Mixing angles and decay constants of η, η', and η_c

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We study the problem of pseudoscalar mixing in an SU(4)×SU(4)×U$_A$(1) breaking framework. In this connection, we derive the condition of a vanishing $c\bar{c}$ component in the η,η' wave functions. We also obtain new constraints obeyed by the mixed decay constants $f^q_{\eta,\eta'}$ $(q=u,s)$, which determine the octet-singlet mixing angles. In the $\eta\eta'\eta_c$ sector we find from a fit to the radiative decay rates of mesons that the presence of a $c\bar{c}$ admixture in η' is small in disagreement with the point of view that the branching ratio of $B\to\eta' K$ is enhanced by $\eta'\eta_c$ mixing. [S0556-2821(99)01417-4]

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I. INTRODUCTION

It is well known that chiral symmetry plays a pivotal role in low-energy particle physics [1]. The QCD Lagrangian with massless quarks is invariant under $SU(3)_L\times SU(3)_R$ and so incorporates chiral symmetry in a natural way. In the physical world, however, chiral symmetry has to be broken in order to generate the observed SU(3) spectra of hadrons and also low-lying mesons such as π, K, and η which satisfy approximate PCAC (particle conservation of axial vector current) conditions. The existence of an additional $I=0$ state, namely, the η', raises an important question concerning the determination of the $\eta\eta'$ mixing angle. The last few years have witnessed several such efforts being made in this direction with most of the calculations (based either on an estimation of one-loop corrections to the pseudoscalar mass spectrum [2], or inclusion of SU(3) breaking in the Gell-Mann–Okubo mass formula [3], or from simple phenomenological considerations of $J/\psi\to(\eta,\eta')\gamma$ decays [4,5]) supporting an appreciably large $\eta\eta'$ mixing angle.

Recently, following the CLEO observation [6] of an anomalously large branching ratio for the inclusive production of η' in the B meson decay $B\to\eta' K$, the composition of η' (and as a result the general problem of pseudoscalar mixing) has come under intense scrutiny. In this regard, a number of models have been advanced [7–12] which look into the possibility of light-heavy quarkonium mixing in addition to the usual $\eta\eta'$. In particular the mixing of η and η' with the $^1S_0$ hyperfine partner of the J/ψ, namely, the η_c state, has been proposed.

In this paper we are concerned with the phenomenology of $\eta\eta'\eta_c$ mixing [11,12] rather than attempt to solve the $B\to\eta' K$ puzzle. Working within a broken SU(4)×SU(4)×U$_A$(1) scheme we fit the mixing angles from the complete list of experimental data now available for the radiative decay rates of 1^{--} and 0^{-+} mesons including not only the J/ψ transitions which are a potential source for the charmonium production but also the recently observed $\phi\to\eta'\gamma$ mode [13]. Indeed we shall demonstrate that these decay widths point to evidence of a large $\eta\eta'$ mixing angle (of about $\sim -22°$) when confronted with other $V\to P\gamma$ or $P\to V\gamma$ rates.

However our model is consistent with an insignificant $c\bar{c}$ leakage in the η' wave function in conformity with current expectations [11,12,14,15]. Exploiting the anomaly relation for the charm quark Yuan and Chao have recently noted [16] that the total mixed amplitude of the $c\bar{c}$ component in η' is rather mild and have conjectured that a relation,

$$2m_c\langle 0|\bar{c}i\gamma_5 c|\eta\eta'\rangle\sim -\frac{\alpha_s}{4\pi}\langle 0|G_{\mu\nu}\tilde{G}_{\mu\nu}|\eta\eta'\rangle, \qquad (1)$$

(where the notations are self-explanatory) should hold well to a reasonable degree of approximation.

This paper is organized as follows: In Sec. II we show that relation (1) can be derived in an SU(4)×SU(4)×U$_A$(1) scheme and argue that if Eq. (1) holds exactly then the $c\bar{c}$ content in η and η' ought to be vanishing. In this section we also write down new constraints satisfied by the mixed decay constants $f^q_{\eta,\eta'}$ $(q=u,s)$ and inquire about a two-parameter mixing scheme of η and η' characterized by a set of octet-singlet mixing parameters. In Sec. III we consider $\eta\eta'\eta_c$ mixing, the light-heavy pseudoscalar mixing angles being estimated using $V\to P\gamma$ and $P\to V\gamma$ decay rates as input. We observe that a certain amount of U(4) breaking is required to get a good fit. Finally in Sec. IV we present a summary of our work.

