Dual parton model motivated parametrization of the hadronic spectra

O. Gaponenko
Institute for Nuclear Research, The Academy of Sciences of Russia, Moscow, Russia
(Received 7 December 1998; published 24 August 1999)

A new parametrization of the nonscaling hadronic spectra, based on the dual parton model, is proposed. Although in the central region this new parametrization resembles the Wdowczyk-Wolfendale formulas, it does not lead to the decrease of the inelasticity coefficient with the rise of energy. The physical grounds of the proposed parametrization are discussed. For the process $p+\bar{p}\to\pi^{\pm}+X$ a simple empirical formula built upon the suggested approach is obtained. [S0556-2821(99)01317-X]

PACS number(s): 13.87.Fh, 12.40.Nn, 13.85.−t

I. INTRODUCTION

The concept of scaling in hadronic processes was introduced as a manifestation of the expectation that in the high energy limit hadronic interactions should not depend on energy and so they become scale invariant. Further experiments had shown, however, that at high energies a large violation of scaling may be still observed in the central region and now a drastic violation of scaling is seen, in fact, at all energies of modern accelerators and with the increase of energy no decline in size of the scaling violation was found.

A rather general parametrization of hadronic spectra with regard to the Feynman scaling violation was proposed by Wdowczyk and Wolfendale (WW) [1–4]:

$$\frac{2E^*}{\sqrt{s}}\frac{d^2N}{dx\,dP_\perp}=k(s;s_1)\left(\frac{s}{s_0}\right)^{\alpha} f\left(x\left(\frac{s}{s_0}\right)^{\alpha};P_\perp\right). \tag{1.1}$$

Here $x=P_\parallel^*/P_0^*$ is Feynman's variable, k and f are some functions, and s_0, s_1 and α are parameters. The formulas of this type (WW formulas) could describe accelerator data very well over a wide range of energy. For instance, for the reaction $p+\bar{p}\to\text{charged particles}+X$ it was found (see [5–7])

$$\int f\left(x\left(\frac{s}{s_0}\right)^{\alpha};P_\perp\right)dP_\perp=\left(1-\left(\frac{s}{s_0}\right)^{\alpha}x\right)^4,$$

$$\alpha=0.26,\quad s_0=3.4\times10^3\ \text{GeV}^2;$$

$$k(s;s_1)\left(\frac{s}{s_0}\right)^{\alpha}=A\left(\frac{s}{s_1}\right)^{\alpha'}, \tag{1.2}$$

$$A=1.67,\quad \alpha'=0.11,\quad s_1=6.3\times10^2\ \text{GeV}^2,$$

and hence

$$\frac{1}{x}\frac{dN^{p+\bar{p}\to\text{charged particles}}}{dx}=A\left(\frac{s}{s_1}\right)^{\alpha'}\left(1-\left(\frac{s}{s_0}\right)^{\alpha}x\right)^4. \tag{1.3}$$

One of the main problems which arises at dealing with the expressions (1.1), (1.2) is that the inelasticity coefficient calculated on the base of these formulas shows a strong tendency to decrease with the rise of energy,

$$K=\int\frac{1}{x}\frac{dN^{p+\bar{p}\to\text{charged particles}}}{dx}dx$$

$$=\frac{1}{5}A\left(\frac{s}{s_1}\right)^{\alpha'}\Big/\left(\frac{s}{s_0}\right)^{\alpha}\underset{s\to\infty}{\to}0, \tag{1.4}$$

but such behavior is in a certain contradiction with the data of various cosmic ray experiments. At different times different attempts were made to ''justify'' the WW formula (see, for example, [5–8]); in so doing rather special additional hypotheses were used sometimes to prevent the predicted fall of the inelasticity.

These difficulties around the WW formulas are, to some extent, a reflection of the complexity of the general situation. QCD, which is considered today as the best candidate for the future theory of strong interactions, cannot provide so far the full description of all hadronic processes. It is known that at available energies a large number of the hadronic events is accounted for by the soft interactions. The QCD perturbation theory cannot be applied under such circumstances and one has to find another approach.

