Coefficient functions and open charm production in deep inelastic scattering

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It is shown that the problem of double counting in open charm production in deep inelastic scattering (DIS) can be solved by using the expression for DIS coefficient functions in terms of two-particle irreducible diagrams. [S0556-2821(99)04415-X]

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Open charm production in deep inelastic scattering (DIS) is a subject of interest from both experimental and theoretical points of view. Recent data from H1 [1] and ZEUS [2] Collaborations have shown that the charm quark contribution is an important component of DIS structure functions.

One of the most predictive methods of calculating the open charm contribution to the structure functions is fixed order perturbative QCD (see [3]). Much effort also has been done in order to formulate a variable flavor number scheme (see [4–7]).

According to the factorization theorem [9,8,10,11], the contribution of charm F_2^c can be represented as follows:[1]

$$\frac{1}{x}F_2^c(Q^2,x)=\int_x^1\frac{dz}{z}\left[C^c(Q^2,\mu^2,z)q_c\left(\mu^2,\frac{x}{z}\right)+C^g(Q^2,\mu^2,z)g\left(\mu^2,\frac{x}{z}\right)\right]. \quad (1)$$

Here C^a are the process-dependent coefficient functions ($a=c,g$), q_c and g being charm and gluon densities of the incoming hadron, and μ is a factorization scale. The gluon coefficient function C^g includes, in particular, a photon-gluon fusion (PGF) contribution which dominates at low scales, $Q^2<m_c^2$, where m_c is a mass of the charm quark. At high Q^2, as was noted in Refs. [4,6], part of the PGF cross section is generated by the evolution of the charm contribution [the first term in Eq. (1)].

To avoid double counting, one has to subtract this contribution. In Ref. [4] this subtraction has been taked into account in lowest order in α_s (in what follows a symbol $\otimes$ means a convolution in variable z):

$$F_2=C_c^{(0)}\otimes q_c-C_c^{(0)}\otimes P_{cg}\otimes g+C_g^{(1)}\otimes g+\mathrm{O}(\alpha_s^2). \quad (2)$$

If $\mu\sim m_c$, we have an approximate cancellation of the first two terms in Eq. (2) and we arrive at the dominance of the PGF mechanism. On the other hand, if $\mu\gg m_c$, the last two terms in Eq. (2) almost cancel and we reproduce the QCD parton model in the leading order.

In fact, in Eq. (2) one should add another order α_s contribution of the form [4]

$$C_c^{(1)}\otimes q_c-C_{c'}^{(0)}\otimes P_{c'c}\otimes q_c, \quad (3)$$

which is numerically less important.

Let us note that the Wilson-type coefficients in Eq. (1) are different from the coefficient functions C_c and C_g in Eqs. (2) and (3). While the former have no infrared or collinear singularities, the same is not true for the latter.

In Ref. [6] the modification of the gluon coefficient function has been proposed:

$$C_g\to C_g'=C_g^{\mathrm{PGF}}-\Delta C_g. \quad (4)$$

By using the well-known PGF expression in the first order in α_s (see, for instance, [13]), $\Delta C_g^{(1)}$ was calculated in [6]. At $Q^2/m_c^2\gg 1$ it looks like

$$\Delta C_g^{(1)}(Q^2,z)=\frac{\alpha_s}{2\pi}\left[P_{cg}(z)\ln\frac{Q^2}{m_c^2}+z(1-z)\right]. \quad (5)$$

The corresponding order α_s^2 expression was also found (with the account of only large logarithmic terms) [6]:

$$\begin{aligned}\Delta C_g^{(2)}=C_c^{(0)}\otimes[(\alpha_s\ln Q^2)^2(P_{cg}^{(0)}\otimes P_{gg}^{(0)}+P_{cc}^{(0)}\otimes P_{cg}^{(0)})\\+\alpha_s^2\ln Q^2P_{cg}^{(1)}]+C_c^{(1)}\otimes\alpha_s\ln Q^2P_{cg}^{(0)}\\+\alpha_sC_g^{(1)}\otimes\alpha_s\ln Q^2P_{gg}^{(0)}.\end{aligned} \quad (6)$$

Both procedures, Eqs. (2) and (4), treat the problem of double counting *by hand* and they cannot be easily generalized to higher orders in α_s. Let us note that expression (6) was obtained with the account of only leading logarithmic contributions in each subtracted term, and the factorization scale was assumed to be large, $\mu^2\simeq Q^2$.