II. U(4)×U(4) FORMALISM

A. The condition of vanishing charmonium in η and η'

To start with let us assume that chiral SU(4)×SU(4)×U$_A$(1) symmetry is solely broken due to the presence of quark mass terms in the QCD Lagrangian. Following Gell-

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Mann, Oakes, and Renner [17] we can write the symmetry breaking piece of the Lagrangian as [18]

$$-L_{\rm SB}=\Sigma c_i u_i\,,\quad i=0,8,15, \tag{2}$$

where the c_i's are related to the quark masses by

$$c_8=(m_u+m_d-2m_s)/\sqrt{3},$$

$$c_{15}=(m_u+m_d-3m_c)/\sqrt{6},$$

$$c_0=(m_u+m_d+m_s+m_c)/\sqrt{2}, \tag{3}$$

and the scalar densities $u_i(\equiv\bar{q}(\lambda^i/2)q)$ along with the pseudoscalars $v_i(\equiv\bar{q}i\gamma_5(\lambda^i/2)q)$ belong to the $(4,\bar{4})+(\bar{4},4)$ representation of U(4)×U(4) symmetry group $q^T=(u,d,s,c)$.

A representative of the estimates of the quark mass ratios shows [19]

$$\frac{2m_s}{m_u+m_d}=25.7\pm 2.6,$$

$$\frac{m_c-m_u}{m_s-m_u}\simeq 9\pm 2. \tag{4}$$

The above result on the ratio of the light quark masses has been obtained from the usual current algebra PCAC relations using various physical inputs like measured masses and coupling constants. That for the c-quark ratio has been deduced using flavor SU(4) assumption for the ψ and ϕ wave functions.

In the presence of $L_{\rm SB}$ the expressions for the divergences of the axial-vector currents $A^i_\mu(\equiv\bar{q}\gamma_\mu i\gamma_5(\lambda^i/2)q)$ within the full structure of QCD read

$$\partial_\mu A^{(i)}_\mu=d^{ijk}c^j v^k+\delta^{io}a, \tag{5}$$

where the gluon anomaly a is given in terms of the field strengths $G^{(1)}_{\mu\nu}$:

$$a=\sqrt{N_f}\,\frac{\alpha_s}{4\pi}G\tilde{G},\quad G\tilde{G}=\frac{1}{\sqrt{2}}G^{(1)}_{\mu\nu}\tilde{G}^{(1)}_{\mu\nu},$$

$$\tilde{G}^{(1)}_{\mu\nu}\equiv\tfrac{1}{2}\in_{\mu\nu\lambda\sigma}G^{(1)}_{\lambda\sigma}, \tag{6}$$

N_f denoting the number of quark flavors and $i=0,\ldots,15$.

Conventionally, the decay constants for the physical states is defined by $\langle 0|A^i_\mu|P(k)\rangle=if_{iP}k_\mu$, implying

$$\langle 0|\partial_\mu A^i_\mu|P\rangle=f_{iP}m_P^2, \tag{7}$$

P representing the pseudoscalars η, η', and η_c whose mixings we shall take into account. The topological charge constants for these states are

$$\langle 0|a|P\rangle=A_P m_P^2,\quad P=\eta,\ \eta',\ \text{and}\ \ \eta_c. \tag{8}$$

Such gluon matrix elements also contribute to the $\eta\eta'\eta_c$ mixing, albeit indirectly, when the divergence of the axial singlet current is taken between the vacuum and isoscalar states.

Yuan and Chau [16] have pointed out that for a typical hadronic mass scale Λ, both η and η' decouple from the heavy quarkonium system $Q\bar{Q}$ for a sufficiently massive Q quark. In particular, for the charm quark, one expects the following bound to hold:

$$f_{c\eta(\eta')}\leqslant O\left(\frac{m_s}{m_c}f_\pi,\frac{\Lambda}{m_c}f_\pi\right)\ll f_\pi \tag{9}$$

for $\Lambda\sim 1$ Gev.

In the context of Eq. (9) if one writes down the divergence condition for $\bar{Q}\gamma_\mu\gamma_5 Q$ from Eq. (5), namely,

$$\partial_\mu(\bar{Q}\gamma_\mu\gamma_5 Q)=2m_Q\bar{Q}i\gamma_5 Q+a \tag{10}$$

and take it between $\langle 0|$ and $|\eta(\eta')\rangle$ states, the condition (1) seems to be justified. Based on Eq. (1) the authors of Ref. [16] have concluded against the possibility of a large magnitude color-singlet ''intrinsic charm'' of η'.