In this work we will employ the dual parton model (DPM). This model was developed during the past years and now it is successfully used for the description of the nucleon-nucleon, nucleon-nucleus and nucleus-nucleus soft hadronic interactions (see, for example, [9] and references therein). In the following sections we will investigate with the help of the DPM the problem of the WW formulas. It will be shown that in our approach the nonscaling parametrization of the data can be given and the fall of the inelasticity can be avoided without use of any additional hypothesis. Unlike the parametrizations (1.1)–(1.3), where a validity of the extrapolation of the initial formulas to a wider region of Feynman's variable is just implied, an application of a reliable theoretical model enables one to check the nature of such extrapolation in the fragmentation regions. It also offers some insight (though in the framework of a model) into the mechanisms and the causes of the Feynman scaling violation.

II. THE PROBLEM FROM THE DPM'S POINT OF VIEW

The violation of the Feynman scaling in the context of the DPM was studied in some detail in [10–12]. It was found out

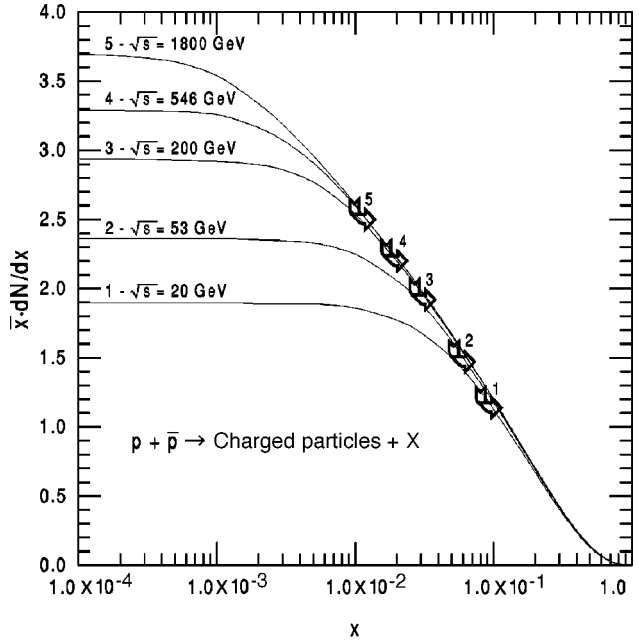

FIG. 1. Inclusive hadronic spectra $\overline{x}(dN/dx)$ as a function of x at different energies [12]. Solid lines are the DPM calculation; by the curved arrows the boundaries (2.2) are sketched.

that the nonscaling rise of $dN/dy|_{y=0}$ (so-called plateau height rise) can be described in the DPM in the following way:

$$\left.\frac{dN}{dy}\right|_{y=0} = a\langle n(s)\rangle\left(1-\frac{b}{\ln(s/s_0)}\right), \tag{2.1}$$

where $\langle n(s)\rangle$ is an average number of the quarks (valence quarks, diquarks, sea quarks) participating in the interaction, and a, b, s_0 are constants. As was explained in [10], the growth of the plateau height in Eq. (2.1) results from the concurrent action of two different factors. The first one is related to the rise with energy of the average number of quarks $\langle n(s)\rangle$ (due to the production of the sea quarks from vacuum), whereas the second factor $[1-b/\ln(s/s_0)]$ describes the violation of scaling in the interactions of individual quarks. In [12,13] it was also derived that starting from the value of Feynman's variable

$$x_0 = \sqrt{\frac{m_\perp}{2\sqrt{s}}} \tag{2.2}$$

the alteration in the production of the secondary hadrons (each possessing the transverse mass $m_\perp$) occurs and the approach to the scaling behavior begins. This situation is illustrated in Fig. 1. Solid curves in the figure are the numerical DPM calculations; by the curved arrows the boundaries (2.2) are sketched. From Fig. 1 it is seen that on the left from the arrows the strong violation of scaling happens, while in the direction of the arrows the curves are tending to have an energy independent form and they almost do not differ from each other in their common areas on the right from the boundaries (2.2). Condition (2.2) has a clear physical sense.

