text{for } \Gamma_B = I, \gamma_5, \gamma_5\gamma_\mu,$$

$$\epsilon_B = +1, \quad \text{for } \Gamma_B = \gamma_\mu, \gamma_\mu\gamma_\nu, \quad \mu < \nu, \tag{5.22}$$

the first of the above conditions implies

$$c_{AB} = 0, \quad \text{for } \epsilon_B = -1. \tag{5.23}$$

Next we require t to be an invariant tensor

$$S^c \otimes S^{\otimes 3} t = t \tag{5.24}$$

for S the said representation of $O(4)_c$ and $S = \gamma_4$, namely,

$$\sum_{AB} c_{AB} S\Gamma_A S^{-1} \otimes S\Gamma_B C^{-1} S^T = \sum_{AB} c_{AB} \Gamma_A \otimes \Gamma_B C^{-1}. \tag{5.25}$$

Using the relations

$$C^{-1} S^T = S^{-1} C^{-1}, \quad C^{-1}\gamma_4^T = -\gamma_4 C^{-1}, \tag{5.26}$$

the above conditions reduce t to the form

$$t = b_1 \gamma_5\gamma_\mu \otimes \gamma_\mu C^{-1} + b_2 \epsilon_{\mu\nu\rho\sigma} \gamma_\mu\gamma_\nu \otimes \gamma_\rho\gamma_\sigma C^{-1}. \tag{5.27}$$

The second of the symmetry conditions (5.20), is actually implied by the first one and Eq. (5.24), for group theoretical reasons. To determine t completely we then need two more conditions. The first one is

$$C_{\tau\alpha}(\lambda_{11}\lambda_{\tau_2\alpha_2}\lambda_{11}) = 0. \tag{5.28}$$

This follows from the observation that, in the representation where γ_5 is diagonal, $C_{\tau\alpha}$ contains at least 5 out of the 6 factors $\psi_{1,\alpha}, \psi_{2,i+2}$, $\alpha = 1, \ldots, 4$, $i = 1,2$. Since according to Eq. (4.8) each of these factors is linear in $\lambda_{1,i} = x_i$, each monomial in the lhs of the above equation contains at least 7 of these factors and must therefore vanish. Hence

$$h_{\tau\alpha,11,\tau_2\alpha_2,11} = 0 \tag{5.29}$$

which yields

$$b_2 = 0. \tag{5.30}$$

The second condition, which determines b_1 is

$$\frac{1}{4}\delta_{\tau\tau'}\delta_{\alpha\alpha'} = -\delta_{\tau'\tau_2}\epsilon_{\tau_1\tau_3}(\gamma_5\gamma_\mu)_{\alpha'\alpha_1} \times (C\gamma_\mu)_{\alpha_2\alpha_3} h_{\tau\alpha\tau_1\alpha_1\tau_2\alpha_2\tau_3\alpha_3}. \tag{5.31}$$

It is obtained by multiplying both sides of Eq. (5.3) by

$$-2/3\epsilon_{a_1a_2a_3}\delta_{\tau'\tau_2}\epsilon_{\tau_1\tau_3}(\gamma_5\gamma_\mu)_{\alpha'\alpha_1}(C\gamma_\mu)_{\alpha_2\alpha_3} \tag{5.32}$$

and by summing over repeated indices. The resulting value of b_1 is

$$b_1 = \frac{1}{96}, \tag{5.33}$$

so that finally, inserting the factor $a^{-3}k^{-1}$ we have

$$h_{\tau\alpha\tau_1\alpha_1\tau_2\alpha_2\tau_3\alpha_3} = \frac{1}{96}a^{-3}k^{-1/2}[\delta_{\tau\tau_2}\epsilon_{\tau_1\tau_3}(\gamma_5\gamma_\mu)_{\alpha_1\alpha}(\gamma_\mu C^{-1})_{\alpha_2\alpha_3} + \delta_{\tau\tau_1}\epsilon_{\tau_2\tau_3}(\gamma_5\gamma_\mu)_{\alpha_2\alpha}(\gamma_\mu C^{-1})_{\alpha_1\alpha_3}]. \tag{5.34}$$

VI. THE FREE ACTION OF THE NUCLEON COMPOSITES

Since the integral over the ψ is equal to the Berezin integral, we can assume for the nucleon the Dirac action

$$S_N(r_N, m_N, V) = a^4 \sum_{xy} \bar{\psi}(x) Q(r_N, m_N, V)_{x,y} \psi(y), \tag{6.1}$$

where

$$Q(r_N, m_N, V)_{x,y} = -\frac{1}{2a}\sum_\mu (r_N - \gamma_\mu) V_\mu(x) \delta_{y,x+\mu} + \left(m_N + \frac{4r_N}{a}\right)\delta_{x,y}. \tag{6.2}$$