We now proceed to establish that the condition (1) can be derived exactly within an SU(4)×SU(4)×U(1) scheme provided the $c\bar{c}$ components in η and η' vanish. To this end let us write down the divergence equations for the currents $i=8,15,0$:

$$\partial_\mu A^{(8)}_\mu=\frac{1}{2\sqrt{3}}(m_u+m_d)(\bar{u}i\gamma_5 u+\bar{d}i\gamma_5 d)-\frac{2}{\sqrt{3}}m_s\bar{s}i\gamma_5 s, \tag{11a}$$

$$\partial_\mu A^{(15)}_\mu=\frac{1}{2\sqrt{6}}(m_u+m_d)(\bar{u}i\gamma_5 u+\bar{d}i\gamma_5 d)+\frac{1}{\sqrt{6}}m_s\bar{s}i\gamma_5 s-\frac{3}{\sqrt{6}}m_c\bar{c}i\gamma_5 c, \tag{11b}$$

$$\partial_\mu A^{(0)}_\mu=\frac{1}{2\sqrt{2}}(m_u+m_d)(\bar{u}i\gamma_5 u+\bar{d}i\gamma_5 d)+\frac{1}{\sqrt{2}}m_s\bar{s}i\gamma_5 s+\frac{1}{\sqrt{2}}m_c\bar{c}i\gamma_5 c+a, \tag{11c}$$

where we have set $N_f=4$. We are then led to the following remarkable sum rule

$$\partial_\mu A^{(0)}_\mu=\sqrt{3}\partial_\mu A^{(15)}_\mu+2\sqrt{2}m_c\bar{c}i\gamma_5 c+a. \tag{12}$$

An interesting feature of Eq. (12) is that it contains no reference to the octet divergence $\partial_\mu A^{(8)}_\mu$.

We now easily see that if Eq. (12) is inserted between the vacuum and $\eta(\eta')$ states then the relations

$$f_{0\eta}m_\eta^2=\sqrt{3}f_{15\eta}m_\eta^2 2\sqrt{2}m_c\langle 0|\bar{c}i\gamma_5 c|\eta\rangle+A_\eta m_\eta^2, \tag{13a}$$

$$f_{0\eta'}m_{\eta'}^2=\sqrt{3}f_{15\eta'}m_{\eta'}^2+2\sqrt{2}m_c\langle 0|\overline{c}i\gamma_5c|\eta'\rangle+A_{\eta'}m_{\eta'}^2 \tag{13b}$$

follow. Invoking the condition (1) on Eqs. (13a) and (13b) implies that the following alignments of the η,η' decay constants,

$$f_{0\eta}=\sqrt{3}f_{15\eta}, \quad f_{0\eta'}=\sqrt{3}f_{15\eta'}, \tag{14}$$

are valid signaling that η and η' are free from any $c\overline{c}$ contamination. On the other hand if Eq. (14) holds (in other words, if the states η and η' are devoid of any charmonium admixtures), then Eq. (1) follows exactly from the relations (13).We now turn to the nonsinglet decay constants of η and η'. As we shall see if the definitions of mixed decay constants $f_\eta^{u,s}$ and $f_{\eta'}^{u,s}$ are invoked as suggested by Refs. [10,20] we get an interesting set of new consistency conditions.

B. New relations for the mixed decay constants of η and η'

For $i=8,15$ it follows from Eqs. (11a), (11b), and (7) that

$$\begin{aligned}\tfrac{1}{2}(m_u+m_d)\langle 0|\overline{u}i\gamma_5u|\eta,\eta'\rangle &= m_c\langle 0|\overline{c}i\gamma_5c|\eta,\eta'\rangle+\frac{1}{2\sqrt{3}}f_{8\eta,\eta'}m_{\eta,\eta'}^2\\ &\quad+\frac{\sqrt{6}}{3}f_{15\eta,\eta'}m_{\eta,\eta'}^2,\end{aligned} \tag{15}$$

$$\begin{aligned}m_s\langle 0|\overline{s}i\gamma_5s|\eta,\eta'\rangle &= m_c\langle 0|\overline{c}i\gamma_5c|\eta,\eta'\rangle\\ &\quad-\frac{1}{\sqrt{3}}f_{8\eta,\eta'}m_{\eta,\eta'}^2+\frac{\sqrt{6}}{3}f_{15\eta,\eta'}m_{\eta,\eta'}^2,\end{aligned} \tag{16}$$

where we assume SU(2) symmetry for the pseudoscalar densities. In Ref. [5] chiral Ward identities in relation to the U(1) problem were comprehensively solved to determine the sizes of the various decay constants and gluonic couplings of η and η'.