The production of hadrons h in soft hadronic interactions is considered in the DPM as the fragmentation into hadrons of the special objects, known in the model as colored chains, stretched between the quarks of the colliding hadrons. The center of a chain with quarks (q_1q_2) at its end is also the central region for the elementary process $(q_1q_2)\to h$. The fastest chain in the forward hemisphere (the hemisphere of the colliding hadron h_1) is one stretched between the fastest quark of the hadron h_1 and the slowest quark of the hadron h_2. The center of such chain is situated, in terms of rapidity, at the point

$$y_0 \approx \frac{1}{2}\ln\frac{\sqrt{s}}{2m_\perp} \tag{2.3}$$

(we took into account that in the c.m. system of colliding hadrons the fastest quark of hadron h_1 bears momentum $P^*_{\max}\approx\sqrt{s}/2$, the slowest quark of h_2 carries $P^*_{\min}\approx 0$, and the fragmentation into the secondary hadrons occurs from the chain elements with the transverse mass $m_\perp$). Of course, the chains should not necessarily be stretched between the fastest and slowest quarks of hadrons h_1 and h_2 since in the dual parton model the momentum of the initial hadron is shared between various quarks so that the probability that a quark possesses a fraction x of the hadron's momentum is described by the momentum distribution function (see, for example, [14–16]). But for any configuration of quarks only the chain between the fastest quark of h_1 and the slowest quark of h_2 has the fastest center which can fragment into the secondary hadrons not being accompanied by the products of fragmentation from the central region of other chains. So in the area $y>y_0$ there is no contribution from the central region of the elementary process $(q_1q_2)\to h$ and only the fragmentation from the hadron h_1 is important, whereas for the rapidities $-y_0<y<y_0$ one can always have contribution into the fragmentation from both colliding hadrons and the central region behavior is peculiar to this area. It must be emphasized that this physically descriptive explanation should not be considered as a derivation of Eq. (2.2). A comprehensive quantitative analysis of this effect and the mathematically strict development of Eq. (2.2) for the case of multi-Reggeon (multi-chain) exchange is given in our paper [12] where one can also find analytical expressions describing the onset of scaling in regions $y>y_0$.

It is easy to see that being rewritten in terms of Feynman's variable, Eq. (2.3) gives Eq. (2.2) which can be thus considered as a ''width'' of the central region in our approach (see [12,13]). The right-hand side of Eq. (2.2) is a function of energy. In order to have a unified description at various energies, one can introduce a certain combination of variables x and s by transposing $s^{1/4}$ into the left side of Eq. (2.2). It might be well to point out that such universal variable $x\times s^{0.25}$ was already used in the UA5 experiments at representing the collider data [17]. It is also interesting to note that the power of s in this combination is very close to the power α in the argument of the function f describing the dependency of the spectra on Feynman's variable in Eqs. (1.1), (1.2) (see discussion in [12]).

In view of the above-stated reasons, it is natural to apply the hadronic spectra parametrization of the following form:

$$\frac{dN(x;s)}{dx} = \varphi(x;s)\tilde{\theta}(x_0-x;m_\perp/\sqrt{s}) + \psi(x)\tilde{\theta}(x-x_0;m_\perp/\sqrt{s}), \tag{2.4}$$

where functions $\varphi(x;s)$ and $\psi(x)$ describe the violation of scaling in the central region and the scaling behavior of the spectra in the fragmentation region correspondingly, while function $\tilde{\theta}$ conducts the change over from the central- to the fragmentation-region behavior. We define this function to be close to zero if its first argument is negative and to be close to unity for the positive argument, so that the change from 0 to 1 happens in the region with the width determined by the value of the second argument of the function. It is convenient to take

$$\tilde{\theta}(x;\Delta x) = \frac{1}{1+e^{-x/|\Delta x|}} \tag{2.5}$$

since this expression complies with the requirements listed above: $\tilde{\theta}(x;\Delta x)\approx 0$ at $x \ll -|\Delta x|$, $\tilde{\theta}(x;\Delta x)\approx 1$ at $x \gg |\Delta x|$; and also has a smooth behavior in the transition region.

In Eq. (2.4) the behavior of the hadronic spectra in the region $x<x_0$ (i.e., in the central region in our terminology) is governed mainly by the function φ and in the complementary region $x>x_0$ the behavior of the spectra is determined by the function ψ; since, according to the conclusions of [12], in the DPM a distinct boundary (2.2) between the central region and the fragmentation regions can be drawn and the characteristic parameter which establishes the approach to the scaling in the fragmentation regions is $m_\perp/\sqrt{s}$, we used $\tilde{\theta}(\pm(x_0-x);m_\perp/\sqrt{s})$ function (2.5). In order to distinguish expression (2.4) from the WW formulas (1.1), further we will designate it as the OG parametrization.