In the above equation r_N is the Wilson parameter

$$0 < r_N \leqslant 1, \tag{6.3}$$

V_μ is the link variable associated to the em field, and m_N is the mass of the nucleon. Obviously V_μ and m_N are diagonal matrices in isospin space. We have adopted the standard conventions

$$\mu \in \{-4, \ldots, 4\},$$

$$\gamma_{-\mu} = -\gamma_\mu,$$

$$V_{-\mu}(x) = V_\mu^+(x-\mu). \tag{6.4}$$

Notice that the above range of values of μ holds only for the wave operator Q. In the sums occurring in the definition of the nucleon composites, and therefore of the transformation functions $\mu \in \{1, \ldots, 4\}$.

Let us consider the free correlation functions

$$\langle \bar{\psi}(x)\psi(y)\rangle = \frac{1}{Z_N}\int [d\bar{\lambda}d\lambda]\bar{\psi}(x)\psi(y) \times \exp[-S_N(r_N, m_N, 1)], \tag{6.5}$$

where the partition function is

$$Z_N = \int [d\bar{\lambda}d\lambda]\exp[-S_N(r_N, m_N, 1)]. \tag{6.6}$$

Of course here

$$[d\bar{\lambda}d\lambda] = \prod_x [d\bar{\lambda}(x)d\lambda(x)]. \tag{6.7}$$

Introducing the ψ as new integration variables we immediately see that the nucleon composites have a canonical propagator

$$\langle \bar{\psi}(x)\psi(y)\rangle = -\frac{1}{a^4}[Q(r_N, m_N, 1)]^{-1}_{yx}. \tag{6.8}$$

It is perhaps worth while noticing that nothing depends at this stage on the parameter k appearing in their definition, but this constant will become the inverse parameter of expansion when we will take into account the QCD action.

VII. PERTURBATION THEORY AND QUARK CONFINEMENT

In this section we use our formalism to set up a perturbative expansion in QCD. Since S_N is an irrelevant operator, it can freely be added to the standard QCD action. We therefore assume as the total action

$$S = S_N + S_G + S_q(r, m_q, Uv), \tag{7.1}$$

where S_G is the pure gluon and em action,

$$S_q(r, m_q, Uv) = a^4 \sum_x \bar{\lambda}(x) Q(r, m_q, Uv)_{x,y}\lambda(y), \tag{7.2}$$

m_q is the quark mass, r the Wilson parameter, which need not in general be equal to that of the nucleons r_N, but it is obviously subject to the same restriction (6.3), U_μ and v_μ are the link variables associated to the gluon and em fields of the quarks, respectively, under the conventions (6.4).

Accordingly the partition function is

$$Z = \int [dU][dv][d\bar{\lambda}d\lambda]\exp[-(S_N + S_G + S_q)]$$

$$= \mathcal{J}\int [dU][dv][d\bar{\psi}d\psi]\exp[-(S_N + S_G + S_q)],$$

$$\mathcal{J} = \left(\prod_x J\right)^2. \tag{7.3}$$

In the last equality we have assumed the nucleons as new integration variables and the quark fields must be understood their functions in the sense specified in Sec. V. The quark action S_q must now be treated as a perturbation

$$Z = \mathcal{J}\int [dU][dv]\exp(-S_G)\int [d\bar{\psi}d\psi] \times \sum_{n=0}^{\infty} \frac{1}{(3n)!}(-S_q)^{3n}\exp(-S_N). \tag{7.4}$$

It should by now be obvious why non cubic powers of S_q do not contribute. Since the factor $(S_q)^{3n}$ yields a factor k^{-n}, because of the dependence on k of the transformation functions, we have an expansion in inverse powers of k. We should emphasize that we are not treating the gauge fields perturbatively.

We will evaluate only the first order contribution to the partition function, where the pions cannot contribute. This splits into two parts

$$-\frac{1}{3!}S_q^3 = T_1 + T_2. \tag{7.5}$$

T_1 comes from the hopping term and T_2 from the ''mass'' term. Obviously there is no interference between the two.