Defining f_8 and f_1 to be decay constants of the eight component of the octet and SU(3) singlet [not to be confused with the SU(4) singlet], respectively, and θ the usual $\eta\eta'$ mixing angle according to

$$|\eta\rangle=\cos\theta|8\rangle-\sin\theta|1\rangle, \quad |\eta'\rangle=\sin\theta|8\rangle+\cos\theta|1\rangle, \tag{17}$$

where $|8\rangle$ and $|1\rangle$ in terms of the quark basis are

$$|8\rangle=\frac{1}{\sqrt{6}}|u\overline{u}+d\overline{d}-2s\overline{s}\rangle, \quad |1\rangle=\frac{1}{\sqrt{3}}|u\overline{u}+d\overline{d}+s\overline{s}\rangle; \tag{18}$$

the results turned out to be [5]

$$\theta=-22°\pm4°, \tag{19}$$

$$f_8=(1.1\pm0.22)f_\pi, \quad f_1=(1.35\pm0.27)f_\pi. \tag{20}$$

In arriving at these estimates the effects of nonzero gluonic matrix elements for η and η' were also considered. The error of uncertainties in f_8 and f_1 are of the same order as one normally expects from SU(3) breaking effects.

The above value of θ agrees well with other estimates from various phenomenological fits to the meson decays [2–4]. Also f_8 is consistent with the findings of chiral perturbation theory and one-loop correction to the pseudoscalar meson masses [2]. However the large errors in the estimate of f_1 reflect that a smaller central value is not ruled out as implied by the analysis of Gilman-Kauffman [4] and Venugopal-Holstein [21]. The latter authors have performed an analysis from $\eta,\eta'\to\gamma\gamma$, and $\eta,\eta'\to\pi\pi\gamma$ decays to obtain $\theta=-22°\pm3.3°$, $f_8=(1.38\pm0.22)f_\pi$, and $f_1=(1.06\pm0.03)f_\pi$. Notice here that f_8 is beset with a large error uncertainty.

Given this background let us consider explicitly the quark contents of the decay constants involving η and η':

$$\langle 0|\overline{q}\gamma_\mu\gamma_5q|\eta,\eta'(k)\rangle=if_{\eta,\eta'}^q k_\mu. \tag{21}$$

Here one has to keep in mind that both the soft part due to quark masses and the hard part due to the gluon anomaly in the axial current divergences contribute to the η,η' masses of which the anomaly portion remains nonvanishing even in the chiral limit. It has been shown that if the anomaly effects are judiciously considered, the matrix element of pseudoscalar density $\overline{s}i\gamma_5s$ undergoes modification in the manner [8,22]:

$$\langle 0|\overline{s}i\gamma_5s|\eta\rangle=\frac{f_\eta^s-f_\eta^u}{2m_s}m_\eta^2, \quad \langle 0|\overline{s}i\gamma_5s|\eta'\rangle=\frac{f_{\eta'}^s-f_{\eta'}^u}{2m_s}m_{\eta'}^2. \tag{22}$$

Substituting Eq. (22) into Eq. (15) gives on subtraction the following equations:

$$2f_\eta^u-f_\eta^s=\sqrt{3}f_{8\eta}, \quad 2f_{\eta'}^u-f_{\eta'}^s=\sqrt{3}f_{8\eta'}. \tag{23}$$

These are new relations for the mixed decay constants for η and η' and may be looked upon as the direct consequences of the PCAC condition (5). In the exact SU(3) symmetry limit where $m_s=m_u=m_d$, $f_{8\eta}=f_{0\eta'}=f_\pi$ and $f_{8\eta'}=f_{0\eta}=0$, it at once follows from the second equation above that $f_{\eta'}^u=\frac{1}{2}f_{\eta'}^s$ as expected naively [11]. It should be mentioned that this relation is consistent with the definitions (22) depicting a correct chiral behavior in the presence of the anomaly.