Now we can use parametrization (2.4) to study the energy dependence of the inelasticity in soft hadronic interactions:

$$\begin{aligned} K &= \int [\varphi(x;s)\tilde{\theta}(x_0-x;m_\perp/\sqrt{s}) + \psi(x)\tilde{\theta}(x-x_0;m_\perp/\sqrt{s})]dx \\ &\approx \int_0^{\sqrt{\langle m_\perp\rangle/(2\sqrt{s})}} \varphi(x;s)dx + \int_{\sqrt{\langle m_\perp\rangle/(2\sqrt{s})}}^1 \psi(x)dx \\ &= K_{centr} + K_{fragm}. \end{aligned} \tag{2.6}$$

Here

$$K_{centr} = \int_0^{\sqrt{\langle m_\perp\rangle/(2\sqrt{s})}} \varphi(x;s)dx \tag{2.7}$$

is the central region part of K and the coefficient

$$K_{fragm} = \int_{\sqrt{\langle m_\perp\rangle/(2\sqrt{s})}}^1 \psi(x)dx \tag{2.8}$$

includes the contribution into K from the fragmentation region. From Eq. (2.1) it follows that

$$K_{centr} \lesssim \int_0^{\sqrt{\langle m_\perp\rangle/(2\sqrt{s})}} \left.\frac{dN}{dy}\right|_{y=0} dx \sim \left(\frac{s}{s_0}\right)^{\alpha'-0.25} \underset{s\to\infty}{\to} 0. \tag{2.9}$$

In the last relation we took into account that at high energies

$$\langle n(s)\rangle \sim (s/s_0)^{\alpha'}, \tag{2.10}$$

where, according to [18],

$$\alpha' \approx 0.11. \tag{2.11}$$

For the coefficient K_{fragm} we obtain from Eq. (2.8)

$$K_{fragm} \approx \int_0^1 \psi(x)dx, \tag{2.12}$$

and thus at high energies we should have

$$K \approx \int_0^1 \psi(x)dx, \tag{2.13}$$

i.e., K is a constant, contrary to the result of Eq. (1.4).

It is noticeable that the energy behavior of K_{centr} in Eq. (2.9) agrees with the energy dependence in Eq. (1.4) [cf. the values of α, α' in Eqs. (1.2), (2.11) and also see the remark in the paragraph next to Eq. (2.3)]. Nevertheless we obviate the disagreement with experiments, caused by the fall of the inelasticity. In our case

$$K_{WW} \sim K_{centr}, \tag{2.14}$$

but the main part of K arises at high energies from the second term in the right-hand side of Eq. (2.6), i.e., from K_{fragm}, rather than from K_{centr}. The origin of this term is a (quasi) scaling behavior of the hadronic spectra in the fragmentation region, the fact which is not taken into account in the WW formulas. The energy behavior of the inelasticity, predicted here with the help of the OG parametrization, is also confirmed by the results of [13].

As we have already mentioned, it is commonly accepted that the dual parton model can describe experimental data well both in central and fragmentation regions over a wide range of energies. Interestingly enough, such an intricate problem as the Feynman scaling violation observed in experiments can be interpreted in the framework of this model in a rather simple and clear way.

Equation (2.4) can be also rewritten in the form

$$\frac{dN(x;s)}{dx} = \Phi(x;s)\tilde{\theta}(x_0-x;m_\perp/\sqrt{s}) + \psi(x), \tag{2.15}$$

where

$$\Phi(x;s) = \varphi(x;s) - \psi(x) \tag{2.16}$$

[we took into account that according to Eq. (2.5) $\tilde{\theta}(x_0-x;m_\perp/\sqrt{s}) + \tilde{\theta}(x-x_0;m_\perp/\sqrt{s}) = 1$]. An obvious physical interpretation can be given for Eq. (2.15). In compliance with this equation the scaling component $\psi(x)$ of the spectra

takes place in the whole kinematic region of Feynman's variable x, whereas at $x<\sqrt{\langle m_\perp\rangle/(2\sqrt{s})}$ [or at $|x|<\sqrt{\langle m_\perp\rangle/(2\sqrt{s})}$ if the secondary hadrons in the backward hemisphere are also considered] there is the nonscaling contribution $\Phi(x;s)$ which determines the shape of the spectra as a function of energy in this area.

In the next section we will illustrate an example of the application of the OG parametrizations to the reaction $p+\bar{p}\to\pi^{\pm}+X$.