To overcome these shortcomings, let us start from a definition of coefficient functions in terms of two-particle irreducible (2PI) amplitudes. In the following we assume that a corresponding object is 2PI in the direction of the iteration (that is in the t channel). We will work in an axial gauges ($n^\mu A_\mu=0$, n_μ being a gauge vector) and follow a scheme developed in Ref. [8].

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[1]Let us note that at very small values of x one has to consider more general $k_\perp$ factorization [12].

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Let D, be a matrix of formal parton distributions [below we define physical parton distribution functions $\tilde{D}$ Eq. (16)]. It obeys a matrix equation

$$D=(I+\hat{V}+\hat{V}^2+\cdots)\hat{D}=(I-\hat{V})^{-1}\hat{D}, \tag{7}$$

where $\hat{D}$ is the 2PI part of D, the 2PI kernel $\hat{V}$ defines the evolution of D and I is a unit matrix. To simplify notations, we dropped a sum both in parton types and spins. Integration in internal momenta are also assumed in Eq. (7).

It is known that in the axial gauge 2PI aplitudes have no singularities associated with a propagation of intermediate physical states [8]. For instance, $\hat{V}(k_n,k_{n-1})$ in Eq. (7) is finite at $k_{n-1}^2\to 0$, but it includes legs, corresponding to momentum k_n, and has a pole in k_n^2 as $k_n^2\to 0$.

Then the DIS structure function has the form

$$F=\hat{F}+\hat{A}(I-\hat{V})^{-1}\hat{D} \tag{8}$$

with $\hat{F}$ being the 2PI part of F, while $\hat{A}$ is the 2PI part of the virtual photon-parton amplitude.

Following [8], let us introduce a projector operator onto physical helicity states ($d_{\mu\nu}$ is a tensor part of the gluon propagator)

$$P_h=\begin{cases}(\hat{k})^{\alpha'\beta'}\left(\dfrac{\hat{n}}{4kn}\right)_{\alpha\beta}, & \text{for a quark line}\\[2ex] \dfrac{1}{2}d^{\mu'\nu'}(-g_{\mu\nu}), & \text{for a gluon line}\end{cases} \tag{9}$$

and another operator which projects onto small virtualities of parton lines[2]

$$P_\mu=\theta(\mu^2-|k^2|). \tag{10}$$

As we will see, our result (22) depends on a product of P_h and P_μ Eq. (23). But at intermediate steps it is convenient to consider these operators separately.

By using matrix identities

$$(I-\hat{V})^{-1}=[I-(I-P_h)\hat{V}]^{-1}(I-P_hV')^{-1},$$

$$(I-P_hV')^{-1}=(I-\tilde{V})^{-1}[I+P_\mu P_h\hat{V}(I-\hat{V})^{-1}] \tag{11}$$

with

$$V'=\hat{V}[I-(I-P_h)\hat{V}]^{-1},\quad \tilde{V}=(I-P_\mu)P_hV', \tag{12}$$

we can rewrite Eq. (8) in the form

$$F=\tilde{A}(I-\tilde{V})^{-1}|_{k_\perp=k_-=0}\tilde{D}+\Delta\tilde{A}(I-\Delta\tilde{V})^{-1}\tilde{D}$$
$$+\tilde{A}(I-\tilde{V})^{-1}|_{k_\perp=k_-=0}\Delta\tilde{V}(I-\Delta\tilde{V})^{-1}\tilde{D}+\hat{F}, \tag{13}$$

where

$$\tilde{A}=\hat{A}[I-(I-P_h)\hat{V}]^{-1} \tag{14}$$

and

$$\Delta\tilde{A}=\tilde{A}-\tilde{A}(q,k)|_{k_\perp=k_-=0},$$

$$\Delta\tilde{V}=\tilde{V}-\tilde{V}(r,k)|_{k_\perp=k_-=0}. \tag{15}$$

Here $k_-=(k^2+k_\perp^2)/kn$.