Ali and Greub [8] have recently employed a two-parameter mixing scenario $\eta'=\eta_8\sin\theta_8+\eta_0\cos\theta_0$, $\eta=\eta_8\cos\theta_8-\eta_0\sin\theta_0$, where $\theta_8\neq\theta_0$, to conduct tests of factorization in charmless nonleptonic $B\to\eta'$ and $B\to\eta$ transitions. Such a mixing scheme induces the following expressions for $f_{\eta,\eta'}^u$ and $f_{\eta,\eta'}^s$:

$$f^u_{\eta'}=\frac{f_8}{\sqrt{6}}\sin\theta_8+\frac{f_1}{\sqrt{3}}\cos\theta_0,$$

$$f^s_{\eta'}=-2\frac{f_8}{\sqrt{6}}\sin\theta_8+\frac{f_1}{\sqrt{3}}\cos\theta_0, \tag{24}$$

$$f^u_{\eta}=\frac{f_8}{\sqrt{6}}\cos\theta_8-\frac{f_1}{\sqrt{3}}\sin\theta_0,\quad f^s_{\eta}=-2\frac{f_8}{\sqrt{6}}\cos\theta_8-\frac{f_1}{\sqrt{3}}\sin\theta_0. \tag{25}$$

It is interesting to note that if the above representations are substituted in our relations (23), we get

$$2\sqrt{2}f_8\cos\theta_8-f_1\sin\theta_0=3f_{8\eta},$$

$$2\sqrt{2}f_8\sin\theta_8-f_1\cos\theta_0=3f_{8\eta'}. \tag{26}$$

Using the estimates (20) for f_8 and f_1 and the SU(3) values for the octet decay constants as mentioned earlier, θ_0 and θ_8 turn out to be

$$\theta_0=-(8.1°\pm1.6°),\quad \theta_8=-(25°\pm5°), \tag{27}$$

in impressive agreement with the determination of Fredman *et al.* [10] from a phenomenological analysis, namely, $\theta_0=-9.2°$ and $\theta_8=-21.2°$. It must however be pointed out that θ_0 is somewhat sensitive to SU(3) breaking effects typified by $f_{8\eta'}$. In the literature various estimates for this quantity have been made taking into account the effect [23,5] of U(1) anomaly and the positivity constraint of the topological susceptibility. Taking say, $f_{8\eta'}=-0.1f_\pi$, the results (26) and (27) predict θ_0 to be $\sim-14°$ at the expense of $\theta_8\sim-31°$.

III. LIGHT-HEAVY QUARKONIUM MIXING IN η, η', AND η_c

A. The mixing scheme

The mixing scheme for $\eta\eta'\eta_c$ may be generated as follows. We assume that both η and η' contain a certain amount of $c\bar{c}$ leakage. This is brought about by a mixing between the SU(3) octet and a superposition of the SU(3) singlet $|1\rangle=|1/\sqrt{3}\Sigma_{i=1}^3\bar{q}_iq_i\rangle$ and the charm-anticharm bound state $|c\bar{c}\rangle$, which we call $|1\rangle'$. Thus η, η', and η_c can be modeled as

$$|\eta\rangle=\cos\theta|8\rangle-\sin\theta|1\rangle,\quad |\eta'\rangle=\sin\theta|8\rangle+\cos\theta|1\rangle', \tag{28}$$

where $|1\rangle'$ is given by

$$|1\rangle'=|1\rangle\cos\theta_c+|c\bar{c}\rangle\sin\theta_c, \tag{29}$$

having the orthogonal partner η_c

$$|\eta_c\rangle=-|1\rangle\sin\theta_c+|c\bar{c}\rangle\cos\theta_c. \tag{30}$$

Since the superposed state $|1\rangle'$ consists of a very small "charm" admixture characterized by the parameter θ_c, θ may be looked upon to a good approximation, as the usual $\eta\eta'$ mixing angle. This point of view is also implicit in the recent works of Ali and Greub [8] and Cheng and Tseng [11]. Since we are working within an U(4)×U(4) formalism it is instructive to represent the SU(3) singlet $|1\rangle$ in terms of the SU(4) basis states:

$$|1\rangle=\frac{\sqrt{3}}{2}|0\rangle+\frac{1}{2}|15\rangle, \tag{31}$$

where $|0\rangle$ stands for SU(4) singlet. In this way, from Eqs. (28), (30), and (31), the η, η', and η_c may be projected in terms of the SU(4) basis vectors $|8\rangle$, $|15\rangle$, and $|0\rangle$.

In our analysis we shall exploit the above mixing structure along with the mapping (31). It needs to be pointed out that the $c\bar{c}$ is not a basis vector in the SU(4) space but can only be expressed as a linear combination $\frac{1}{2}(|0\rangle-\sqrt{3}|15\rangle)$. This means that if Eq. (31) is imposed upon the mixing scheme Eqs. (28)–(30), η, η', and η_c can be orthogenal states up to first order in θ_c only. It does turn out as we shall presently see that Eqs. (27)–(29) are consistent with a small value of θ_c.