III. A CONCRETE EXAMPLE: PROCESS $p+\bar{p}\to\pi^{\pm}+X$

A detailed comparison of the DPM's spectra with the experimental data for the process $p+\bar{p}\to\pi^{\pm}+X$ can be found, for instance, in [9,19,20]. It was proved that the model reproduces data very well in the central region and also in the fragmentation regions of the reaction. In the present section we will apply the results of the dual parton model in order to determine functions $\varphi(x;s)$ and $\psi(x)$ in Eq. (2.4) for this case. The consequences obtained in such way can be used then for predictions of the hadronic spectra in the regions of x and s where experimental data are yet poorly known or unavailable.

Since one of the important properties of the hadronic spectra in the central region resides in the growth with energy of the central plateau height, it is reasonable to present function $\varphi(x;s)$ as

$$\varphi(x;s)=h(s)f(x;s) \tag{3.1}$$

where $h(s)$ is the plateau height. The results of the DPM calculations for $h(s)$ and their fit by the analytical expression

$$h(s)=1.26\ln^{1/2}(s/s_0)-1.2,\qquad s_0=1\ \mathrm{GeV}^2 \tag{3.2}$$

are shown in Fig. 2. As is seen from the figure, such approximation is in a close agreement with the results of the numerical calculations. It is pertinent to note, however, that this simple approximation can be applied in a limited range of energies only, since at asymptotically high energies the rise of the central plateau height obeys the exponential function (2.10). Nonetheless, Eq. (3.2) can be applied for all energies available now in accelerator experiments merely as a good parametrization [as much as, for example, the expression $h(s)=a\ln(s/s_0)+b$ also often used at description of the height of the central plateau in experiment, see [17]].

Function $f(x;s)$ in Eq. (3.1) describes the behavior of the hadronic spectra in the central region. For this region it is more appropriate to present the spectra against the rapidity $y[=\mathrm{arcsinh}(x\sqrt{s}/2m_\perp)]$ rather than against Feynman's variable x. Another thing which should also be taken into account is that the interval of variable x (respectively y) where function f has to be specified depends on energy, see Eq. (2.2) [respectively, Eq. (2.3)]. In order to deal with the function defined in the single area of the function's first argument, it makes sense to represent $f(x;s)$ in the following way:

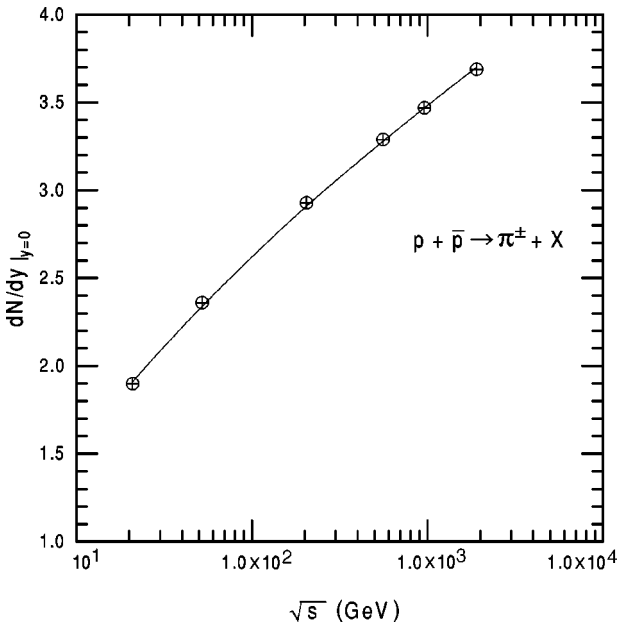

FIG. 2. The height of the central plateau vs energy. Crossed circles are the results of the DPM calculation; solid line is their approximation by the expression (3.2).

$$f(x;s)=g(y(x)/y_0;s), \tag{3.3}$$

for, according to Eq. (2.4), it is sufficient to know function g only for $|y|/y_0\lesssim 1$ (though the kinematically allowed region is $|y|/y_0\lesssim 2$: it is simple to see that $y_0\approx y_{\max}/2$). In Fig. 3 we present the results of the numerical calculations of function $g(y(x)/y_0;s)$. It is interesting that, referring to the figure, all curves have approximately a uniform shape at different energies and so one may assume, at least at high energies, that function g does not depend explicitly on energy:

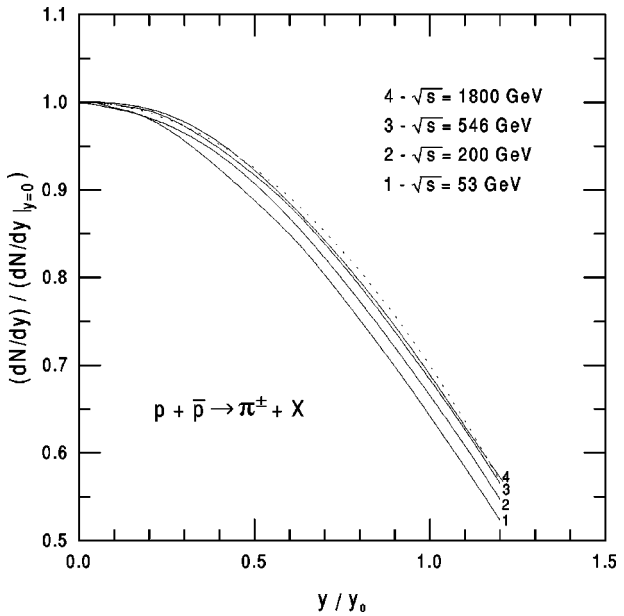

FIG. 3. $[dN(y;s)/dy]/\{[dN(y;s)/dy]|_{y=0}\}$ as a function of y/y_0, see text. Solid curves are the DPM results; by the dashed line approximation (3.4) is shown.

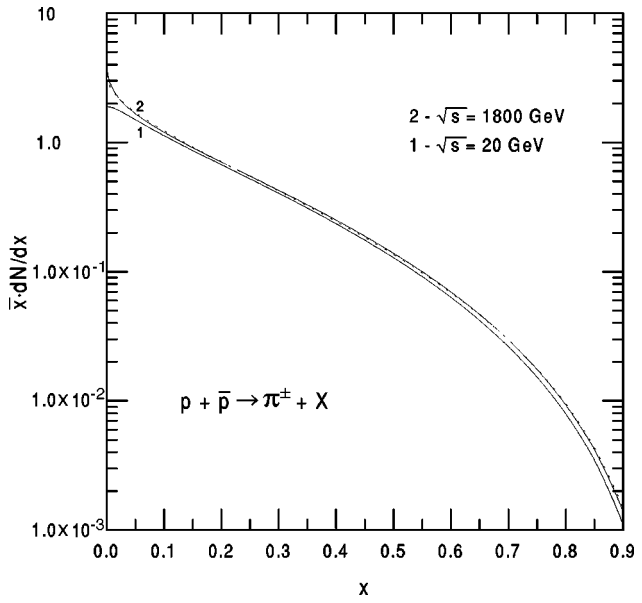

FIG. 4. Hadronic spectra in the fragmentation region. Solid lines are the results of the DPM numerical calculations; dashed curve is the analytical fit (3.5).

$g(y(x)/y_0;s)=g(y(x)/y_0)$ [yet it depends on energy via $y_0(s)$]. It was found that a simple approximation can be used for this function

$$g(y(x)/y_0)=1-0.3(y(x)/y_0)^2, \tag{3.4}$$

and this parametrization (dashed line in Fig. 3) is in a good agreement with the results of the precise calculations.

In Fig. 4 the hadronic spectra in the fragmentation region are shown at different energies. As is evident from the picture, the validity of the Feynman scaling can be accepted here (at least as a first approximation, for analysis of the nonscaling corrections to the spectra in the fragmentation regions see [12]). By the dashed line we plotted the analytical expression

$$\psi(x)=3.3(1-\sqrt{x})^{2.6} \tag{3.5}$$

which, as it is seen from the figure, approximates the results of the calculations very well.

Summing up the results of the present section, finally for the process $p+\bar{p}\to\pi^{\pm}+X$ we obtain the following OG parametrization:

$$\frac{1}{x}\frac{dN(x;s)}{dx}=h(s)g(y(x)/y_0)\tilde{\theta}(x_0-x;m_{\perp}/\sqrt{s}) + \psi(x)\tilde{\theta}(x-x_0;m_{\perp}/\sqrt{s}), \tag{3.6}$$

where functions $\tilde{\theta}$, h, g, ψ are defined by the expressions (2.5), (3.2), (3.4), (3.5) correspondingly and quantities x_0, y_0 are given in Eqs. (2.2), (2.3). We do not show here a special plot for Eq. (3.6) since its spectra are barely perceptible from the corresponding curves in Fig. 1 at every value of x, including the areas near x_0.