In Eq. (13) we defined the quantity

$$\tilde{D}=[I+P_\mu P_h\hat{V}(I-\hat{V})^{-1}]\hat{D}. \tag{16}$$

The quantity $\Delta\tilde{A}(I-\Delta\tilde{V})^{-1}$ in the second term in Eq. (13) has no contributions of the type $\alpha_s^n(\ln Q^2)^k$. As far as one is interested in logarithms of Q^2, it is possible to omit this term in the coefficient function. The third term in Eq. (13), $\Delta\tilde{V}(I-\Delta\tilde{V})^{-1}\tilde{D}$, is of the oder of Λ^2/μ^2, where Λ is a typical hadron scale. The nonperturbative scale Λ arises as a result of integration of the kernel $\hat{V}$ with $\hat{D}$ (which describes initial parton distributions inside the nucleon). The quantity $\hat{F}$ in Eq. (13) is related to higher twist effects and it is suppressed by a factor Λ^2/Q^2.

So, at high Q^2 and $\mu^2\gg\Lambda^2$ we get

$$F=C\tilde{D}, \tag{17}$$

where

$$C=\tilde{A}(I-\tilde{V})^{-1}(q,k)|_{k_\perp=k_-=0} \tag{18}$$

with $\tilde{V}$ and $\tilde{A}$ defined above [see Eqs. (12), (14)].

Let us note that the quantity $\tilde{D}$ Eq. (16) is of the sum of the terms which looks like the following (after an integration in transverse components of momenta k_i):

$$\int^{\mu^2}dk_n^2\int^{k_n^2}dk_{n-1}^2\int\frac{dz_{n-1}}{z_{n-1}}\hat{V}\left(z_{n-1},\frac{k_{n-1}^2}{k_n^2}\right)\int^{k_{n-1}^2}dk_{n-2}^2$$
$$\times\int\frac{dz_{n-2}}{z_{n-2}}\hat{V}\left(z_{n-2},\frac{k_{n-1}^2}{k_{n-2}^2}\right)\ldots\int^{k_1^2}dk_0^2$$
$$\times\int\frac{dz_0}{z_0}\hat{V}\left(z_0,\frac{k_0^2}{k_1^2}\right)\hat{D}(z_0,k_0^2,p^2), \tag{19}$$

where $z=k_{n+1}n/k_nn$. On the other hand, from an analogous formula in [8] we get terms of the type

[2]Another possibility is to use an operator P_ε which extracts poles in ε in $\overline{\text{MS}}$ renormalization scheme (see, for instance, [14]). We prefer to use the operator P_μ in order to have clear physical meaning for the scale μ (see below).

$$\int^{\mu^2} dk_n^2 \int^{k_n^2} dk_{n-1}^2 \int \frac{dz_{n-1}}{z_{n-1}} U\left(z_{n-1}, \frac{k_{n-1}^2}{k_n^2}\right) \int^{k_{n-1}^2} dk_{n-2}^2 \times \int \frac{dz_{n-2}}{z_{n-2}} U\left(z_{n-2}, \frac{k_{n-1}^2}{k_{n-2}^2}\right) \ldots \int \frac{dz_0}{z_0} U\left(z_0, \frac{k_0^2}{k_1^2}\right), \tag{20}$$

where

$$U = P_\mu P_h \hat{V}[I-(I-P_h)\hat{V}]^{-1}. \tag{21}$$

As one can easily see from Eqs. (16) and (19), $\widetilde{D}$ has the meaning of the distribution of partons whose virtualities vary up to μ^2. That is why we can consider C in Eq. (17) to be a coefficient function. It can be represented in another (equivalent) form:

$$C = \hat{A}[I-(I-K)\hat{V}]^{-1}(q,k)|_{k_\perp = k_- = 0}. \tag{22}$$

Here

$$K = P_\mu P_h. \tag{23}$$

So, the operator $(I-K)=(I-P_h)+(I-P_\mu)P_h$ in Eq. (22) projects onto states without any collinear singularities. It acts on the full expression on the right [10]:

$$[I-(I-K)\hat{V}]^{-1} = I + (I-K)\hat{V} + (I-K)[\hat{V}(I-K)\hat{V}] + \cdots. \tag{24}$$

The expression for $C_g^{(1)}$, calculated with the use of formula (22), coincides with the α_s order result from Ref. [15].