B. Radiative decays and U(4) breakings

Radiative decay rates involving the conventional low-lying pseudoscalars π^0, η, and η' and the vectors ρ^0, ω, ϕ, and J/ψ provide important clues towards our understanding of the mechanism of isoscalar mixings. The $V\to P\gamma$ decay width is given by [24]

$$\Gamma(V\to P\gamma)=\frac{1}{32\pi^2m_v}\left[\frac{1\vec{p}_v1^3}{m_v}\right]\sum_{\text{spins}}\int\mu^2d\Omega_v, \tag{32}$$

where the matrix element $\mu=g_{vp\gamma}\in_{\mu\nu\lambda\sigma}\partial^\mu\in^\nu_v\partial^\lambda\in^\sigma_\gamma$, $g_{vp\gamma}$ is the coupling constant, $\in_v$, $\in_\gamma$ are the polarization vectors, and $\vec{p}_v$ is the center of mass momentum. In terms of the branching ratio Eq. (32) gives

$$B(V\to P\gamma)=|A|^2xp_v^3, \tag{33}$$

where A, the amplitude, is of the form $A=\text{const}\times g_{vp\gamma}$. The branching ratio of the related process $B(P\to V\gamma)$ is a factor of 3 larger.

We assume that J/ψ, ω, and ϕ are ideally mixed being a pure $c\bar{c}$, a pure $(u\bar{u}+d\bar{d})/\sqrt{2}$, and a pure $s\bar{s}$ state, respectively. In terms of the SU(4) vectors these are

$$|\omega\rangle=\frac{1}{\sqrt{3}}|8\rangle+\frac{1}{\sqrt{6}}|15\rangle+\frac{1}{\sqrt{2}}|0\rangle,$$

$$|\phi\rangle=-\frac{2}{\sqrt{3}}|8\rangle+\frac{1}{\sqrt{12}}|15\rangle+\frac{1}{2}|0\rangle,$$

$$|J/\psi\rangle=-\frac{\sqrt{3}}{2}|15\rangle+\frac{1}{2}|0\rangle. \tag{34}$$

Now using vector dominance, we can write [25,26]

$$A(\phi_i \to \gamma\gamma) = \sum_v \frac{e}{f_v} A(\phi_i \to v\gamma), \tag{35}$$

where $i=8$, 15, 0 and ϕ_i corresponds to some 0^{-+} state. Since the vector meson-photon couplings are related by

$$\frac{e}{f_\rho}:\frac{e}{f_\omega}:\frac{e}{f_\phi}:\frac{e}{f_\psi}=1:\tfrac{1}{3}:-\frac{\sqrt{2}}{3}:\frac{2\sqrt{2}}{3}, \tag{36}$$

where f_v's are defined according to $\Gamma(V\to e^+e^-) = \frac{1}{3}\alpha^2 m_v (f_v^2/4_x)^{-1}$ it at once follows from Eq. (35) that [26]

$$A(\phi_8\to\rho\gamma):A(\phi_8\to\omega\gamma):A(\phi_8\to\phi_\gamma):A(\phi_8\to\psi\gamma)=1:\tfrac{1}{3}:\frac{2\sqrt{2}}{3}:0, \tag{37a}$$

$$A(\phi_{15}\to\rho\gamma):A(\phi_{15}\to\omega\gamma):A(\phi_{15}\to\phi\gamma):A(\phi_{15}\to\psi\gamma)=1:\tfrac{1}{3}:-\tfrac{2}{3}:-2\sqrt{2}, \tag{37b}$$

$$A(\phi_0\to\rho\gamma):A(\phi_0\to\omega\gamma):A(\phi_0\to\phi\gamma):A(\phi_0\to\psi\gamma)=1:\tfrac{1}{3}:-\frac{\sqrt{2}}{3}:\frac{2\sqrt{2}}{3}. \tag{37c}$$

In Eq. (37), we have used the standard SU(4) description for the electromagnetic current

$$V_\mu^{(em)} = V_\mu^{(3)} + \frac{1}{\sqrt{3}} V_\mu^{(8)} - \sqrt{\frac{2}{3}} V_\mu^{(15)} + \frac{\sqrt{3}}{3} V_\mu^{(0)}. \tag{38}$$