IV. DISCUSSION AND CONCLUSIONS

In this work with the help of the dual parton model we have constructed the nonscaling parametrization of the inclusive hadronic spectra, valid in the full kinematic region of Feynman's variable x. It was shown that though in the central region this new parametrization resembles the WW parametrization, it is free from the main problems typical for the WW formulas and does not lead to the fall of the inelasticity coefficient. This new parametrization was deduced only from the consistent physical consideration based on the DPM and did not employ special additional hypotheses to prevent the decrease of the inelasticity with the rise of energy. Contrary to the WW parametrization, in our approach a strong violation of scaling in the central region transforms into the scaling behavior in the fragmentation regions. The physical reasons for such behavior and the causes and mechanisms of the scaling violation in the central regions were discussed. For the reaction $p+\bar{p}\to\pi^{\pm}+X$ a simple empirical formula built upon the proposed type of parametrization was given.

It should be emphasized that the results of this work differ from the conclusions made in [5–8]. We argue, however, that the results of papers [5–8] were inferred from the central region data or the fragmentation region data ($x\approx 0.3$) of UA7 [21] and C jets [22] where several assumptions were made to transform initial experimental data into the expected distributions of charged pions. In fact, the UA7 Collaboration confirmed the validity of an approximate scaling in the fragmentation regions. A subsequent analysis performed in [23] showed that the features of the very high-energy interactions recorded with the emulsion chambers can be well described within the scaling approach, supporting thereby the idea of change from the strongly violated scaling to the scaling behavior in the fragmentation regions.

ACKNOWLEDGMENT

I am thankful to Professor E. V. Bugaev (Institute for Nuclear Research, The Academy of Sciences of Russia, Moscow, Russia) for many helpful discussions.

[1] J. Wdowczyk and A. W. Wolfendale, Nuovo Cimento A **54**, 433 (1977).
[2] J. Wdowczyk and A. W. Wolfendale, Nature (London) **306**, 347 (1983).
[3] J. Wdowczyk and A. W. Wolfendale, J. Phys. G **10**, 257 (1984).
[4] J. Wdowczyk and A. W. Wolfendale, J. Phys. G **13**, 411 (1987).

[5] A. Ohsawa, Prog. Theor. Phys. **84**, 50 (1990).
[6] A. Ohsawa and K. Sawayanagi, Phys. Rev. D **45**, 3128 (1992).
[7] J. Bellandi F. *et al.*, Prog. Theor. Phys. **83**, 58 (1990).
[8] A. Ohsawa, Prog. Theor. Phys. **92**, 1005 (1994).
[9] A. Capella, U. Sukhatme, C.-I. Tan, and J. Tran Thanh Van, Phys. Rep. **236**, 225 (1994).
[10] E. V. Bugaev and O. N. Gaponenko, Z. Phys. C **69**, 135 (1995).
[11] E. V. Bugaev and O. N. Gaponenko, J. Phys. G **23**, 1093 (1997).
[12] E. V. Bugaev and O. N. Gaponenko, Phys. Rev. D **58**, 032001 (1998).
[13] E. V. Bugaev and O. N. Gaponenko, Eur. Phys. J. C **5**, 103 (1998).
[14] A. Capella and J. Tran Thanh Van, Z. Phys. C **10**, 249 (1981).
[15] P. Aurenche and F. W. Bopp, Phys. Lett. **114B**, 363 (1982).
[16] A. Capella and J. Tran Thanh Van, Phys. Lett. **114B**, 450 (1982).
[17] UA5 Collaboration, G. J. Alner *et al.*, Z. Phys. C **33**, 1 (1986).
[18] A. Capella and J. Tran Thanh Van, Z. Phys. C **18**, 85 (1983).
[19] A. Capella, U. Sukhatme, and J. Tran Thanh Van, Z. Phys. C **3**, 329 (1980).
[20] A. Capella, U. Sukhatme, and J. Tran Thanh Van, Phys. Lett. **119B**, 220 (1982).
[21] E. Pare *et al.*, Phys. Lett. B **242**, 531 (1990).
[22] N. Arate, Nucl. Phys. **B211**, 189 (1983).
[23] C. G. S. Costa, F. Halzen, and C. Salles, Phys. Rev. D **52**, 3890 (1995).