Starting from Eq. (18) or (22), we can represent the DIS coefficient function C in a form analogous to Eq. (4):

$$C = (C^{\rm PhP} - \Delta C)(q,k)|_{k_\perp = k_- = 0}, \tag{25}$$

where

$$C^{\rm PhP} = \hat{A}(I-\hat{V})^{-1} \tag{26}$$

is a ‘‘naive’’ expression for virtual photon-parton coefficient function that does not take into account the evolution included in parton distributions $\widetilde{D}$ Eq. (16). In particular, if we consider a parton to be a gluon, we have $C_g^{\rm PhP} = C^{\rm PGF}$.

From Eqs. (25), (22), and (26) we get the following expression for ΔC:

$$\Delta C = \widetilde{C} K \hat{V}(I-\hat{V})^{-1}, \tag{27}$$

where $\widetilde{C}(q,k)$ is given by formula (22) but without imposing a condition $k_\perp = k_- = 0$. In deriving Eq. (27) we used a matrix identity

$$(I-\hat{V})^{-1} - [I-(I-K)\hat{V}]^{-1} = [I-(I-K)\hat{V}]^{-1} K \hat{V}(I-\hat{V})^{-1}. \tag{28}$$

Let us note that both $C^{\rm PhP}(q,k)$ Eq. (26) and $\Delta C(q,k)$ Eq. (27) contain, in general, singularities at $k^2=0$, while their difference, $C(q,k)$ Eq. (22), does not.

The formula (27) enables us to calculate ΔC_g in any fixed order in α_s. Let the coefficient function C and the kernel $\hat{V}$ have, respectively, the expansions:

$$C = C^{(0)} + C^{(1)} + \cdots \tag{29}$$

and

$$\hat{V} = \hat{V}^{(0)} + \hat{V}^{(1)} + \cdots. \tag{30}$$

Then we get

$$\Delta C_g^{(1)} = \widetilde{C}_c^{(0)} K \hat{V}_{cg}^{(1)} \tag{31}$$

and

$$\Delta C_g^{(2)} = \widetilde{C}_c^{(0)} K[\hat{V}_{cg}^{(1)}\hat{V}_{gg}^{(1)} + \hat{V}_{cc}^{(1)}\hat{V}_{cg}^{(1)}] + \widetilde{C}_c^{(0)} K \hat{V}_{cg}^{(2)} + \widetilde{C}_c^{(1)} K \hat{V}_{cg}^{(1)} + \widetilde{C}_g^{(1)} K \hat{V}_{gg}^{(1)}. \tag{32}$$

The kernels $\hat{V}$ can be related with parton splitting functions $(a,b=q,g)$ [14]:

$$\hat{V}_{ab}^{(n)}(z,r^2,k^2) = \frac{1}{r^2}\frac{\alpha_s}{2\pi}\hat{P}_{ab}^{(n-1)}\left(z,\frac{k^2}{r^2}\right), \tag{33}$$

where the off-shell splitting function has the form:

$$\hat{P}_{ab} = \left(\frac{\alpha_s}{2\pi}\right)\hat{P}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{P}_{ab}^{(1)} + \cdots. \tag{34}$$

In particular, in the leading logarithmic approximation, we have

$$\hat{V}_{ab}^{(1)}(z,r^2,k^2=0) = \frac{1}{r^2}\frac{\alpha_s}{2\pi}P_{ab}^{(0)}(z), \tag{35}$$

where $P_{ab}^{(0)}(z)$ is a leading order Altarelli-Parisi splitting function.

If we put $\mu^2 \simeq Q^2$ in Eq. (32) and save $(\alpha_s \ln Q^2)^2$ and $\alpha_s^2 \ln Q^2$ contributions, all the terms in expression (6) for ΔC_g can be reproduced (taking into account slight difference between our definitions of $C^{(n)}$ and $\Delta C^{(n)}$ and those from Ref. [6]). Indeed, from Eq. (33) we conclude that $C_c^{(1)} K \hat{V}_{cg}^{(1)} \simeq C_c^{(1)} \otimes \alpha_s \ln Q^2 P_{cg}^{(1)}$, etc., where coefficient functions $C_a^{(n)} = C_a^{(n)}(\alpha_s(Q^2))$ have no logarithms of Q^2.

However, the exact formula (32) results in additional contributions which are absent in Eq. (6). In particular, due to power corrections in $\hat{V}_{ab}^{(1)}$ [see Eq. (33)] the term $C_c^{(0)} K[\hat{V}_{cg}^{(1)}\hat{V}_{gg}^{(1)} + \hat{V}_{cc}^{(1)}\hat{V}_{cg}^{(1)}]$ contains a nonleading contribution $(\alpha_s^2 \ln Q^2)$ in addition to a leading one $[(\alpha_s \ln Q^2)^2]$.