Denoting the U(4) symmetry breaking parameters by α and β, namely,

$$\alpha = \frac{A(\phi_8\to\rho\gamma)}{A(\phi_0\to\rho\gamma)}\left(=\sqrt{\frac{2}{3}}\right), \quad \beta = \frac{A(\phi_{15}\to\rho\gamma)}{A(\phi_0\to\rho\gamma)}\left(=\frac{1}{\sqrt{3}}\right), \tag{39}$$

where the figure in the parenthesis corresponds to the U(4)-exact value, the amplitude ratios (37) induce

$$A(\omega\to\pi^\circ\gamma)=3A(\rho^\circ\to\pi^\circ\gamma), \quad A(\eta'\to\rho\gamma)=3A(\eta'\to\omega\gamma), \tag{40a}$$

$$\frac{A(\omega\to\eta\gamma)}{A(\omega\to\pi^\circ\gamma)}=\frac{1}{9}\,\frac{A(\rho^\circ\to n\gamma)}{A(\rho^\circ\to\pi^\circ\gamma)}, \tag{40b}$$

$$\frac{A(J/\psi\to\eta'\gamma)}{A(J/\psi\to\eta\gamma)}=-\cot\theta, \tag{40c}$$

$$\frac{A(\phi\to\eta'\gamma)}{A(\phi+\eta\gamma)}=\frac{4\alpha\sin\theta-\cos\theta(\sqrt{3}\cos\theta_c+\sin\theta_c)-\sqrt{2}\beta\cos\theta(\cos\theta_c-\sqrt{3}\sin\theta_c)}{4\alpha\cos\theta+\sin\theta(\sqrt{3}\cos\theta_c+\sin\theta_c)+\sqrt{2}\beta\sin\theta(\cos\theta_c-\sqrt{3}\sin\theta_c)}. \tag{40d}$$

While Eqs. (40a) and (40b) are the familiar vector dominance relations supported by experiment, the right-hand side (RHS) of (40c)–(40d) are nontrivial constraints for the corresponding decay ratios. Notice that Eqs. (40a) and (40b) do not involve the mixing angles explicitly. That the mixing factors cancel out cleanly when the two ratios are compared as in Eq. (40b) is a consequence of the U(4) symmetry assumption for the vector states inspired by the almost ideal structure of J/ψ, ω, and φ mesons. It is useful to note that $\eta'\to\rho^\circ\gamma$, which is one of the processes in which CLEO finds [27] a strong signal for the $B^+\to\eta' K^+$, remains uninfluenced by any $\eta\eta'$ or $\eta'\eta_c$ mixing effects when taken against the decay $\eta'\to\omega\gamma$ for an ideal vector nonet.

Tied with Eq. (40) is another constraint coming from the branching ratio of $J/\psi\to\eta_c\gamma$. Experimentally $B(J/\psi\to\eta_c\gamma):B(J/\psi\to\eta'\gamma)$ is predicted to be (1.3±0.4):(0.43±0.31), whereas in the framework of Eqs. (37) and (39)

$$\frac{A(J/\psi\to\eta_c\gamma)}{A(J/\psi\to\eta'\gamma)}=\frac{-(\sqrt{3}\sin\theta_c-\cos\theta_c)+3\beta(\sin\theta_c+\sqrt{3}\cos\theta_c)}{\cos\theta(\sqrt{3}\cos\theta_c+\sin\theta_c)-3\beta\cos\theta(\cos\theta_c-\sqrt{3}\sin\theta_c)}, \tag{40e}$$

where the right-hand side if U(4) is exact ($\beta=1/\sqrt{3}$) reduces to the form $(\tan\theta_c\cos\theta)^{-1}$ in Ref. [8]. So we have three equations [Eqs. (40c)–(40e)] to be solved to determine α, β, θ, and θ_c. With $\psi\rightarrow(\eta,\eta')\gamma$ rates directly yielding θ, it does turn out that the experimental branching ratios [13,3] $B(\phi\rightarrow\eta'\gamma):B(\phi\rightarrow\eta\gamma)=9.5^{+5.2}_{-4.0}\times10^{-3}$ and $B(\psi\rightarrow\eta'\gamma):B(\psi\rightarrow\eta\gamma)=(0.43\pm0.03):(0.086\pm0.008)$ favor β close to its U(4) value of $1/\sqrt{3}$ for small θ_c provided α is allowed to vary. Indeed we find that a consistent fit emerges for

$$\theta\approx-21.9°,\quad \alpha\approx0.96,\quad \beta\approx0.57,\quad \theta_c\approx0.01, \tag{41}$$