Let us start from T_1

$$T_1 = \frac{1}{2^3}\frac{1}{3!}a^9 \sum_x \sum_\mu \bar{\lambda}^{a_1}_{\tau_1\alpha_1}(x)\bar{\lambda}^{a_2}_{\tau_2\alpha_2}(x)\bar{\lambda}^{a_3}_{\tau_3\alpha_3}(x) \times U_\mu^{a_1b_1}(x)U_\mu^{a_2b_2}(x)U_\mu^{a_3b_3}(x)v_{\mu,\tau_1}(x)v_{\mu,\tau_2}(x)v_{\mu,\tau_3}(x) \times (r-\gamma_\mu)_{\alpha_1\beta_1}(r-\gamma_\mu)_{\alpha_2\beta_2}(r-\gamma_\mu)_{\alpha_3\beta_3} \times \lambda^{b_1}_{\tau_1\beta_1}(x+\mu)\lambda^{b_2}_{\tau_2\beta_2}(x+\mu)\lambda^{b_3}_{\tau_3\beta_3}(x+\mu), \tag{7.6}$$

which has been written using the fact that, to generate a nucleon field, the positions of the quarks must coincide with one another. Now we will use the expressions (5.6) and (5.34) for the transformation functions and the relations

$$\epsilon^{a_1a_2a_3}U_\mu^{a_1b_1}U_\mu^{a_2b_2}U_\mu^{a_3b_3} = \epsilon^{b_1b_2b_3}. \tag{7.7}$$

Moreover we observe that the product of the em fields acting over the quarks generates the electromagnetic field acting over the nucleons according to

$$\epsilon_{\tau_2\tau_3}\nu_{\mu,\tau_1}\nu_{\mu,\tau_2}\nu_{\mu,\tau_3}=\epsilon_{\tau_2\tau_3}V_{\mu,\tau_1} \quad \text{(no summation)}. \tag{7.8}$$

Since the sum over color indices gives a factor 6, performing also the sum over isospin indices we get

$$T_1\sim\frac{1}{2^3}\frac{1}{3!}\frac{12}{96^2}k^{-1}a^3\sum_x\sum_\mu\bar{\psi}_{\tau\alpha}(x)V_\mu(x)\psi_{\tau\beta}(x+\mu)\times\{2(r\gamma_\nu\gamma_\rho+\gamma_\nu\gamma_\mu\gamma_\rho)\mathrm{Tr}[\gamma_\rho(r+\gamma_\mu)\gamma_\nu(r-\gamma_\mu)]+\gamma_\nu\gamma_5(r-\gamma_\mu)\gamma_\rho(r+\gamma_\mu)\gamma_\nu(r-\gamma_\mu)\gamma_5\gamma_\rho\}_{\alpha\beta}. \tag{7.9}$$

We remind the reader that the sign $\sim$ means equality under Berezin integrals. The sums over ν and ρ finally give

$$T_1\sim\frac{3}{8}\frac{1}{24^2}(2+r^2)k^{-1}a^4\sum_x\sum_\mu\frac{1}{2a}\bar{\psi}(x)\times(r'_N-\gamma_\mu)V_\mu(x)\psi(x+\mu), \tag{7.10}$$

where

$$r'_N=r\frac{2r^2+1}{2+r^2}. \tag{7.11}$$

Notice that also r'_N satisfies the restriction (6.3), and in particular $r'_N=0,1$ for $r=0,1$.

In a similar way we get

$$T_2\sim-\frac{3}{8}\frac{1}{24^2}(2+r^2)k^{-1}a^4\sum_x\bar{\psi}\left(m'_N+\frac{4r'_N}{a}\right)\psi, \tag{7.12}$$

where

$$m'_N=\frac{4}{(2+r^2)}\left[2a^2\left(m_q+\frac{4r}{a}\right)^3-r(2r^2+1)\frac{1}{a}\right]. \tag{7.13}$$

In conclusion the total first order contribution can be written as a pure renormalization of the electromagnetic action of the nucleons

$$(S_N)_1=-\frac{3}{8}\frac{1}{24^2}(2+r^2)k^{-1}S_N(r'_N,m'_N,V). \tag{7.14}$$

Some comments concerning higher order terms. One can deduce by inspection that we will get a nucleon-nucleon interaction (quartic in the composites), to second order. In such a term there can be no contribution from the gluons. This will appear to fourth order in a configuration where there are nucleon composites at the vertices of a plaquette. But, as explained at the beginning, before going to higher order we must include the pions.

We can also evaluate the quark-quark correlation function. It is obvious that at order n the quarks can move only by n lattice spacings. Consider for instance the quark propagator at first order

$$\langle\bar{\lambda}^a(x)\Gamma^{ab}_{xy}\lambda^b(y)\rangle=\frac{1}{Z}\mathcal{J}\int[dU][dv]\exp(-S_G)\times\int[d\bar{\psi}][d\psi]\bar{\lambda}^a(x)\Gamma^{ab}_{xy}\lambda^b(y)\frac{1}{2}S_q^2\times\exp(-S_N). \tag{7.15}$$