Let us note that expression (27) and, consequently, Eqs. (31) and (32) do not have factorized forms as the right-hand sides of Eqs. (2) and (6) do. Namely, the integration of $\tilde{C}(q,l)$ or $\tilde{C}^{(n)}(q,l)$ in the momenta l with the right part of the corresponding expression should be done. In particular, due to this integration in Eq. (31) we get a constant term in $\Delta C_c^{(1)}(q,k)$ in addition to a large logarithmic term. It depends both on a ratio μ^2/Q^2 and μ_0^2/Q^2, where $\mu_0^2=-k^2$ [simultaneously, we have to set $k^2=-\mu_0^2$ in $C^{\mathrm{PhP}}(q,k)$]. At $\mu^2 \simeq Q^2 \gg m_c^2$ and $k^2=0$ we get the constant term which is different from $z(1-z)$ in Eq. (5).

Fortunately, we are not forced to deal with the quantity ΔC [Eq. (27)] as we have derived the formula (22) which enables us to calculate C_g and C_c in any order in strong coupling without getting double counting in the coefficient functions.

In the present paper we have studied the structure function F_2. However, our formulas (22), (27) may be also applied to the longitudinal deep inelastic structure function F_L.

In conclusion let us note that the problem of double counting should also exist for Wilson coefficients for light quarks C^q considered in higher orders in α_s. In such a case the double counting means that one and the same term is accounted for both in the coefficient function C^q and in a distribution function $\tilde{D}_q$. Formula (18) [Eq. (22)] enables one to separate diagrams describing C^q from those included in the evolution of the quark distribution $\tilde{D}_q$. For instance, in the lowest order in α_s we should get a term analogous to Eq. (4).

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[1] H1 Collaboration, C. Adloff *et al.*, Z. Phys. C **72**, 593 (1996).
[2] ZEUS Collaboration, J. Breitweg *et al.*, Phys. Lett. B **407**, 402 (1997).
[3] M. Glück, E. Reya, and M. Stratmann, Nucl. Phys. **B422**, 37 (1994); A. V. Kisselev and V. A. Petrov, Z. Phys. C **75**, 277 (1997).
[4] M. A. G. Aivazis, F. I. Olness, and W.-K. Tung, Phys. Rev. D **50**, 3085 (1994); **50**, 3102 (1994).
[5] R. S. Thorne and R. G. Roberts, Phys. Rev. D **57**, 6871 (1998); Phys. Lett. B **421**, 303 (1998).
[6] A. D. Martin *et al.*, Eur. Phys. J. C **2**, 287 (1998).
[7] S. Kretzer and I. Schienbein, Phys. Rev. D **58**, 094035 (1998).
[8] R. K. Ellis *et al.*, Phys. Lett. **78B**, 281 (1978); Nucl. Phys. **B152**, 285 (1979).
[9] A. V. Radyushkin, Phys. Lett. **69B**, 245 (1977); H. D. Politzer, Nucl. Phys. **B129**, 301 (1977); D. Amati, R. Petronzio, and G. Veneziano, *ibid.* **B140**, 54 (1978); C. T. Sachrajda, Phys. Lett. **73B**, 281 (1978); S. Libby and G. Sterman, Phys. Rev. D **18**, 3252 (1978); **18**, 4737 (1978); A. H. Mueller, *ibid.* **18**, 3705 (1978).
[10] G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. **B175**, 437 (1980).
[11] J. C. Collins, D. E. Soper, and G. Sterman, in *Perturbative Quantum Chromodynamics*, edited by A. H. Mueller (World Scientific, Singapore, 1989).
[12] S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B **242**, 97 (1990); Nucl. Phys. **B366**, 657 (1991).
[13] E. Witten, Nucl. Phys. **B104**, 445 (1976); M. A. Shifman, A. I. Vanshtein, and V. I. Zakharov, *ibid.* **B136**, 157 (1978); M. Glück and E. Reya, Phys. Lett. **83B**, 98 (1979).
[14] S. Catani and F. Hautmann, Nucl. Phys. **B427**, 475 (1994).
[15] P. Mathews and V. Ravindran, Int. J. Mod. Phys. A **11**, 2783 (1996).