where α is away from the U(4)-exact value by a little over 15%. Interestingly, the mixing angle θ obtained as above proves to be very compatible with the decay rates of $\eta\rightarrow\gamma\gamma$ and $\eta'\rightarrow\gamma\gamma$ which predict [4,3,28] $\theta=-(23°\pm3°)$. It needs mention that the experimental group of Ref. [13] had utilized the ratio of the branching fraction of $\eta\rightarrow\gamma\gamma$ and $\pi°\rightarrow\gamma\gamma$ to arrive at the estimate of $B(\phi\rightarrow\eta'\gamma)/B(\phi\rightarrow\eta\gamma')$ quoted above.

One of the consequences of Eq. (41) is that from the expression of the two photon decay width of

$$\eta_c\rightarrow\gamma\gamma{:}\Gamma(\eta_c\rightarrow\gamma\gamma)=\frac{4(4\pi\alpha)^2f_{\eta_c}^2}{81\pi m_{\eta_c}}=7.5^{+1.6}_{-1.4}\ \text{keV},$$

implying $f_{\eta_c}=411\,\text{MeV}$, the charm content of the η' emerges as

$$|f_{\eta'}^{(c)}|\approx|\cos\theta\tan\theta_c f_{\eta_c}|\approx4\ \ \text{MeV}. \tag{42}$$

It suggests that the presence of the $c\bar{c}$ leakage in η' is rather small in agreement with recent calculations [8,11,14,15]. Indeed from the form-factor data the upper limit of $f_{\eta'}^c$ has been predicted to be only 15 MeV [14]. A nonrelativistic quark model estimate also favors $f_{\eta'}^c$ around 6 MeV [15]. Obviously these values, including the present one of Eq. (41), falls far short of $|f_{\eta'}^c|\sim140\,\text{MeV}$ needed to [7] explain the data on $B^\pm\rightarrow\eta'K^\pm$ which is supposed to be triggered by the transition $b\rightarrow c\bar{c}+s\rightarrow\eta'+s$. It is not our purpose to give an answer to this discrepancy; we only note that $\eta'\eta_c$ mixing appears an unlikely possibility to explain the anomalous rate for $B\rightarrow\eta'K$.

The nature of our estimates for the U(4) violating parameters α and β, although being determined in the context of $\eta\eta'\eta_c$ mixing for the first time, come hardly as a surprise. Previous determination from the J/ψ decays into a vector and a pseudoscalar have suggested that nonet symmetry violations can be large [29]. A study of the decays of the charmed mesons D_u and D_s into two pseudoscalars has also revealed moderate breakings in nonet symmetry [30]. In keeping with these trends we find that U(4) symmetry is broken too, with most of the breaking being borne by the parameter α. That the other parameter β stays close to its U(4)-exact value is a consequence of an interplay between the smallness of θ_c and a much larger value for the $\eta\eta'$ mixing angle θ. These conspire to restrict β close to $1/\sqrt{3}$, the experimental values for the decay widths of $J/\psi\rightarrow\eta_c\gamma$ and $\phi\rightarrow\eta'\gamma$ not allowing much room for deviation.

IV. SUMMARY

In this paper we have carried out an analysis of $\eta\eta'$ mixing in particular and $\eta\eta'\eta_c$ in general within a broken U(4) $\times$U(4) scheme. We have established that the vanishing of the matrix element of a linear combination of pseudoscalar density $ci\gamma_5c$ and gluonic strength $(\alpha_s/4\pi)G_{\mu\nu}\tilde{G}_{\mu\nu}$ between $\langle0|$ and $|\eta,\eta'\rangle$, states point to the absence of any $c\bar{c}$ admixtures in these states. We have obtained new constraints for the decay constants $f^q_{\eta,\eta'}$ $(q=u,s)$ which are new and of interest. These have been exploited to obtain SU(3) octet and singlet mixing angles. We have also shown that a simple mixing scheme for η, η', and η_c gives a complete description of $V\rightarrow P\gamma$ and $P\rightarrow V\gamma$ decay rates (including the most recently observed $\phi\rightarrow\eta'\gamma$) corresponding to a $\eta\eta'$ mixing of $\sim-22°$ in agreement with its currently accepted value. However, $c\bar{c}$ leakage in η' (and also η) has proved to be small suggesting that $\eta'\eta_c$ mixing is an unlikely explanation for the anomaly of $B\rightarrow\eta'K$ decay.

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