Notice that we have connected the quarks by a string of gluons Γ, because otherwise the correlation function would vanish because of gauge invariance. Proceeding as in the case of the nucleons we find that this correlation funtion is different from zero only for $y=x,x+\mu$. Our perturbative series results to be a weak coupling expansion for the nucleons, but a strong coupling expansion for the quarks. To establish quark confinement in the present context, we should carry out this expansion to infinite order, whereas obviously we will calculate physical processes only to finite order. Our approach, however, has the desirable property that in such calculations quarks are never produced, a result which is only compatible with quark confinement. It follows that in the present expansion there appear no poles whatsoever of the quarks, and the situation is similar in the expansion for the pions [4]. We therefore wonder whether we can forget altogether the Wilson term of the quarks (but not, of course, the Wilson term of the nucleons). A convincing assessement of this issue requires the study of the anomaly, a problem which is under investigation.

VIII. THE WEAK INTERACTIONS OF THE NUCLEONS

In this section we study the electroweak interactions of the nucleons. Because of the well known difficulties with chirality, however, in the present form our calculations can only be regarded as a further illustration of the potentiality of our approach in the reconstruction of the structures appearing in the action of barionic composites. For this purpose it is sufficient to restrict ourselves to the charge changing weak interactions neglecting the Wilson term for the quarks. The corresponding action for the quarks is

$$S_q^{ch}=i\frac{1}{2}g\cos\theta a^4\sum_x\sum_{h=1,2}j^h_\mu(x)W^h_\mu, \tag{8.1}$$

where θ is the Cabibbo angle, W_μ are the intermediate vector bosons, and $j^h_\mu(x)$ is the charge changing quark current

$$j^h_\mu(x)=\frac{1}{2}\left[\bar{\lambda}(x)\tau^h\gamma_\mu\frac{1}{2}(1-\gamma_5)\lambda(x+\mu)+\bar{\lambda}(x+\mu)\tau^h\gamma_\mu\frac{1}{2}(1-\gamma_5)\lambda(x)\right]. \tag{8.2}$$

The above expression for the quark current is obtained by writing an action invariant under the weak isospin and retain-

ing the first two terms in the expansion of the related link variables. Also the (irrelevant) action of the nucleons must be invariant under the weak isospin transformations, so that it contains the term

$$S_\psi^{ch} = i\frac{1}{2} g \cos\theta a^4 \sum_x \sum_{h=1,2} J_\mu^h(x) W_\mu^h, \tag{8.3}$$

with the corresponding charge changing nucleon current

$$J_\mu^h(x) = \frac{1}{2}\Big[\overline{\psi}(x)\tau^h \gamma_\mu \frac{1}{2}(1-\gamma_5)\psi(x+\mu) + \overline{\psi}(x+\mu)\tau^h \gamma_\mu \frac{1}{2}(1-\gamma_5)\psi(x)\Big]. \tag{8.4}$$

The first order QCD correction to this action comes obviously from a term quadratic in S_q

$$(S_\psi^{ch})_1 = \frac{1}{2}[S_q(0,m_q,1)]^2 i\frac{1}{2} g \cos\theta a^4 \sum_x \sum_{h=1,2} j_\mu^h(x) W_\mu^h(x). \tag{8.5}$$

We have suppressed the em and the gluon fields for simplicity, but it is easy to check that at this order these fields do not contribute.

Following our procedure, we find

$$\begin{aligned}(S_\psi^{ch})_1 \sim{}& \frac{6}{8}\frac{1}{96^2} k^{-1} i\frac{1}{2} g\cos\theta a^4 \\ &\times \sum_x \sum_{h=1,2} W_\mu^h(x) \overline{h}_{\tau\alpha,\tau_1\alpha_1\tau_2\alpha_2\tau_3\alpha_3} (\tau^h)_{\tau_3\sigma_3} \\ &\times \Big[\gamma_\mu \frac{1}{2}(1-\gamma_5)\Big]_{\alpha_3\beta_3} (\gamma_\mu)_{\alpha_1\beta_1} (\gamma_\mu)_{\alpha_2\beta_2} \\ &\times h_{\sigma\beta\tau_1\beta_1\tau_2\beta_2\sigma_3\beta_3} \frac{1}{2}[\overline{\psi}_{\tau\alpha}(x)\psi_{\sigma\beta}(x+\mu) \\ &+ \overline{\psi}_{\tau\alpha}(x+\mu)\psi_{\sigma\beta}(x)].\end{aligned} \tag{8.6}$$

By using the expression of the structure functions and by performing the sums over isospin and Dirac indices we find that the first order contribution to the charge changing weak interaction of the nucleons is a pure renormalization

$$(S_\psi^{ch})_1 \sim \frac{3}{8}\frac{1}{24^2} k^{-1} S_\psi^{ch}. \tag{8.7}$$

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